CS 200: Machine Learning
Final Exam
Date: Spring 2020

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This exam is worth 200 points in total:

- You have up to 120 minutes to complete the test, and then 15 extra minutes to submit your answers as a pdf on Canvas.
- This exam is open book, but not open notes or open internet.
- NOTE: For all problems involving calculations - set up the calculation, and evaluate it if it just involves basic arithmetic operations. You do not need to fully evaluate it if it involves square roots, logs, matrix inverses, etc.
- If you are asked to define or explain something, use your own words (do not just copy a definition out of the book).

Turn the page now to start Problem 1.

Before you turn in your exam:

Sign the statement below. If you cannot, plan to talk with me within the next two days.

I have neither given nor received inappropriate assistance on this exam.

Signed: Jared Kelnhofer

1. Calculations, etc... (5 pts each)

a) Given two vectors:

$$\vec{v}=\begin{bmatrix}2\\3\\-1\end{bmatrix} \quad \text{and} \quad \vec{w}=\begin{bmatrix}1\\0\\4\end{bmatrix}$$

$(2\cdot 1)+(3\cdot 0)+(-1\cdot 4) = 2+0-4 = \boxed{-2}$

what is the dot product of v and w?

b) Given the matrices:

$$\mathbf{A}=\begin{bmatrix}1 & 2 & -2\\0 & 3 & 1\end{bmatrix} \quad \text{and} \quad \mathbf{B}=\begin{bmatrix}-1 & 2\\-2 & 1\\0 & 1\end{bmatrix}$$

$[1\ 2\ -2]\cdot[-1\ -2\ 0]$, $[0\ 3\ 1]\cdot[-1\ -2\ 0]$

$[1\ 2\ -2]\cdot[2\ 1\ 1]$, $[0\ 3\ 1]\cdot[2\ 1\ 1]$

what is the result of multiplying B x A?

$= [-1\ -4], [-6]$

$[2+2-2], [3+1]$

$$\boxed{\begin{bmatrix}-5, & -6\\2, & 4\end{bmatrix}}$$

c) Given a multivariate linear regression model with:

$$\theta=\begin{bmatrix}2\\3\\-1\end{bmatrix} \quad \text{and} \quad x_1=\begin{bmatrix}3\\2\end{bmatrix}$$

$Y_1 = \theta\cdot X_1 = (2\cdot 3)+(3\cdot 2)+(-1) = \boxed{11}$

what is the predicted y_1 output value?

d) Given:

$Y_2 = \theta\cdot X_2 = (2\cdot 1)+(3\cdot 0)+(-1) = \boxed{5}$

$$x_2=\begin{bmatrix}1\\0\end{bmatrix}$$

what is the predicted y_2 output value for the same model as in question "c)"?

e) Given the multivariate regression model of question "c)", and the output values of y_1 and y_2, with corresponding target values of y_1 target = 20 and y_2 target = 12, what is the mean squared error?

$$MSE = \frac{1}{m}\sum_{i=1}^{m}\left(h(x^i)-y^i\right)^2 = \frac{1}{2}\left((11-20)+(5-12)\right)^2$$

$$= \frac{1}{2}\left(-9+(-7)\right)^2 = \frac{1}{2}(-16)^2 = \frac{1}{2}(256) = \boxed{128}$$

f) Given a polynomial regression model with:

$$\theta = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix} \quad \text{and} \quad x=3$$

$$y = 2\cdot 3^2 + 3\cdot 3^1 + (-1\cdot 3^0) = 18 + 9 - 1 = \boxed{26}$$

determine the predicted value y.

g) Assume we have a 3 class classifier, with a softmax probability distribution output. Assume the classes are numbered [0 .. 2]. Given a classifier output, **p**, and corresponding labels, **y**:

good

$$p = \begin{bmatrix} 0.1 & 0.2 & 0.7 \\ 0.0 & 0.8 & 0.2 \end{bmatrix} \quad \text{and} \quad y = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

better

K = num classes

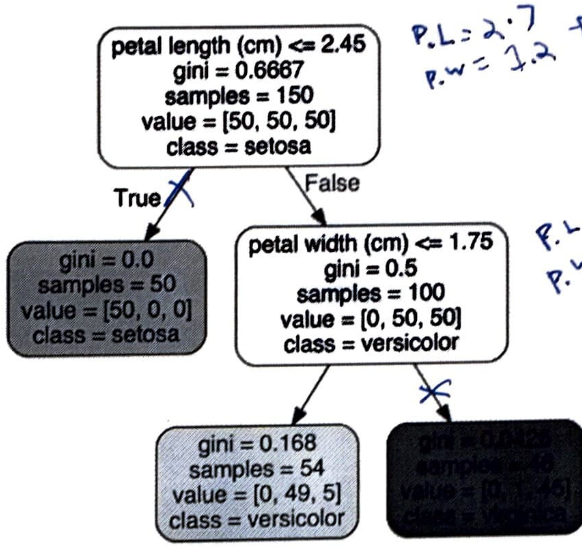
$y_k^{(i)}$ is 1

Set up the calculation for the cross-entropy.

