KORENI: $\int \frac{dx}{\sqrt{x^2-a^2}} = \ln\left(x + \sqrt{x^2-a^2}\right)$, $\int \frac{dx}{\sqrt{x^2+a^2}} = \ln\left(x+\sqrt{x^2+a^2}\right)$, $\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\left(\frac{x}{a}\right)$, $\int$

$\int \sqrt{a^2+x^2}\,dx = \frac{1}{2}\left(x\sqrt{a^2+x^2} + a^2 \ln\left(\sqrt{a^2+x^2}+x\right)\right)$, $\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left(x\sqrt{a^2-x^2} + a^2 \arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right)\right)$

$\int \sqrt{x^2-a^2}\,dx = \frac{1}{2}\left(x\sqrt{x^2-a^2} - a^2 \ln\left(x+\sqrt{x^2-a^2}\right)\right)$

OBRATNE VR. KVADRATOV: $\int \frac{dx}{a^2x^2+b^2} = \frac{1}{ab}\arctan\left(\frac{ax}{b}\right)$, $\int \frac{dx}{b^2-a^2x^2} = \frac{1}{2ab}\left(\ln\left(\frac{ax}{b}+1\right) - \ln\left(1-\frac{ax}{b}\right)\right)$

RACIONALNE FUNKCIJE: $\int \frac{A}{x-a}dx = A\ln(x-a)$ ; $\int \frac{A}{(x-a)^n}dx = \frac{-A}{(n-1)}\cdot\frac{1}{(x-a)^{n-1}}$

$D = b^2-4ac < 0$:

$$\int \frac{Bx+C}{x^2+bx+c}dx = \frac{B}{2}\ln|x^2+bx+c| + \frac{2C-Bb}{\sqrt{-D}}\arctan\left(\frac{2x+b}{\sqrt{-D}}\right)$$

$$\int \frac{Bx+C}{(x^2+bx+c)^2}dx = \frac{(2C-Bb)x+(bC-2Bc)}{(-D)(x^2+bx+c)} + \frac{2(2C-Bb)}{(-D)(\sqrt{-D})}\arctan\left(\frac{2x+b}{\sqrt{-D}}\right)$$

TRIGONOMETRIČNE: $t = \tan\left(\frac{x}{2}\right) \Rightarrow \sin(x) = \frac{2t}{1+t^2}$, $\cos(x) = \frac{1-t^2}{1+t^2}$, $dx = \frac{2}{1+t^2}dt$ $\qquad \int \frac{1}{\cos(x)^2} = \tan(x)$

↳ Če sin in cos nastopata v int. kot potenci več od 1 → drugjni koti. $\qquad \int \frac{1}{\sin(x)^2} = ctg(x)^2$

↳ $\int \sin^p(x)\cos^q(x)\,dx$ : q lih : $t = \cos(x)$; p lih : $t = \sin(x)$ $\qquad$ p in q sode : double angle.

$\int \frac{dx}{\sin(x)} = \log\left|\tan\left(\frac{x}{2}\right)\right|$ $\qquad \cancel{\int \frac{dx}{\sin(x)} = \log\left(\sin\left(\frac{x}{2}\right)\right) - \log\left(\cos\left(\frac{x}{2}\right)\right) - \log\left(\tan\left(\frac{x}{2}\right)\right)}$

POSEBNE RACIONALNE:

- $\int R\left(x, \left(\frac{ax+b}{cx+d}\right)^{\frac{m}{n}}\right)dx \rightarrow t = \left(\frac{ax+b}{cx+d}\right)^{\frac{1}{n}}$ je substitucija, $x = \frac{-dt^n+b}{ct^n-a}$
- $\int \frac{p(x)}{\sqrt{ax^2+bx+c}}dx = \tilde{p}(x)\sqrt{ax^2+bx+c} + \int \frac{C}{\sqrt{ax^2+bx+c}}$, kjer je $\deg(\tilde{p})+1 = \deg(p)$ in $C \in \mathbb{R}$.
- $\int R\left(x, \sqrt{ax^2+bx+c}\right) \rightarrow$ $a>0$ : $\sqrt{a}(x-u) = \sqrt{ax^2+bx+c}$; $a<0$ : $\sqrt{a}(x-x_1)u = \sqrt{ax^2+bx+c}$; $x_1$ je ničla $ax^2+bx+c=0$
- $\int \frac{dx}{(x+\alpha)^n\sqrt{ax^2+bx+c}} \rightarrow t = \frac{1}{x+\alpha}$

