状态估计 hw5

Tuesday, May 26, 2020 0:48

1. (6.6.1) Show that $u^{\wedge}v \equiv -v^{\wedge}u$ for any two 3×1 colns u and v

$$u^{\wedge}=\begin{bmatrix}0 & -u_3 & u_2\\ u_3 & 0 & -u_1\\ -u_2 & u_1 & 0\end{bmatrix}$$

$$v^{\wedge}=\begin{bmatrix}0 & -v_3 & v_2\\ v_3 & 0 & -v_1\\ -v_2 & v_1 & 0\end{bmatrix}$$

$$\therefore\ LHS = u^{\wedge}v=\begin{bmatrix}0 & -u_3 & u_2\\ u_3 & 0 & -u_1\\ -u_2 & u_1 & 0\end{bmatrix}\begin{bmatrix}v_1\\ v_2\\ v_3\end{bmatrix}$$

$$=\begin{bmatrix}-u_3v_2+u_2v_3\\ u_3v_1-u_1v_3\\ -u_2v_1+u_1v_2\end{bmatrix}$$

$$RHS=-v^{\wedge}u=\begin{bmatrix}0 & v_3 & -v_2\\ -v_3 & 0 & v_1\\ v_2 & -v_1 & 0\end{bmatrix}\begin{bmatrix}u_1\\ u_2\\ u_3\end{bmatrix}$$

$$=\begin{bmatrix}v_3u_2-v_2u_3\\ -v_3u_1+v_1u_3\\ v_2u_1-v_1u_2\end{bmatrix}$$

$\because$ LHS = RHS

$\therefore$ Q.E.D.

2. (6.6.2) Show that $C^{-1}=C^T$ from

$$C=\cos\theta I+(1-\cos\theta)aa^T+\sin\theta a^{\wedge}$$

$\because\ (a^{\wedge})^T=-a^{\wedge}$

$\therefore\ C^T=\cos\theta I+(1-\cos\theta)aa^T-\sin\theta a^{\wedge}$

$\therefore$ to prove $C^{-1}=C^T$

is to prove $C\cdot C^T=I$

$$\therefore RHS = C\cdot C^T=(c\theta I+(1-c\theta)aa^T+s\theta a^{\wedge})\cdot(c\theta I+(1-c\theta)aa^T-s\theta a^{\wedge})$$

$$=c^2\theta I+c\theta(1-c\theta)aa^T-\cancel{c\theta s\theta a^{\wedge}}$$

$$+c\theta(1-c\theta)aa^T+(1-c\theta)^2\left(\sum_{i=1}^{n}a_i^2\right)aa^T-\cancel{s\theta(1-c\theta)aa^Ta^{\wedge}}$$

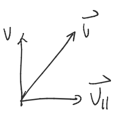
$$+\cancel{s\theta c\theta a^{\wedge}}+s\theta(1-c\theta)\cancel{a^{\wedge}aa^T}-s^2\theta a^{\wedge 2}$$

$$(\ a^{\wedge}a=\underline{0}\qquad a^Ta^{\wedge}=\underline{0}^T\ )$$

$$=c^2\theta I+2c\theta(1-c\theta)aa^T+s^2\theta\left(\sum_{i=1}^{n}a_i^2\right)aa^T-s^2\theta a^{\wedge 2}$$

$$=c^2\theta I+(2c\theta-2c^2\theta+s^2\theta\left(\sum_{i=1}^{n}a_i^2\right)aa^T-s^2\theta a^{\wedge 2}=I$$

let $aa^T=\dfrac{aa^T}{a^Ta}=P_{\parallel}$, a is unit vector,



$v=v_{\parallel}+v_{\perp}$, $v_{\parallel}=P_{\parallel}v$, $v_{\parallel}\perp v_{\perp}$

$\therefore\ v_{\perp}=v-v_{\parallel}=-a\times(a\times v)$

and $a\times v=a^{\wedge}v$ $\quad a\times(a\times v)=a^{\wedge 2}v$

$\therefore\ C\cdot v=\cos\theta v+(1-\cos\theta)aa^Tv+\sin\theta a^{\wedge}v$

$$=\cos\theta v+(1-\cos\theta)v_{\parallel}+\sin\theta a^{\wedge}v$$

$$\ldots(1-c\theta)(v+a^{\wedge 2}v)+s\theta a^{\wedge}v$$

$$= \cos\theta\, v + (1-\cos\theta)\, v_{\parallel} + \sin\theta\, a^{\wedge} v$$

$$= c\theta\, v + (1-c\theta)(v + a^{\wedge 2} v) + s\theta\, a^{\wedge} v$$

$$= (c\theta + (1-c\theta)(1 + a^{\wedge 2}) + s\theta\, a^{\wedge})\, v$$

$$= (\cancel{c\theta} + 1 + a^{\wedge 2} - \cancel{c\theta} - c\theta\, a^{\wedge 2} + s\theta\, a^{\wedge})\, v$$

$$= (1 + s\theta\, a^{\wedge} + (1-c\theta)\, a^{\wedge 2})\, v$$

$$\therefore\ C = 1 + s\theta\, a^{\wedge} + (1-c\theta)\, a^{\wedge 2}$$

$$C^T = 1 - s\theta\, a^{\wedge} + (1-c\theta)\, a^{\wedge 2} \qquad ((a^{\wedge})^T = -a^{\wedge},\ (a^{\wedge} a^{\wedge})^T = a^{\wedge T} a^{\wedge T} = (a^{\wedge})^2)$$

