MATH 3322 Quiz 4 T1B

Apr 27, 2022

1.(30 pts) Identify all possible eigenvalues for the Householder matrix.

2.(40 pts)

(1)(10 pts) How power iteration get an estimation of eigenvalue from $\boldsymbol{v}^k = \dfrac{\boldsymbol{A}^k\boldsymbol{x}}{\|\boldsymbol{A}^k\boldsymbol{x}\|}$

(2)(30 pts) Let

$$\boldsymbol{A} = \begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix}, \quad \boldsymbol{u}^0 = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

Apply power iteration to $\boldsymbol{A}$ with initial eigenvector estimate $\boldsymbol{u}^0$. Calculate λ^1 and $\boldsymbol{u}^1$, where λ^1 is the estimation of eigenvalue in the first iteration.

3.(30 pts). Suppose $\boldsymbol{A} \in \mathbb{R}^{m\times m}$ is symmetric postive definite .The rayleigh quotient of a vector $\boldsymbol{x} \in \mathbb{R}^m$ is a function from $\mathbb{R}^m$ to $\mathbb{R}$

$$r(\boldsymbol{x}) = \frac{\boldsymbol{x}^T\boldsymbol{A}\boldsymbol{x}}{\boldsymbol{x}^T\boldsymbol{x}}$$

(1) Prove that

$$\frac{\partial}{\partial x_j}\boldsymbol{x}^T\boldsymbol{A}\boldsymbol{x} = 2(\boldsymbol{A}\boldsymbol{x})_j$$

(2) Prove that the eigenvectors of $\boldsymbol{A}$ are the statioary points of $r(\boldsymbol{x})$.
($\boldsymbol{x}_0$ is a statioary point of $r(\boldsymbol{x})$ if $\nabla r(\boldsymbol{x}_0) = 0$)