

# Montessori Math Album

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## Montessori Math Album (3-6) - Outline

### 1. Introduction to Montessori Math (3-6)

- **Philosophy** (How children learn math through hands-on materials)
- **Purpose** (Preparing the mathematical mind through sensorial and practical life foundations)
- **Key Principles** (Isolation of difficulty, control of error, repetition, and self-discovery)

### 2. Numeration (Early Counting & Number Recognition)

- **2.1 Number Rods** (Quantity & Symbol Association)
- **2.2 Sandpaper Numbers** (Tactile number formation)
- **2.3 Number Rods** (Symbol Association)
- **2.4 Spindle Boxes** (Zero as a placeholder, counting)
- **2.5 Cards & Counters** (Odd & even introduction)
- **2.6 Memory Game of Numbers** (Reinforcing numeral-quantity link)

### 3. The Decimal System (Introduction to Place Value)

- **3.1 Golden Beads (Concrete Quantity)**
  - 3.1.1 Introduction to Units, Tens, Hundreds, Thousands
  - 3.1.2 Formation of Numbers (Building quantities)
- **3.2 Stamp Game** (Transition to semi-concrete representation)
- **3.3 Bead Cabinet** (Linear & Skip Counting)
- **3.4 Teen Boards** (Symbol and Quantity)

### 4. Operations (Addition, Subtraction, Multiplication, Division)

- **4.1 Linear Counting – 100 and 1000 Chains** (Static & Dynamic)
- **4.2 Stamp Game Operations** (More abstract representation)
- **4.3 Small Bead Frame** (Abstraction of addition/subtraction)
- **4.4 Multiplication & Division with Bead Bars**

### 5. Memorization of Math Facts

- **5.1 Addition Strip Board**
- **5.2 Subtraction Strip Board**
- **5.3 Multiplication & Division Boards**
- **5.4 Finger Charts (Addition)**
- **5.5 Finger Charts (Subtraction)**
- **5.6 Finger Charts (Multiplication)**

## **6. Fractions (Introduction)**

- **6. 1 Fraction Insets &**
- **6.2 Fraction Skittles (Sensorial introduction)**

# Philosophy

## A. The Montessori Approach to Mathematics

Maria Montessori believed that children have a "**mathematical mind**"—an innate ability to understand order, sequence, patterns, and quantities. Unlike traditional education, where math is often taught through rote memorization, Montessori math is:

- **Hands-on and sensory-based** – Children learn through manipulation of concrete materials.
- **Self-directed** – Lessons are individualized, allowing children to progress at their own pace.
- **Sequential and logical** – Each concept builds upon the previous one, moving from concrete → symbolic → abstract.
- **Joyful and discovery-based** – Children experience the thrill of mathematical understanding through exploration.

## Key Principles in Montessori Math:

1. **"From the Hand to the Mind"** – Children must physically touch, count, and arrange materials before internalizing abstract concepts.
2. **Isolation of Difficulty** – Each material focuses on one skill (e.g., Number Rods for quantity, Sandpaper Numerals for symbols).
3. **Control of Error** – Materials are self-correcting, allowing children to independently recognize and fix mistakes.
4. **Repetition & Mastery** – Children repeat activities until they achieve confidence and fluency.

## B. The Role of Sensorial & Practical Life in Math Readiness

Before formal math lessons, Montessori children develop foundational skills through:

- **Practical Life** (order, precision, fine motor control)
- **Sensorial Work** (discrimination of size, length, weight, and gradation)

**Example:**

- The **Pink Tower** (Sensorial) indirectly prepares for understanding **size and dimension**, which later connects to **decimal system hierarchy**.
- **Pouring & Sorting** (Practical Life) develops the **pincer grip** needed for handling small beads in math work.

## C. How Children Learn Math in Montessori (3-6)

### 1. Stage 1: Numeration (0-10)

- **Materials:** Number Rods, Sandpaper Numerals, Spindle Boxes, Cards & Counters
- **Objective:** Associate quantity with symbol, recognize numbers, introduce zero.

### 2. Stage 2: The Decimal System (Place Value)

- **Materials:** Golden Beads, Stamp Game, Bead Cabinet
- **Objective:** Understand units, tens, hundreds, thousands through hands-on grouping.

### 3. Stage 3: Operations (Concrete → Abstract)

- **Materials:** Golden Beads (static/dynamic addition), Stamp Game, Bead Frames
- **Objective:** Perform +, -, ×, ÷ first with manipulatives, then transition to paper.

### 4. Stage 4: Memorization & Abstraction

- **Materials:** Strip Boards, Finger Charts, Word Problems
- **Objective:** Master math facts without relying on physical materials.

## D. The Teacher's Role in Montessori Math

- **Observer** – Recognize when a child is ready for the next challenge.
- **Guide** – Present lessons clearly, then step back to allow independent practice.
- **Facilitator** – Ensure the environment has all necessary materials and extensions.

### Quote from Maria Montessori:

*"The hands are the instruments of man's intelligence."*

# Purpose

The purpose of this Montessori Mathematics Album is to provide a comprehensive and scientifically grounded guide to the sequential presentation of mathematical concepts to children aged 3 to 6 years. Rooted in Dr. Maria Montessori's philosophy, this album is designed to nurture the child's innate "**mathematical mind**"—the natural human tendency to explore, quantify, and find order in the world.

## 1. Cultivating the Mathematical Mind

Maria Montessori observed that young children possess a natural inclination toward precision, classification, and numerical patterns. This album serves to:

- **Awaken** the child's inherent curiosity for numbers and relationships
- **Satisfy** the child's need for order through structured, logical materials
- **Develop** analytical thinking through hands-on exploration

*"The child's mind is mathematical, and it is founded on exactness."*

— *The Absorbent Mind*

## 2. Bridging the Concrete to the Abstract

The materials and presentations in this album follow a meticulously designed progression that allows the child to:

- **Experience** mathematical concepts sensorially (e.g., feeling the weight of a thousand bead cube)
- **Associate** quantities with symbols (e.g., Sandpaper Numerals, Number Rods and Cards)
- **Internalize** operations through repetition and manipulation (e.g., Golden Bead addition, Stamp Game)
- **Memorize** essential facts through patterns and relationships (e.g., Addition Strip Board, Multiplication Bead Board)

This structured yet flexible approach ensures that children **do not merely memorize** but **deeply understand** mathematical principles before transitioning to abstract work.

## 3. Fostering Independence and Problem-Solving

Each material is designed with **built-in control of error**, empowering the child to:

- **Self-correct** without adult intervention (e.g., limited spindles in the Spindle Boxes)
- **Discover** mathematical properties through experimentation (e.g., exchanging ten units for one ten in Golden Beads)
- **Apply** knowledge to real-life situations (e.g., measuring, counting, and fair-sharing with Fraction Insets)

## 4. Preparing for Lifelong Mathematical Thinking

Beyond arithmetic, this album lays the foundation for:

- **Algebraic reasoning** (e.g., seeing patterns in the Bead Cabinet chains)
- **Geometric understanding** (e.g., exploring shapes and spatial relationships with Metal Insets)
- **Logical deduction** (e.g., solving word problems with the Stamp Game)

The ultimate goal is not just computational skill but **mathematical fluency**—the ability to think critically, recognize relationships, and approach problems with confidence.

## 5. Aligning with AMS Standards

This album adheres to the **American Montessori Society's** guidelines by:

- ✓ Presenting materials in the **authentic Montessori sequence**
- ✓ Ensuring each lesson includes **clear objectives, control of error, and extensions**
- ✓ Balancing **teacher guidance** with **child-led discovery**
- ✓ Providing **adaptations** for different learning styles

## Conclusion

This Mathematics Album is more than a collection of lessons—it is a **scientifically designed pathway** that respects the child's developmental stages, nurtures a love for learning, and builds an unshakable foundation for future mathematical exploration.

*"The essence of mathematics is not to make simple things complicated, but to make complicated things simple."*

— Dr. Maria Montessori

# Sequence Overview

## *I. Foundational Preparation (2.5-4 years)*

### Sensorial Prelude

- **Materials:** Pink Tower, Brown Stair, Red Rods
- **Key Presentations:**
  1. Grading by size/dimension
  2. Pairing identical quantities
- **Purpose:** Develops visual discrimination and establishes concepts of order, sequence, and relative magnitude that underpin mathematical thinking

### Practical Life Connections

- **Key Activities:** Pouring, Spooning, Sorting
- **Mathematical Value:**
  - Refines pincer grip for writing numerals
  - Establishes one-to-one correspondence
  - Develops left-to-right work patterns

## *II. Number Foundations (3-4.5 years)*

### Quantity Recognition

1. **Number Rods**
  - Stage 1: Matching quantities to lengths
  - Stage 2: Three-period lesson for numeral association
  - Extension: Memory game with randomized rods

### Symbol Introduction

1. **Sandpaper Numerals**
  - Tactile tracing protocol
  - Sound/symbol association
  - Transition to writing in sand tray

## Integration

### 1. Cards & Counters

- Systematic placement protocol
- Odd/even discovery
- Control of error through exact quantity

## *III. Decimal System Mastery (4-5.5 years)*

## Hierarchical Understanding

### 1. Golden Bead Material

- Initial presentation: Category isolation (units, tens, hundreds, thousands)
- Quantity formation: Building 4-digit numbers
- Reading practice: Verbalizing place value

## Operations Progression

### 1. Static Addition

- Layout protocol
- Verbalization patterns ("2 thousands plus 3 thousands...")

### 2. Dynamic Addition

- Exchange procedure demonstration
- Error recovery techniques

### 3. Subtraction & Multiplication

- Specialized presentation techniques
- Inverse operation demonstrations

## *IV. Linear Counting & Memorization (4.5-6 years)*

## Bead Cabinet Work

### 1. Short Chains

- Arrow placement technique
- Squaring demonstrations
- Transition to written recording

### 2. Long Chains

- Decade counting patterns
- Cubing demonstrations
- Comparative analysis (square vs. cube)

## Operation Boards

### 1. Addition/Subtraction Strip Boards

- Color-coding utilization
  - Missing addend exercises
- 2. Multiplication/Division Boards**
- Finger chart integration
  - Speed challenges

## *V. Advanced Applications (5.5-6.5 years)*

### Fraction Studies

- 1. Fraction Insets**
- Nomenclature introduction
  - Equivalence demonstrations
  - First operations protocol

### Geometric Foundations

- 1. Metal Insets**
- Shape tracing progression
  - Dimension vocabulary
  - Preparation for geometry

### Developmental Flowchart

Sensorial → Numeration → Decimals → Operations → Memorization → Fractions  
 ↓            ↓            ↓            ↓            ↓            ↓  
 Pre-Writing → 1-10 Mastery → Place Value → +×÷ → Fact Fluency → Applications

### Pedagogical Rationale

#### Scientific Basis

- Neurological research confirms the effectiveness of:
  - Tactile learning for concept retention
  - Sequential skill-building for mathematical understanding
  - Movement integration for cognitive development

### Montessori Philosophy

- Alignment with observed developmental planes
- Respect for individual learning trajectories
- Preparation for cosmic education

### AMS Standards Compliance

- Material progression follows AMS guidelines:
  - 3.4.2 (Concrete to Abstract)
  - 5.1 (Isolation of Difficulty)
  - 7.3 (Auto-education)

## **Implementation Framework**

### **Observation Guidelines**

- Readiness indicators for each stage
- Red flag warnings
- Advanced learner adaptations

### **Presentation Protocol**

- Standardized presentation steps
- Language precision requirements
- Common error anticipations

### **Assessment Benchmarks**

- Mastery indicators for each material
- Transition readiness criteria
- Record-keeping templates

## **Sequence Adaptations**

### **For Younger Children (3-4)**

- Extended sensorial preparation
- Modified presentation pacing
- Additional pre-numeration activities

### **For Advanced Learners**

- Accelerated progression options
- Enrichment extensions
- Cross-curricular connections

# Key Principles

The Montessori approach to mathematics is rooted in the understanding that children possess an innate "**mathematical mind**"—a natural tendency to explore, quantify, and discover patterns in their environment. Dr. Maria Montessori observed that this cognitive predisposition thrives when children engage with purposeful materials that transform abstract concepts into tangible, hands-on experiences.

This album is structured around **five foundational principles** that guide the presentation and mastery of mathematical concepts in the 3-6 classroom. These principles ensure that children not only learn arithmetic but also develop a deep, joyful, and meaningful understanding of mathematics as a universal language of logic and order.

## 1. The Mathematical Mind: Nurturing Innate Potential

### Core Idea:

Children are born with a natural inclination to classify, sequence, and measure. Montessori materials are designed to **awaken and refine** this inherent ability, allowing mathematical understanding to unfold organically.

### Implementation:

- **Sensorial Foundations:** Early materials (e.g., Number Rods, Spindle Boxes) isolate quantitative concepts like length, quantity, and gradation.
- **Precision in Language:** Teachers use exact terms (e.g., "*This is three*" rather than "*This is the number three*") to reinforce clarity.
- **Exploration-Based Learning:** Children discover mathematical properties (e.g., commutativity in addition) through manipulation rather than rote instruction.

### Montessori Quote:

*"The child's mind is mathematical, and it is based on the exactness of observation."*

— *The Absorbent Mind*

## 2. Concrete → Abstract Progression: Building Understanding Layer by Layer

**Core Idea:**

Children master concepts by first interacting with **physical materials**, then transitioning to symbolic representation, and finally achieving mental abstraction.

**Stages of Progression:****1. Sensorial (Ages 3-4):**

- *Example:* Feeling the weight of a thousand-cube vs. a unit bead to grasp magnitude.

**2. Concrete (Ages 4-5):**

- *Example:* Solving  $2,453 + 1,231$  using Golden Beads.

**3. Pictorial (Ages 5-6):**

- *Example:* Using the Stamp Game to represent the same problem with colored tiles.

**4. Abstract (Ages 6+):**

- *Example:* Solving  $3,684 \div 4$  on paper without materials.

**Why It Matters:**

This sequenced approach prevents gaps in understanding and ensures children internalize concepts before memorizing facts.

**3. Isolation of Difficulty: Mastering One Skill at a Time****Core Idea:**

Each Montessori material isolates **a single mathematical concept** to avoid cognitive overload.

**Examples:**

Material	Isolated Concept	Control of Error
Sandpaper Numerals	Symbol recognition	Tactile feedback from tracing
Spindle Boxes	Quantity-to-symbol matching	Exactly 45 spindles
Stamp Game	Place value in operations	Color-coded columns

**Teacher's Role:**

- Present one difficulty at a time (e.g., introduce addition before subtraction).
- Observe when a child is ready to combine skills (e.g., dynamic addition with exchanges).

**4. Auto-Education: The Child as an Active Learner****Core Idea:**

Materials are designed for **self-directed discovery** and **self-correction**, fostering independence and problem-solving.

### **Key Features:**

- **Control of Error:**
  - *Example:* The Spindle Boxes include exactly 45 spindles—no more, no less—so the child recognizes miscounts.
- **Repetition:** Children work with materials until they achieve mastery, not until an adult intervenes.
- **Choice:** Children select work based on interest, deepening engagement.

### **Montessori Quote:**

*"The greatest sign of success for a teacher is to say, 'The children are now working as if I did not exist.'"*

## **5. Movement and Memory: The Body as a Learning Tool**

### **Core Idea:**

Physical interaction with materials **anchors abstract concepts** in the child's memory.

### **How It Works:**

- **Tracing:** Sandpaper Numerals link numeral shapes to muscle memory.
- **Bead Manipulation:** Exchanging 10 units for 1 ten in Golden Beads reinforces place value.
- **Full-Body Engagement:** Walking the 1000-chain builds a sensory understanding of large quantities.

### **Neuroscience Connection:**

Kinesthetic learning activates multiple brain regions, enhancing retention and recall.

## **Conclusion: The Harmony of Principles**

These five principles—the **mathematical mind, concrete-to-abstract progression, isolation of difficulty, auto-education, and movement-based learning**—work synergistically to create a transformative mathematical experience. Together, they ensure that children:

Develop **true comprehension** (not just memorization)

Cultivate **confidence and joy** in problem-solving

Build a **lifelong foundation** for advanced mathematics

### **Final Quote:**

*"The goal of early childhood education should be to activate the child's own natural desire to learn."*

— Maria Montessori

# Numeration in the Montessori 3-6 Classroom

## The Foundation of Mathematical Understanding

Numeration—the ability to recognize, represent, and work with quantities and their symbols—is the cornerstone of mathematical development in the Montessori early childhood environment. Dr. Maria Montessori observed that children possess a natural sensitivity to numbers during the first plane of development (ages 3-6), which she termed the "**mathematical mind.**" This album section presents a systematic approach to nurturing this innate capacity through:

- **Sensorial exploration** of quantity
- **Precise language** for numerical concepts
- **Sequential materials** that isolate key skills

*"The hands are the instruments of man's intelligence. Through touch and movement, the child builds a concrete understanding of number."*

— *The Discovery of the Child*

## Key Principles of Numeration Work

### 1. From Concrete to Symbolic

- **Stage 1:** Quantity alone (Number Rods)
- **Stage 2:** Symbol alone (Sandpaper Numerals)
- **Stage 3:** Integrated quantity & symbol (Cards & Counters)

### 2. Isolation of Difficulty

Each material focuses on **one** key skill:

Material	Isolated Concept	Control of Error
Number Rods	Pure quantity	Fixed length differences
Spindle Boxes	Zero concept	Exactly 45 spindles provided
Memory Game	Random quantity-symbol matching	Self-verifying design

### 3. Movement-Based Learning

- Tracing numerals (tactile memory)
- Carrying materials (gross motor reinforcement)
- Left-to-right work patterns (pre-literacy preparation)

## Developmental Sequence

*Phase 1: Quantity Recognition (3-4 years)*

### Materials & Objectives:

1. **Number Rods**
  - Distinguishes lengths representing 1-10
  - Develops visual discrimination of quantity
2. **Spindle Boxes**
  - Introduces 0 as a quantitative reality
  - Reinforces one-to-one correspondence

### Teacher's Role:

- Use **exact language**: "*This is five.*" (Not "This is the number five")
- Observe for **readiness signs**: Consistent quantity recognition, ability to work left-to-right

*Phase 2: Symbol Association (3.5-4.5 years)*

### Materials & Objectives:

1. **Sandpaper Numerals**
  - Tactile imprinting of numeral forms
  - Prepares for numeral writing
2. **Numerical Cards**
  - Symbol recognition in isolation
  - Matching to quantities

### Critical Technique:

- **Three-period lesson structure:**
  1. Naming: "*This is seven.*"
  2. Recognition: "*Show me nine.*"
  3. Recall: "*What is this?*"

### *Phase 3: Integration (4-5 years)*

#### **Materials & Objectives:**

1. **Cards & Counters**
  - Odd/even discovery through patterning
  - Final verification of 1-10 mastery
2. **Memory Game of Numbers**
  - Randomized quantity-symbol matching
  - Transition to operational work

#### **AMS Standard Alignment:**

- 3.4.2:** Materials progress from concrete to abstract  
**5.1.3:** Built-in control of error  
**7.2:** Supports individual pacing

#### **Special Considerations**

##### **For Younger Children (3-3.5 years):**

- Begin with only 1-5 quantities
- Extra sensorial preparation (e.g., pairing/sorting extensions)

##### **For Advanced Learners:**

- Early introduction of teen numbers
- Spontaneous exploration of larger quantities

#### **Common Challenges & Solutions:**

<b>Issue</b>	<b>Montessori Response</b>
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Numeral reversals *"The curve opens this way"* (tracing)

Skipping quantities *"Let's count together while touching"*

Confusing 6/9 *"The circle closes at the bottom for six"*

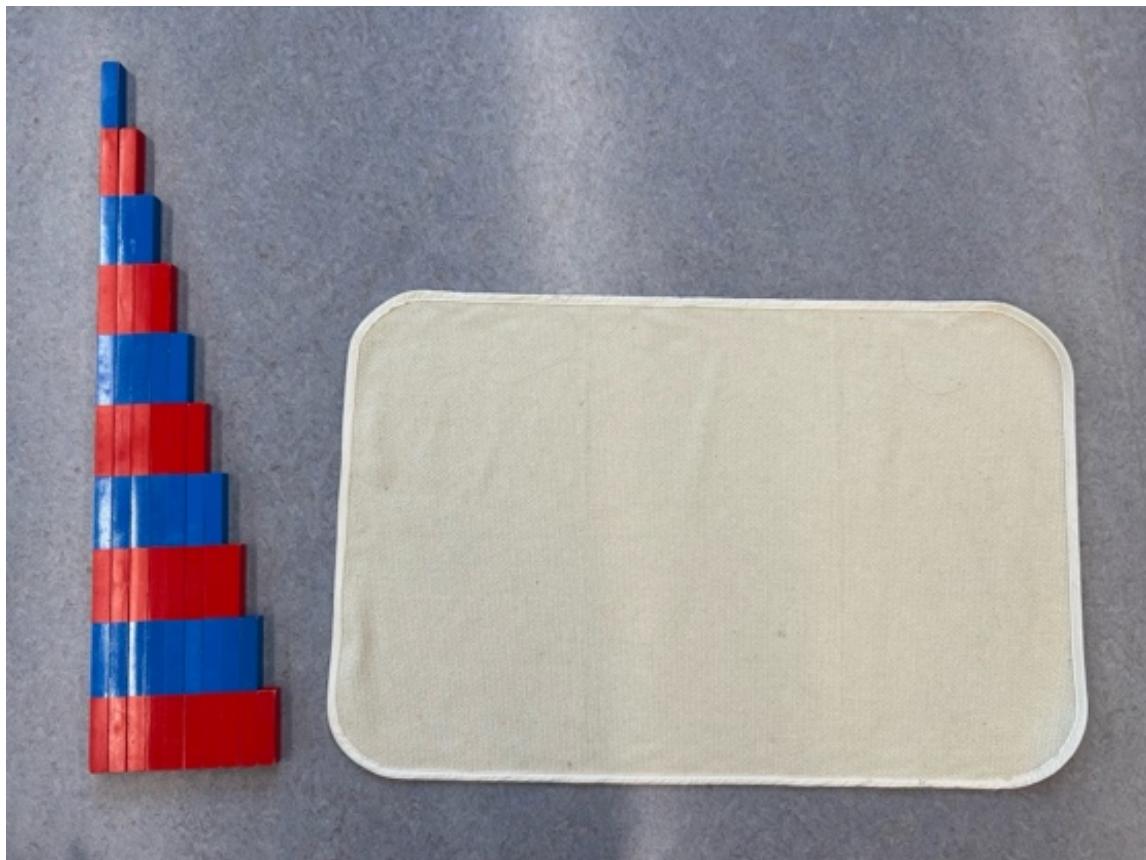
#### **Purpose & Outcomes**

Through this carefully sequenced numeration work, children will:

- Develop **true number sense** (not rote counting)
- Build **cognitive frameworks** for future operations
- Experience **mathematics as joyful discovery**

## 2.1 Number Rods

(Concrete Introduction to Quantity 1-10)



### A. Material List:

- ✓ **Set of 10 Number Rods** (Red and blue alternating segments, increasing in length from 10cm to 100cm)
- ✓ **Floor mat or large table** (For orderly arrangement)

### B. Purpose:

- To introduce the child to **quantity 1-10** through visual and tactile discrimination.
- To prepare for **symbolic representation** (later linked with Sandpaper Numerals).

- To develop the child's **sense of order, sequence, and coordination**.

