图像处理 hw6

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1. $$\sigma^2=\frac{1}{(2a+1)(2b+1)}\sum_{s=-a}^{a}\sum_{t=-b}^{b}\Big(\big[g(x+s,y+t)-w(x,y)\cdot\eta(x+s,y+t)\big]-\big[\overline{g(x,y)}-w(x,y)\overline{\eta(x,y)}\big]\Big)^2$$

let $$\frac{\partial\sigma^2}{\partial w(x,y)}\equiv 0$$

$$LHS=\frac{1}{(2a+1)(2b+1)}\sum_{s=-a}^{a}\sum_{t=-b}^{b}2\Big(\big[g(x+s,y+t)-w(x,y)\eta(x+s,y+t)\big]-\big[\overline{g(x,y)}-w(x,y)\overline{\eta(x,y)}\big]\Big)\cdot\big(-\eta(x+s,y+t)+\overline{\eta(x,y)}\big)$$

$$=\frac{1}{(2a+1)(2b+1)}\sum_{s=-a}^{a}\sum_{s=-b}^{b}2\Big(\big[\overline{\eta(x,y)}-\eta(x+s,y+t)\big]w(x,y)+g(x+s,y+t)-\overline{g(x,y)}\Big)\cdot\big(-\eta(x+s,y+t)+\overline{\eta(x,y)}\big)$$

$$=\frac{2}{(2a+1)(2b+1)}\sum_{s=-a}^{a}\sum_{s=-b}^{b}\Big(\underbrace{-\big[\overline{\eta(x,y)}-\eta(x+s,y+t)\big]^2w(x,y)}_{A}+\underbrace{\big[g(x+s,y+t)-\overline{g(x,y)}\big]\cdot\big[-\eta(x+s,y+t)+\overline{\eta(x,y)}\big]}_{B}\Big)$$

$$A=\frac{-2w(x,y)}{(2a+1)(2b+1)}\sum_{s=-a}^{a}\sum_{s=-b}^{b}\Big(\overline{\eta(x,y)}^2+\eta(x+s,y+t)^2-2\overline{\eta(x,y)}\eta(x+s,y+t)\Big)$$

$$=-2w(x,y)\Big(\overline{\eta(x,y)}^2+\overline{\eta(x,y)^2}-2\overline{\eta(x,y)}^2\Big)$$

$$=2w(x,y)\Big(\overline{\eta(x,y)}^2-\overline{\eta(x,y)^2}\Big)$$

$$B=\frac{2}{(2a+1)(2b+1)}\sum_{s=-a}^{a}\sum_{t=-b}^{b}\Big(-g(x+s,y+t)\eta(x+s,y+t)+g(x+s,y+t)\overline{\eta(x,y)}+\overline{g(x,y)}\eta(x+s,y+t)-\overline{g(x,y)}\,\overline{\eta(x,y)}\Big)$$

$$=2\Big(-\overline{g(x,y)\eta(x,y)}+2\overline{g(x,y)}\,\overline{\eta(x,y)}-\overline{g(x,y)}\,\overline{\eta(x,y)}\Big)$$

$$=2\Big(\overline{g(x,y)}\,\overline{\eta(x,y)}-\overline{g(x,y)\eta(x,y)}\Big)$$

$$\therefore LHS=2w(x,y)\Big(\overline{\eta(x,y)}^2-\overline{\eta(x,y)^2}\Big)+2\Big(\overline{g(x,y)}\,\overline{\eta(x,y)}-\overline{g(x,y)\eta(x,y)}\Big)\equiv 0$$

$$\therefore w(x,y)\equiv\frac{\overline{g(x,y)\eta(x,y)}-\overline{g(x,y)}\,\overline{\eta(x,y)}}{\overline{\eta(x,y)^2}-\overline{\eta(x,y)}^2}$$

2. ① 垂直方向匀速运动，$t=T_1$

② 水平方向匀速运动，$t=T_2$

空域平移性质：

$$\mathfrak{J}\big[f(x-x_0,y-y_0)\big]=F(u,v)e^{-j2\pi(ux_0+vy_0)}$$

$$g(x,y)=\int_0^{T_1}f(x,y-y_0(t))\,dt+\int_0^{T_2}f(x-x_0(t),y-y_0(T_1))\,dt$$

$$G(u,v)=\mathfrak{J}(g(x,y))=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}g(x,y)\exp(-j2\pi(ux+vy))\,dx\,dy$$

$$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\Big(\int_0^{T_1}f(x,y-y_0(t))\,dt\Big)\exp(-j2\pi(ux+vy))\,dx\,dy$$

$$+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\Big(\int_0^{T_2}f(x-x_0(t),y-y_0(T_1))\,dt\Big)\exp(-j2\pi(ux+vy))\,dx\,dy$$

$$=A+B$$

$$A=\int_0^{T_1}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}f(x,y-y_0(t))\exp(-j2\pi(ux+vy))\,dx\,dy\,dt$$

$$= \int_0^{T_1} \mathfrak{F}\left(f(x, y - y_0(t))\right) dt$$

$$= \int_0^{T_1} F(u,v)\, e^{-j2\pi(v y_0(t))}\, dt$$

$$= F(u,v) \underbrace{\int_0^{T_1} e^{-j2\pi(v y_0(t))}\, dt}_{H_1(u,v)}$$

If $y_0(t) = \frac{at}{T_1}$

$$H_1(u,v) = \int_0^{T_1} \exp\left(-j2\pi vat/T_1\right) dt$$

$$= \frac{jT_1}{2\pi va} \exp\left(-j2\pi vat/T_1\right) \Big|_0^{T_1}$$

$$= \frac{jT_1}{2\pi va}\left(e^{-j2\pi va} - 1\right) \qquad e^{-j\pi va + j\pi va}$$

$$= \frac{jT_1}{2\pi va} e^{-j\pi va}\left(e^{-j\pi va} - e^{j\pi va}\right)$$

$$= \frac{jT_1}{2\pi va} e^{-j\pi va}\left(\cancel{\cos \pi va} - j\sin\pi va - \cancel{\cos\pi va} - j\sin\pi va\right)$$

$$= \frac{T_1}{\pi va} e^{-j\pi va} \sin(\pi va)$$

$$B = \int_0^{T_2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x - x_0(t), y - y_0(T_1)) \exp(-j2\pi(ux + vy))\, dx\, dy\, dt$$

$$= \int_0^{T_2} \mathfrak{F}\left(f(x - x_0(t), y - y_0(T_1))\right) dt$$

$$= \int_0^{T_2} F(u,v) \exp\left(-j2\pi(ux_0(t) + vy_0(T_1))\right) dt$$

$$= F(u,v) \exp(-j\pi va) \cdot \int_0^{T_2} \exp(-j2\pi u x_0(t))\, dt$$

let $x_0(t) = \frac{bt}{T_2}$

$$B = F(u,v) \exp(-j2\pi va) \frac{T_2}{\pi ub} e^{-j\pi ub} \sin(\pi ub)$$

$$\therefore G(u,v) = F(u,v)\left[\frac{T_1}{\pi va} e^{-j\pi va} \sin(\pi va) + \exp(-j2\pi va) \frac{T_2}{\pi ub} e^{-j\pi ub} \sin(\pi ub)\right]$$

$$\therefore H(u,v) = \frac{T_1}{\pi va} e^{-j\pi va} \sin(\pi va) + \frac{T_2}{\pi ub} e^{-j\pi(ub + 2va)} \sin(\pi ub)$$