状态估计 hw3

Wednesday, May 6, 2020 14:02

1. (P17)

Suppose x_k is gaussian:

$p(x_k|\check{x}_0, v_{1:k}, y_{0:k}) = \mathcal{N}(\hat{x}_k, \hat{P}_k)$ $\quad w_k \sim \mathcal{N}(0, Q_k)$ $\quad n_k \sim \mathcal{N}(0, R_k)$ $\quad p(x,y) = p(x|y)p(y) = p(y|x)p(x)$

Linearization of NL system:

$x_k = f(x_{k-1}, v_k, w_k) \approx \check{x}_k + F_{k-1}(x_{k-1} - \hat{x}_{k-1}) + w_k'$ (F_{k-1}, G_k are jacobians.)

$y_k = g(x_k, n_k) \approx \check{y}_k + G_k(x_k - \check{x}_k) + n_k'$

and $\check{x}_k = f(\hat{x}_{k-1}, v_k, 0)$, $F_{k-1} = \left.\frac{\partial f(x_{k-1}, v_k, w_k)}{\partial x_{k-1}}\right|_{\hat{x}_{k-1}, v_k, 0}$, $\check{y}_k = g(\check{x}_k, 0)$, $G_k = \left.\frac{\partial g(x_k, n_k)}{\partial x_k}\right|_{\check{x}_k, 0}$

$w_k' = \left.\frac{\partial f(x_{k-1}, v_k, w_k)}{\partial w_k}\right|_{\hat{x}_{k-1}, v_k, 0} \cdot w_k$ $\qquad n_k' = \left.\frac{\partial g(x_k, n_k)}{\partial n_k}\right|_{\check{x}_k, 0} \cdot n_k$

$$\underbrace{p(x_k|\check{x}_0, v_{1:k}, y_{0:k})}_{\mathcal{N}(\hat{x}_k, \hat{P}_k)} = \eta \underbrace{p(y_k|x_k)}_{\mathcal{N}(\check{y}_k + G_k(x_k - \check{x}_k), R_k')}$$

$$\times \underbrace{\int \underbrace{p(x_k|x_{k-1}, v_k)}_{\mathcal{N}(\check{x}_k + F_{k-1}(x_{k-1} - \hat{x}_{k-1}), Q_k')} \underbrace{p(x_{k-1}|\check{x}_0, v_{1:k-1}, y_{0:k-1})}_{\mathcal{N}(\hat{x}_{k-1}, \hat{P}_{k-1})} dx_{k-1}}_{I}$$

$g(x) \approx \mu_y + G(x - \mu_x) + n'$

$\mu_y = g(\mu_x)$

$\because$ from lecture 1:

let $p(\bar{y}) = \int_{-\infty}^{\infty} p(\bar{y}|\bar{x}) p(\bar{x}) d\bar{x}$, $p(\bar{y}|\bar{x}) = \mathcal{N}(g(\bar{x}), R)$, $g(\bar{x}) = \mu_y + G(\bar{x} - \mu_x)$, $G = \left.\frac{\partial g(\bar{x})}{\partial \bar{x}}\right|_{\bar{x} = \mu_x}$

$$p(\bar{y}) = \eta \int_{-\infty}^{\infty} \exp\left(-\tfrac{1}{2}(\bar{y} - (\mu_y + G(\bar{x} - \mu_x)))^T R^{-1} (\bar{y} - (\mu_y + G(\bar{x} - \mu_x)))\right) \cdot \exp\left(-\tfrac{1}{2}(\bar{x} - \mu_x)^T \Sigma_{\bar{x}\bar{x}}^{-1} (\bar{x} - \mu_x)\right) d\bar{x}$$

$$= \eta \exp\left(-\tfrac{1}{2}(\bar{y} - \mu_y) R^{-1} (\bar{y} - \mu_y)\right) \times \int_{-\infty}^{\infty} \exp\left(-\tfrac{1}{2}(\bar{x} - \mu_x)^T (\Sigma_{\bar{x}\bar{x}}^{-1} + G^T R^{-1} G)(\bar{x} - \mu_x)\right) \cdot \exp\left((\bar{y} - \mu_y)^T R^{-1} G (\bar{x} - \mu_x)\right) d\bar{x}$$