#### C. Presentation Steps:

1. **Invite the child** to carry the rods one by one to the mat (starting with the shortest).
2. **Arrange randomly**, then slowly **build the stair** (left-aligned, like the Red Rods from Sensorial).
3. **Trace each rod** while counting segments aloud: "*One. One, two. One, two, three...*"
4. **Isolate the last segment** (blue) when counting to emphasize the new unit.
5. **Verify by counting** each rod again.
6. **Mix and invite the child** to rebuild independently.

#### D. Control of Error:

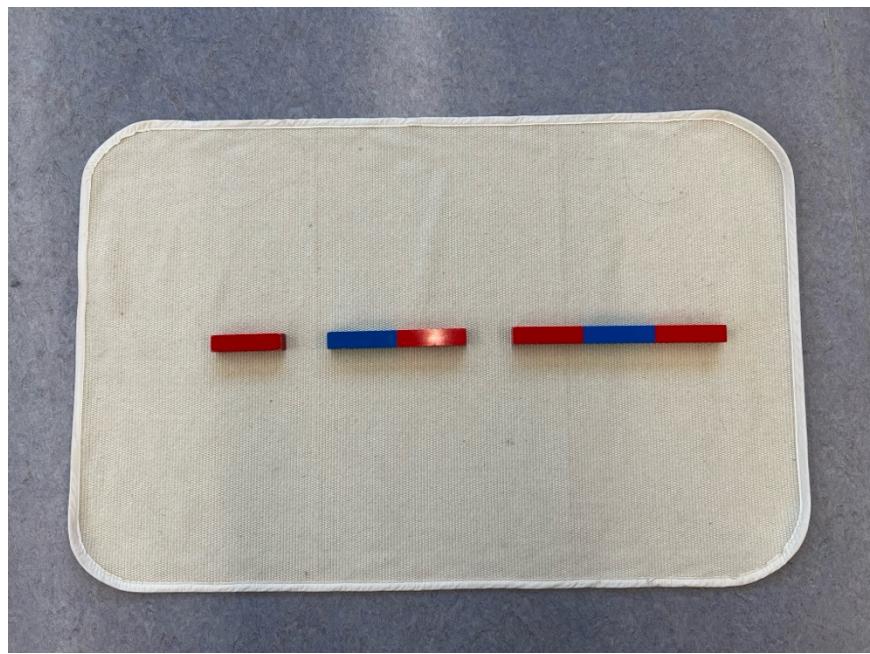
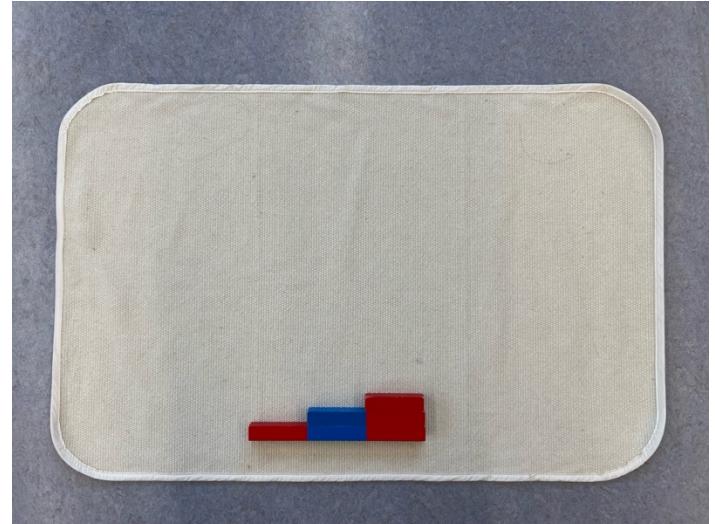
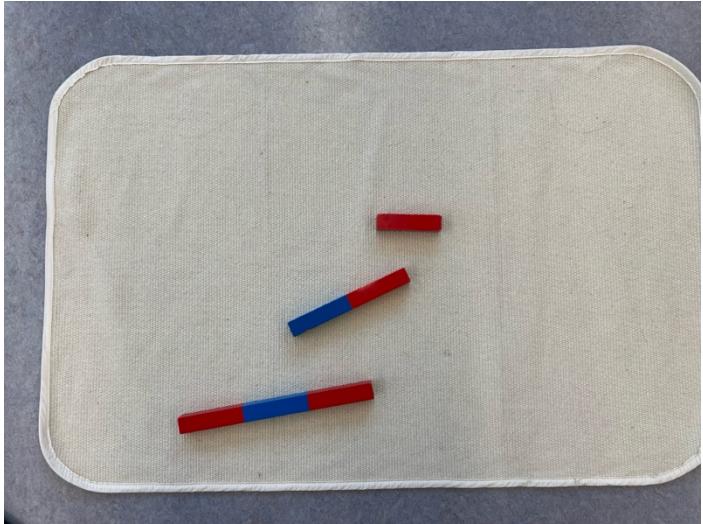
- Visual: Incorrect stair formation will appear disordered.
- Tactile: Child feels uneven gaps if rods are misaligned.

#### E. Extensions/Variations:

- **3-Period Lesson** (For younger children):
  - "Show me 3." → "What is this?" → "Point to 5."
- **Blindfolded Matching** (Advanced sensorial challenge).
- **Combine with numerals** (After Sandpaper Numbers are introduced).

#### Why This Lesson Comes First:

- The Number Rods bridge the **Sensorial (Red Rods)** and **Math** curricula, leveraging the child's prior experience with length discrimination.
- It lays the foundation for **all future math work** (e.g., the concept of "10" prepares for the decimal system).



## 2.2 Sandpaper Numbers

(Tactile Introduction to Number Symbols 0-9)



### A. Materials:

- ✓ **Sandpaper Numerals** (0-9 on individual wooden tablets)
- ✓ **Small tray**
- ✓ **Floor mat or table workspace**

### B. Direct Purpose:

- To develop **muscle memory** for number formation through tactile experience
- To associate **number names** with their **symbolic representation**
- To prepare for writing numerals

## C. Presentation Steps:

### 1. Preparation:

- Carry the tray to the mat and place it in the upper left corner
- Arrange numerals randomly on the mat (avoid alphabetical order)

### 2. Three-Period Lesson (Focus on 3 numerals at a time):

#### Period 1: Naming ("This is...")

- Select one numeral (e.g., "3")
- Trace with **dominant hand index/middle fingers** using proper stroke order
- Say clearly: "*This is three.*"
- Invite child to trace

#### Period 2: Recognition ("Show me...")

- Ask: "*Show me five.*" "*Trace two.*"
- Observe if child uses correct finger technique

#### Period 3: Recall (Only if child shows mastery)

- Point to numeral: "*What is this?*"

### 3. Repeat with remaining numerals in sets of 3

## D. Control of Error:

- **Tactile:** Rough sandpaper guides correct formation
- **Visual:** Teacher demonstrates proper tracing technique
- **Auditory:** Clear verbal reinforcement of number names

## E. Extensions/Variations:

1. **Blindfold Tracing:** Enhance tactile discrimination
2. **Numeral Hunt:** Match to quantities in the environment
3. **Salt/Sand Tray:** Practice writing after tracing

## F. Notes for AMS Album:

- **Prerequisite:** Child should have experience with Number Rods
- **Follow-up:** Later combine with Spindle Boxes (for zero)

- **Language Tip:** Always say "*This is the symbol for five*" rather than "*This is five*" to distinguish symbol from quantity

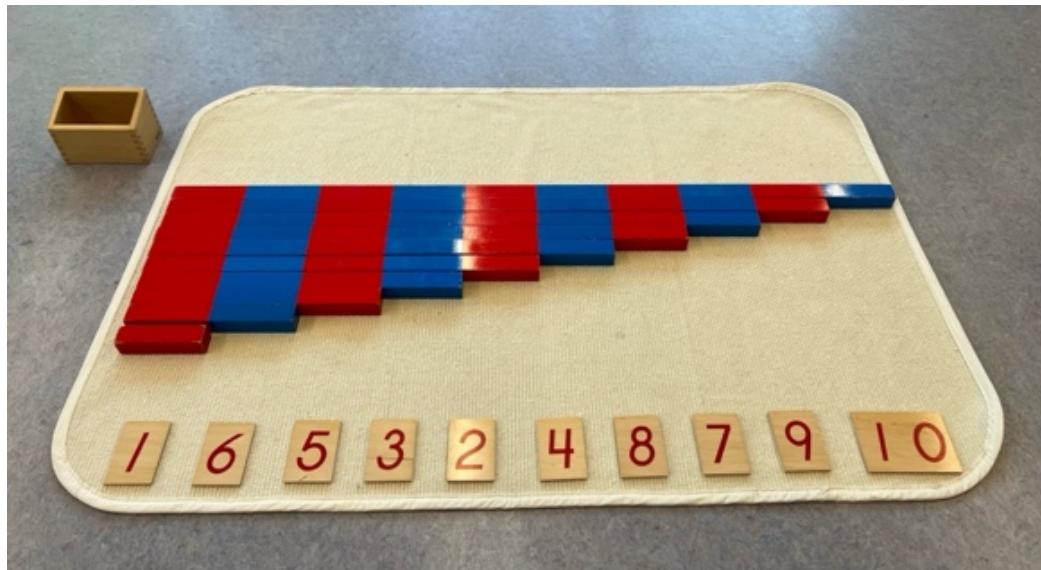
#### G. Key Montessori Principle:

*"What the hand does, the mind remembers."* The tactile experience creates neural pathways for number recognition and future writing.



## 2.3 Number Rods

(Ages 3.5-5 | Prerequisites: Red Rods & Sandpaper Numerals 0-9)



Presentation 1: Introduction to Fixed Quantities (1-10)

**A. Materials:**

- ✓ Number Rods (1-10)
- ✓ Floor mat

**B. Direct Purpose:**

- To associate quantity names with fixed lengths
- To develop visual discrimination of increasing quantities

**C. Presentation Steps:**

1. **Invitation**

- "Today we'll work with the Number Rods. Let's bring them to the mat."
- Carry rods one-by-one (shortest to longest)

2. **Building the Stair**

- Arrange rods randomly on mat

- Slowly construct stair (left-aligned) while counting each rod:
  - \*"One" (10cm rod)
  - "One, two" (20cm rod)
  - ...up to "One-ten" (100cm rod)\*

### 3. Isolating New Units

- After placing each rod, point to new blue segment:  
*"This is three."*

### 4. Verification

- Re-count entire stair together
- Mix rods and invite child to rebuild

## D. Control of Error:

- Visual: Misaligned rods break stair pattern
- Kinesthetic: Child feels length discrepancies

## Presentation 2: Quantity-to-Symbol Association

### A. Materials:

- Number Rods
- Large wooden numeral cards 1-10
- Mat

### B. Direct Purpose:

- To link concrete quantities with abstract symbols
- To reinforce numeral recognition

### C. Presentation Steps:

#### 1. Preparation

- Child builds Number Rod stair independently

#### 2. Three-Period Lesson

- **Period 1 (Naming):**

Trace rod "3" while counting, then trace numeral:  
*"This quantity is three. This symbol says 'three'."*

- **Period 2 (Recognition):**

*"Show me the rod for 'five'. Place 'two' by its rod."*

- **Period 3 (Recall):**

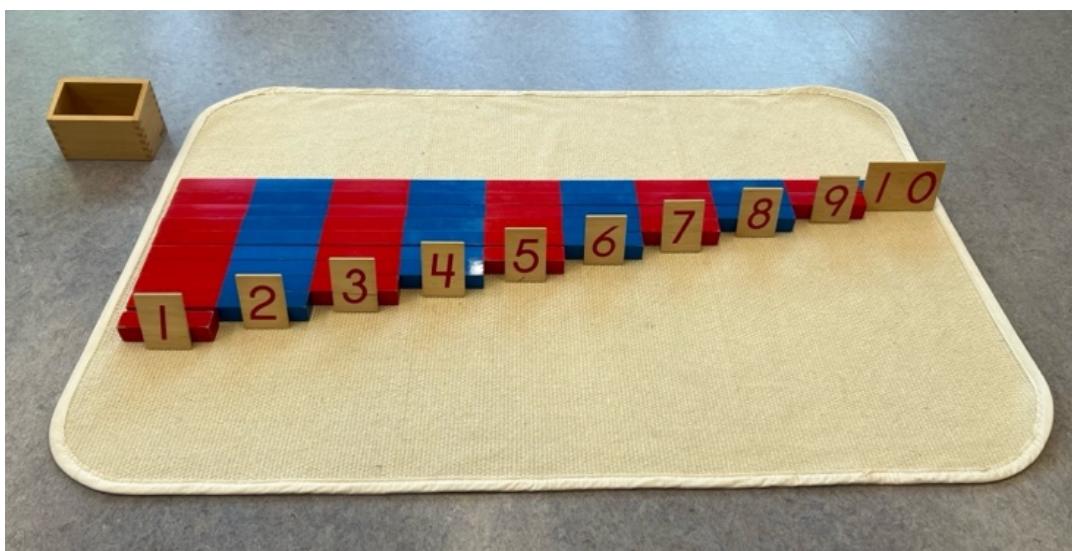
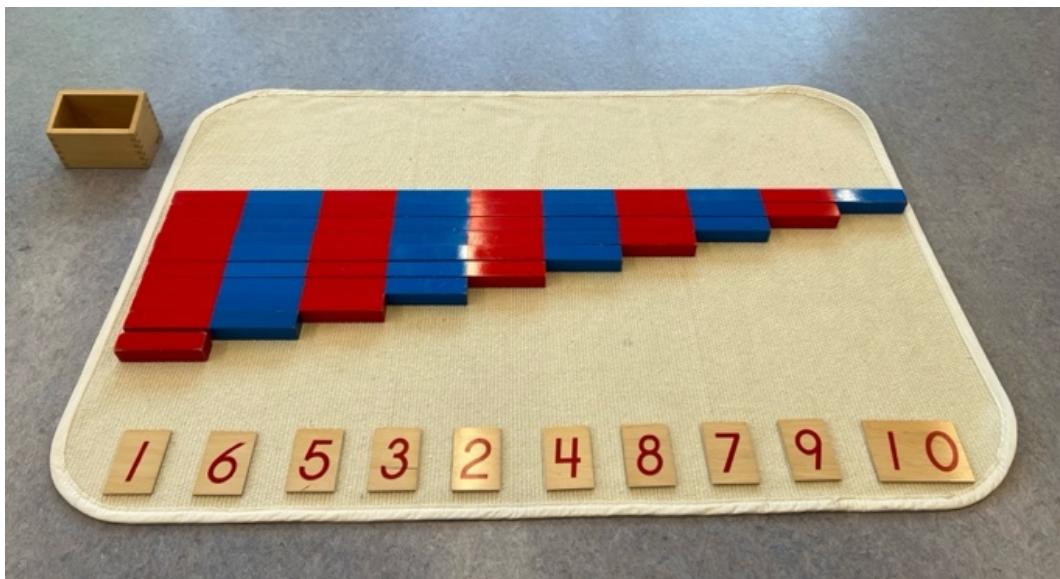
*"What number is this?" (Point to rod/card)*

### 3. Full Matching

- Child pairs all rods 1-10 with corresponding numeral cards

#### D. Control of Error:

- Numeral cards must match rod lengths exactly
- Extra/missing cards become obvious



## 2.4 Spindle Boxes

(Ages 3.5-5 | Prerequisites: Number Rods & Sandpaper Numerals 0-9)



### A. Materials:

- **Wooden spindle box** with 10 compartments labeled 0-9
- **45 identical spindles** (or wooden sticks/bundling straws)
- **Small basket** to hold spindles

### B. Direct Purposes:

1. Reinforce **one-to-one correspondence** for quantities 1-9
2. Introduce **zero as an empty set**
3. Develop **number conservation** (understanding "5" is always five objects)

## C. Presentation Steps:

### 1. Preparation

- Place spindle box centered on table
- Position basket of spindles to the right

### 2. Demonstration

1. Point to compartment "1":
  - *"This is one."* Place **1 spindle** while counting aloud
2. Continue sequentially with compartments 2-4:
  - *"One, two."* (Place 2 spindles)
  - *"One, two, three."* (Place 3 spindles)
3. **Pause at "0":**
  - *"Zero means nothing. We put nothing here."* Leave compartment empty
4. Invite child to complete compartments 5-9

### 3. Verification

- Re-count compartments 1-4 together
- Ask: *"Why does zero have no spindles?"*

## D. Control of Error:

**Physical Limit:** Exactly 45 spindles provided (prevents counting errors)

**Visual:** Empty "0" compartment stands out

**Kinesthetic:** Child feels extra/missing spindles when replacing them

## E. Extensions/Variations:

### 1. Blindfold Counting

- Child places spindles by touch only

### 2. Numeral Matching

- Add movable numeral cards to label compartments

### 3. Zero Hunt

- Find examples of "nothing" in the classroom (empty cup, blank paper)

## F. Teacher Notes:

### Key Language:

- Always say: "*Zero means no spindles*" (not "Zero is nothing")
- Emphasize **grouping**: "*Five is all these spindles together.*"

### Observation Signs of Mastery:

- Child consistently leaves "0" empty
- Can recount spindles without prompting
- Shows understanding that numbers represent fixed quantities

### Link to Prior Work:

- Builds on **Number Rods** by transitioning:  
Fixed quantities → Loose quantities

### Preparation for Next Steps:

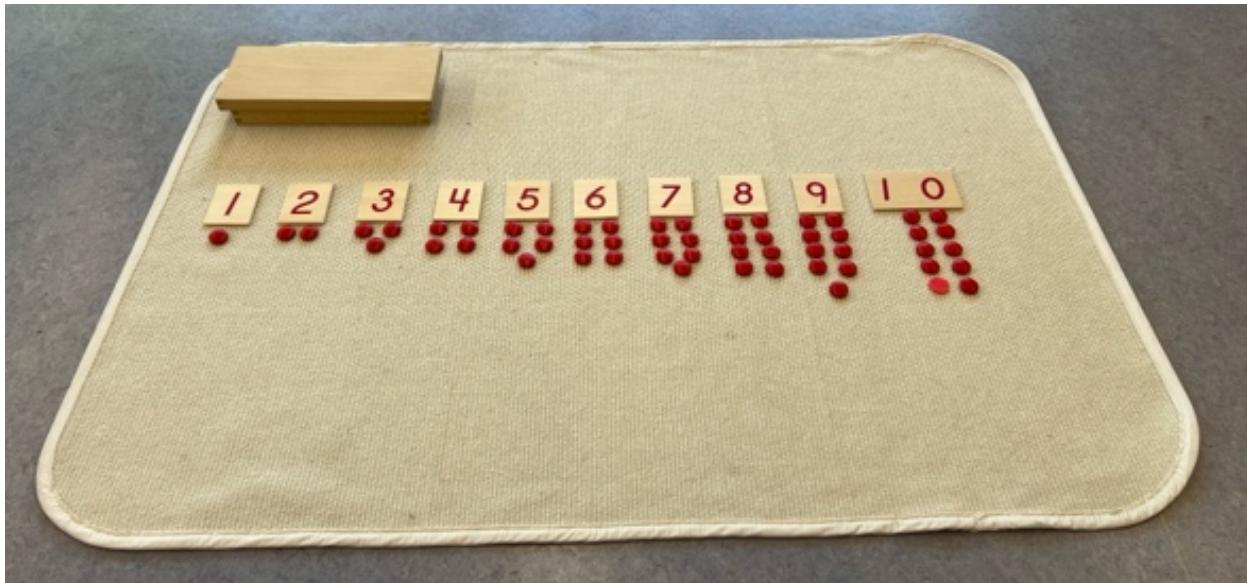
- Leads directly to **Cards & Counters** (odd/even)
- Later connects to **Golden Bead System** (place value)

## G. Common Mistakes & Solutions

Issue	Montessori-Aligned Response
Child skips counting	<i>"Let's touch each spindle as we count."</i>
Leaves "0" empty	<i>"Zero means we put nothing here. Can you show me nothing?"</i>
Miscounts	<i>"I notice we have extra spindles. Let's check together."</i>



## 2.5 Cards & Counters



(Ages 4-5 | Prerequisites: Spindle Boxes & Numeral Recognition 0-9)

### A. Materials:

- ✓ **Wooden numeral cards 1-10** (or printed cards matching Sandpaper Numerals font)
- ✓ **55 identical counters** (e.g., glass gems, wooden chips)
- ✓ **Felt-lined tray** (to organize materials)

### B. Direct Purposes:

1. Reinforce **quantity-to-symbol association**
2. Introduce **odd/even classification** through patterning
3. Develop **left-to-right work habits** (pre-reading preparation)

## C. Presentation Steps:

### 1. Preparation

- Arrange numeral cards vertically in random order on left side of mat
- Place counters in a bowl on the right

### 2. Demonstration

1. Select card "1":
  - *"This is one."* Place **1 counter** beneath it (centered)
2. Continue to "2":
  - Place counters side-by-side: *"One, two."*
3. **Pause at "3":**
  - *"Let's line them up like this."* Place two counters side-by-side, third centered below
4. Invite child to complete 4-10

### 3. Odd/Even Discovery

- Point to "4": *"These counters have partners. Four is even."*
- Point to "5": *"This counter is alone. Five is odd."*

## D. Control of Error:

### ✓ Visual Patterning:

- Even numbers form perfect pairs
- Odd numbers have a single unpaired counter

### ✓ Quantitative:

- Exactly 55 counters provided (ensures accuracy)

## E. Extensions/Variations:

### 1. Advanced Sorting

- Label two baskets "Odd" and "Even"
- Child sorts numeral cards into correct basket

## 2. Creative Counting

- Use themed counters (acorns in autumn, shells in summer)

## 3. Movement Game

- Child hops **once** for odd numbers, **claps twice** for evens

## F. Teacher Notes:

### Key Language:

- "*Even numbers have friends. Odd numbers have a loner.*"
- Avoid over-explaining – let the pattern reveal itself

### Observation Signs of Mastery:

- Child verbalizes why a number is odd/even
- Consistently places counters in correct patterns
- Notices own errors when counters are mismatched