$$\therefore\ RHS = C \cdot C^T = (1 + s\theta\, a^{\wedge} + (1-c\theta)\, a^{\wedge 2}) \cdot (1 - s\theta\, a^{\wedge} + (1-c\theta)\, a^{\wedge 2})$$

$$= 1 - \cancel{s\theta\, a^{\wedge}} + (1-c\theta)\, a^{\wedge 2} + \cancel{s\theta\, a^{\wedge}} - s^2\theta\, a^{\wedge 2} + \cancel{s\theta(1-c\theta)\, a^{\wedge 3}}$$
$$+ (1-c\theta)\, a^{\wedge 2} - \cancel{s\theta(1-c\theta)\, a^{\wedge 3}} + (1-c\theta)^2\, a^{\wedge 4}$$

$$= 1 + 2(1-c\theta)\, a^{\wedge 2} - s^2\theta\, a^{\wedge 2} + (1-c\theta)^2\, a^{\wedge 4}$$

$$= 1 + (1-c\theta)^2\, a^{\wedge 4} + 2(1-c\theta)\, a^{\wedge 2} + (1-c^2\theta)\, a^{\wedge 2}$$

$$= 1$$

$\therefore$ Q.E.D.

3. (6.6.3) Show that $(Cv)^{\wedge} \equiv C v^{\wedge} C^T$ for any 3×1 col v and rot mat C

let $C = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$ $\quad v = [v_1\ v_2\ v_3]^T$

$$LHS = \begin{bmatrix} C_{11}v_1 + C_{12}v_2 + C_{13}v_3 \\ C_{21}v_1 + C_{22}v_2 + C_{23}v_3 \\ C_{31}v_1 + C_{32}v_2 + C_{33}v_3 \end{bmatrix}^{\wedge}$$

$$= \begin{bmatrix} 0 & -C_{31}v_1 - C_{32}v_2 - C_{33}v_3 & C_{21}v_1 + C_{22}v_2 + C_{23}v_3 \\ C_{31}v_1 + C_{32}v_2 + C_{33}v_3 & 0 & -C_{11}v_1 - C_{12}v_2 - C_{13}v_3 \\ -C_{21}v_1 - C_{22}v_2 - C_{23}v_3 & C_{11}v_1 + C_{12}v_2 + C_{13}v_3 & 0 \end{bmatrix}$$

$$RHS = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \begin{bmatrix} 0 & -v_3 & v_2 \\ v_3 & 0 & -v_1 \\ -v_2 & v_1 & 0 \end{bmatrix} \begin{bmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{23} \\ C_{13} & C_{23} & C_{33} \end{bmatrix}$$

$$= \begin{bmatrix} C_{12}v_3 - C_{13}v_2 & -C_{11}v_3 + C_{13}v_1 & C_{11}v_2 - C_{12}v_1 \\ C_{22}v_3 - C_{23}v_2 & -C_{21}v_3 + C_{23}v_1 & C_{21}v_2 - C_{22}v_1 \\ C_{32}v_3 - C_{33}v_2 & -C_{31}v_3 + C_{33}v_1 & C_{31}v_2 - C_{32}v_1 \end{bmatrix} \cdot \begin{bmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{23} \\ C_{13} & C_{23} & C_{33} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & (\underbrace{C_{13}C_{22} - C_{12}C_{23}}_{-C_{31}})v_1 + (\underbrace{C_{11}C_{23} - C_{13}C_{21}}_{-C_{32}})v_2 + (\underbrace{C_{12}C_{21} - C_{11}C_{22}}_{-C_{33}})v_3, & (\underbrace{C_{13}C_{32} - C_{12}C_{33}}_{C_{21}})v_1 + (\underbrace{C_{11}C_{33} - C_{13}C_{31}}_{C_{22}})v_2 + (\underbrace{C_{12}C_{31} - C_{11}C_{32}}_{C_{23}})v_3 \\ (\underbrace{C_{12}C_{23} - C_{13}C_{22}}_{C_{31}})v_1 + (\underbrace{C_{13}C_{21} - C_{11}C_{23}}_{C_{32}})v_2 + (\underbrace{C_{11}C_{22} - C_{12}C_{21}}_{C_{33}})v_3, & 0, & (\underbrace{C_{23}C_{32} - C_{22}C_{33}}_{-C_{11}})v_1 + (\underbrace{C_{21}C_{33} - C_{23}C_{31}}_{-C_{12}})v_2 + (\underbrace{C_{22}C_{31} - C_{21}C_{32}}_{-C_{13}})v_3 \\ (\underbrace{C_{12}C_{33} - C_{13}C_{32}}_{-C_{21}})v_1 + (\underbrace{C_{13}C_{31} - C_{11}C_{33}}_{-C_{22}})v_2 + (\underbrace{C_{11}C_{32} - C_{12}C_{31}}_{-C_{23}})v_3, & (\underbrace{C_{22}C_{33} - C_{23}C_{32}}_{C_{11}})v_1 + (\underbrace{C_{23}C_{31} - C_{21}C_{33}}_{C_{12}})v_2 + (\underbrace{C_{21}C_{32} - C_{22}C_{31}}_{C_{13}})v_3, & 0 \end{bmatrix}$$

$$= LHS$$

$\therefore$ Q.E.D.