Aimbot

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Problem 1. Given $T(0), T(-1) \in \mathbb{R}^2$ and $v \in \mathbb{R}$ with $v > 0$. If possible, calculate $t \geqslant 0$ and $T(t) \in \mathbb{R}^2$ assuming T is a line and

$$\frac{|T(t)|}{v} = t$$

holds.

Solution.

With $T := T(0), T' := T(-1), D := T - T'$ the system of equations is derived:

$$\begin{aligned}
T(t) &= T + t(T - T') \\
T(t)_x &= T_x + t\,D_x && (1)\\
T(t)_y &= T_y + t\,D_y && (2)\\
\frac{|T(t)|}{v} &= t \\
|T(t)| &= t\,v \\
T(t)_x^2 + T(t)_y^2 &= t^2 v^2 && (3)
\end{aligned}$$

Solving for t with:

$$\begin{aligned}
(T_x + t\,D_x)^2 + (T_y + t\,D_y)^2 &= t^2 v^2 \\
T_x^2 + 2t\,T_x D_x + t^2 D_x^2 + T_y^2 + 2t\,T_y D_y + t^2 D_y^2 - t^2 v^2 &= 0 \\
\Bigg(\underbrace{\underbrace{D_x^2 + D_y^2}_{=|T(0)-T(1)|^2} - v^2}_{a}\Bigg) t^2 + \Bigg(\underbrace{2T_x D_x + 2T_y D_y}_{b}\Bigg) t + \Bigg(\underbrace{T_x^2 + T_y^2}_{c=|T(0)|^2}\Bigg) &= 0
\end{aligned}$$

Assuming $c = 0$

$$t = 0$$

Assuming $v > |T(0) - T(1)|$

$$\begin{aligned}
t &= \frac{-b \pm \sqrt{b^2 - 4a\,c}}{2a} \\
a < 0\colon\ &= \frac{-b - \sqrt{b^2 - 4a\,c}}{2a} \\
&= \frac{-(2T_x D_x + 2T_y D_y) - \sqrt{(2T_x D_x + 2T_y D_y)^2 - 4(D_x^2 + D_y^2 - v^2)\,(T_x^2 + T_y^2)}}{2(D_x^2 + D_y^2 - v^2)} \\
&= \frac{-(T_x D_x + T_y D_y) - \sqrt{(T_x D_x + T_y D_y)^2 - \overbrace{(D_x^2 + D_y^2 - v^2)}^{<0}\,\overbrace{(T_x^2 + T_y^2)}^{>0}}}{(D_x^2 + D_y^2 - v^2)}
\end{aligned}$$

Assuming $v < |T(0) - T(1)|, b < 0, b^2 \geqslant 4a\,c$

$$\begin{aligned}
t &= \frac{-b \pm \sqrt{b^2 - 4a\,c}}{2a} \\
a > 0\colon\ &= \frac{-b - \sqrt{b^2 - 4a\,c}}{2a}
\end{aligned}$$

Assuming $v = |T(0) - T(1)|, b < 0$

$$t = -\frac{c}{b}$$