### Link to Prior Work:

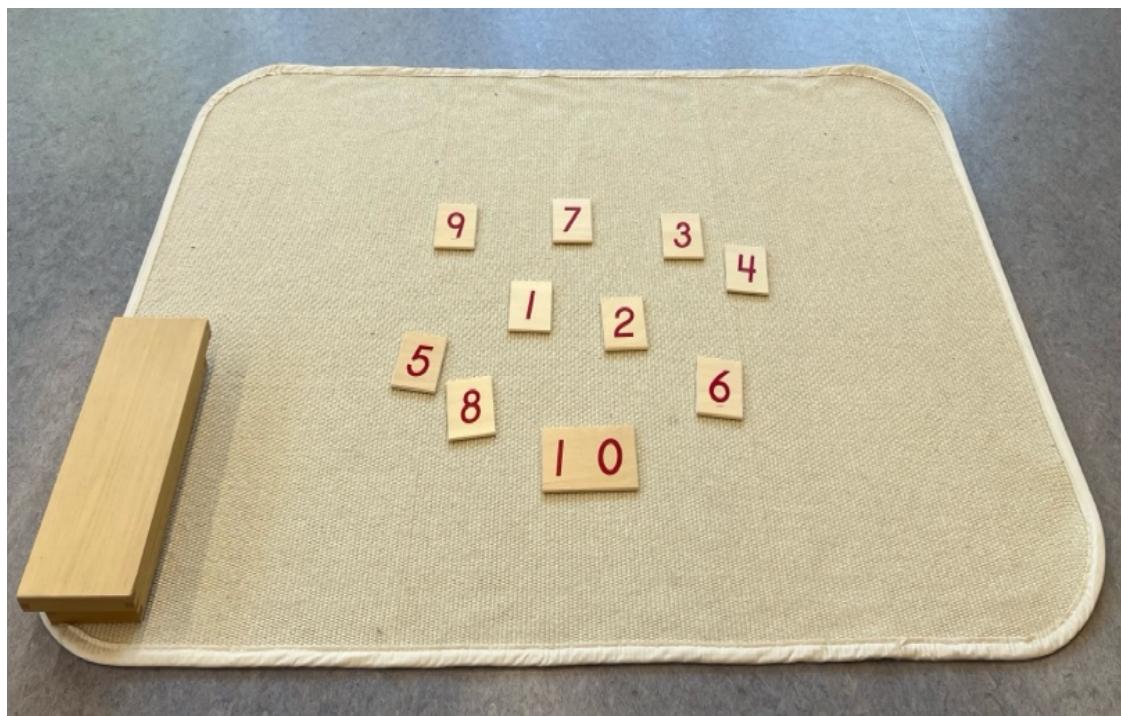
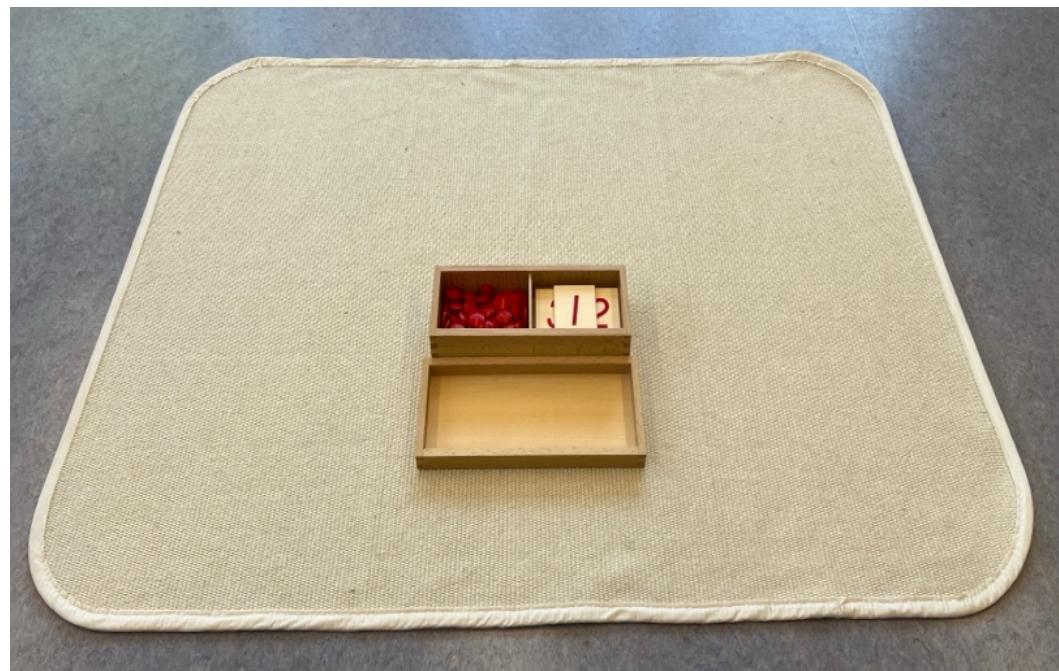
- Builds on **Spindle Boxes** by adding:
  - Horizontal patterning
  - Mathematical classification

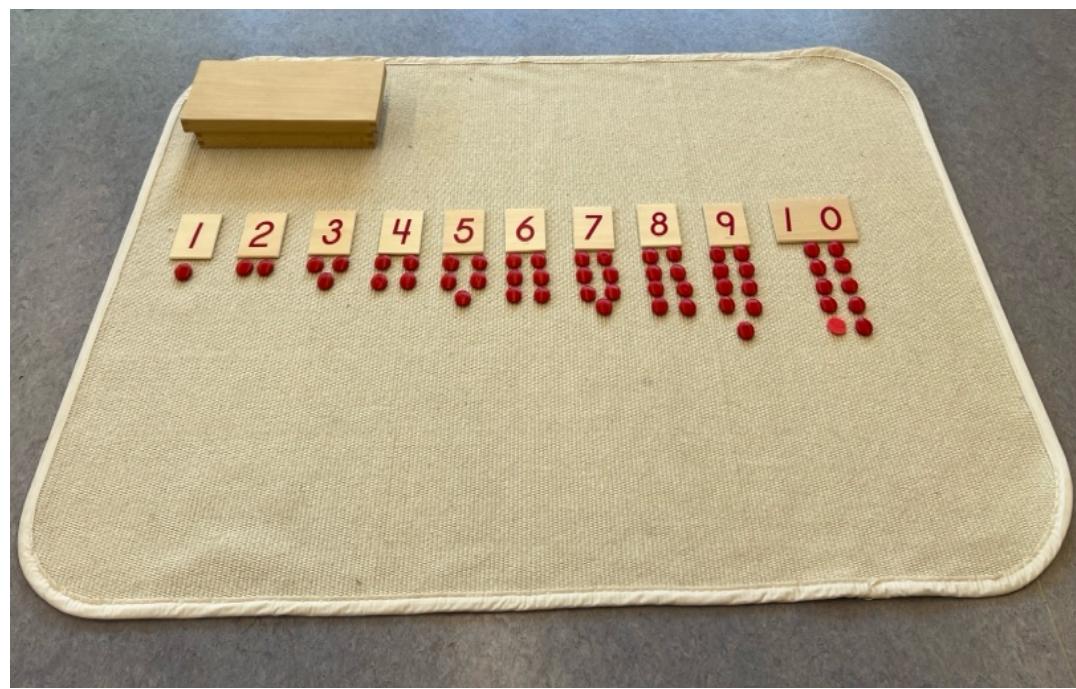
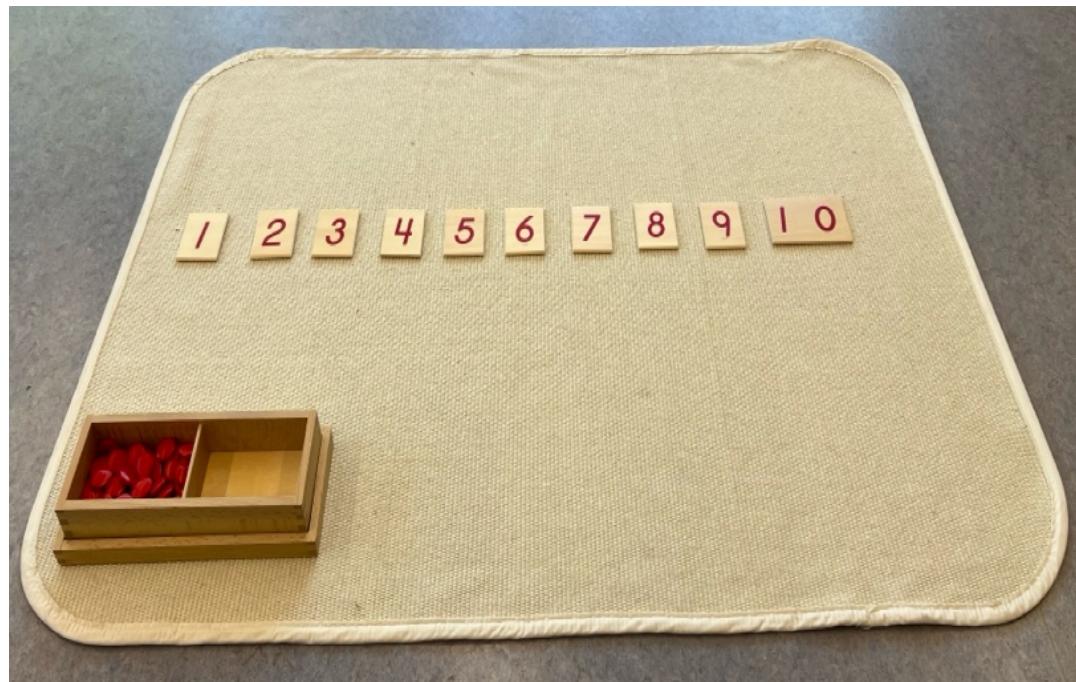
### Preparation for Next Steps:

- Prepares for **Addition Strip Board** (odd/even sums)
- Later connects to **Skip Counting** with Bead Chains

## G. Common Mistakes & Solutions

Issue	Montessori-Aligned Response
Counters placed randomly	"Let's line them up like we did with '2'."
Misclassifies odd/even	"Does this counter have a partner?" (Point to lone counter)
Skips numbers	"I notice we missed four. Let's check together."





## 2.6 Memory Game of Numbers



(Ages 4-5 | Prerequisites: Cards & Counters & Numeral Recognition 0-10)

### A. Materials:

- ✓ **Two identical sets of numeral cards 0-10** (Set A: red, Set B: blue)
- ✓ **55 counters** (matching previous work)
- ✓ **Large felt mat** (for organized spacing)

### B. Direct Purposes:

1. Strengthen **instant quantity recognition** (subitizing)
2. Develop **working memory** through matching
3. Prepare for **mental math** by internalizing quantities

## C. Presentation Steps:

### 1. Setup

- Spread **red numeral cards** face-up in a vertical line (left side)
- Place **blue numeral cards** face-down in a stack (right side)
- Counters placed in bowl at top

### 2. Demonstration

1. Turn over first blue card (e.g., "5")
  - *"Five needs five counters."* Place counters beneath card
2. Find its red match:
  - *"Let's find another five!"* Scan red cards
  - Place matching pair together

### 3. Child's Turn

- Child flips blue cards one by one
- For "0": *"Zero needs...?"* (Child responds: "Nothing!")

## D. Control of Error:

### Quantitative:

- Total counters (55) prevent mismatches
- Child recounts if uncertain

### Visual:

- Mismatched quantities are obvious when paired

## E. Extensions/Variations:

### 1. Advanced Memory Mode

- Flip **both sets** face-down for true memory challenge

### 2. Movement Integration

- Place red cards across room – child runs to fetch matches

### **3. Sound Pairing**

- Add bell/chime to ring correct number (e.g., ding 3 times for "3")

#### F. Teacher Notes:

#### **Key Language:**

- "*This is the symbol for seven. How many counters does it need?*"
- "*Zero is special because...*" (Pause for child to complete)

#### **Observation Signs of Mastery:**

- Child matches quantities within 3 seconds
- Self-corrects before touching counters
- Verbalizes thought process ("*Eight is two less than ten!*")

#### **Link to Prior Work:**

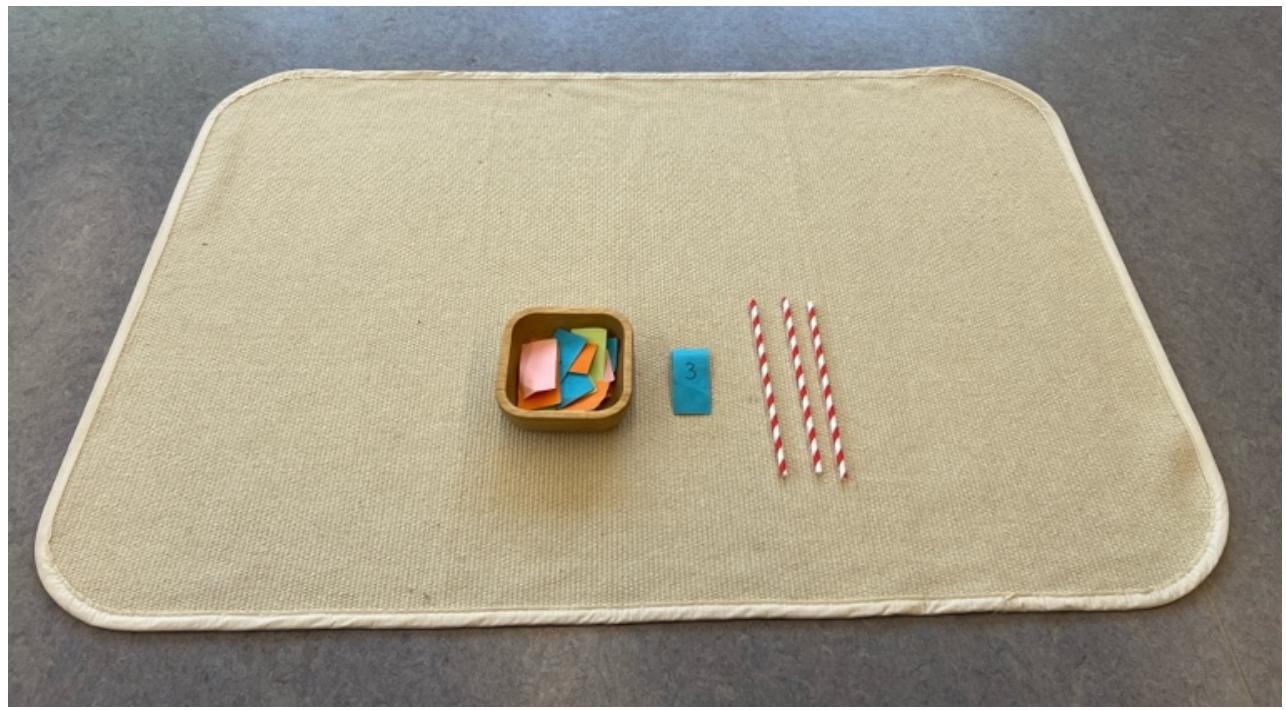
- Builds on **Cards & Counters** by:
  - Removing linear order
  - Adding memory challenge

#### **Preparation for Next Steps:**

- Directly leads to **Addition Strip Board**
- Later connects to **Bead Cabinet** skip counting

#### G. Common Mistakes & Solutions

Issue	Montessori Response
Miscounts quickly " <i>Let's touch each counter as we count.</i> "	
Forgets zero	" <i>What did we learn about zero in Spindle Boxes?</i> "
Confuses 6/9	" <i>The circle points to the bottom in six.</i> " (Trace numeral)



### 3. Introduction to the Decimal System

*(A Foundational Approach to Place Value and Hierarchical Mathematics)*

#### The Significance of the Decimal System

The decimal system is the cornerstone of modern mathematics and a fundamental component of the Montessori 3-6 curriculum. Dr. Maria Montessori recognized that children possess a natural sensitivity to hierarchical order and categorization, making the early childhood years the ideal time to introduce place value concepts. This album section presents a **developmentally appropriate, sensorial-based** approach to:

- **Concretizing** the base-ten number system
- **Establishing** the relationship between quantity and symbol
- **Preparing** for all mathematical operations

*"The decimal system is the golden key that unlocks the universe of mathematics."*

— *The Discovery of the Child*

#### Key Principles of Decimal System Work

##### *1. The Three-Hierarchy Approach*

Montessori materials present the decimal system through **three interconnected levels** of understanding:

Level	Material Example	Key Concept
Sensorial	Golden Beads	Physical experience of quantity
Symbolic	Large Number Cards	Abstract representation
Integrated	Stamp Game	Bridging concrete and abstract

## *2. The Power of Ten*

Each material emphasizes the **critical relationships**:

- 10 units = 1 ten
- 10 tens = 1 hundred
- 10 hundreds = 1 thousand

## *3. Movement-Based Learning*

- **Bead exchanges** (physical regrouping)
- **Quantity formation** (building numbers)
- **Verbal reinforcement** (naming hierarchies)

# **Developmental Sequence**

*Phase 1: Introduction to Quantity (4-4.5 years)*

## **Materials & Objectives:**

1. **Golden Bead Material**
  - Initial Presentation: Isolating categories (units, tens, hundreds, thousands)
  - Key Experience: Feeling the weight difference between a unit bead and thousand cube
2. **Quantity Formation**
  - Building numbers up to 9,999
  - Verbalizing place value: "*3 thousands, 4 hundreds, 2 tens, and 1 unit*"

## **Teacher's Role:**

- Use **precise language**: "*This is one thousand*" (not "This is a thousand cube")
- Demonstrate **careful handling** to convey respect for mathematical materials

*Phase 2: Symbol Introduction (4.5-5 years)*

## **Materials & Objectives:**

1. **Large Number Cards**
  - Color-coding: units (green), tens (blue), hundreds (red), thousands (green)
  - Composition of numbers: Layering cards to form 4-digit numbers

## 2. Combined Quantity & Symbol

- Matching golden beads to number cards
- Reading numbers up to 9,999

### *Phase 3: Operations (5-6 years)*

#### Progressive Complexity:

1. **Static Addition** (no exchanges)
2. **Dynamic Addition** (with exchanges)
3. **Subtraction, Multiplication, and Division**

#### Critical Progression:

- Begin with golden beads → transition to stamp game → move toward abstraction

### Special Considerations

#### For Younger Children (4-4.5 years):

- Focus only on quantity formation
- Use simplified language: "*Let's count how many tens we have*"

#### For Advanced Learners:

- Introduce the concept of "ten thousands"
- Explore number formation beyond 9,999

#### Common Challenges & Solutions:

Challenge	Montessori Response
Confusing place values	<i>"Let's rebuild the number together"</i>
Skipping hierarchical steps	<i>"Always start with the thousands"</i>
Difficulty with exchanges	<i>"Watch how ten units become one ten"</i> (physical demonstration)

### Purpose & Outcomes

Through this systematic decimal system work, children will:

Develop a **concrete understanding** of place value

Master the **language of large numbers**

Build **problem-solving skills** for all operations

Gain **confidence** in working with complex mathematics

*"The child who holds a thousand bead cube in their hand will never fear large numbers."*

## 3.1.1 Introduction to Golden Beads

(First Presentation – Quantity Only)



### Material:

Complete set of Golden Beads:

- Individual unit beads (approx. 7mm spheres)
  - Ten-bars (10 units strung on wire)
  - Hundred-squares (10 ten-bars wired together)
  - Thousand-cube (10 hundred-squares layered)
- Felt-lined tray for units and tens  
Large work mat (minimum 3'x3')

### **Prerequisites:**

- Mastery of Number Rods 1-10
- Experience with Teens/Tens work
- Ability to count reliably to 100

## **1. Preparation**

### **A. Invitation:**

*"Today I want to show you something special about how numbers grow big."*

### **B. Material Transport:**

- Demonstrate carrying:
  - Thousand cube with two hands (emphasizing weight)
  - Hundred squares flat on palms
  - Ten-bars in one hand
  - Unit beads in small dish

### **C. Arrangement:**

- Place categories vertically on mat (left to right):
  1. Thousands (green)
  2. Hundreds (red)
  3. Tens (blue)
  4. Units (green)

## **2. Presentation**

### **A. Naming the Categories**

1. **Isolate one unit bead:**
  - Roll between fingers: *"This is one unit."*
  - Place alone in units column
2. **Show ten-bar:**
  - Finger-count beads: *"One, two...ten. Ten units make one ten."*
  - Place vertically in tens column
3. **Introduce hundred-square:**
  - Point to ten-bars: *"One ten, two tens...ten tens make one hundred."*
  - Place horizontally in hundreds column

4. **Reveal thousand-cube:**
  - Lift with both hands: "*Ten hundreds make one thousand!*"
  - Place prominently in thousands column

## B. Three-Period Lesson

1. **Naming:**
  - "*This is one thousand. This is one hundred. This is one ten. This is one unit.*"
2. **Recognition:**
  - "*Show me one hundred. Where is one ten?*"
3. **Recall:**
  - "*What is this?*" (Point to thousand cube)

## C. Building Quantities

1. Form "2,341":
  - Take 2 thousand cubes
  - 3 hundred squares
  - 4 ten-bars
  - 1 unit bead
2. Verbalize:  
*"Two thousands, three hundreds, four tens, and one unit: two thousand three hundred forty-one."*

## 3. Exploration

### A. Child's Turn:

- Invite child to build their own number
- Observe handling:
  - Proper carrying technique
  - Left-to-right sequence
  - Category alignment

### B. Language Reinforcement:

- Use consistent terminology:
  - "*How many hundreds do you have?*"

- "What comes after thousands?"

## 4. Control of Error

### Visual/Tactile Checks:

- ✓ Thousand cube should be heaviest
- ✓ Hundred squares lie flat (cannot stand vertically)
- ✓ Ten-bars must contain exactly 10 beads

### Quantitative Limits:

- Only 9 of each category provided
- Prevents formation of incorrect quantities

## 5. Purpose & Key Points

### Direct Aim:

- Introduce hierarchical nature of decimal system
- Establish vocabulary of place value

### Indirect Aim:

- Preparation for all four operations
- Development of mathematical reasoning

### Essential Notes:

Always name categories before quantities ("one ten" not "ten")

Maintain exact spatial arrangement (thousands at top/left)

Use slow, deliberate movements when exchanging

## 6. After the Lesson

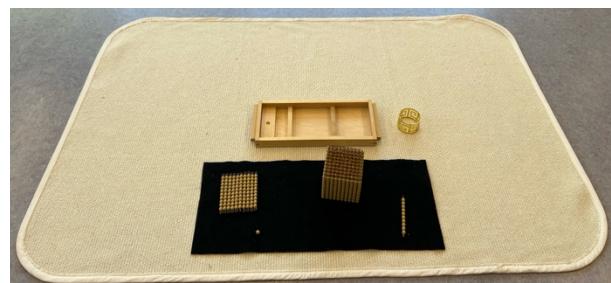
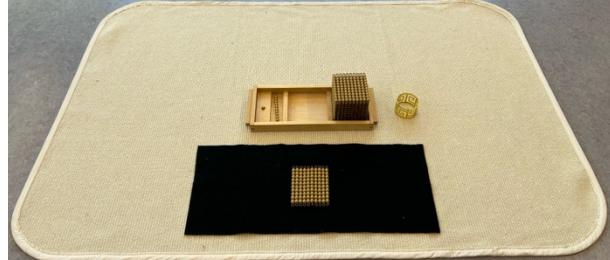
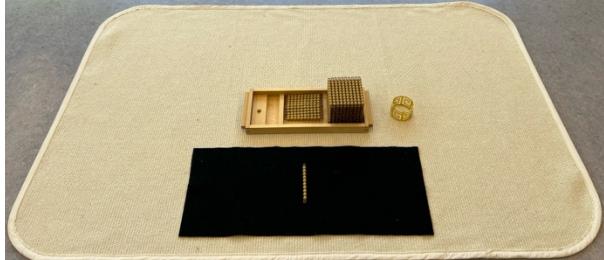
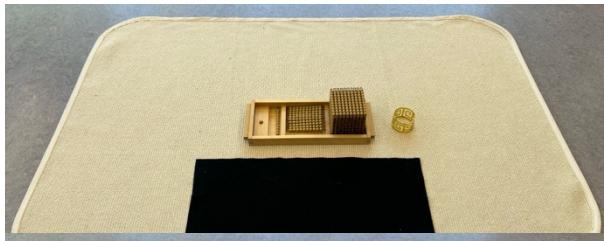
### Observations to Record:

- Does child handle thousand cube with appropriate care?
- Can child verbalize place values independently?
- Any confusion between hundreds and thousands?