$$J(p) = -\frac{1}{3}\sum_{i=1}^{2}\sum_{k=1}^{3} y_k^{(i)} \log\left(\hat{p}_k^{(i)}\right)$$

$$\boxed{J(p) = -\frac{1}{3}\sum_{i=1}^{2}\sum_{k=1}^{3} \log\left(\hat{p}_k^{(i)}\right)}$$

h) Determine the output of the decision tree shown below for an input iris with petal length = 2.7 and petal width of 1.2. (one word)



P.L = 2.7 → False
P.W = 1.2

P.L = 2.7 → False
P.W = 1.2

$\boxed{\text{versicolor}}$

i) Given a 4 class tree classifier with a node with the following number of instances from each class: [4, 2, 1, 5], calculate the Gini impurity.

$\frac{4}{12} = P(0)$ $\frac{2}{12} = P(1)$ $\frac{1}{12} = P(2)$ $\frac{5}{12} = P(5)$

$$G(node) = 1 - \sum_{k=1}^{n} P_{i,k}^{2} = 1 - \left(\left(\frac{4}{12}\right)^2 + \left(\frac{2}{12}\right)^2 + \left(\frac{1}{12}\right)^2 + \left(\frac{5}{12}\right)^2\right) = 1 - (0.3944)$$

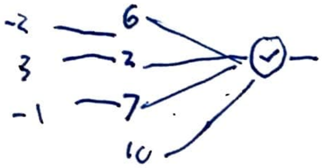
$$\boxed{\approx 0.680555}$$

2) Single Artificial Neuron (10 pts.)

Given a <u>single</u> neuron w/:

input x values = $\begin{bmatrix} -2 \\ 3 \\ -1 \end{bmatrix}$ weights = $\begin{bmatrix} 6 \\ 2 \\ 7 \end{bmatrix}$ and bias = 10

and ReLU activation function, calculate the resulting output.



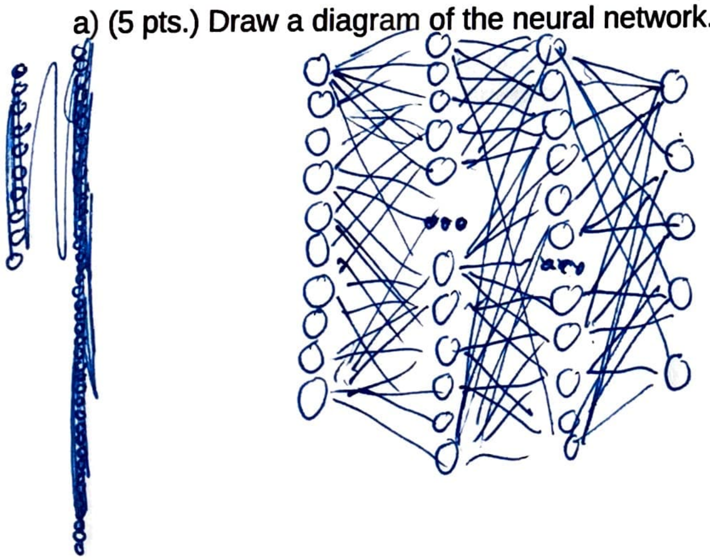
-12 + 6 - 7 + 10 = -3

reLu(-3) = $\boxed{0}$

3) Deep Neural Network

Assume you are given a fully connected deep neural network with 10 inputs, 2 hidden layers, and a 5 output softmax layer. The first hidden layer has 100 neurons, and the second hidden layer has 50 neurons.

a) (5 pts.) Draw a diagram of the neural network.



or $\boxed{10}$ - $\boxed{100}$ - $\boxed{50}$ - $\boxed{5}$

b) (10 pts.) Assuming an input dataset with 1,000 instances, show all of the tensors (2-D matrices in this case) corresponding to the weights and data as it flows through the network.

not really sure how to do this one. I guess I'd split all nodes into their own matrix

~~Input~~

Input Shape: (10, 1)

Hidden 1 Shape: (100, 1)

Hidden 2 Shape: (50, 1)

Output Shape: (5, 1)

Progess/temp values Shape: (10, batch size, 4) so you can track all states of input.

c) (10 pts.) Calculate the total number of weights in the network. $\boxed{6250}$

4) Convolution Filters

a) (10 pts.) Given a 3x3 convolution filter acting on a gray-scale image that is 4x4, where the convolution filter weights are specified by:

1	0
0	1

I'll just treat the bottom right index as the "chooser" not sure how this gets placed over image.

and the image pixels are:

0	0	0	0	0
0	2	3	4	0
0	4	3	2	0
0	3	5	0	0
0	0	0	0	0

2	3	4
4	5	5
3	9	3

$\begin{matrix} 0 & 0 \\ 0 & 2 \end{matrix} \rightarrow \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} \rightarrow$

$(0 \cdot 1) + (0 \cdot 0) + (0 \cdot 0) + (2$

$= 2$

Calculate the output, where the zero padding is specified as 'SAME'.