补全平方项

$$= \eta \exp\left(-\tfrac{1}{2}(\bar{y} - \mu_y) R^{-1} (\bar{y} - \mu_y)\right) \times \int_{-\infty}^{\infty} \exp\left(-\tfrac{1}{2}(\bar{x} - \mu_x)^T (\Sigma_{\bar{x}\bar{x}}^{-1} + G^T R^{-1} G)(\bar{x} - \mu_x)\right) \cdot \exp\left((\bar{y} - \mu_y)^T F^T (G^T R^{-1} G + \Sigma_{\bar{x}\bar{x}}^{-1})(\bar{x} - \mu_x)\right)$$
$$\cdot \exp\left(-\tfrac{1}{2}\left((F(\bar{y} - \mu_y))^T (G^T R^{-1} G + \Sigma_{\bar{x}\bar{x}}^{-1}) F(\bar{y} - \mu_y)\right)\right) \cdot \exp\left(\tfrac{1}{2}\left((F(\bar{y} - \mu_y))^T (G^T R^{-1} G + \Sigma_{\bar{x}\bar{x}}^{-1}) F(\bar{y} - \mu_y)\right)\right)$$

$$= \eta \exp\left(-\tfrac{1}{2}(\bar{y} - \mu_y) R^{-1} (\bar{y} - \mu_y)\right) \cdot \exp\left(\tfrac{1}{2}(\bar{y} - \mu_y)^T F^T (G^T R^{-1} G + \Sigma_{\bar{x}\bar{x}}^{-1}) F(\bar{y} - \mu_y)\right) \cdot \underbrace{\int_{-\infty}^{\infty} \exp\left(-\tfrac{1}{2}((\bar{x} - \mu_x) - F(\bar{y} - \mu_y))^T (G^T R^{-1} G + \Sigma_{\bar{x}\bar{x}}^{-1}) ((\bar{x} - \mu_x) - F(\bar{y} - \mu_y))\right) d\bar{x}}_{\text{与}\,\eta\,\text{合并}}$$

$$= \rho \exp\left(-\tfrac{1}{2}(\bar{y} - \mu_y)^T \underbrace{(R^{-1} - F^T (G^T R^{-1} G + \Sigma_{\bar{x}\bar{x}}^{-1}) F)}_{SWM} (\bar{y} - \mu_y)\right)$$

$$= \rho \exp\left(-\tfrac{1}{2}(\bar{y} - \mu_y)^T (R + G \Sigma_{xx} G^T)^{-1} (\bar{y} - \mu_y)\right)$$

$\Rightarrow \bar{y} \sim \mathcal{N}(\mu_{\bar{y}}, \Sigma_{\bar{y}\bar{y}}) = \mathcal{N}(g(\mu_{\bar{x}}), R + G \Sigma_{\bar{x}\bar{x}} G^T)$

$\therefore$ let $\bar{y} \equiv x_k$, $\bar{x} \equiv x_{k-1}$, $g(\bar{x}) \equiv \check{x}_k + F_{k-1}(x_{k-1} - \hat{x}_{k-1})$, $R \equiv Q_k'$, $\mu_{\bar{x}} = E(x_{k-1}) = \hat{x}_{k-1}$

$G \equiv \left.\frac{\partial g(\bar{x})}{\partial \bar{x}}\right|_{\bar{x} = \mu_{\bar{x}}} = \left.\frac{\partial g(x_{k-1})}{\partial x_{k-1}}\right|_{x_{k-1} = \hat{x}_{k-1}} = F_{k-1}$, $\Sigma_{\bar{x}\bar{x}} = \Sigma_{x_{k-1}x_{k-1}} = \hat{P}_{k-1}$

$$\therefore \Rightarrow \begin{cases} \mu_{x_k} \equiv \mu_{\bar{y}} = g(\mu_{\bar{x}}) = g(\mu_{x_{k-1}}) = g(\hat{x}_{k-1}) = \check{x}_k \\ \Sigma_{x_k x_k} \equiv \Sigma_{\bar{y}\bar{y}} = Q_k' + F_{k-1} \Sigma_{x_{k-1}x_{k-1}} F_{k-1}^T = Q_k' + F_{k-1} \hat{P}_{k-1} F_{k-1}^T \end{cases}$$

$$\therefore I = \mathcal{N}(\mu_{x_k}, \Sigma_{x_k x_k}) = \mathcal{N}(\check{x}_k, Q_k' + F_{k-1} \hat{P}_{k-1} F_{k-1}^T)$$