## Next Steps:

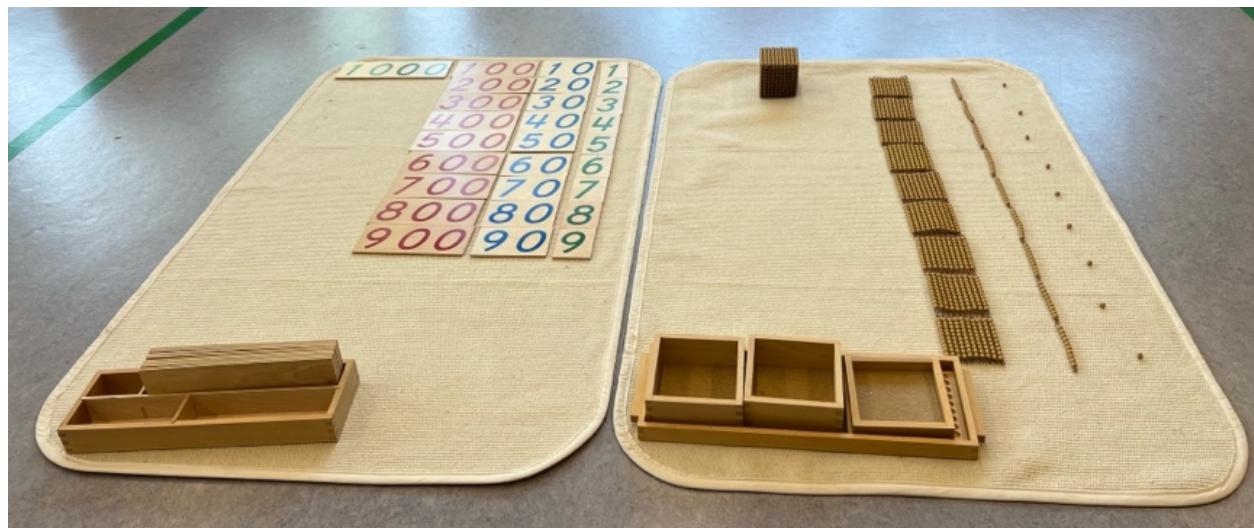
- Forming quantities with numeral cards
- Static addition with golden beads

*"The golden beads transform the abstract decimal system into a tangible reality the child can hold and explore."*



## 3.1.2 Formation of Numbers

(Ages 4.5-5.5 | Prerequisites: Introduction to Golden Beads)



### A. Materials:

- ✓ **Full Golden Bead set** (units, tens, hundreds, thousands)
- ✓ **Large floor mat** with place value sections
- ✓ **Wooden numeral cards** 1-9, 10-90, 100-900, 1000-9000
- ✓ **Tray** for numeral cards

### B. Direct Purposes:

1. Combine categories to form **composite quantities**
2. Reinforce **hierarchical relationships** in the decimal system
3. Prepare for **reading and writing** large numbers

## C. Presentation Steps:

### 1. Setting Up

1. Arrange place value sections (Units, Tens, Hundreds, Thousands from right to left)
2. Place numeral cards in stacks by category (units cards nearest child)

### 2. Forming a Quantity (e.g., 2,413)

#### 1. Thousands:

- "Let's make two thousand."
- Take **2 thousand-cubes**, place in thousands section
- Select "2000" card, place below beads

#### 2. Hundreds:

- "Now we need four hundred."
- Take **4 hundred-squares**, place in hundreds section
- Select "400" card, place below

#### 3. Tens & Units:

- Repeat with **1 ten-bar** ("10" card)
- **3 unit beads** ("3" card)

#### 4. Verification:

- Point left-to-right: "*Two thousand, four hundred, one ten, three units: 2,413.*"

## D. Control of Error:

### ✓ Visual:

- Numeral cards must match bead quantities in each category
- Color-coding (green=units, blue=tens, red=hundreds, green=thousands)

### ✓ Kinesthetic:

- Thousand-cube's weight vs. unit beads reinforces magnitude

### ✓ Quantitative:

- Limited beads prevent incorrect formations (only 9 per category)

## E. Extensions:

### 1. "Mystery Quantity" Game

- Teacher forms a quantity; child names it

### 2. Writing Practice

- Child copies formed quantities on place value paper

### 3. Movement Variation

- Place categories across room – child "shops" for correct beads

## F. Teacher Notes:

### Key Language:

- Always name full categories:  
*"You've selected three hundreds, not just three."*
- Emphasize **left-to-right** reading:  
*"We say two thousand before four hundred."*

### Observation Signs of Mastery:

- Consistently aligns beads with correct numeral cards
- Self-corrects misplaced categories
- Verbalizes place value: *"This is the hundreds place."*

### Link to Next Work:

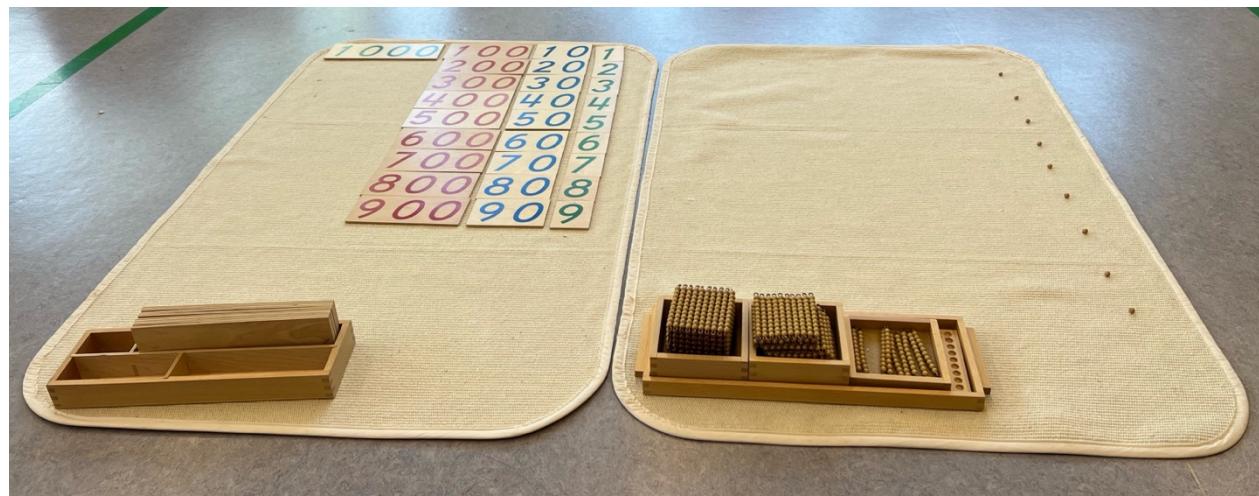
- Directly prepares for:
  1. **Golden Bead Addition (Static)**
  2. **Bank Game (Exchanges)**

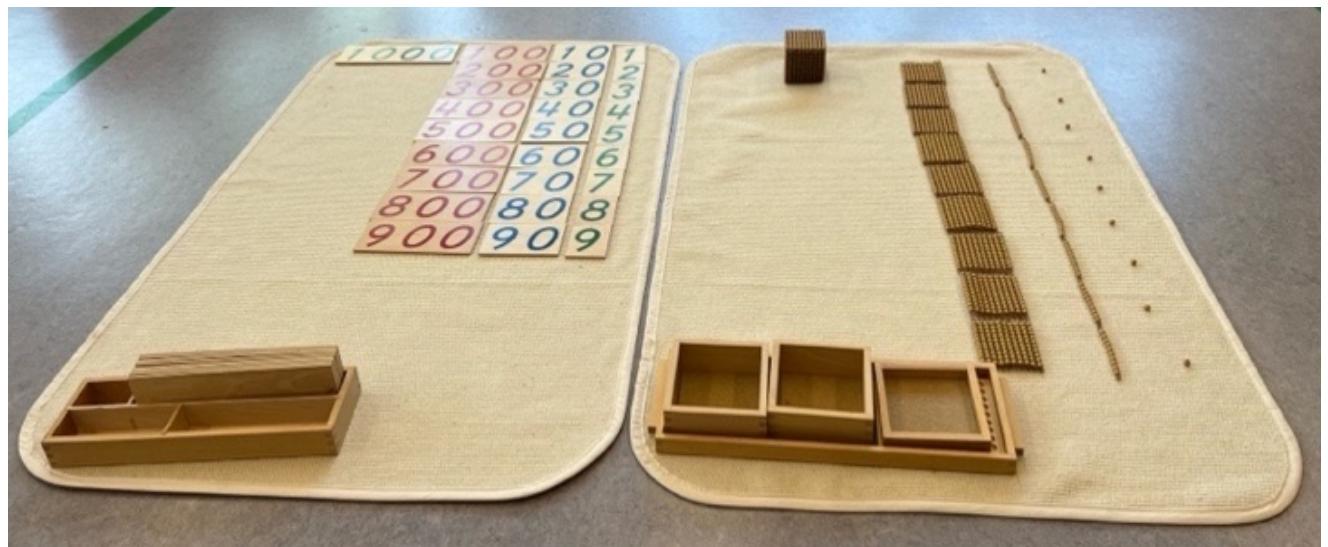
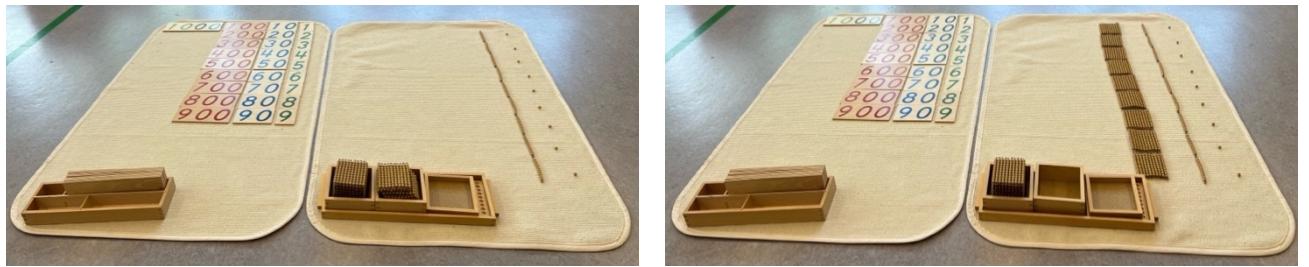
## G. Common Mistakes & Solutions

Issue	Montessori Response
Confuses 105 vs. 150	"Let's check – are the tens or hundreds larger?" (Compare bead amounts)
Omits zero categories	"For 3,050, we leave the tens place empty." (Use placeholder card)
Misaligns cards	"Watch how the units card always goes farthest right."

## AMS Album Enhancements:

1. **Photograph Tip:** Capture child's hands placing the thousand-cube with its card
2. **Scientific Note:** This work satisfies the **sensitive period for order** through precise categorization
3. **Quote:**  
*"Through these beads, the child discovers that numbers are built hierarchically – like a tree growing branches."*





## 3.2 Stamp Game

(Ages 5.5-6.5 | Prerequisites: Golden Bead Static & Dynamic Addition)



*(First Presentation – Static Addition)*

**Material:**

✓ **Stamp Game Box** containing:

- Colored tiles: Green (units 1-9), Blue (tens 10-90), Red (hundreds 100-900), Green (thousands 1000-9000)
- Skittles (counters for division)
- +, -, ×, ÷, and = operation signs
  - ✓ **Small felt mat** (12"x18" with grid lines preferred)
  - ✓ **Pencil and paper** for recording
  - ✓ **Control chart** (for verification)

**Prerequisites:**

- Mastery of Golden Bead operations (static addition)
- Recognition of all numeral symbols 1-9000
- Understanding of place value columns

## 1. Preparation

### A. Invitation:

*"You've worked with the golden beads. Now we'll use these special stamps to do big math!"*

### B. Material Setup:

1. Place mat vertically with columns (right to left):
  - **Units** (green)
  - **Tens** (blue)
  - **Hundreds** (red)
  - **Thousands** (green)
2. Open box showing color-coded tiles:
  - *"Green for units, blue for tens – just like our golden beads."*

## 2. Presentation: Static Addition (No Exchanges)

**Sample Problem:**  $2,413 + 1,322 = \underline{\hspace{2cm}}$

### Step 1: Forming the Addends

1. Build first addend (2,413):
  - Take **2** thousand tiles
  - **4** hundred tiles
  - **1** ten tile
  - **3** unit tiles
  - Place in top-right of mat
2. Build second addend (1,322) below first:
  - **1** thousand
  - **3** hundreds
  - **2** tens
  - **2** units

### Step 2: Adding Columns

1. **Slide tiles together** vertically:
  - \*"Let's add the units first:  $3 + 2 = 5$ ."\*
  - Push 3 and 2 unit tiles together, count total
  - Place = sign, record "5" on paper
2. Repeat for each category:
  - Tens:  $1 + 2 = 3$
  - Hundreds:  $4 + 3 = 7$
  - Thousands:  $2 + 1 = 3$

### Step 3: Verifying the Sum

- Read final answer: "*Two thousand plus one thousand is three thousand...*"
- Confirm with control chart

## 3. Key Demonstrations

### A. Color Coding:

- "*The colors help us keep each place value separate – just like the golden beads.*"

### B. Proper Tile Handling:

- Demonstrate flipping tiles to show blank side when unused
- "We always work from right (units) to left (thousands)."

### C. Recording Work:

- Model writing:

$$\begin{array}{r}
 2,413 \\
 +1,322 \\
 \hline
 3,735
 \end{array}$$

## 4. Control of Error

### Visual Checks:

- ✓ Tiles must align in exact columns
- ✓ No category exceeds 9 tiles (for static addition)
- ✓ Color consistency with golden beads

### Mathematical Verification:

- Re-count combined tiles
- Compare to control chart

## 5. Purpose & Notes

### Direct Aims:

- Transition from concrete (beads) to semi-abstract (tiles)
- Reinforce place value in operations
- Develop mental calculation strategies

### Teacher Tips:

#### Language Precision:

- "*Three hundred tiles*" (not "three hundreds")

#### Error Handling:

- If child miscounts: "*Let's touch each tile as we count.*"

#### Pacing:

- First presentation should be <15 minutes

## 6. After the Lesson

### Observations to Record:

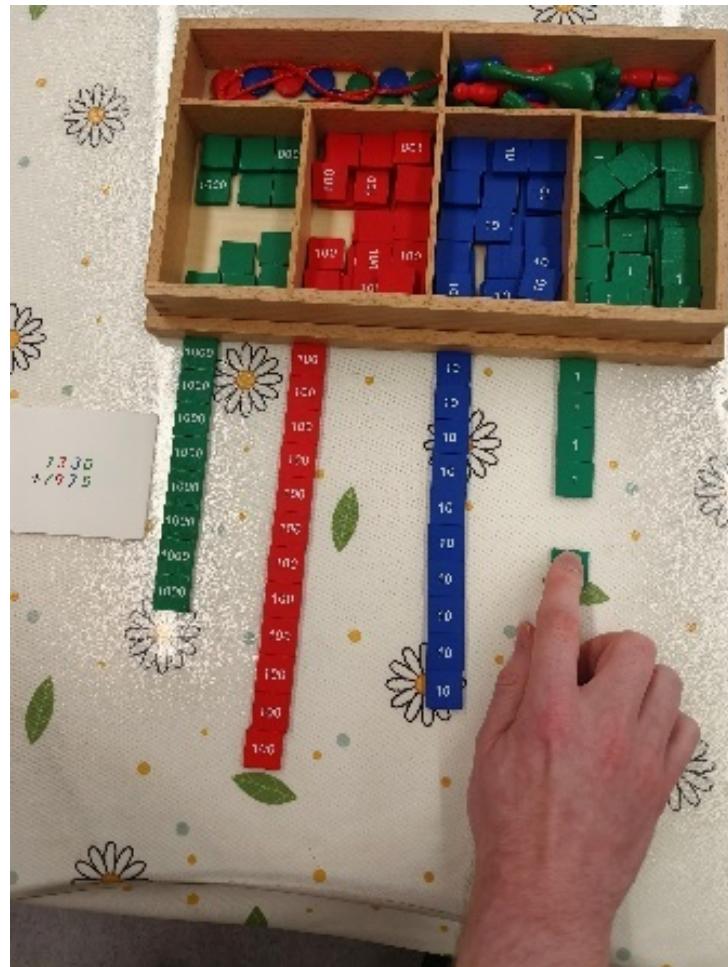
- Does child maintain column alignment?
- Can child verbalize the addition process?
- Any confusion between place values?

### Next Steps:

- Dynamic addition (with exchanges)
- Subtraction with stamp game

*"The stamp game is the child's bridge from holding quantities to imagining them."*

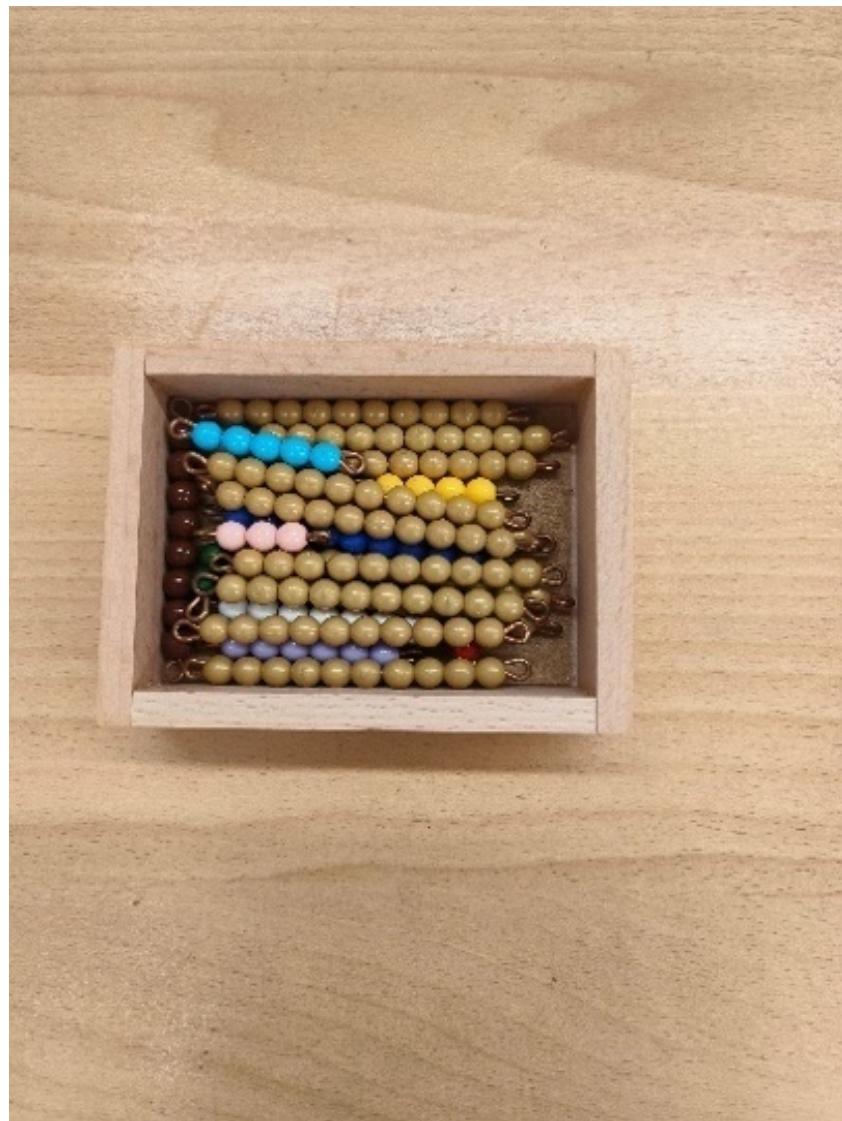




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### 3.3 Bead Cabinet (Linear & Skip Counting)

(Ages 3.5-5 | Prerequisite: Sandpaper Numerals 1-10, Number Rods 1-10)



## A. Materials

- **Teen bead bars** (9 golden 10-bars + colored bead bars 1-9)
- **Wooden "10" board or mat with "10+" notation**
- **Numeral cards 11-19**
- **Small felt tray** for bead bars

## B. Direct Purpose

1. Master **quantity and symbol association** for 11-19
2. Understand **teen numbers as "10 + x"**
3. Develop **sequential counting** beyond 10

## C. Presentation

### 1. Building Teen Quantities

1. Place golden 10-bar vertically: "*This is ten.*"
2. Add colored bead bar horizontally to the right:
  - "*Ten and one make eleven.*" (Place "11" card)
  - "*Ten and two make twelve.*"
3. Continue through 19, emphasizing:  
*"Ten and [x] make [teen number]."*

### 2. Three-Period Lesson

#### Period 1 (Naming):

- "*This is eleven. This is twelve.*"

#### Period 2 (Recognition):

- "*Show me fifteen. Where is thirteen?*"

#### Period 3 (Recall):

- "*What is this?*" (Point to 17 beads)

### **3. Verification**

- Child reassembles quantities while saying:  
*"Ten and seven make seventeen."*

## **D. Control of Error**

#### **Visual:**

- Golden 10-bar + colored beads = unmistakable teen structure
- Numeral cards must match bead combinations

#### **Tactile:**

- Bead bars fit exactly in "10 + x" formation

#### **Quantitative:**

- No quantity exceeds 19 beads

## **E. Extensions**

### **1. "Bring Me" Game**

- "*Bring me ten and six.*"

### **2. Mystery Bag**

- Child identifies teen number by feeling beads

### **3. Card Matching**

- Match loose beads to pre-made teen quantity cards

## **F. Teacher Notes**

### **Key Language:**

- Always use "ten and" phrasing (not "eleven is 10+1")
- Emphasize "make": "*Ten and three make thirteen.*"

### **Observation Signs:**

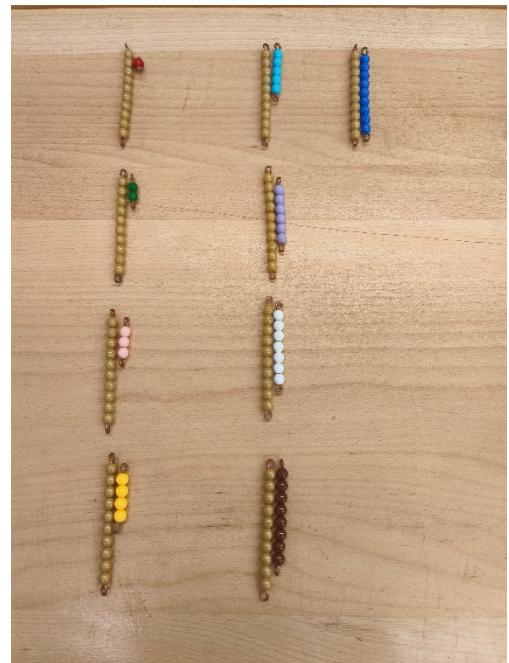
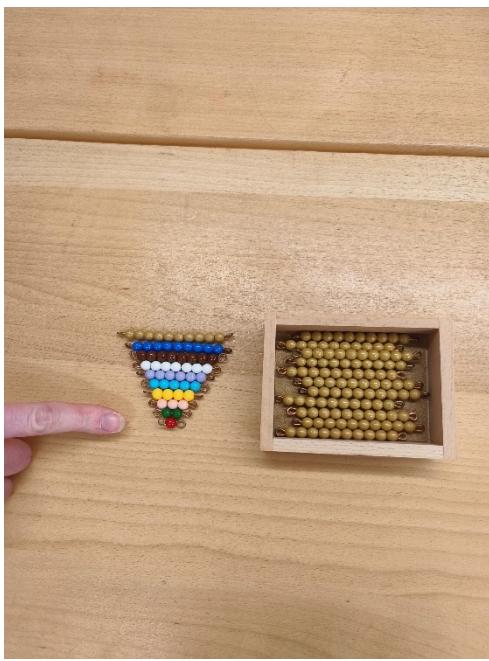
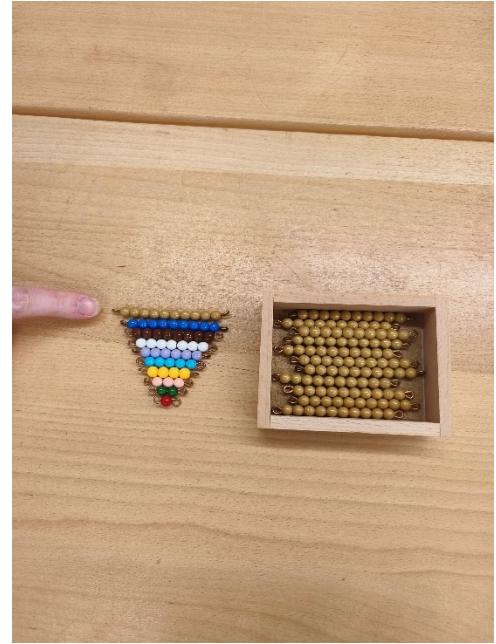
- Consistently forms correct "10 + x" combinations
- Names all teens correctly in Period 3