GEOMETRIJA INTEGRALA:

1) $F = (x(t), y(t))$ je vr. odv. $\rightarrow l(F) = \int_a^b \sqrt{\dot{x}(t)^2+\dot{y}(t)^2}\,dt$

2) Ploščine: • Ploščina med funkcijama: $\int_a^b |f(x)-g(x)|dx$

- Ploščina območja podanega s krivuljo: $F = (x,y)$ je zv. odv.:
  - $y(t) \geq 0$ in $x(a) = \min\{x(t) \mid t\in[a,b]\}$, $x(b) = \max\{x(t) \mid t \in [a,b]\}$ ↳ $pl(F) = \int_a^b y(t)\dot{x}(t)\,dt$
  - Analogno velja za $x(t) \geq 0$ ...
  - $pl(F) = \frac{1}{2}\int_a^b \left(x(t)\dot{y}(t) - \dot{x}(t)y(t)\right)dt$
- Ploščina polarno podane krivulje: $\alpha$ začetni in $\beta$ končni kot: $pl(F) = \int_\alpha^\beta r(\varphi)^2\,d\varphi$

3) Površina vrtenine:

funkcija: $P = 2\pi\int_a^b f(x)\sqrt{1+f'(x)^2}\,dx$

parametrično: $P = 2\pi\int_a^b y(t)\sqrt{\dot{x}(t)^2+\dot{y}(t)^2}\,dt$

polarno: $P = 2\pi\int_\alpha^\beta r(\varphi)\sin(\varphi)\sqrt{r(\varphi)^2+\dot{r}(\varphi)^2}\,d\varphi$

4) Volumen vrtenine:

funkcija: $V = \pi\int_a^b f(x)^2\,dx$

parametrično: $V = \pi\int_a^b y(t)^2\,\dot{x}(t)\,dt$

polarno: $V = \pi\int_\alpha^\beta -r(\varphi)^3\sin(\varphi)^3\,d\varphi$

$\boxed{\begin{aligned}\sin(A)+\sin(B) &= 2\sin\left(\tfrac{A+B}{2}\right)\cos\left(\tfrac{A-B}{2}\right)\\ \cos(A)+\cos(B) &= 2\cos\left(\tfrac{A+B}{2}\right)\cos\left(\tfrac{A-B}{2}\right)\\ \cos(A)-\cos(B) &= -2\sin\left(\tfrac{A+B}{2}\right)\sin\left(\tfrac{A-B}{2}\right)\end{aligned}}$

$f$ je int. na $[a,b]$

- $s<1 \Rightarrow \int_a^b \frac{f(x)}{(x-a)^s}dx$ obstaja
- $s \geq 1$ in $\exists m \in \mathbb{R}$ : $\begin{matrix} f(x) > m \\ f(x) < -m \end{matrix}$ $\forall x \in [a,b]$

$$\Rightarrow \int_a^b \frac{f(x)}{(x-a)^s}dx = \infty$$

$g$ je zvezna na $[a,\infty)$

- $g$ omejena in $p>1$ : $\int_a^\infty \frac{g(x)}{x^p}dx$ konv.
- $\exists m>0$ : $\begin{matrix} g(x) \geq m \\ g(x) \leq -m \end{matrix}$ $\forall x$ in $p \leq 1$ je $\int_a^\infty \frac{g(x)}{x^p}dx = \infty$

Ponavadi uporabljamo z limitami $f$ in $g$!

L'Hospital:

$f,g$ odvedljivi na $(a,b)$ ter $\forall x \in (a,b)$: $g(x) \neq 0$ in $g'(x) \neq 0$. Če je $\lim_{x \to a} f(x) = \lim_{x \to a} g(x) = 0$ in če obstaja $\lim_{x \to a} \frac{f'(x)}{g'(x)}$, potem je enako $\lim_{x \to a} \frac{f(x)}{g(x)}$

Taylor: $T_{n,a}(x) = \sum_{i=0}^{n} \frac{f^{(i)}(a)}{i!}(x-a)^i$

$f \in C^{n+1}(I)$ (odvedljivost), $I$ je odprt interval, ki vsebuje $a$.