$$\therefore \underbrace{p(x_k|\check{x}_0, v_{1:k}, y_{0:k})}_{\mathcal{N}(\hat{x}_k, \hat{P}_k)} = \eta \underbrace{p(y_k|x_k)}_{\mathcal{N}(\check{y}_k + G_k(x_k - \check{x}_k), R_k')} \cdot \underbrace{\int p(x_k|x_{k-1}, v_k) p(x_{k-1}|\check{x}_0, v_{1:k-1}, y_{0:k-1}) dx_{k-1}}_{\mathcal{N}(\check{x}_k, F_{k-1}\hat{P}_{k-1}F_{k-1}^T + Q_k')}$$

$$RHS = \eta \exp\left(-\tfrac{1}{2}(y_k - \check{y}_k - G_k(x_k - \check{x}_k))^T R_k'^{-1} (y_k - \check{y}_k - G_k(x_k - \check{x}_k))\right) \cdot \exp\left(-\tfrac{1}{2}(x_k - \check{x}_k)^T \underbrace{(F_{k-1}\hat{P}_{k-1}F_{k-1}^T + Q_k')^{-1}}_{\check{P}_k^{-1}} (x_k - \check{x}_k)\right)$$

$$= \eta \exp\Big[-\tfrac{1}{2}(y_k^T R_k'^{-1} - \check{y}_k^T R_k'^{-1} - x_k^T G_k^T R_k'^{-1} + \check{x}_k^T G_k^T R_k'^{-1}) \cdot (y_k - \check{y}_k - G_k x_k + G_k \check{x}_k)$$
$$-\tfrac{1}{2}(x_k^T \check{P}_k^{-1} - \check{x}_k^T \check{P}_k^{-1})(x_k - \check{x}_k)\Big]$$

$$= \eta \exp\Big[-\tfrac{1}{2}\big(y_k^T R_k'^{-1} y_k - \check{y}_k^T R_k'^{-1} y_k - x_k^T G_k^T R_k'^{-1} y_k + \check{x}_k^T G_k^T R_k'^{-1} y_k$$
$$- y_k^T R_k'^{-1} \check{y}_k + \check{y}_k^T R_k'^{-1} \check{y}_k + x_k^T G_k^T R_k'^{-1} \check{y}_k - \check{x}_k^T G_k^T R_k'^{-1} \check{y}_k$$
$$- y_k^T R_k'^{-1} G_k x_k + \check{y}_k^T R_k'^{-1} G_k x_k + x_k^T G_k^T R_k'^{-1} G_k x_k - \check{x}_k^T G_k^T R_k'^{-1} G_k x_k$$
$$+ y_k^T R_k'^{-1} G_k \check{x}_k - \check{y}_k^T R_k'^{-1} G_k \check{x}_k - x_k^T G_k^T R_k'^{-1} G_k \check{x}_k + \check{x}_k^T G_k^T R_k'^{-1} G_k \check{x}_k$$
$$+ x_k^T \check{P}_k^{-1} x_k - \check{x}_k^T \check{P}_k^{-1} x_k - x_k^T \check{P}_k^{-1} \check{x}_k + \check{x}_k^T \check{P}_k^{-1} \check{x}_k\big)\Big]$$

$$LHS = \exp\left(-\tfrac{1}{2}(x_k - \hat{x}_k)^T \hat{P}_k^{-1} (x_k - \hat{x}_k)\right)$$
$$= \exp\left(-\tfrac{1}{2}(x_k^T \hat{P}_k^{-1} - \hat{x}_k^T \hat{P}_k^{-1})(x_k - \hat{x}_k)\right)$$
$$= \exp\left(-\tfrac{1}{2}(x_k^T \hat{P}_k^{-1} x_k - \hat{x}_k^T \hat{P}_k^{-1} x_k - x_k^T \hat{P}_k^{-1} \hat{x}_k + \hat{x}_k^T \hat{P}_k^{-1} \hat{x}_k)\right)$$