### **Link to Next Work:**

- Leads directly to **Ten Boards** (formal symbol association)
- Prepares for **Tens Beads (20-90)**

## **G. Common Errors**

<b>Issue</b>	<b>Montessori Response</b>
Skips "14-16" sequence	<i>"Let's count together: thirteen, fourteen, fifteen..."</i>
Confuses 12/21	<i>"The '1' comes first in teens"</i> (Trace numerals)
Forgets golden 10-bar	<i>"How do we start every teen number?"</i>



## 3.4 Teen Boards – Symbol & Quantity Association

(Ages 4-5 | Prerequisites: Teen Beads, Sandpaper Numerals 1-9)



## A. Materials

- **Teen Boards** (wooden frames with "10" printed)
- **Wooden numeral cards** 1-9 (for slot-in digits)
- **Teen bead quantities** (golden 10-bar + colored bead bars 1-9)
- **Felt-lined tray** for numerals
- **Control chart** (pre-made 11-19 display)

## B. Direct Purposes

1. Associate **symbols with quantities** for 11-19
2. Reinforce "**10 + x**" **structure** of teen numbers
3. Develop **number formation** skills

## C. Presentation

### 1. Setting Up

1. Place Teen Board horizontally: "*This says ten.*"
2. Slide "1" numeral into slot: "*Ten and one make...*" (Pause for child to say "eleven")
3. Place corresponding beads (10-bar + 1-bar) below

### 2. Three-Period Lesson

#### Period 1 (Naming):

- Build 11-13: "*Ten and three make thirteen.*"

#### Period 2 (Recognition):

- "*Show me 'twelve'. Where is 'fifteen'?*"

#### Period 3 (Recall):

- "*What is this?*" (Point to 14 formation)

### 3. Independent Practice

- Child builds all teen numbers while verbalizing:  
*"Ten and seven make seventeen."*

## D. Control of Error

### ✓ Visual:

- Beads must match numeral combination
- Control chart verifies correct matches

### ✓ Physical:

- Numeral cards fit only in correct slots
- Requires exact bead quantities

## E. Extensions

### 1. Missing Numeral Game

- Omit one numeral card; child identifies gap

### 2. Bead-to-Board Matching

- Pre-make quantities; child finds correct board position

### 3. Tactile Reinforcement

- Trace sandpaper numerals before slotting in

## F. Teacher Notes

### Key Language

- Consistently use:  
*"Ten [point to board] and [slide numeral] make [child says result]."*
- Never shortcut to *"This is thirteen"* – always show construction

### Observation Signs

- Smooth numeral card insertion
- Spontaneous self-correction using control chart
- Verbalizes full "10 + x" equation

## Link to Next Work

- Leads directly to **Ten Boards (20-90)**
- Prepares for **Colored Bead Bars** addition

## G. Common Errors

Issue	Solution
Reverses digits (14 as 41)	<i>"The '1' comes first in teens"</i> (Trace numeral)
Skips verbalization	<i>"Tell me while you build"</i>
Confuses 12/13 beads	<i>"Let's recount your ten-bar"</i>

*"The child who constructs thirteen will never doubt its place between twelve and fourteen."*



# 4. Introduction to Mathematical Operations

(A Comprehensive Approach to Addition, Subtraction, Multiplication, and Division)

## *The Foundational Role of Operations in Montessori Mathematics*

In the Montessori 3-6 curriculum, the four primary mathematical operations are not merely computational techniques but fundamental expressions of the child's developing reasoning ability. Dr. Maria Montessori designed a sequential, sensorimotor approach that allows children to:

- **Discover** operational concepts through manipulation
- **Internalize** mathematical relationships through repetition
- **Master** abstract calculation through graduated materials

*"The operations of arithmetic are the keys that unlock the child's understanding of the quantitative world."*

— *Psych arithmetic*

## Core Pedagogical Principles

### *1. The Pathway from Concrete to Abstract*

All operations follow this developmental progression:

Stage	Material Example	Cognitive Focus
Sensorial	Golden Beads	Physical experience of quantity
Concrete	Stamp Game	Symbol-quantity association
Transitional	Bead Frames	Visual patterns
Abstract	Finger Charts	Mental calculation

### *2. Interconnected Learning*

- **Addition & Subtraction:** Inverse relationship emphasized through "checking" work
- **Multiplication & Division:** Demonstrated as repeated addition/subtraction

- **Common Language:** Consistent use of "exchanging" across all operations

### *3. Auto-Educative Design*

Each material contains:

- **Control of Error:** Physical limits (e.g., only 9 unit stamps)
- **Self-Verification:** Ability to check work against simpler problems

## **Operation-Specific Introductions**

### *I. Addition: The Mathematics of Combining*

#### **Developmental Purpose:**

- Understand totality as the union of parts
- Master place value through exchanges

#### **Key Materials Sequence:**

1. **Golden Beads**
  - Static:  $2,341 + 3,412$
  - Dynamic:  $1,576 + 2,687$  (with exchanges)
2. **Stamp Game**
  - Introduces standard notation
3. **Small Bead Frame**
  - Bridges to abstract calculation

#### **Unique Montessori Approach:**

Children physically experience that:

*"When we add, each category must stay in its proper place."*

### *II. Subtraction: The Mathematics of Difference*

#### **Developmental Purpose:**

- Comprehend removal as inverse of addition
- Develop problem-solving flexibility

#### **Key Materials Sequence:**

1. **Golden Beads**
  - Static: 5,763 - 2,341
  - Dynamic: 4,032 - 1,857 (with borrowing)
2. **Subtraction Strip Board**
  - Reveals patterns in fact families

**Critical Language:**

*"We start subtracting from the units because we might need to break a ten."*

*III. Multiplication: The Mathematics of Replication*

**Developmental Purpose:**

- Recognize multiplication as efficient addition
- Build foundations for algebraic thinking

**Key Materials Sequence:**

1. **Bead Chains**
  - 5-chain demonstrates  $5 \times$  tables
2. **Multiplication Bead Board**
  - Forms visual arrays ( $3 \times 4$  rectangle)
3. **Bank Game**
  - Large number multiplication

**Sensorial Insight:**

*"Three taken four times makes the same rectangle as four taken three times."*

*IV. Division: The Mathematics of Distribution*

**Developmental Purpose:**

- Understand fair sharing (partitive division)
- Develop measurement concepts (quotative division)

**Key Materials Sequence:**

1. **Golden Beads with Skittles**
  - $45 \div 3$ : Sharing among 3 "people"
2. **Unit Division Board**
  - Memorization through grouping
3. **Racks & Tubes**

- Hierarchical division

### **Real-World Connection:**

*"Division helps us solve problems like sharing cookies or making equal teams."*

## **The Teacher's Role in Operations Work**

### *Presentation Essentials*

#### **1. Sequential Demonstration**

- Show complete process first
- Isolate challenging steps (e.g., exchanges)

#### **2. Language Precision**

- *"We exchange ten units for one ten"* (not "carry the one")
- Avoid anthropomorphism (*"The numbers want to..."*)

#### **3. Observation Points**

- Readiness for new operation: 90% accuracy with current work
- Mastery signs: Spontaneous self-correction

### *Common Challenges & Solutions*

<b>Operation</b>	<b>Typical Difficulty</b>	<b>Montessori Intervention</b>
Addition	Place value misalignment	<i>"Let's rebuild the number together"</i>
Subtraction	Borrowing confusion	Physical demonstration with beads
Multiplication	Memorization resistance	Bead cabinet pattern discovery
Division	Remainder understanding	Sharing real objects (e.g., marbles)

## **Alignment with AMS Standards**

- ✓ **Standard 5.2.3:** Materials demonstrate mathematical properties concretely
- ✓ **Standard 7.1.2:** Presentations follow logical, developmentally-appropriate sequence
- ✓ **Standard 3.4.1:** Child progresses from concrete to abstract understanding

### **Documentation Requirements:**

- Include photos showing:
  - Proper hand position during exchanges
  - Correct problem setup
- Provide sample observation records

## Purpose and Outcomes

Through this operations sequence, children develop:

- **Mathematical Fluency:** Fact mastery through understanding
- **Problem-Solving Skills:** Multiple approaches to challenges
- **Cognitive Flexibility:** Seeing relationships between operations

*"The materials are but the keys; the child's mind is the locksmith."*

## 4.1 Linear Counting: 100 & 1000 Chains

(Ages 5-6 | Prerequisites: Short Chains 1-10, Teen Boards.)



## A. Materials

- ✓ **100 Chain** (10 linked 10-bars, gold beads)
- ✓ **1000 Chain** (100 linked 10-bars, gold beads)
- ✓ **Arrow labels:**
  - For 100 chain: 10, 20, 30...100 (blue)
  - For 1000 chain: 100, 200...1000 (red)
- ✓ **Bead squares ( $10^2$ ) and cube ( $10^3$ )**
- ✓ **Floor mat** (4'x6' for 1000 chain)

## B. Direct Purposes

1. Master **sequential counting** to 1000
2. Develop **decade transition** fluency
3. Sensorial understanding of **square/cube quantities**

## C. Presentation

### 1. 100 Chain (10x10)

1. Unfold chain completely on mat
2. Count by 10s while placing blue arrows:
  - "10 [place arrow], 20, 30...100"
3. Form square: "*Ten tens make one hundred – a square!*"
4. Verify by recounting arrows

### 2. 1000 Chain (10x100)

1. Lay chain in zigzag pattern to fit mat
2. Count by 100s with red arrows:
  - "100 [place arrow], 200...1000"
3. Form ten 100-squares, then stack into cube:
  - "Ten hundreds make one thousand – a cube!"

### 3. Three-Period Reinforcement

**Period 1:** "This says two hundred." (Point to arrow)

**Period 2:** "Find where four hundred goes."

**Period 3:** "What is this number?" (Point to 700)

## D. Control of Error

### ✓ Visual:

- Arrows must align with every 10th/100th bead
- 100-chain forms perfect 10x10 square

### ✓ Kinesthetic:

- Weight difference between 100-square and 1000-cube
- Chain tension at arrow points

### ✓ Quantitative:

- Exactly 100/1000 beads when correctly coiled

## E. Extensions

### 1. Labeling Race

- Time how quickly child can place all 100-chain arrows

### 2. Skip Counting

- Jump/clap while counting by 10s or 100s

### 3. Golden Bead Comparison

- Match 1000-chain to 1000-cube from decimal system

## F. Teacher Notes

### Key Language

- Always count **left-to-right**

- Emphasize transitions:  
*"After 90 comes 100, after 900 comes 1000."*

## Observation Signs

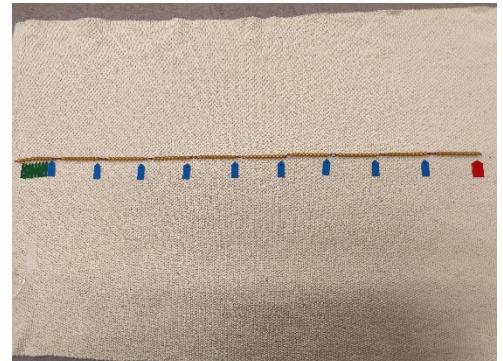
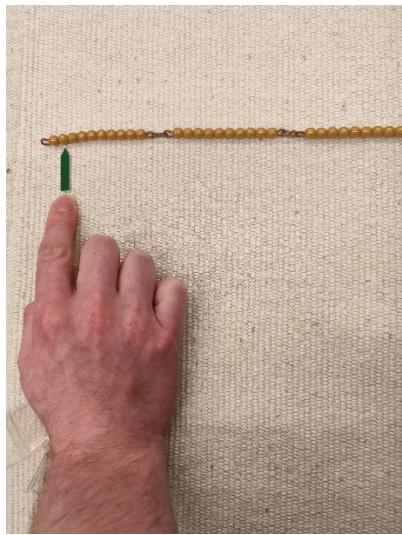
- Consistently places arrows correctly
- Verbalizes decade transitions
- Shows awe when forming 1000-cube

## Link to Future Work

- Prepares for **Multiplication Bead Board**
- Enables **Stamp Game** with larger numbers

## G. Common Errors

Issue	Solution
Miscounts by 10s	<i>**Touch each 10-bar as we count**</i>
Misaligns 1000-chain	<i>"Let's stretch it straight first"</i>
Forgets square/cube	<i>"What happens when we fold ten tens?"</i>



## 4.2 Stamp Game Operations

### (More abstract representation)

**Age:** 5.5-6.5 years

**Prerequisite:** Mastery of Golden Bead operations (addition/subtraction with exchanges)

#### Materials

**Stamp Game Box** containing:

- Green unit tiles (1-9)
  - Blue ten tiles (10-90)
  - Red hundred tiles (100-900)
  - Green thousand tiles (1000-9000)
- Operation symbols** (+, -, ×, ÷, =)  
**Skittles** (for division)  
**Control chart** (for verification)  
**Pencil/paper** for recording

#### Presentation 1: Static Addition (No Exchanges)

**Example Problem:**  $2,413 + 1,322$

##### *Step 1: Setting Up*

1. Lay mat vertically with place value columns (units → thousands)
2. Build first addend:
  - 2 thousand tiles
  - 4 hundred tiles
  - 1 ten tile
  - 3 unit tiles
3. Build second addend below with spacing

##### *Step 2: Adding*

1. **Combine columns** right-to-left:
  - \*"3 units + 2 units = 5 units"\* (push tiles together)

2. Repeat for tens, hundreds, thousands
3. Record equation:

3. 2,413
4. +1,322
- 5.
6.  $\underline{3,735}$

#### Control of Error:

- No column should exceed 9 tiles
- Verify with Golden Beads

### Presentation 2: Dynamic Addition (With Exchanges)

**Example Problem:** 1,576 + 2,687

#### *Key Demonstration:*

1. **Units column:**  $6 + 7 = 13$ 
  - Exchange 10 units  $\rightarrow$  1 ten tile
  - Place leftover 3 units
2. **Record exchange:** Write small "1" above tens column
3. Continue through all columns

#### Verbal Cue:

"When we get 10 in any column, we exchange for 1 of the next higher category."

### Presentation 3: Static Subtraction

**Example Problem:** 5,763 - 2,431

#### *Process:*

1. Build minuend (5,763)
2. **Remove subtrahend tiles** column-by-column
3. Read remainder: "3,332"

#### Critical Point:

Always start subtracting from the **units** column

## Presentation 4: Dynamic Subtraction (With Borrowing)

Example Problem:  $4,032 - 1,857$

*Borrowing Procedure:*

1. **Units column:** Can't remove 7 from 2
  - o Borrow 1 ten → exchange for 10 units
  - o Now 12 units - 7 units
2. Continue through all columns

**Language:**

"We break down 1 hundred into 10 tens, just like in the Golden Beads."

## Presentation 5: Multiplication

Example Problem:  $2,413 \times 2$

*Method:*

1. Build multiplicand ( $2,413$ )
2. **Duplicate quantity** for multiplier
  - o "We take  $2,413$  two times"
3. Combine and exchange as needed

**Extension:**

Introduce  $\times$  symbol and recording

## Presentation 6: Division

Example Problem:  $4,864 \div 2$

*Sharing Method:*

1. Place 2 skittles (divisor) below mat
2. **Distribute tiles equally:**
  - o "4 thousands shared by 2 gives each 2 thousands"
3. Record quotient: "2,432"

## **Key Teaching Points**

**Color Coding Consistency:** Matches Golden Beads (green=units, blue=tens, etc.)

**Error Control:**

- Physical limit of 9 tiles per category
- Final answer must match control chart

**Progression:**

- Begin with static problems
- Introduce exchanges only after mastery

## **Observation Tips:**

- Does child maintain place value alignment?
- Can child verbalize the exchange process?

## **AMS Standards Alignment**

**5.2.4:** Materials maintain hierarchical structure

**7.3.1:** Child works independently after demonstration

**3.4.3:** Gradual movement toward abstraction

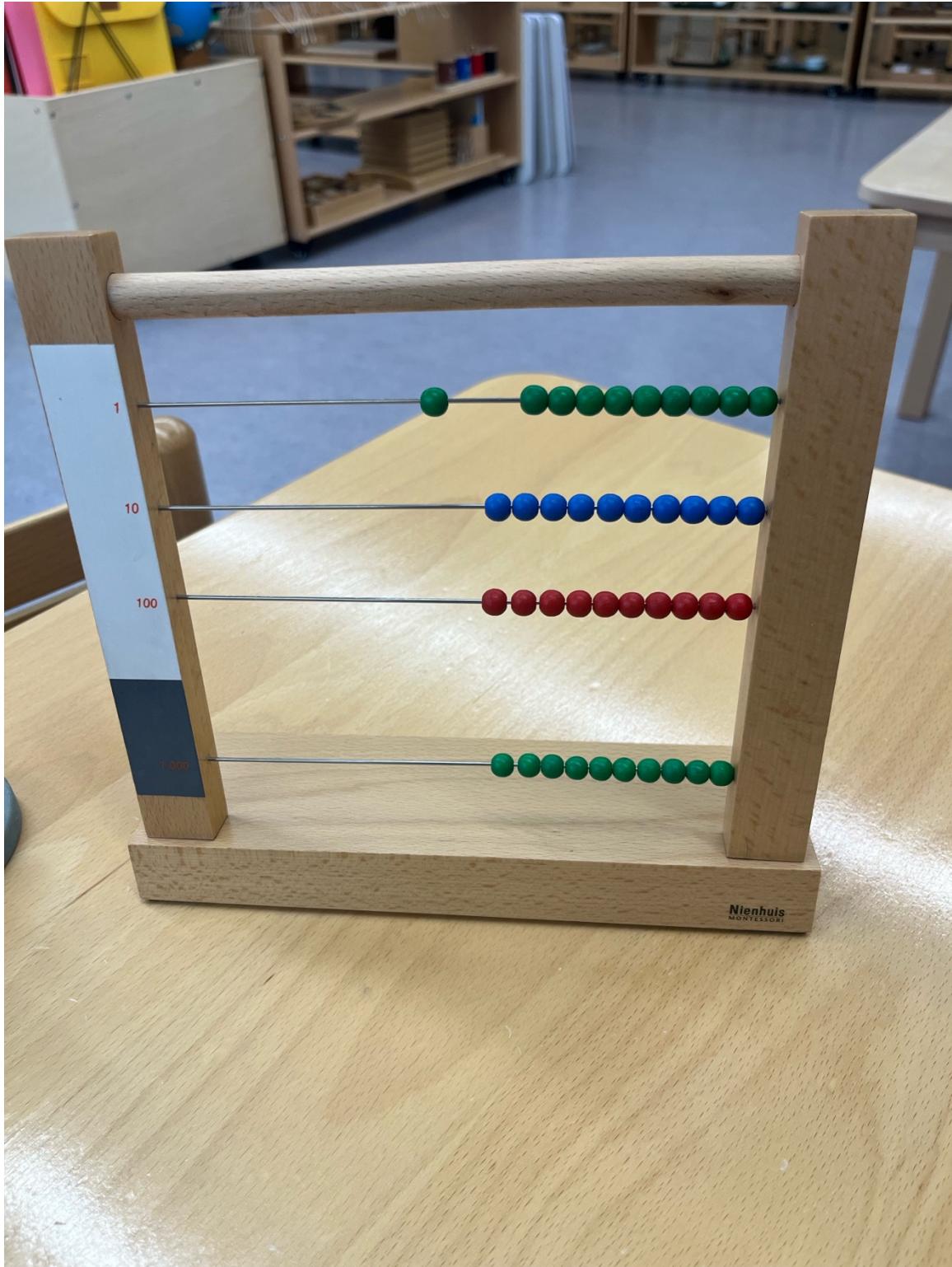
**Next Step:** Small Bead Frame (fully abstract calculation)

## **Extension Ideas:**

1. **Word Problems:** Create real-life scenarios
2. **Missing Addend:** Cover part of equation for child to solve
3. **Peer Teaching:** Child demonstrates to younger students

This structured approach ensures children internalize operations before transitioning to pure symbolic work.

## 4.3 Small Bead Frame



### *1. Invitation & Naming the Material*

- "Today, we will work with the Small Bead Frame. It helps us add and subtract large numbers quickly!"
- Show the frame and name each wire:
  - **Top wire (green):** Thousands (1,000)
  - **Second wire (blue):** Hundreds (100)
  - **Third wire (red):** Tens (10)
  - **Bottom wire (green):** Units (1)

### *2. Setting Up the Frame*

- Slide all beads to the **right** (starting position).
- Explain: "When all beads are on the right, the number is **zero**."

### *3. Representing a Number*

- Example: **2,431**
  - **Thousands:** Slide 2 beads to the left.
  - **Hundreds:** Slide 4 beads to the left.
  - **Tens:** Slide 3 beads to the left.
  - **Units:** Slide 1 bead to the left.
- Have the child read the number: "Two thousand, four hundred, thirty-one."

### *4. Resetting the Frame*

- Slide all beads back to the right.
- Let the child practice forming different numbers.

