状态估计第八讲作业讲评

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题目

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证明:

$$
\begin{aligned}
z\left(x_{jk}\right) &\approx \left(1+\epsilon_k^{\wedge}+\frac{1}{2}\epsilon_k^{\wedge}\epsilon_k^{\wedge}\right) T_{\mathrm{op},k}\left(p_{\mathrm{op},j}+D\zeta_j\right) \\
&\approx T_{\mathrm{op},k}p_{\mathrm{op},j}+\epsilon_k^{\wedge}T_{\mathrm{op},k}p_{\mathrm{op},j}+T_{\mathrm{op},k}D\zeta_j \\
&\quad+\frac{1}{2}\epsilon_k^{\wedge}\epsilon_k^{\wedge}T_{\mathrm{op},k}p_{\mathrm{op},j}+\epsilon_k^{\wedge}T_{\mathrm{op},k}D\zeta_j \\
&= z\left(x_{\mathrm{op},jk}\right)+Z_{jk}\delta x_{jk}+\frac{1}{2}\sum_i 1_i \underbrace{\delta x_{jk}^{\mathrm{T}}\mathcal{Z}_{ijk}\delta x_{jk}}_{\text{标量}}
\end{aligned}
$$

其中:

$$
\begin{aligned}
z\left(x_{\mathrm{op},jk}\right) &= T_{\mathrm{op},k}p_{\mathrm{op},j} \\
Z_{jk} &= \left[\begin{array}{cc} \left(T_{\mathrm{op},k}p_{\mathrm{op},j}\right)^{\odot} & T_{\mathrm{op},k}D \end{array}\right] \\
\mathcal{Z}_{ijk} &= \left[\begin{array}{cc} 1_i^{\circledcirc}\left(T_{\mathrm{op},k}p_{\mathrm{op},j}\right)^{\odot} & 1_i^{\circledcirc}T_{\mathrm{op},k}D \\ \left(1_i^{\circledcirc}T_{\mathrm{op},k}D\right)^{\mathrm{T}} & 0 \end{array}\right]
\end{aligned}
$$

解题过程（方法一）

展开:

$$z(x_{jk}) \approx (1+\epsilon_k^\wedge+\frac{1}{2}\epsilon_k^\wedge\epsilon_k^\wedge)T_{op,k}(p_{op,j}+D\zeta_j)$$

$$\approx T_{op,k}p_{op,j}+\underbrace{\epsilon_k^\wedge T_{op,k}p_{op,j}+T_{op,k}D\zeta_j}_{一次项}+\underbrace{\frac{1}{2}\epsilon_k^\wedge\epsilon_k^\wedge T_{op,k}p_{op,j}+\epsilon_k^\wedge T_{op,k}D\zeta_j}_{二次项}$$

验证一次项:

$$Z_{jk}\delta x_{j,k} = [\,(T_{op,k}p_{op,j})^\odot \quad T_{op,k}D\,]\begin{bmatrix}\epsilon_k\\ \zeta_j\end{bmatrix}$$

$$= (T_{op,k}p_{op,j})^\odot\epsilon_k + T_{op,k}D\zeta_j$$

解题过程（方法一）

又因为作业7.5.7:

$$x=\begin{bmatrix} u \\ v \end{bmatrix},\ x^{\wedge}=\begin{bmatrix} v^{\wedge} & u \\ 0^{T} & 0 \end{bmatrix},\ p=\begin{bmatrix} \epsilon \\ \eta \end{bmatrix},\ p^{\odot}=\begin{bmatrix} \eta^{1} & -\epsilon^{\wedge} \\ 0^{T} & 0^{T} \end{bmatrix}$$

$$x^{\wedge}p=\begin{bmatrix} v^{\wedge} & u \\ 0^{T} & 0 \end{bmatrix}\cdot\begin{bmatrix} \epsilon \\ \eta \end{bmatrix}=\begin{bmatrix} v^{\wedge}\epsilon+u\eta \\ 0 \end{bmatrix}$$

$$p^{\odot}x=\begin{bmatrix} \eta^{1} & -\epsilon^{\wedge} \\ 0^{T} & 0^{T} \end{bmatrix}\cdot\begin{bmatrix} u \\ v \end{bmatrix}=\begin{bmatrix} u\eta-\epsilon^{\wedge}v \\ 0 \end{bmatrix}=\begin{bmatrix} u\eta+v^{\wedge}\epsilon \\ 0 \end{bmatrix}=x^{\wedge}p$$