(1.) x_k 一次项:

$$I_k = -\tfrac{1}{2}\big(-x_k^T G_k^T R_k'^{-1} y_k + x_k^T G_k^T R_k'^{-1} \check{y}_k - y_k^T R_k'^{-1} G_k x_k + \check{y}_k^T R_k'^{-1} G_k x_k$$
$$- \check{x}_k^T G_k^T R_k'^{-1} G_k x_k - x_k^T G_k^T R_k'^{-1} G_k \check{x}_k - \check{x}_k^T \check{P}_k^{-1} x_k - x_k^T \check{P}_k^{-1} \check{x}_k\big)$$

$$I_k' = -\tfrac{1}{2}(-x_k^T \hat{P}_k^{-1} \hat{x}_k - \hat{x}_k^T \hat{P}_k^{-1} x_k)$$

$$I_k = -\frac{1}{2}\Big(\underline{-x_k^T G_k^T R_k'^{-1} y_k} + \underline{x_k^T G_k^T R_k'^{-1}\check{y}_k} - y_k^T R_k'^{-1} G_k x_k + \check{y}_k^T R_k'^{-1} G_k x_k$$
$$- \check{x}_k^T G_k^T R_k'^{-1} G_k x_k - \underline{x_k^T G_k^T R_k'^{-1} G_k \check{x}_k} - \check{x}_k^T \check{P}_k^{-1} x_k - \underline{x_k^T \check{P}_k^{-1}\check{x}_k}\Big)$$

[illegible]

$$= -\frac{1}{2} x_k^T\left(-G_k^T R_k'^{-1} y_k + G_k^T R_k'^{-1}\check{y}_k - G_k^T R_k'^{-1} G_k \check{x}_k - \check{P}_k^{-1}\check{x}_k\right)$$
$$-\frac{1}{2}\left(-y_k^T R_k'^{-1} G_k + \check{y}_k^T R_k'^{-1} G_k - \check{x}_k G_k^T R_k'^{-1} G_k - \check{x}_k^T \check{P}_k^{-1}\right) x_k$$

$\therefore$ $I_k = I_k'$

$$\Rightarrow G_k^T R_k'^{-1}(y_k - \check{y}_k) + \underbrace{\left(G_k^T R_k'^{-1} G_k + \check{P}_k^{-1}\right)}_{\left(\check{P}_k - \check{P}_k G_k^T (R_k' + G_k \check{P}_k G_k^T)^{-1} G_k \check{P}_k\right)^{-1}} \check{x}_k = \hat{P}_k^{-1}\hat{x}_k$$

and let $K_k = \check{P}_k G_k^T (R_k' + G_k \check{P}_k G_k^T)^{-1}$

$$\Rightarrow \underline{G_k^T R_k'^{-1}(y_k - \check{y}_k) + (\check{P}_k - K_k G_k \check{P}_k)^{-1}\check{x}_k = \hat{P}_k^{-1}\hat{x}_k} \quad ①$$

(2) x_k = 次项:

$$J_k = \frac{1}{2}\left(x_k^T G_k^T R_k'^{-1} G_k x_k + x_k^T \check{P}_k^{-1} x_k\right) \qquad J_k' = -\frac{1}{2} x_k^T \hat{P}_k^{-1} x_k$$
$$= -\frac{1}{2} x_k^T \left(G_k^T R_k'^{-1} G_k + \check{P}_k^{-1}\right) x_k$$

$J_k = J_k'$

$$\Rightarrow \underline{G_k^T R_k'^{-1} G_k + \check{P}_k^{-1}} = \hat{P}_k^{-1} \quad ②$$
$$\left(\check{P}_k^{-1} + G_k^T R_k'^{-1} G_k\right)^{-1} \cdot G_k^T R_k'^{-1} = \hat{P}_k G_k^T R_k'^{-1}$$

SMW(3)
$$\Rightarrow \hat{P}_k G_k^T R_k'^{-1} = \check{P}_k G_k^T (R_k' + G_k \check{P}_k G_k^T)^{-1} = K_k$$

from ②:
$$\check{P}_k^{-1} = \hat{P}_k^{-1} - G_k^T R_k'^{-1} G_k$$
$$= \hat{P}_k^{-1}(1 - \underbrace{\hat{P}_k G_k^T R_k'^{-1}}_{K_k} G_k)$$
$$= \hat{P}_k^{-1}(1 - K_k G_k)$$
$$\Rightarrow \underline{\hat{P}_k^{-1} = \check{P}_k^{-1}(1 - K_k G_k)^{-1} = \left[(1 - K_k G_k)\check{P}_k\right]^{-1}} \quad ③$$