## Presentation 2: Addition on the Small Bead Frame

### 1. Simple Addition (No Exchanges)

- Example: **1,243 + 3,512 = ?**
  - Set the first number: **1,243**
  - **Add thousands:** Slide 3 more beads (total: 4).
  - **Add hundreds:** Slide 5 more beads (total: 7).
  - **Add tens:** Slide 1 more bead (total: 4).
  - **Add units:** Slide 2 more beads (total: 5).
  - Final answer: **4,755**

### 2. Addition with Exchanges (Carrying Over)

- Example: **1,573 + 2,469 = ?**
  - Set **1,573** on the frame.
  - Add **2,469:**
    - **Thousands:** +2 → 3 (no exchange).
    - **Hundreds:** +4 → 9 (no exchange).
    - **Tens:** +6 → **13** (exchange 10 tens for 1 hundred).
      - Move all tens back, add 1 to hundreds (now 10 hundreds).
      - Exchange 10 hundreds for 1 thousand (now 4 thousands).
    - **Units:** +9 → 12 (exchange 10 units for 1 ten).
  - Final answer: **4,042**

## Presentation 3: Subtraction on the Small Bead Frame

### 1. Simple Subtraction (No Borrowing)

- Example: **4,755 – 1,243 = ?**
  - Set **4,755** on the frame.
  - Subtract:
    - **Thousands:** Slide 1 bead back → 3.
    - **Hundreds:** Slide 2 beads back → 5.
    - **Tens:** Slide 4 beads back → 1.
    - **Units:** Slide 3 beads back → 2.
  - Final answer: **3,512**

### 2. Subtraction with Exchanges (Borrowing)

- Example: **4,042 – 2,469 = ?**
  - Set **4,042** on the frame.
  - Subtract:
    - **Thousands:** –2 → 2.
    - **Hundreds:** –4 → 0 (**but we need to subtract 6!**)
      - Borrow 1 thousand → 10 hundreds (now 10 hundreds).
      - Subtract 6 → 4 hundreds left.
    - **Tens:** –6 → 3 (**but we have 4 tens, so subtract 6?**)
      - Borrow 1 hundred → 10 tens (now 14 tens).
      - Subtract 6 → 8 tens left.
    - **Units:** –9 → 2 (**but we have 2 units, so borrow 1 ten → 12 units.**)
      - Subtract 9 → 3 units left.
  - Final answer: **1,573**

### Extensions & Follow-Up Work:

**Worksheet Practice** – Provide addition/subtraction problems for independent work.

**Word Problems** – "If you have 2,345 apples and buy 1,267 more, how many do you have?"

**Game:** Partner work—one child sets a number, the other solves.

### **Assessment:**

Observe if the child can:

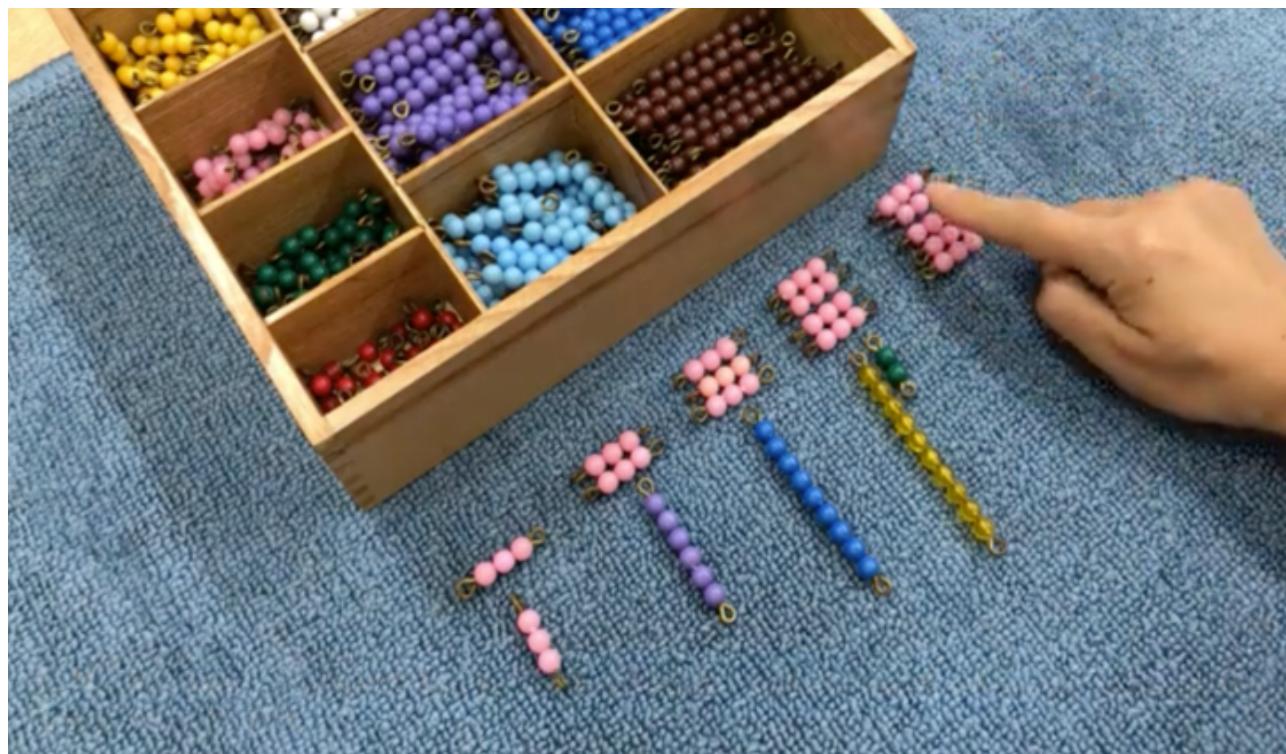
- Correctly slide beads for addition/subtraction.
- Perform exchanges (carrying/borrowing) accurately.
- Read and write the final number correctly.

### **Conclusion:**

The Small Bead Frame bridges concrete counting (Golden Beads) to abstract arithmetic, preparing children for pencil-and-paper calculations.

## **4.4 Bead Bars**

### **Multiplication & Division**



**Age Group:** 3-6 years

**Area:** Mathematics

**Materials:**

- Colored Bead Bars (1-10)
- Golden Bead Material (for reinforcement)
- Printed multiplication & division equations (optional)
- Small number cards

**Lesson Objectives:**

1. Introduce multiplication as **repeated addition** using bead bars.
2. Introduce division as **sharing equally** using bead bars.
3. Develop a concrete understanding of basic multiplication (up to  $10 \times 10$ ) and simple division.

**Prerequisites:**

Child has mastered **addition & subtraction** with bead bars.

Child can **count and recognize quantities** (1-10) with bead bars.

Child understands **one-to-one correspondence** in counting.

# Presentation 1:

## Multiplication as Repeated Addition

### 1. Introduction to Multiplication

- "Multiplication is adding the same number many times."
- Example:  $3 \times 4 = ?$ 
  - Lay out **three 4-bars (yellow)**.
  - Count:  $"4 + 4 + 4 = 12"$ \*
  - Show that  $3 \times 4 = 12$

### 2. Exploring Multiplication Families

- $5 \times 2 = ? \rightarrow$  Lay out **five 2-bars (red)** and count:  $"2 + 2 + 2 + 2 + 2 = 10"$ \*
- $4 \times 3 = ? \rightarrow$  Lay out **four 3-bars (pink)** and count:  $"3 + 3 + 3 + 3 = 12"$ \*

### 3. Recording Work (Optional)

- Child writes equations in a math journal.
- Example:

Multiplication	Bead Bars Used	Total
$2 \times 5$	Two 5-bars (light blue)	10
$3 \times 4$	Three 4-bars (yellow)	12

# Presentation 2: Division as Equal Sharing

### 1. Introduction to Division

- "Division means sharing equally."
- Example:  $8 \div 2 = ?$ 
  - Take an **8-bar (brown)** and say: *"If we share 8 between 2 friends, how many does each get?"*
  - Split into **two 4-bars (yellow)** → Each gets **4**.

## 2. Exploring Division with Remainders (Advanced)

- Example: **10 ÷ 3 = ?**
  - Take a **10-bar (gold)** and try to split into 3 equal parts.
  - Each gets **3**, and **1 remains**.
  - Explain: "*3 goes into 10 three times, with 1 left over.*"

## 3. Recording Work (Optional)

- Child writes division problems in a math journal.
- Example:

Division	Bead Bars Used	Each Gets	Remainder
----------	----------------	-----------	-----------

$6 \div 3$	One 6-bar (purple)	2	0
------------	--------------------	---	---

$7 \div 2$	One 7-bar (white)	3	1
------------	-------------------	---	---

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# Extensions & Follow-Up Work

**Bead Board Multiplication** (for structured practice)

**Word Problems** – "If 4 children have 3 apples each, how many apples are there in total?"

**Memory Game** – Match bead bars to multiplication/division equations.

**Group Work** – One child creates a multiplication problem, another solves it.

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# Assessment:

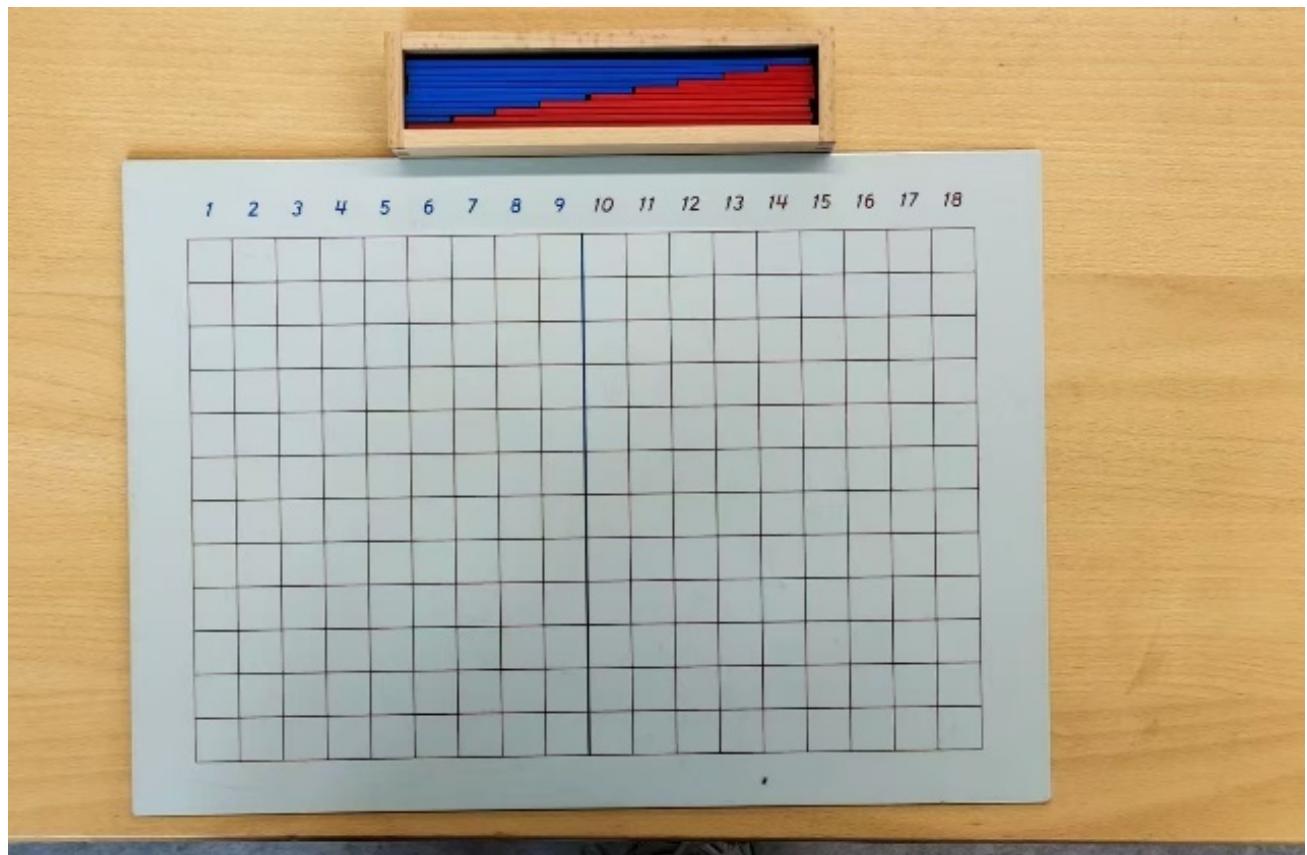
Observe if the child can:

- Correctly lay out bead bars for multiplication/division.
- Verbally explain the concept ("Multiplication is adding the same number again and again").
- Solve simple word problems using bead bars.



## 5.1: Addition & Subtraction Strip Board

(Ages 5-6 | Prerequisites: Snake Game, Number Rods)



## A. Materials

- **Blue wooden board** with 12 rows and 18 columns
- **Red wooden strips** (1-9)
- **Blue wooden strips** (1-9)
- **Control Chart #1** (pre-filled sums)
- **Equation slips** (e.g.,  $5 + 3 = \underline{\hspace{1cm}}$ )
- **Pencil and paper**

## B. Direct Purposes

1. Memorize **essential addition combinations**
2. Discover **commutative property** ( $3+5 = 5+3$ )
3. Transition from **manipulative to visual calculation**

## C. Presentation

### 1. First Problem (Demonstration)

1. Select equation: \*"Let's solve  $4 + 5$ ."\*
2. Place **blue 4-strip** vertically along left edge
3. Place **red 5-strip** horizontally across top
4. Trace intersection: \*" $4 + 5 = 9$ "\* (verify with Control Chart)

### 2. Child's Turn

1. Child selects equation slip (e.g.,  $7 + 2$ )
2. Repeats process independently
3. Records answers on paper:
3.  $7$
4.  $+2$
5.  $\underline{\hspace{1cm}}$
6.  $9$

### 3. Three-Period Reinforcement

**Period 1:** \*"This shows  $6 + 3 = 9$ ."\*

**Period 2:** \*"Find where  $5 + 4$  is."\*

**Period 3:** \*"What is  $8 + 1$ ?"\*

## D. Control of Error

### ✓ Visual:

- Control Chart verifies all sums
- Strip colors guide proper alignment

### ✓ Kinesthetic:

- Strips fit only in correct positions
- Finger tracing confirms intersections

### ✓ Mathematical:

- Maximum sum is 18 (board limits)

## E. Extensions

### 1. "Hidden Sum" Game

- Cover control chart numbers with sticky notes

### 2. Speed Challenge

- Time how many problems child solves in 3 minutes

### 3. Equation Writing

- Create original problems for friends to solve

## F. Teacher Notes

### Key Language

- Always phrase as: "   plus    equals   "
- Highlight patterns: \*"Notice how 3+4 and 4+3 both make 7!"\*

### Observation Signs

- Consistently matches strips to correct sum

- Begins recalling facts without strips
- Self-corrects using control chart

## Link to Prior Work

- Applies **Snake Game's** "making 10" strategy
- Uses same color coding as **Number Rods**

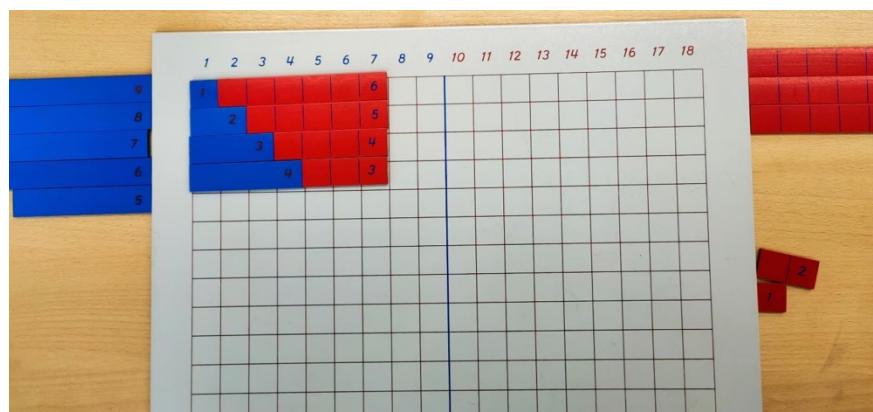
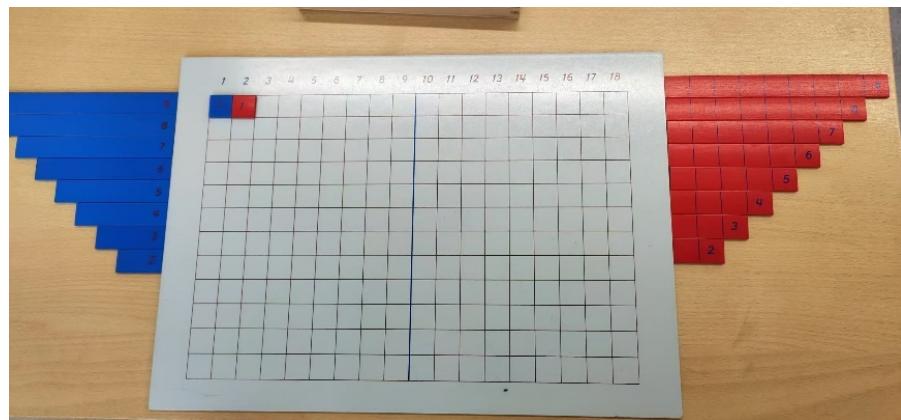
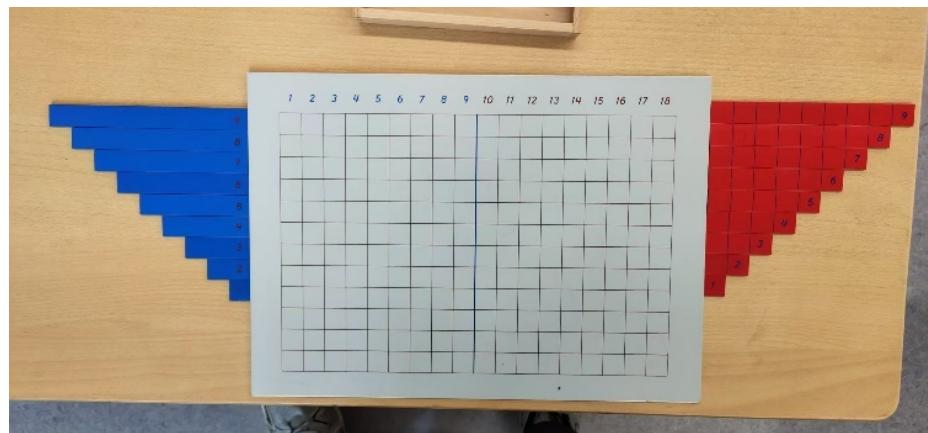
## Preparation for Next Steps

- Leads to **Subtraction Strip Board**
- Prepares for **Multiplication Bead Board**

## G. Common Mistakes

Issue	Solution
Misaligns strips <i>"Blue goes down, red goes across"</i>	
Confuses 6/9 <i>"The circle closes at the bottom for 6"</i>	
Skips recording <i>"Let's write what your hands showed"</i>	

*"The strips are the child's last physical bridge before pure mental arithmetic."*



## 5.2 Subtraction Strip Board

(Ages 5-6 | Prerequisites: Addition Strip Board, Number Rods)



## A. Materials

- **White wooden board** with 18 columns and 9 rows
- **Blue strips** (1-9, labeled with minuends 9-18)
- **Red strips** (1-9, for subtrahends)
- **Control Chart** (pre-filled differences)
- **Equation slips** (e.g.,  $12 - 5 = \underline{\hspace{1cm}}$ )
- **Gray blank strips** (for remainders)

## B. Direct Purposes

1. Master **subtraction as inverse addition**
2. Develop **mental calculation** strategies
3. Reinforce **number relationships** (minuend - subtrahend = difference)

## C. Presentation

### 1. First Problem (Demonstration)

1. Select equation: \*"Let's solve  $14 - 6$ ."\*
2. Place **blue 14-strip** vertically (left edge)
3. Slide **red 6-strip** under 14 until it stops:
  - "Six subtracted from fourteen leaves..."
4. Place **gray strip** to measure remainder: \*" $14 - 6 = 8$ "\*

### 2. Child's Independent Work

1. Child selects equation slip (e.g.,  $16 - 9$ )
2. Repeats process, verifies with control chart
3. Records answers:
3.    16
4.    - 9
5.    

---
6.    7

### **3. Three-Period Reinforcement**

**Period 1:** \*"This shows  $15 - 7 = 8$ ."\*

**Period 2:** \*"Find where  $17 - 9$  is."\*

**Period 3:** \*"What is  $12 - 4$ ?"\*

---

#### **D. Control of Error**

##### **Physical:**

- Red strip cannot slide beyond minuend length
- Gray strip exactly fills the gap

##### **Visual:**

- Control chart confirms all answers
- Color coding matches addition board (red=subtraction)

##### **Mathematical:**

- Remainder always  $\leq 9$  (limited by board design)

#### **E. Extensions**

##### **1. "Missing Minuend" Game**

- Provide difference and subtrahend (e.g.,  $\underline{\quad} - 5 = 4$ )

##### **2. Timed Challenges**

- Solve 5 problems before sand timer runs out

##### **3. Story Problems**

- *"You have 17 apples and eat 8. How many remain?"*

#### **F. Teacher Notes**

##### **Key Language**

- Use precise terms: "*minuend*," "*subtrahend*," "*difference*"
- Connect to addition: \*"If  $11 - 6 = 5$ , then  $5 + 6 = \dots$ !"\*

## Observation Signs

- Notices inverse relationships independently
- Uses gray strips without prompting
- Self-corrects using control chart

## Link to Prior Work

- Applies **Addition Strip Board's** color coding
- Reverses **Snake Game's** making-10 process

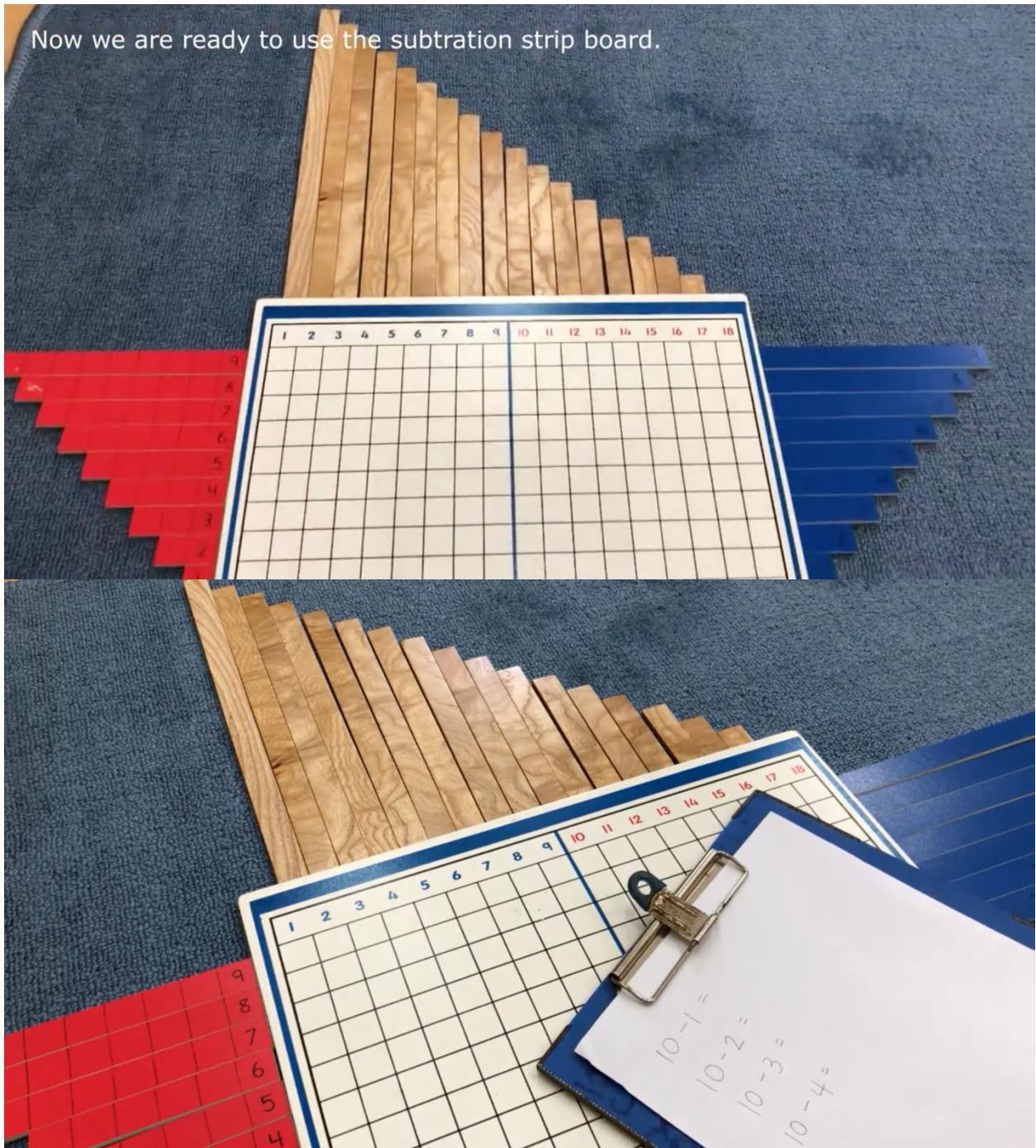
## Next Steps

- Leads to **Multiplication Bead Board**
- Prepares for **Division with Racks & Tubes**

## G. Common Errors

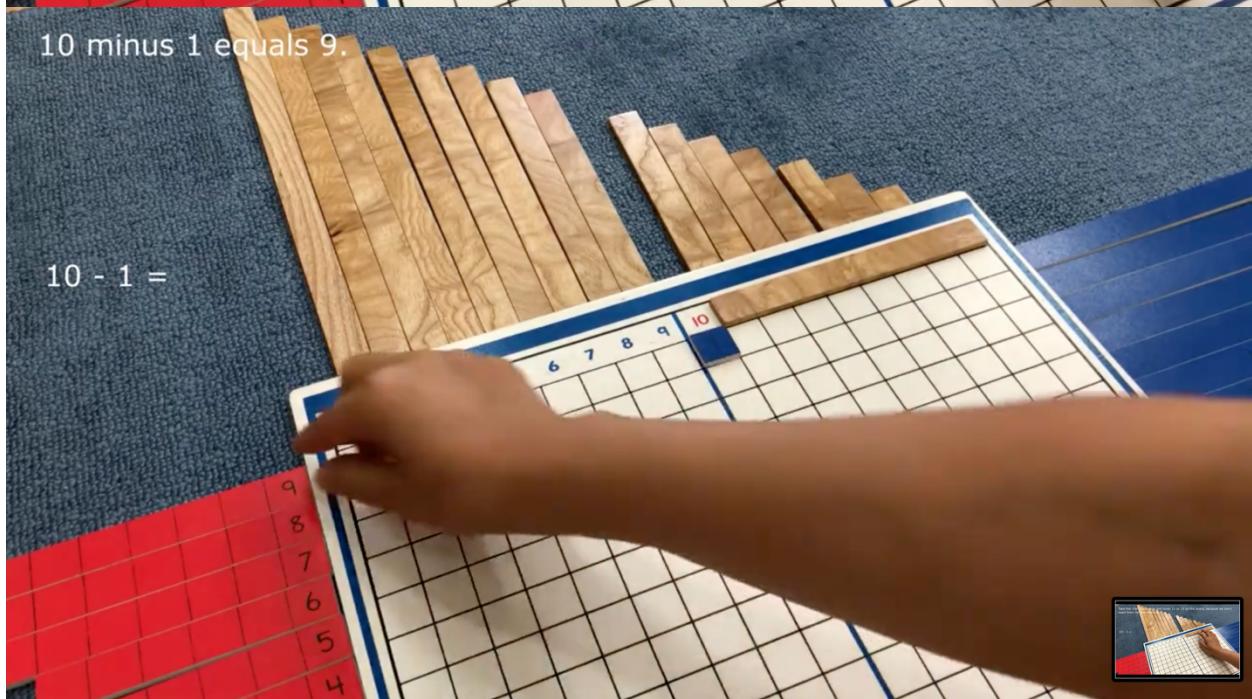
Issue	Solution
Misaligns red strip	<i>"Start at the minuend's top edge"</i>
Confuses minuend/subtrahend	<i>"The big number goes on the left"</i>
Forgets remainder	<i>"What's left after taking away?"</i> (Point to gap)

Now we are ready to use the subtraction strip board.