Za vsak $x \in I$ obstaja $c$ med $a$ in $x$, da je

$R_{n,a}(x) = f(x) - T_n(x)$. $\quad R_{n,a}(x) = \frac{f^{(n+1)}(c)}{(n+1)!}(x-a)^{n+1}$

Pri parametrično podanih krivuljah je $k(x) = \frac{\dot{y}(t)}{\dot{x}(t)}$, enako pri polarnih

SKICIRANJE GRAFOV:

① ničle, def. obm., pole, limite na robu $D_f$, asimptote

② ODVOD: ničle, pot. [illegible], tangente na robu $D_f$

$e^x = \sum_{i=0}^\infty \frac{1}{i!}x^i \quad \forall x \in \mathbb{R}$ $\qquad \cos(x) = \sum_{k=0}^\infty \frac{(-1)^k}{(2k)!}x^{2k} \quad \forall x$

$\sin(x) = \sum_{i=0}^\infty \frac{(-1)^i}{(2i+1)!}x^i \quad \forall x \in \mathbb{R}$ $\qquad \ln(1+x) = \sum_{k=0}^\infty \frac{(-1)^k}{k+1}x^{k+1} \quad \forall |x|<1$

$(1+x)^\alpha = \sum_{k=0}^\infty \binom{\alpha}{k}x^k$ ; $|x|<1$ in $\binom{\alpha}{k} = \frac{\alpha \ldots (\alpha-k+1)}{k!}$

Razvoj log. okoli a: $\quad |\frac{x-a}{a}|<1 \Rightarrow 0<x<2a$

$$\log(x) = \log(a) + \sum_{i=0}^\infty \frac{(-1)^i}{i+1}\left(\frac{x-a}{a}\right)^{i+1}$$

POTENČNE VRSTE:

① obstaja $R \in [0,\infty]$, da $\sum a_n(x-c)^n$ konvergira na $x \in (c-R, c+R)$ ter divergira za $|x-c|>R$, v krajiščih pa ne vemo.

↳ na $r<R$, $x \in (c-r, c+r)$ vrsta konv. enak. in absolutno.

↳ Če $\sum a_n(x-c)^n$ konv. pri $x = \pm R$, potem je vrsta tam zvezna [Abel].

↳ $\frac{1}{R} = \limsup_{n\to\infty} \sqrt[n]{|a_n|}$

$f: [0,1] \to \mathbb{R}$ zvezna. Velja:

$$\int_0^\pi x f(\sin(x))dx = \frac{\pi}{2}\int_0^\pi f(\sin(x))dx = \pi \int_0^{\frac{\pi}{2}} f(\sin(x))dx$$

$$\int_0^{2\pi} \sin(x)^2 dx = \int_0^{2\pi} \cos(x)^2 dx = \pi$$

$f_n: [a,b] \to \mathbb{R}$ ; $f_n \in C^1[a,b]$. Če $f_n' \to g$ enakomerno in $f_n(x)$ konvergira v vsaj eni točki, potem $f_n \to f$ enakomerno in

$$f'(x) = \lim_{n\to\infty} f_n'(x)$$

$\sum_{n=0}^\infty x^n = \frac{1}{1-x}$ ; $e^x = \sum_{n=0}^\infty \frac{x^n}{n!}$ ; $\cos(x) = \sum_{n=0}^\infty \frac{(-1)^n}{(2n)!}x^{2n}$

$\sin(x) = \sum_{n=0}^\infty \frac{(-1)^n}{(2n+1)!}x^{2n+1}$ ; $\ln(1+x) = \sum_{n=1}^\infty \frac{(-1)^{n-1}}{n}x^n$

↳ na $\mathbb{R}$

$(1+x)^\alpha = \sum_{n=0}^\infty \binom{\alpha}{n}x^n$ in $\binom{\alpha}{n} = \frac{\alpha(\alpha-1)\ldots(\alpha-n+1)}{n!} = \prod_{k=1}^n \frac{\alpha-k+1}{k}$

Ostali na $|x|<1$

$f_n \in C[a,b]$ ; $f_n \to f$ enakomerno, potem

$$\lim_{n\to\infty} \int_a^b f_n(x)dx = \int_a^b f(x)dx$$