Note: "gray-scale" means just one band, not rgb

✓

b) (5 pts.) If I was trying to find '+'s in a gray-scale image, what would two useful 3x3 convolution filters be? (draw the 2 filters with weight values)

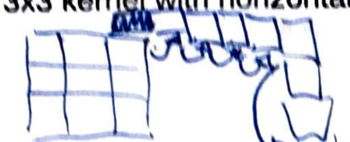
0	1	0
1	1	1
0	1	0

1	0	1
0	0	0
1	0	1

small '+' shapes.

5) Neural Net Parameters, Special Layers

a) (5 pts) What is the reduction in size that would result from applying a pooling layer with a 3x3 kernel with horizontal and vertical strides of 4, acting on a gray-scale image? (a number)



Vertical height is $\frac{1}{4}$ original size

b) (5 pts.) Why would this kernel size be unusual given these stride parameters? (one sentence)

You skip a pixel area each stride because the kernel is smaller than the stride.

6) Recurrent Neural Networks (15 pts.)

Given a recurrent neural network with a single neuron and the following values, what is the output y(t) for 0 <= t <=4?

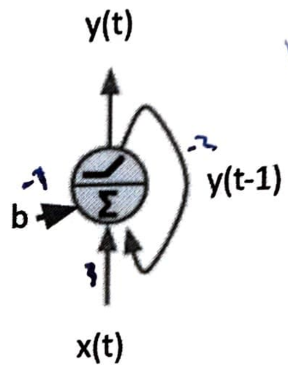
$x(t) = [5,4,3,2,1]$ starting at $t = 0$

$y(t) = 0$ for all $t < 0$

$w_x = 3$

$w_y = -2$

$w_b = -1$



$y(t) = \phi(W_x^T x_{(t)} + W_y^T y_{(t-1)} + b)$

~~ϕ = relu~~ ~~$\text{relu}([3]^T [5,4,3,2,1] + [-2]^T$~~

$12 - 1 + 2$

$9 - 1 + (-26)$

$-26 - 8 = -34$

$5 + (68)$

$2 + (-146)$

$\boxed{-1}$

$y(0-1) = 0 \quad x = 0$

~~$y(1) = (-1 \cdot -2) + (3 \cdot 1) - 1$~~

$y(0) = -1$

$y(1) = 13$

$y(2) = -34$

$y(3) = 73$

$y(4) = -144$

7) Reinforcement Learning Agents (10 pts.)

Given a reinforcement learning based agent with a policy to control a Roomba (i.e. a vacuuming robot), what are some possibilities for quantifying its:

environment -
- Floor type
- Indoors/outdoors
- Cleaned already?

observations -
- distance from start
- distance to object in all directions
- Moving objects
- floor gradient

actions -
- Turn on vaccum
- Turn off vacuum
- Rotate (num degrees)
- move forward
- move backwards

rewards -

8) Reinforcement Learning (20 pts.)

If you had a function, `my_policy(state)`, that took in a state as as an input parameter and returned an action (like the hard-coded policy function we used for the cart-pole example) - describe in pseudo-code the process you would use to train a neural net to implement an approximation of the same policy. (roughly 5 sentences)

I would begin by generating a lot of data in order to find out what actions can be taken. I would then treat the problem as a classification problem, and train the neural net to choose an action based on an input state. I would use the data generated by inputting lots of states into the function as the training data. Because of the Universal Approximation Theorem, I would know that the function could likely be approximated with a neural net.

9) Reinforcement Learning Discounted Future Returns

Assume that we've simulated a cart-pole episode and that it lasted 3 time steps, with the following rewards:

time	reward
0	4
1	3
2	1

a) (10 pts.) Assuming a value for gamma = 0.95, set up the calculations for the Discounted Future Returns for each of the 3 time steps.

$\cancel{1 + (0.95 \times 3) + ((0.95)^2 \times 1)}$

$$\boxed{((0.95)^0 \cdot 4) + ((0.95)^1 \cdot 3) + ((0.95)^2 \cdot 1)}$$

b) (10 pts.) In general, what does a value of gamma = 0 mean? What does a value of gamma = 1 mean? (2 sentences)

A gamma of 1 means your future actions matter a lot.

A gamma of 0 means future actions really don't matter.

11) Q Learning (10 pts.)

Given

$$Q(s,a)=\begin{bmatrix} 4.2 & 6.7 \\ 3.7 & 2.3 \\ 9.8 & 1.2 \end{bmatrix}$$

for states **s** ϵ {0,1,2} and actions **a** ϵ {0,1} what is the corresponding optimal policy **π(s)**?

Choosing the action with the highest Q-Value.

12) Q Learning (20 pts.)
Say that we have the Q(s,a) given in the previous problem, and we want to improve our estimate of it by training based on two experiences in our replay buffer:

$(s,a,r,s')_0 = (2,1,1,0)$ and $(s,a,r,s')_1 = (1,0,0,2)$

assume that we're using a value of gamma = 0.8, and alpha = 0.5

Calculate what the new Q(s,a) will be as a result of using Q-Learning in this case to update it based on these two experiences.

I'm honestly lost on this one.