Take the 10th blank strip and cover 11 to 18 on the board, because we don't need them for this problem.

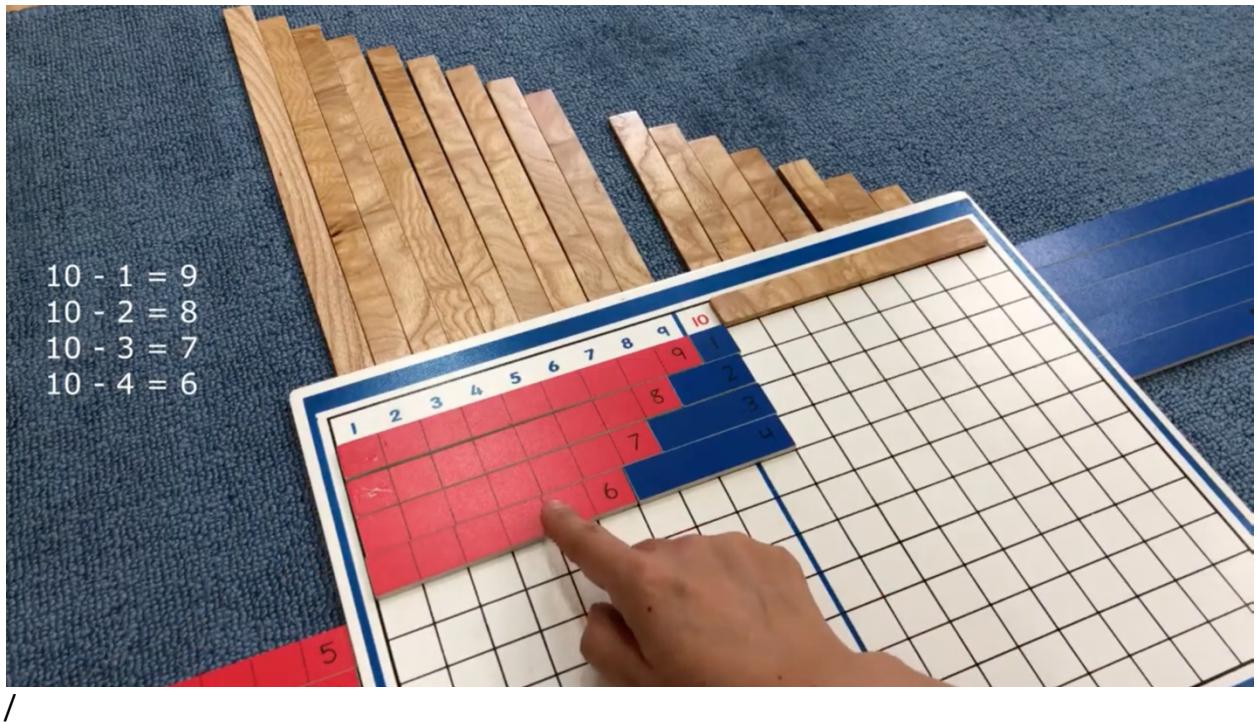
$$10 - 1 =$$





$$\begin{aligned}10 - 1 &= 9 \\10 - 2 &= 8 \\10 - 3 &= \end{aligned}$$

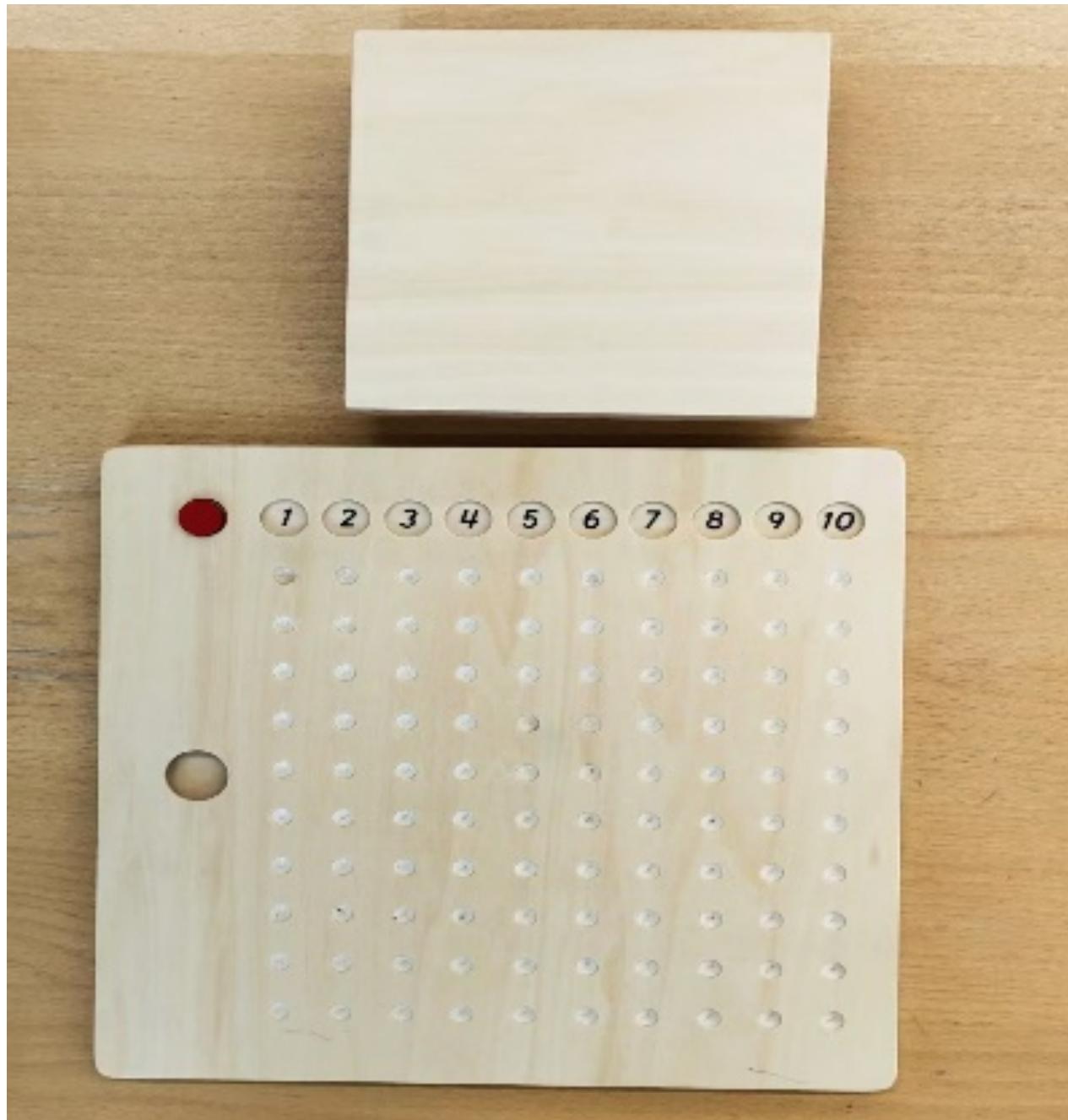




/

## 5.3 Multiplication Bead Board

(Ages 5.5-6.5 | Prerequisites: Addition & Subtraction Strip Boards, Skip Counting with Bead Chains)



### A. Materials

- ✓ Perforated wooden board (10x10 grid)
- ✓ 100 red beads in a box

- ✓ **White number tiles** 1-10 (for multipliers)
- ✓ **Red counter** (to mark rows)
- ✓ **Control chart** (multiplication table)

## B. Direct Purposes

1. Discover **multiplication as repeated addition**
2. Memorize **essential math facts** through patterns
3. Prepare for **long multiplication**

## C. Presentation

### 1. First Problem (e.g., 4 x 3)

1. Place "4" tile at board top
2. Insert 4 beads in 1st row: "*4 one time is 4.*"
3. Add 4 beads to 2nd row: "*4 two times is 8.*"
4. Add 4 beads to 3rd row: "*4 three times is 12.*"
5. Slide counter to 3: "*4 multiplied by 3 equals 12.*"

### 2. Child's Independent Work

1. Child selects equation (e.g.,  $6 \times 4$ )
2. Builds vertically while verbalizing:  
\*" $6 + 6 = 12, +6 = 18, +6 = 24$ "\*
3. Verifies with control chart

### 3. Three-Period Reinforcement

**Period 1:** \*"*This shows  $5 \times 2 = 10$ .*"\*

**Period 2:** "*Make 7 multiplied by 3.*"

**Period 3:** "*What is  $8 \times 2$ ?*"

## D. Control of Error

### ✓ Visual:

- Beads form perfect rectangles ( $4 \times 3 = 12$  beads)
- Control chart confirms answers

✓ **Kinesthetic:**

- Only 100 beads provided (prevents overflow)
- Counter physically limits multipliers to 10

✓ **Mathematical:**

- Consistent patterns (e.g., 5s end in 5/0)

## E. Extensions

### 1. "Bead Graphing"

- Lay beads horizontally to show commutativity ( $3 \times 4$  vs.  $4 \times 3$ )

### 2. Equation Writing

- Record problems and answers in multiplication booklet

### 3. Dice Game

- Roll dice for random multipliers

## F. Teacher Notes

### Key Language

- Use all three phrasings interchangeably:  
\*"4 three times" / "4 multiplied by 3" / "4 x 3"\*

### Observation Signs

- Begins predicting answers after 2nd row
- Notices patterns (e.g., all 9s sums digits = 9)
- Uses control chart only for verification

### Link to Prior Work

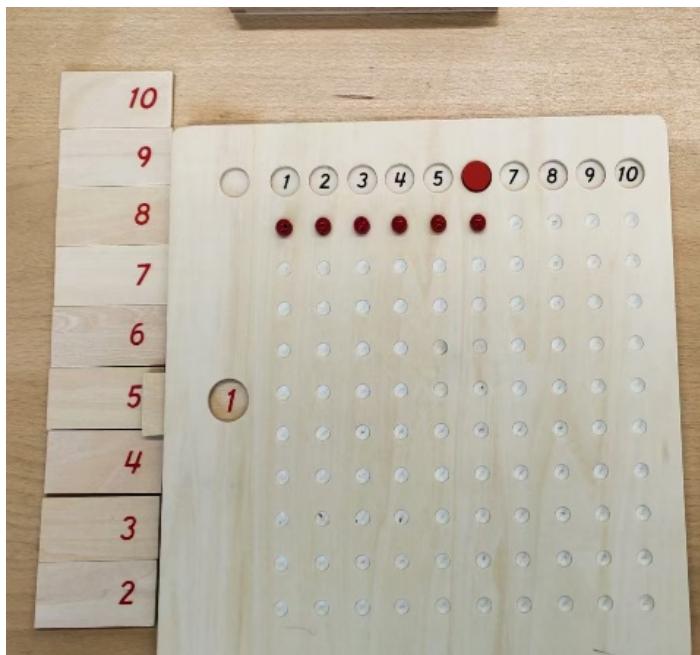
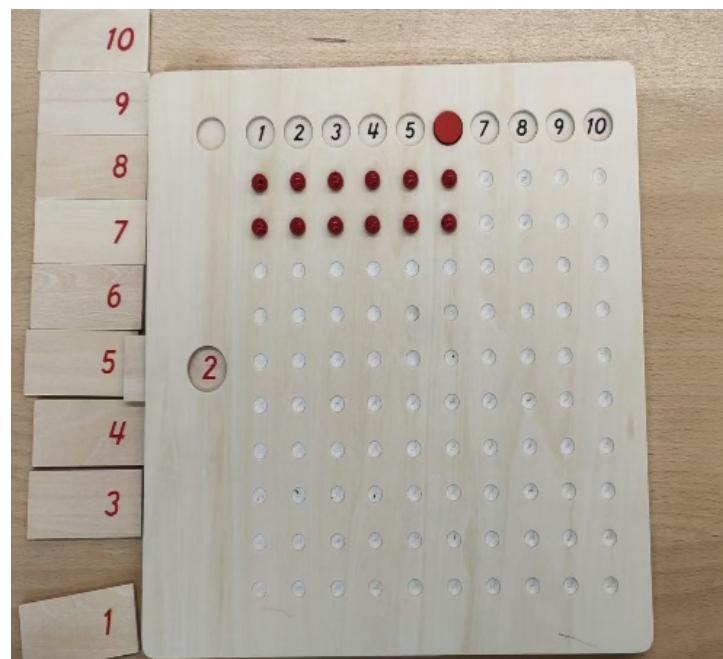
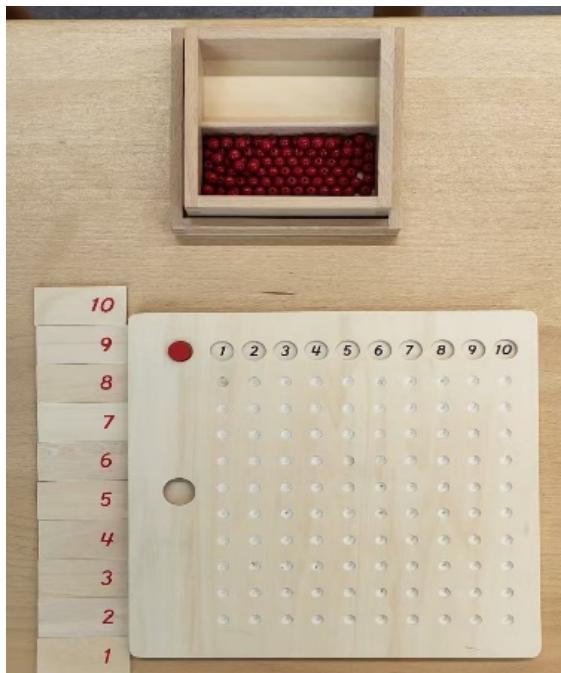
- Applies **Skip Counting** from bead chains
- Uses **Addition Strip Board's** color coding

## Next Steps

- Leads to **Stamp Game Multiplication**
- Prepares for **Decanomial Bead Box**

## G. Common Errors

Issue	Solution
Miscounts rows	<i>"Move the counter after each row"</i>
Confuses 6x7/7x6	<i>"Rotate the board to see it both ways"</i>
Loses place	<i>"Point to each bead as you count"</i>



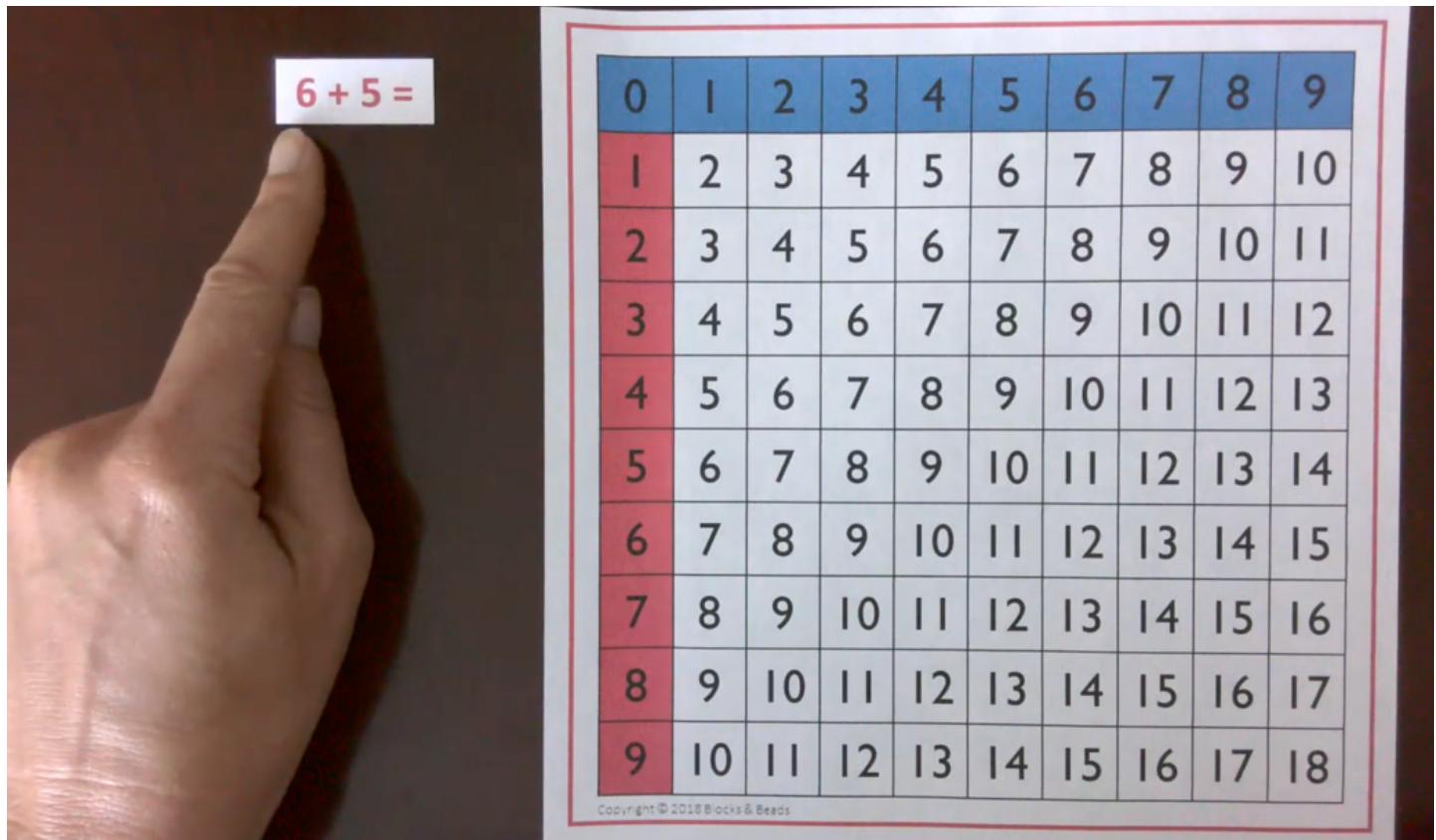
**MULTIPLICATION TABLE OF  
SIX**

with the natural series of numbers from 1 to 10

---

$6 \times 1 =$	<u>6</u>
$6 \times 2 =$	<u>12</u>
$6 \times 3 =$	<u>18</u>
$6 \times 4 =$	_____
$6 \times 5 =$	_____
$6 \times 6 =$	_____
$6 \times 7 =$	_____
$6 \times 8 =$	_____
$6 \times 9 =$	_____
$6 \times 10 =$	_____

## 5.4 Addition Finger Chart



**Materials:** Addition Finger Chart 1 (full grid with all sums filled in)

**Presentation:**

- Introduce: "*This chart helps us remember addition facts quickly.*"
- Demonstrate:
  - Choose an equation (e.g., **4 + 5**).
  - Place left index finger on **4** (left column), right index finger on **5** (top row).
  - Slide fingers to meet at **9**.
- Child practices with equations like **3 + 6, 7 + 2**.

