状态估计 hw4

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1.(5.5.1)

Consider Discret-time system

$$\begin{cases} x_k = x_{k-1} + u_k + \bar{u}_k \\ d_k = x_k \end{cases} \quad A=1 \quad B=1 \quad C=1$$

$\bar{u}_k$ is an unknown input bias.

set up augmented-state system and determine if it's observable.

Augmented-state $x_k' = [x_k \ \bar{u}_k]^T$

and assume $\bar{u}_k = \bar{u}_{k-1} + s_k \quad s_k \sim N(0, w)$

$$\begin{cases} x_k' = \begin{bmatrix} A & B \\ 0 & 1 \end{bmatrix} x_{k-1}' + \begin{bmatrix} B \\ 0 \end{bmatrix} u_k + \begin{bmatrix} B \\ 1 \end{bmatrix} s_k \\ y_k = [1 \ \ 0] x_k' \end{cases}$$

$$x_k = Ax_{k-1} + B(u_k + \bar{u}_k)$$
$$= Ax_{k-1} + B\bar{u}_k + Bu_k$$
$$= Ax_{k-1} + B(\bar{u}_{k-1} + s_k) + Bu_k$$

$$\Rightarrow \begin{cases} x_k' = \underbrace{\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}}_{A'} x_{k-1}' + \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{B'} u_k + \begin{bmatrix} 1 \\ 1 \end{bmatrix} s_k \\ y_k = \underbrace{[1, 0]}_{C'} x_k' \end{cases}$$

$$O' = \left[C' \ \ C'A' \ \ \cdots \ \ C'A'^{(N+U-1)} \right]^T, \quad U=1 \quad N=1$$

$$= \left[C' \ \ C'A' \right]^T$$

$$= \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$\mathrm{rank}(O') = 2 = N+U$

$\therefore$ It's observable

2.(5.5.2) Consider discrete time system

$$\begin{cases} x_k = x_{k-1} + v_k, \\ v_k = v_{k-1} + a_k \\ d_{1,k} = x_k, \end{cases}$$

$$x_k = x_{k-1} + v_k$$
$$= x_{k-1} + v_{k-1} + a_k$$

$A = [1 \ \ 1 \ \ldots$ $B = [1 \ldots$ $\ldots [1 \ \ 0 \ldots$

$$\begin{cases} v_k = v_{k-1} + a_k & = x_{k-1} + v_{k-1} + a_k \\ d_{1,k} = x_k, & A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \\ d_{2,k} = x_k + \bar{d}_k \end{cases}$$

Augmented State $x_k' = [x_k, v_k, \bar{d}_k]^T$

and assume $\bar{d}_k = \bar{d}_{k-1} + s_k$, $s_k \sim N(0, W)$

$$\therefore \begin{cases} x_k' = \underbrace{\begin{bmatrix} A & 0 \\ 0 & 1 \end{bmatrix}}_{A'} x_{k-1}' + \underbrace{\begin{bmatrix} B \\ 0 \end{bmatrix}}_{B'} u_k + \begin{bmatrix} 0 \\ 1 \end{bmatrix} s_k \\ y_k = \underbrace{[C, 1]}_{C'} x_k' \end{cases}$$

$$\Rightarrow \begin{cases} x_k' = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} x_{k-1}' + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} u_k + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} s_k \\ y_k = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix} x_k' \end{cases}$$

$$\therefore O' = [C', C'A', \dots, C'A'^{(N+U-1)}]^T \quad N=2, U=1$$

$$= [C', C'A', C'A'^2]^T$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 2 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

$\text{rank}(O') = 3 = N + U$

$\therefore$ It's observable.

3. (5.5.3) RANSAC

get iteration k for set of $n=3$ inlier pts with probability $p = 0.999$, each has probability $w = 0.1$ of being inlier.

probability $w=0.1$ of being inlier.

① probability to pick all points as inliers for 1 iteration:

$$P_0 = w^n$$

② probability to pick at least 1 point as outlier for 1 iteration:

$$P_1 = 1 - P_0 = 1 - w^n$$

③ probability to pick at least 1 point as outlier for each iteration in k iterations

(i.e. not a single iteration has all points as inlier)

$$P_2 = P_1^k = C_k^k (1-w^n)^k (w^n)^0 = (1-w^n)^k$$

④ probability to pick all points as inlier for at least one iteration in k iterations:

$$p = 1 - P_2 = 1 - (1-w^n)^k$$

$$\therefore \quad 1 - p = (1-w^n)^k$$

$$\ln(1-p) = k \ln(1-w^n)$$

$$k = \frac{\ln(1-p)}{\ln(1-w^n)}$$

plug in w, n, p

$$\Rightarrow k = 6904.3 \approx 6904$$