所以很容易知道:

$$(T_{op,k}p_{op,j})^{\odot}\epsilon_{k}=\epsilon_{k}^{\wedge}T_{op,k}p_{op,j}$$

解题过程（方法一）

验证二次项:

$$
\begin{aligned}
\frac{1}{2}\sum_i \mathbf{1}_i \underbrace{\delta \boldsymbol{x}_{jk}^{\mathrm{T}} \boldsymbol{\mathcal{Z}}_{ijk} \delta \boldsymbol{x}_{jk}}_{\text{标量}} &= \frac{1}{2}\sum_i 1_i \begin{bmatrix} \boldsymbol{\epsilon}_k \\ \boldsymbol{\zeta}_j \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} 1_i^{\circledcirc}(\boldsymbol{T}_{\mathrm{op},k}\boldsymbol{p}_{\mathrm{op},j})^{\odot} & 1_i^{\circledcirc}\boldsymbol{T}_{\mathrm{op},k}\boldsymbol{D} \\ (1_i^{\circledcirc}\boldsymbol{T}_{\mathrm{op},k}\boldsymbol{D})^{\mathrm{T}} & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{\epsilon}_k \\ \boldsymbol{\zeta}_j \end{bmatrix} \\
&= \frac{1}{2}\sum_i 1_i \left[\boldsymbol{\epsilon}_k^{\mathrm{T}} 1_i^{\circledcirc}(\boldsymbol{T}_{\mathrm{op},k}\boldsymbol{p}_{\mathrm{op},j})^{\odot} + \boldsymbol{\zeta}_j^{\mathrm{T}}(1_i^{\circledcirc}\boldsymbol{T}_{\mathrm{op},k}\boldsymbol{D})^{\mathrm{T}} \quad \boldsymbol{\epsilon}_k^{\mathrm{T}} 1_i^{\circledcirc}\boldsymbol{T}_{\mathrm{op},k}\boldsymbol{D}\right] \begin{bmatrix} \boldsymbol{\epsilon}_k \\ \boldsymbol{\zeta}_j \end{bmatrix} \\
&= \frac{1}{2}\sum_i 1_i \left(\boldsymbol{\epsilon}_k^{\mathrm{T}} 1_i^{\circledcirc}(\boldsymbol{T}_{\mathrm{op},k}\boldsymbol{p}_{\mathrm{op},j})^{\odot}\boldsymbol{\epsilon}_k + \boldsymbol{\zeta}_j^{\mathrm{T}}(1_i^{\circledcirc}\boldsymbol{T}_{\mathrm{op},k}\boldsymbol{D})^{\mathrm{T}}\boldsymbol{\epsilon}_k + \boldsymbol{\epsilon}_k^{\mathrm{T}} 1_i^{\circledcirc}\boldsymbol{T}_{\mathrm{op},k}\boldsymbol{D}\boldsymbol{\zeta}_j\right) \\
&= \frac{1}{2}\sum_i 1_i \left(\boldsymbol{\epsilon}_k^{\mathrm{T}} 1_i^{\circledcirc}(\boldsymbol{T}_{\mathrm{op},k}\boldsymbol{p}_{\mathrm{op},j})^{\odot}\boldsymbol{\epsilon}_k + 2\boldsymbol{\epsilon}_k^{\mathrm{T}} 1_i^{\circledcirc}\boldsymbol{T}_{\mathrm{op},k}\boldsymbol{D}\boldsymbol{\zeta}_j\right) \\
&= \frac{1}{2}\sum_i 1_i \left(\underbrace{1_i^{\mathrm{T}}\boldsymbol{\epsilon}_k^{\wedge}\boldsymbol{\epsilon}_k^{\wedge}\boldsymbol{T}_{\mathrm{op},k}\boldsymbol{p}_{\mathrm{op},j} + 2\cdot 1_i^{\mathrm{T}}\boldsymbol{\epsilon}_k^{\wedge}\boldsymbol{T}_{\mathrm{op},k}\boldsymbol{D}\boldsymbol{\zeta}_j}_{\text{根据书中公式}(7.159)\boldsymbol{\xi}^{\wedge}\boldsymbol{p}\equiv\boldsymbol{p}^{\odot}\boldsymbol{\xi},\boldsymbol{p}^{\mathrm{T}}\boldsymbol{\xi}^{\wedge}\equiv\boldsymbol{\xi}^{\mathrm{T}}\boldsymbol{p}^{\odot}}\right) \\
&= \frac{1}{2}\underbrace{\sum_i 1_i 1_i^{\mathrm{T}}}_{=\mathbf{1}}\left(\boldsymbol{\epsilon}_k^{\wedge}\boldsymbol{\epsilon}_k^{\wedge}\boldsymbol{T}_{\mathrm{op},k}\boldsymbol{p}_{\mathrm{op},j} + 2\boldsymbol{\epsilon}_k^{\wedge}\boldsymbol{T}_{\mathrm{op},k}\boldsymbol{D}\boldsymbol{\zeta}_j\right) \\
&= \frac{1}{2}\boldsymbol{\epsilon}_k^{\wedge}\boldsymbol{\epsilon}_k^{\wedge}\boldsymbol{T}_{\mathrm{op},k}\boldsymbol{p}_{\mathrm{op},j} + \boldsymbol{\epsilon}_k^{\wedge}\boldsymbol{T}_{\mathrm{op},k}\boldsymbol{D}\boldsymbol{\zeta}_j = \text{左边}
\end{aligned}
$$

