**Algorithm 1:** XGBoost Feature Importance with SHAP Values

**Input:** Dataset $\mathcal{D} = \{(\mathbf{x}_i, y_i^{(J)}, y_i^{(F)})\}_{i=1}^N$, where $y^{(J)}$ is judge score, $y^{(F)}$ is fan share

**Input:** Feature set $\mathcal{F} = \{f_1, f_2, \ldots, f_p\}$ including age, industry indicators, week, partner encoding

**Input:** XGBoost hyperparameters: $T$ trees, max depth $d$, learning rate $\eta$

**Output:** Feature importance rankings for judges and fans

// **Data Preprocessing**;

Encode categorical partner IDs: $f_{\text{partner}} \leftarrow \text{factorize(partner)}$;

Log-transform fan share: $y^{(F)} \leftarrow \log(y^{(F)} + \epsilon)$, $\epsilon = 10^{-6}$;

Remove samples with missing values in $\mathcal{F}$;

// **Model Training**;

**for** *target* $y \in \{y^{(J)}, y^{(F)}\}$ **do**

  Initialize model $\hat{y}^{(0)} = 0$;

  **for** $t = 1$ **to** $T$ **do**

    Compute gradients: $g_i = \frac{\partial \ell(y_i, \hat{y}_i^{(t-1)})}{\partial \hat{y}_i^{(t-1)}}$;

    Compute Hessians: $h_i = \frac{\partial^2 \ell(y_i, \hat{y}_i^{(t-1)})}{\partial (\hat{y}_i^{(t-1)})^2}$;

    Fit regression tree $f_t$ by optimizing:;

    $\mathcal{L}^{(t)} = \sum_{i=1}^N [g_i f_t(\mathbf{x}_i) + \frac{1}{2} h_i f_t^2(\mathbf{x}_i)] + \Omega(f_t)$;

    Update: $\hat{y}^{(t)} = \hat{y}^{(t-1)} + \eta \cdot f_t(\mathbf{x})$;

  **end**

  Store trained model $\mathcal{M}_y$;

**end**

// **SHAP Value Computation (TreeExplainer)**;

**for** *each model* $\mathcal{M} \in \{\mathcal{M}_J, \mathcal{M}_F\}$ **do**

  **for** *each sample* $\mathbf{x}_i$ **do**

    **for** *each feature* $f_j \in \mathcal{F}$ **do**

      Compute SHAP value via tree path decomposition:;

      $\phi_j(\mathbf{x}_i) = \sum_{S \subseteq \mathcal{F} \setminus \{j\}} \frac{|S|!(p-|S|-1)!}{p!} [f_{\mathcal{M}}(S \cup \{j\}) - f_{\mathcal{M}}(S)]$;

    **end**

  **end**

  Store SHAP matrix $\Phi \in \mathbb{R}^{N \times p}$;

**end**

// **Feature Importance Ranking**;

**for** *each feature* $f_j$ **do**

  Compute mean absolute SHAP: $I_j = \frac{1}{N} \sum_{i=1}^N |\phi_j(\mathbf{x}_i)|$;

**end**

Rank features by $I_j$ in descending order;

**return** Importance rankings $\mathcal{R}^{(J)}$, $\mathcal{R}^{(F)}$ for judge and fan models;

**Algorithm 2:** Linear Mixed-Effects Model for Pro Partner Random Effects

**Input:** Dataset $\mathcal{D}$ with outcomes $y$, fixed effects $\mathbf{X}$, grouping factor (partner) $g$

**Input:** Model specification: $y = \mathbf{X}\boldsymbol{\beta} + \mathbf{Z}\mathbf{u} + \boldsymbol{\varepsilon}$

**Output:** Fixed effects $\hat{\boldsymbol{\beta}}$, random effects $\hat{\mathbf{u}}$, variance components

// **Model Specification**;

Fixed effects: $\mathbf{X} = [\text{Age}, \text{Industry}_1, \ldots, \text{Industry}_k, \text{Week}, \text{StageRatio}]$;