$\therefore$ plug ③ in ①:
$$G_k^T R_k'^{-1}(y_k - \check{y}_k) + \hat{P}_k^{-1}\check{x}_k = \hat{P}_k^{-1}\hat{x}_k$$
$$\Rightarrow \hat{x}_k = \check{x}_k + \underbrace{\hat{P}_k G_k^T R_k'^{-1}}_{K_k}(y_k - \check{y}_k)$$
$$\Rightarrow \underline{\hat{x}_k = \check{x}_k + K_k(y_k - \check{y}_k)} \quad ④$$

$\therefore$ EKF:

Predict:
$$\check{P}_k = F_{k-1}\hat{P}_{k-1}F_{k-1}^T + Q_k'$$
$$\check{x}_k = f(\hat{x}_{k-1}, v_k, 0)$$

Update:
$$K_k = \check{P}_k G_k^T (G_k \check{P}_k G_k^T + R_k')^{-1}$$
$$\hat{P}_k = (1 - K_k G_k)\check{P}_k$$
$$\hat{x}_k = \check{x}_k + K_k(y_k - g(\check{x}_k, 0))$$

2 (P27) sigma points (total $2L+1$)

$LL^T = \Sigma_{xx}$ Cholesky decomp.

$$x_0 = \mu_x$$
$$x_i = \mu_x + \sqrt{L+k}\, L(c_i) \qquad i = 1, \ldots, L$$
$$x_{i+L} = \mu_x - \sqrt{L+k}\, L(c_i)$$

and $\mu_x = \sum_{i=0}^{2L} \alpha_i x_i$ ① where $\alpha_i = \begin{cases} \frac{k}{L+k} & i=0 \\ \frac{1}{2}\frac{1}{L+k} & \text{otherwise} \end{cases}$

$$= \sum^{2L} \alpha_i (x_i - \mu_x)(x_i - \mu_x)^T \quad ②$$

and $\mu_x = \sum_{i=0}^{2L} a_i x_i$ ①   where $a_i = \begin{cases} \frac{k}{L+k} & i=0 \\ \frac{1}{2}\frac{1}{L+k} & \text{otherwise} \end{cases}$

$\Sigma_{xx} = \sum_{i=0}^{2L} a_i (x_i - \mu_x)(x_i - \mu_x)^T$ ② $L = \dim(\mu_x)$

① plug in a_i:

$$RHS = \frac{k}{L+k}\mu_x + \frac{1}{2}\frac{1}{L+k}\sum_{i=1}^{L}(\mu_x + \sqrt{L+k}\,L(c_i)) + \frac{1}{2}\frac{1}{L+k}(\mu_x - \sqrt{L+k}\,L(c_i))$$

$$= \frac{k}{L+k}\mu_x + \frac{1}{2}\frac{2L\mu_x}{L+k} + \frac{1}{2(L+k)}\sum_{i=1}^{L}(\sqrt{L+k}\,L(c_i) - \sqrt{L+k}\,L(c_i))$$

$$= \frac{k+L}{L+k}\mu_x = \mu_x = LHS$$

$\therefore$ ① proved.

② plug in a_i:

$$RHS = \frac{k}{L+k}(\mu_x - \mu_x)(\mu_x - \mu_x)^T + \frac{1}{2(L+k)}\sum_{i=1}^{L}(\mu_x + \sqrt{L+k}\,L(c_i) - \mu_x)\cdot(\mu_x + \sqrt{L+k}\,L(c_i) - \mu_x)^T$$

$$+ \frac{1}{2(L+k)}\sum_{i=1}^{L}(\mu_x - \sqrt{L+k}\,L(c_i) - \mu_x)\cdot(\mu_x - \sqrt{L+k}\,L(c_i) - \mu_x)^T$$

$$= \frac{L+k}{2(L+k)}\left(\sum_{i=1}^{L} L(c_i)L(c_i)^T + \sum_{i=1}^{L} L(c_i)L(c_i)^T\right)$$

$$= \sum_{i=1}^{L} L(c_i)L(c_i)^T$$

$$= L \cdot L^T = \Sigma_{xx} = LHS$$

$\therefore$ ② proved.