解题过程（方法二）

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证明一次项:

$$\boldsymbol{\epsilon}_k^{\wedge}\boldsymbol{T}_{\mathrm{op},k}\boldsymbol{p}_{\mathrm{op},j} + \boldsymbol{T}_{\mathrm{op},k}\boldsymbol{D}\boldsymbol{\zeta}_j = (\boldsymbol{T}_{\mathrm{op},k}\boldsymbol{p}_{\mathrm{op},j})^{\odot}\boldsymbol{\epsilon}_k + \boldsymbol{T}_{\mathrm{op},k}\boldsymbol{D}\boldsymbol{\zeta}_j$$

$$= \left[(\boldsymbol{T}_{\mathrm{op},k}\boldsymbol{p}_{\mathrm{op},j})^{\odot} \quad \boldsymbol{T}_{\mathrm{op},k}\boldsymbol{D}\right]\begin{bmatrix}\boldsymbol{\epsilon}_k \\ \boldsymbol{\zeta}_j\end{bmatrix}$$

证明二次项:

$$\frac{1}{2}\left(\boldsymbol{\epsilon}_k^{\wedge}(\mathbf{T}_{\mathrm{op},k}\mathbf{p}_{\mathrm{op},j})^{\odot}\boldsymbol{\epsilon}_k + 2\boldsymbol{\epsilon}_k^{\wedge}\mathbf{T}_{\mathrm{op},k}\mathbf{D}\boldsymbol{\zeta}_j\right) = \frac{1}{2}\sum_{i=1}^{4}\mathbf{1}_i(\mathbf{1}_i^T\boldsymbol{\epsilon}_k^{\wedge}(\mathbf{T}_{\mathrm{op},k}\mathbf{p}_{\mathrm{op},j})^{\odot}\boldsymbol{\epsilon}_k + 2\cdot\mathbf{1}_i^T\boldsymbol{\epsilon}_k^{\wedge}\mathbf{T}_{\mathrm{op},k}\mathbf{D}\boldsymbol{\zeta}_j)$$

$$= \frac{1}{2}\sum_{i=1}^{4}\mathbf{1}_i(\boldsymbol{\epsilon}_k^T\mathbf{1}_i^{\circledcirc}(\mathbf{T}_{\mathrm{op},k}\mathbf{p}_{\mathrm{op},j})^{\odot}\boldsymbol{\epsilon}_k + 2\cdot\underbrace{\boldsymbol{\epsilon}_k^T\mathbf{1}_i^{\circledcirc}\mathbf{T}_{\mathrm{op},k}\mathbf{D}\boldsymbol{\zeta}_j}_{\text{scale}})$$

$$= \frac{1}{2}\sum_{i=1}^{4}\mathbf{1}_i(\boldsymbol{\epsilon}_k^T\mathbf{1}_i^{\circledcirc}(\mathbf{T}_{\mathrm{op},k}\mathbf{p}_{\mathrm{op},j})^{\odot}\boldsymbol{\epsilon}_k + \boldsymbol{\epsilon}_k^T\mathbf{1}_i^{\circledcirc}\mathbf{T}_{\mathrm{op},k}\mathbf{D}\boldsymbol{\zeta}_j + \boldsymbol{\zeta}_j^T(\mathbf{1}_i^{\circledcirc}\mathbf{T}_{\mathrm{op},k}\mathbf{D})^T\boldsymbol{\epsilon}_k)$$

$$= \frac{1}{2}\sum_{i=1}^{4}\mathbf{1}_i\left[\boldsymbol{\epsilon}_k^T \quad \boldsymbol{\zeta}_j^T\right]\underbrace{\begin{bmatrix}\mathbf{1}_i^{\circledcirc}(\mathbf{T}_{\mathrm{op},k}\mathbf{p}_{\mathrm{op},j})^{\odot} & \mathbf{1}_i^{\odot}\mathbf{T}_{\mathrm{op},k}\mathbf{D} \\ (\mathbf{1}_i^{\odot}\mathbf{T}_{\mathrm{op},k}\mathbf{D})^T & \mathbf{0}\end{bmatrix}}_{\boldsymbol{\mathcal{Z}}_{ijk}}\begin{bmatrix}\boldsymbol{\epsilon}_k \\ \boldsymbol{\zeta}_j\end{bmatrix}$$

感谢各位聆听！

Thanks for Listening