Random effects: $\mathbf{u}_g \sim \mathcal{N}(0, \sigma_u^2)$ for each partner $g$;

Residuals: $\varepsilon_i \sim \mathcal{N}(0, \sigma^2)$;

// **Parameter Estimation (REML)**;

Construct marginal covariance: $\mathbf{V} = \mathbf{Z}\mathbf{G}\mathbf{Z}^\top + \mathbf{R}$;

Maximize restricted log-likelihood:;

$\ell_R(\boldsymbol{\theta}) = -\frac{1}{2}\left[\log|\mathbf{V}| + \log|\mathbf{X}^\top\mathbf{V}^{-1}\mathbf{X}| + (\mathbf{y} - \mathbf{X}\hat{\boldsymbol{\beta}})^\top\mathbf{V}^{-1}(\mathbf{y} - \mathbf{X}\hat{\boldsymbol{\beta}})\right]$;

Estimate $\hat{\sigma}_u^2$, $\hat{\sigma}^2$ via Powell optimization;

// **Extract Partner Effects**;

Compute BLUPs: $\hat{\mathbf{u}} = \mathbf{G}\mathbf{Z}^\top\mathbf{V}^{-1}(\mathbf{y} - \mathbf{X}\hat{\boldsymbol{\beta}})$;

Compute ICC: $\rho = \frac{\hat{\sigma}_u^2}{\hat{\sigma}_u^2 + \hat{\sigma}^2}$;

// **Interpretation**;

**for** *each partner g* **do**

  **if** $\hat{u}_g > 0$ **then** Partner adds positive effect (“star power”);

  **else** Partner has negative baseline effect;

**end**

**return** $\hat{\boldsymbol{\beta}}$, $\{\hat{u}_g\}$, ICC $\rho$;

**Algorithm 3:** Comparative Feature Effect Analysis: Judges vs Fans

**Input:** SHAP matrices $\Phi^{(J)}$, $\Phi^{(F)}$ from Algorithm 1
**Input:** Feature matrix $\mathbf{X}$, feature names $\mathcal{F}$
**Output:** Comparative effect analysis for each feature
// **Global Importance Comparison**;
**for** *each feature* $f_j \in \mathcal{F}$ **do**
  $I_j^{(J)} = \text{mean}(|\Phi_{:,j}^{(J)}|)$ ; `// Judge importance`
  $I_j^{(F)} = \text{mean}(|\Phi_{:,j}^{(F)}|)$ ; `// Fan importance`
  $\Delta_j = I_j^{(F)} - I_j^{(J)}$ ; `// Relative fan preference`
**end**
// **Industry-Specific Effects**;
**for** *each industry indicator* $f_{ind} \in \{Athlete, Actor, Model, \ldots\}$ **do**
  Select samples: $\mathcal{S} = \{i : x_{i,\text{ind}} = 1\}$;
  $\bar{\phi}_{\text{ind}}^{(J)} = \text{mean}(\Phi_{\mathcal{S},\text{ind}}^{(J)})$ ; `// Judge effect`
  $\bar{\phi}_{\text{ind}}^{(F)} = \text{mean}(\Phi_{\mathcal{S},\text{ind}}^{(F)})$ ; `// Fan effect`
  **if** $\bar{\phi}_{ind}^{(J)} < 0$ ***and*** $\bar{\phi}_{ind}^{(F)} > 0$ **then**
    Flag: Industry favored by fans, penalized by judges;
  **end**
**end**
// **Age Dependence Analysis**;
Fit polynomial: $\phi_{\text{age}}^{(J)}(x) = \beta_0 + \beta_1 x + \beta_2 x^2$;
Fit polynomial: $\phi_{\text{age}}^{(F)}(x) = \gamma_0 + \gamma_1 x + \gamma_2 x^2$;
Optimal age (judges): $x_J^* = -\beta_1/(2\beta_2)$;
Optimal age (fans): $x_F^* = -\gamma_1/(2\gamma_2)$;
**return** Importance rankings, industry effects, age optima;