3. (4.6.1) state $s_k = [x_k \ y_k \ \theta_k]^T$ $y_k = [r_k \ \phi_k]^T$

system: $s_k = s_{k-1} + T\begin{bmatrix} \cos\theta_{k-1} & 0 \\ \sin\theta_{k-1} & 0 \\ 0 & 1 \end{bmatrix}\left(\begin{bmatrix} v_k \\ \omega_k \end{bmatrix} + w_k\right)$ $w_k \sim N(0, Q)$

$y_k = \begin{bmatrix} \sqrt{x_k^2 + y_k^2} \\ \operatorname{atan2}(-y_k, -x_k) - \theta_k \end{bmatrix} + n_k$ $n_k \sim N(0, R)$

Jacobians: F_{k-1}, G_k

$$F_{k-1} = \frac{\partial f(x_{k-1}, v_k, w_k)}{\partial x_{k-1}}\bigg|_{\hat{x}_{k-1}, v_k, 0}$$

$$= \begin{bmatrix} \frac{\partial f_1}{\partial x_{k-1}} & \frac{\partial f_1}{\partial y_{k-1}} & \frac{\partial f_1}{\partial \theta_{k-1}} \\ \frac{\partial f_2}{\partial x_{k-1}} & \frac{\partial f_2}{\partial y_{k-1}} & \frac{\partial f_2}{\partial \theta_{k-1}} \\ \frac{\partial f_3}{\partial x_{k-1}} & \frac{\partial f_3}{\partial y_{k-1}} & \frac{\partial f_3}{\partial \theta_{k-1}} \end{bmatrix}\Bigg|_{\hat{x}_{k-1}, v_k, 0}$$

$$= \begin{bmatrix} 1 & 0 & -Tv_k\sin\theta_{k-1} \\ 0 & 1 & Tv_k\cos\theta_{k-1} \\ 0 & 0 & 1 \end{bmatrix}\Bigg|_{\hat{x}_{k-1}, v_k, 0}$$

$$= \begin{bmatrix} 1 & 0 & -Tv_k\sin\hat{\theta}_{k-1} \\ 0 & 1 & Tv_k\cos\hat{\theta}_{k-1} \\ 0 & 0 & 1 \end{bmatrix}$$

$$G_k = \frac{\partial g(x_k, n_k)}{\partial x_k}\bigg|_{\check{x}_k, 0}$$ $\operatorname{atan}(x)' = \frac{1}{1+x^2}$

$$= \begin{bmatrix} \frac{\partial g_1}{\partial x_k} & \frac{\partial g_1}{\partial y_k} & \frac{\partial g_1}{\partial \theta_k} \\ \frac{\partial g_2}{\partial x_k} & \frac{\partial g_2}{\partial y_k} & \frac{\partial g_2}{\partial \theta_k} \end{bmatrix}\Bigg|_{\check{x}_k, 0}$$

$$= \begin{bmatrix} x_k(x_k^2+y_k^2)^{-\frac{1}{2}} & y_k(x_k^2+y_k^2)^{-1/2} & 0 \\ \frac{-y_k}{x_k^2+y_k^2}, & \frac{x_k}{x_k^2+y_k^2}, & -1 \end{bmatrix}\Bigg|_{\check{x}_k, 0}$$

$$= \begin{bmatrix} \check{x}_k(\check{x}_k^2+\check{y}_k^2)^{-\frac{1}{2}} & \check{y}_k(\check{x}_k^2+\check{y}_k^2)^{-\frac{1}{2}} & 0 \\ -\frac{\check{y}_k}{\check{x}_k^2+\check{y}_k^2}, & \frac{\check{x}_k}{\check{x}_k^2+\check{y}_k^2}, & -1 \end{bmatrix}$$

$$w_k' = \frac{\partial f(x_{k-1}, v_k, w_k)}{\partial w_k}\bigg|_{\hat{x}_{k-1}, v_k, 0} w_k$$

$$= \begin{bmatrix} T\cos\theta_{k-1} & 0 \\ T\sin\theta_{k-1} & 0 \\ 0 & 1 \end{bmatrix}\Bigg|_{\hat{x}_{k-1}, v_k, 0} \cdot w_k$$

$$= \begin{bmatrix} T\cos\hat{\theta}_{k-1} & 0 \\ T\sin\hat{\theta}_{k-1} & 0 \\ 0 & 1 \end{bmatrix} w_k$$

$$= M \cdot w_k$$

$$n_k' = \frac{\partial g(x_k, n_k)}{\partial n_k}\bigg|_{\hat{x}_k, 0} n_k$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\bigg|_{\hat{x}_k, 0} n_k$$

$$= n_k$$

$\therefore R_k' = R_k$

$\therefore\ Q_k' = M Q_k M^T$

(from Lecture 1)