## 2. Addition Finger Chart 2 (Blank for Active Recall)

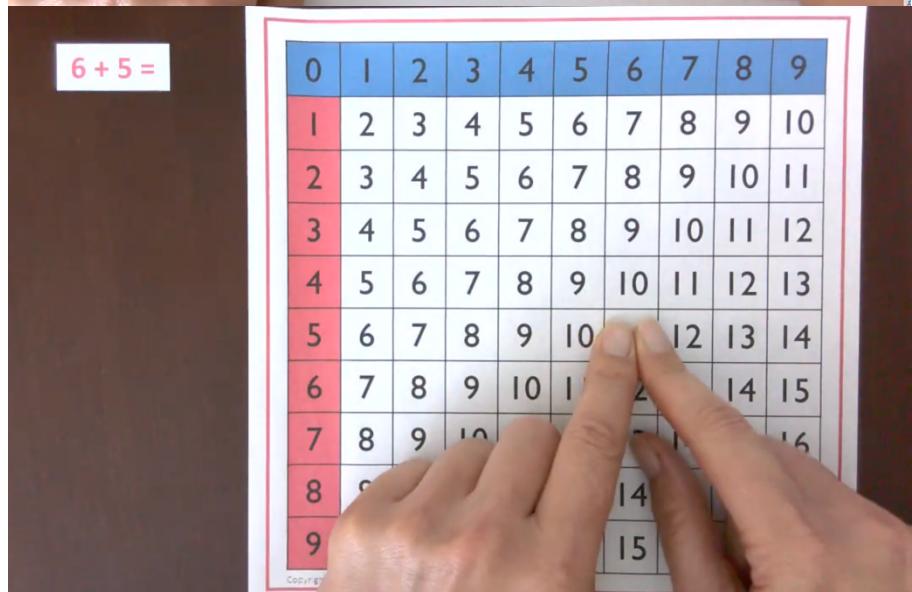
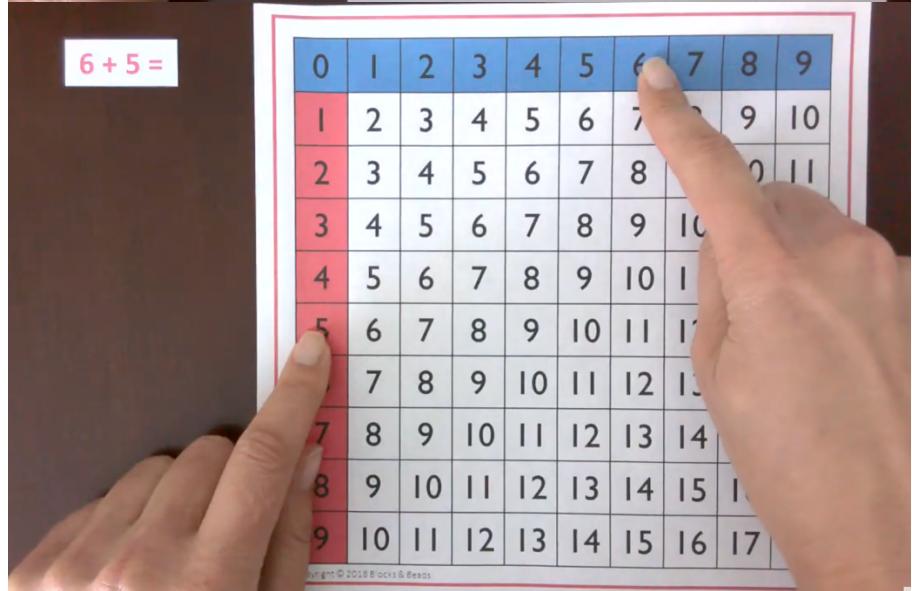
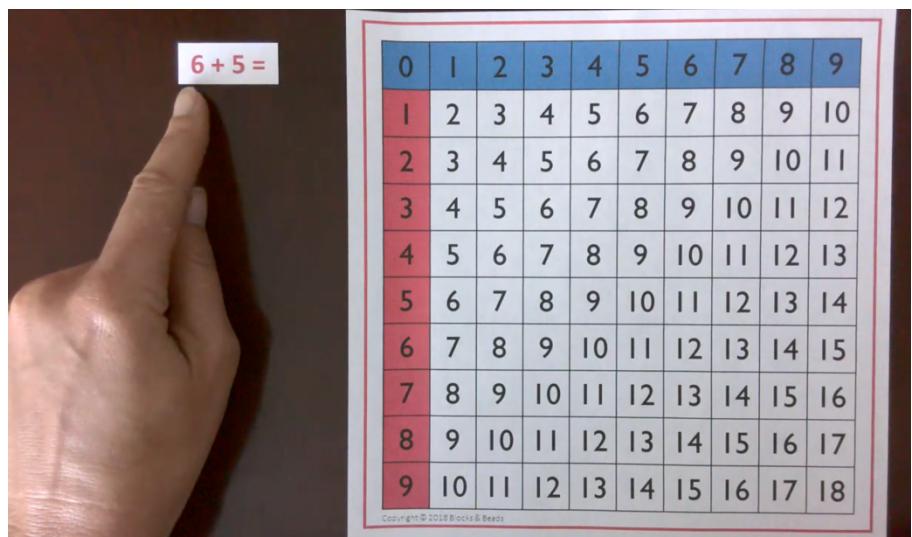
**Materials:** Addition Finger Chart 2 (blank grid with only headers)

**Presentation:**

- "*Now, let's test our memory!*"
- Child selects an equation (e.g., **8 + 3**), writes the sum in the correct cell.
- **Control Chart** is used to check answers.

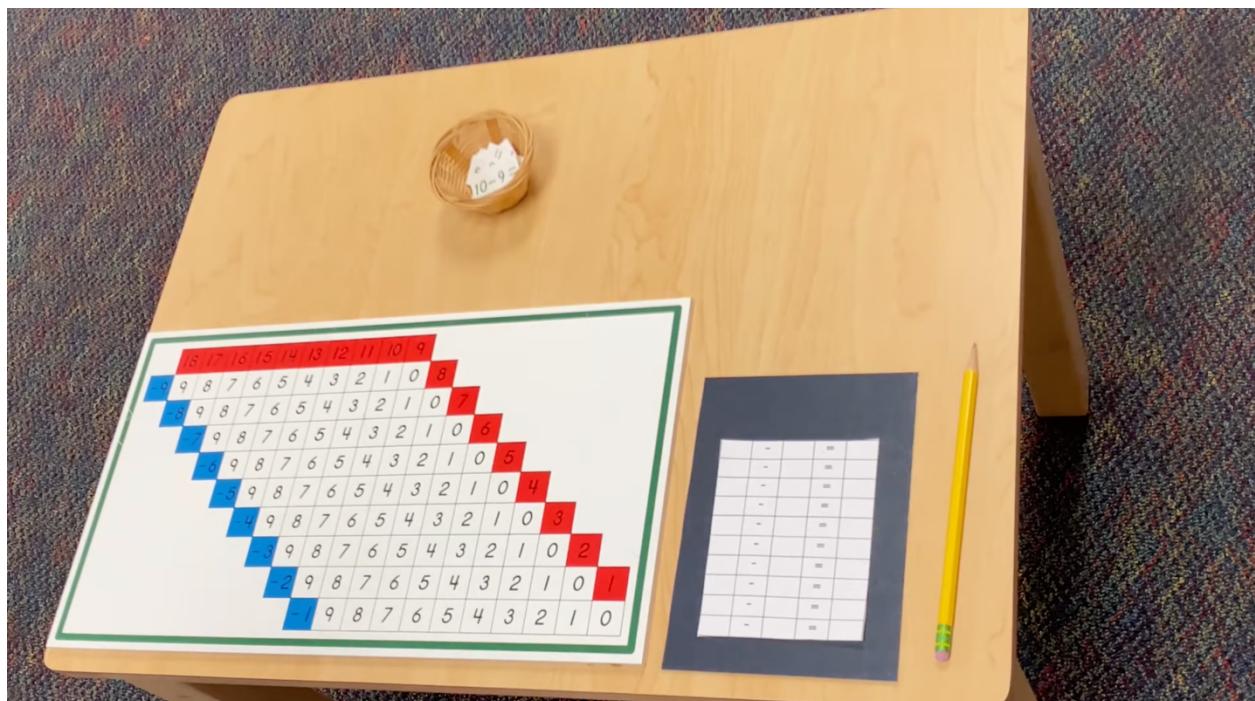
**Follow-Up:**

- Timed drills (optional for advanced learners).
- Game: "Race the Clock" (solve 5 equations in 1 minute).



## 5.5 Subtraction Finger Charts

### 1. Subtraction Finger Chart 1 (Sequential Order)



**Materials:** Subtraction Finger Chart 1 (full grid)

**Presentation:**

- "Just like addition, subtraction has patterns to memorize."
- Demonstrate:
  - Equation:  $9 - 4$
  - Left finger on **9** (left column), right finger on **4** (top row).
  - Slide to meet at **5**.
- Child practices  $7 - 2$ ,  $10 - 6$ .

## 2. Subtraction Finger Chart 2 (Blank for Active Recall)

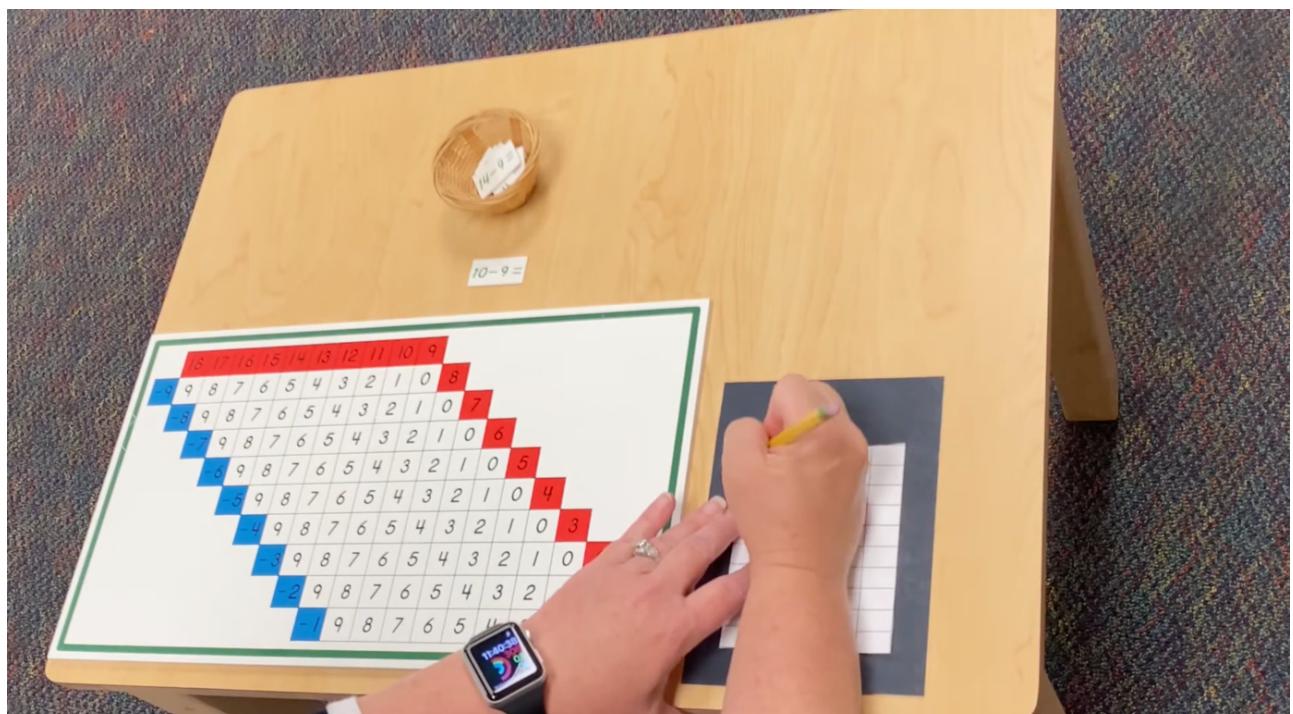
**Materials:** Subtraction Finger Chart 2 (blank grid)

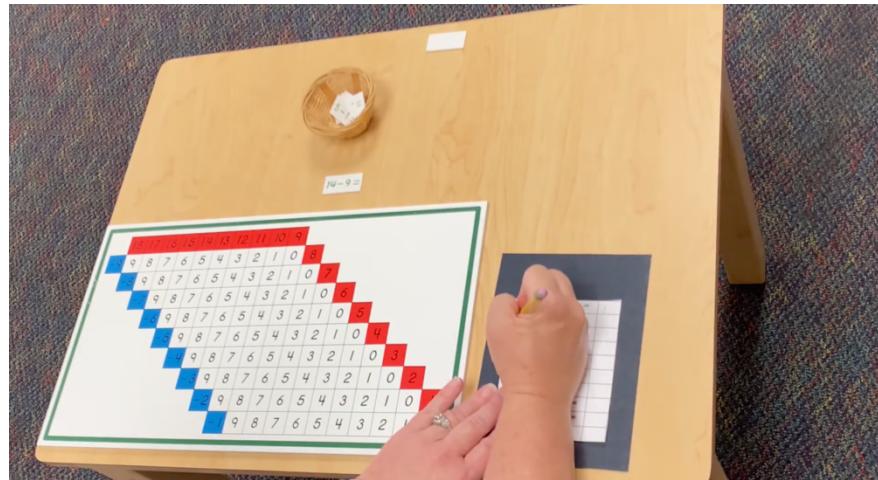
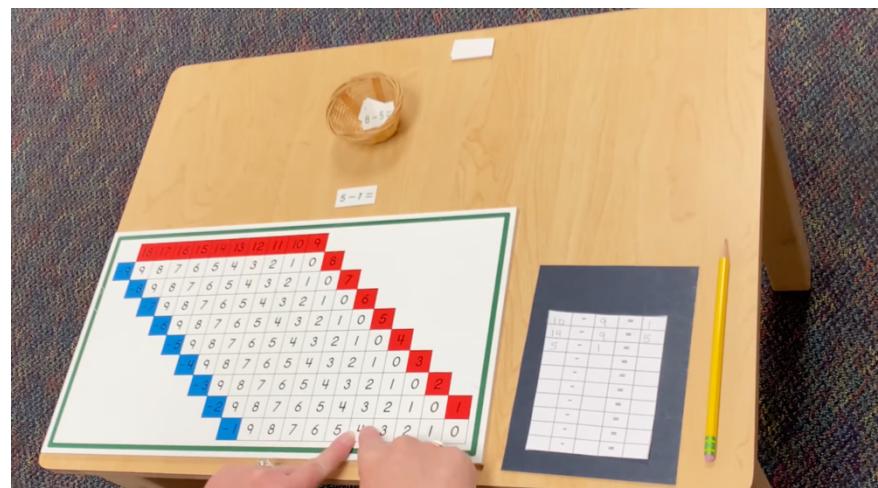
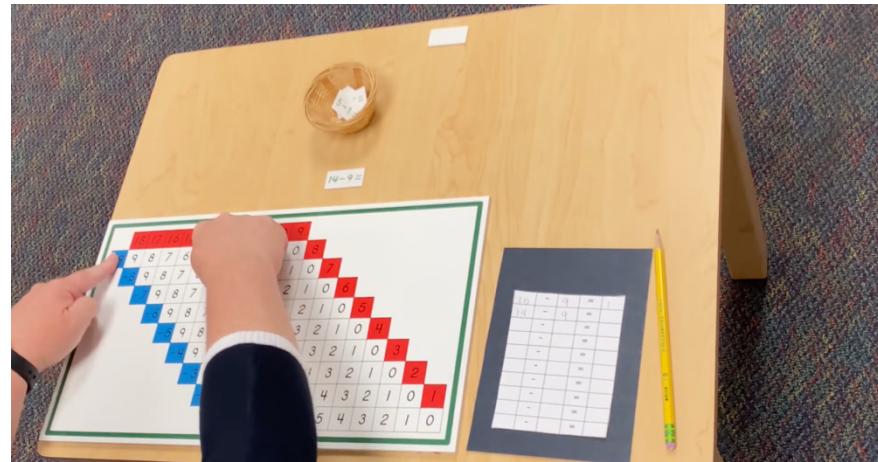
**Presentation:**

- Child fills in missing differences (e.g.,  $8 - 5 = ?$ ).
- Uses **Control Chart** to self-correct.

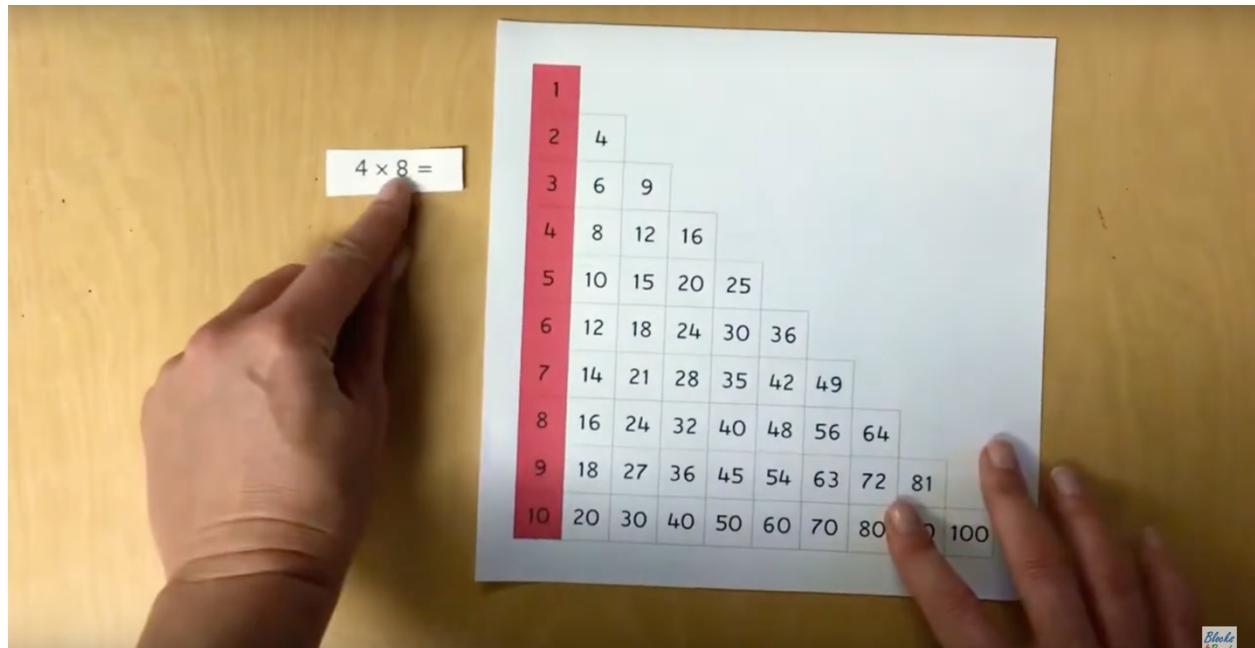
**Follow-Up:**

- Word problems: "If you had 10 cookies and ate 3, how many are left?"





## 5.6 Multiplication Finger Charts



**Materials:** Multiplication Finger Chart 1 (full grid)

**Presentation:**

- *"Let's find patterns in multiplication!"*
- Demonstrate:
  - Equation: **6 × 4**
  - Left finger on **6** (left column), right finger on **4** (top row).
  - Slide to meet at **24**.
- Child practices **3 × 7, 5 × 9**.

## 2. Multiplication Finger Chart 2 (Blank for Active Recall)

**Materials:** Multiplication Finger Chart 2 (blank grid)

**Presentation:**

- Child writes products from memory (e.g., **7 × 8 = 56**).
- Checks work with **Control Chart**.

**Follow-Up:**

- Skip-counting reinforcement (e.g., chant "6, 12, 18, 24...").
- Game: "Multiplication Bingo."

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Assessment & Differentiation:

✓ **Observation:** Can the child locate answers without counting?

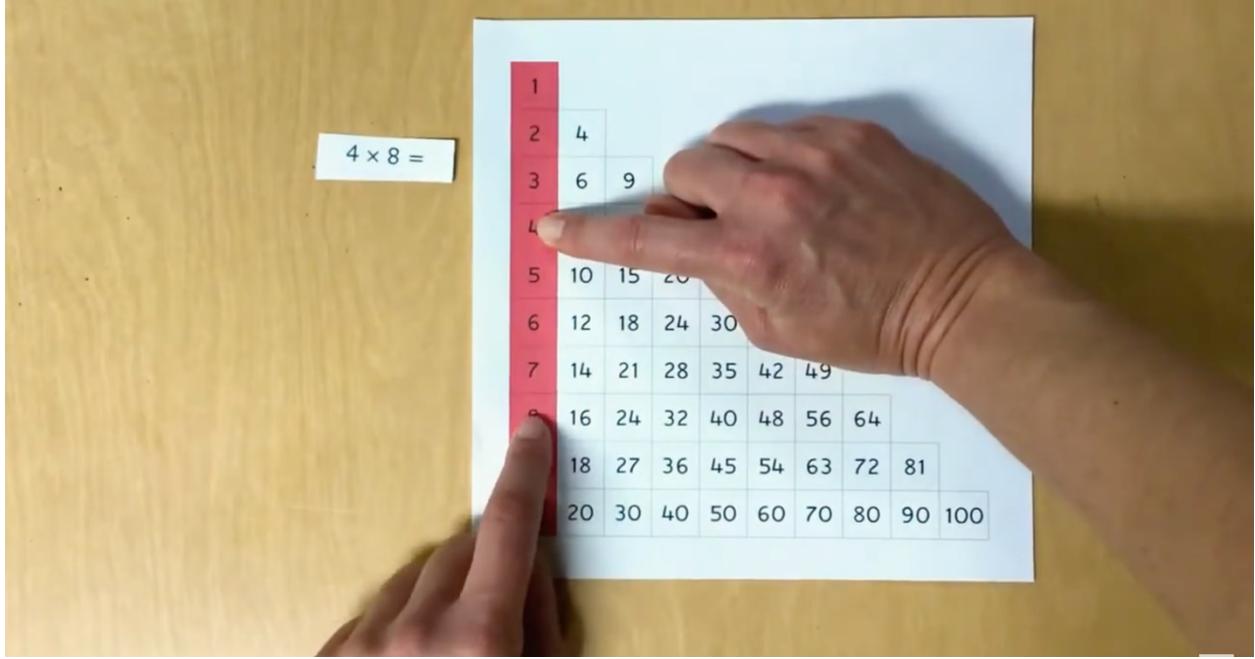
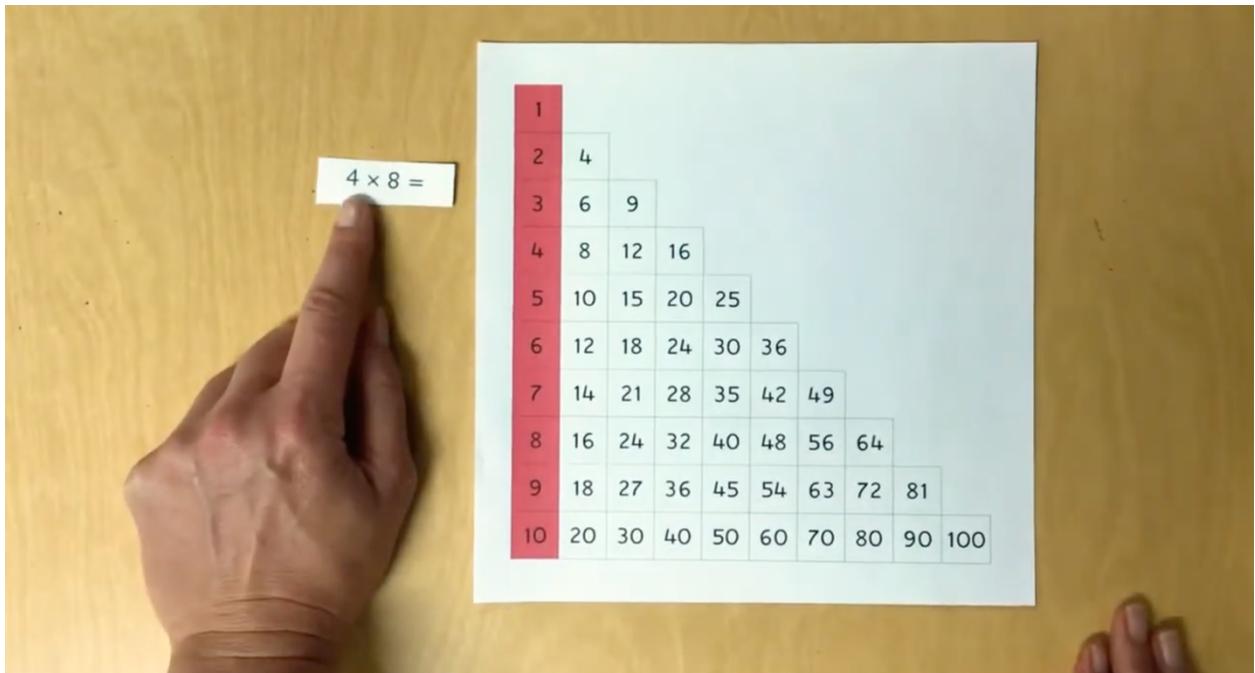
✓ **Advanced:** Mix operations (e.g., solve  $4 + 5$ , then  $9 - 4$ ).

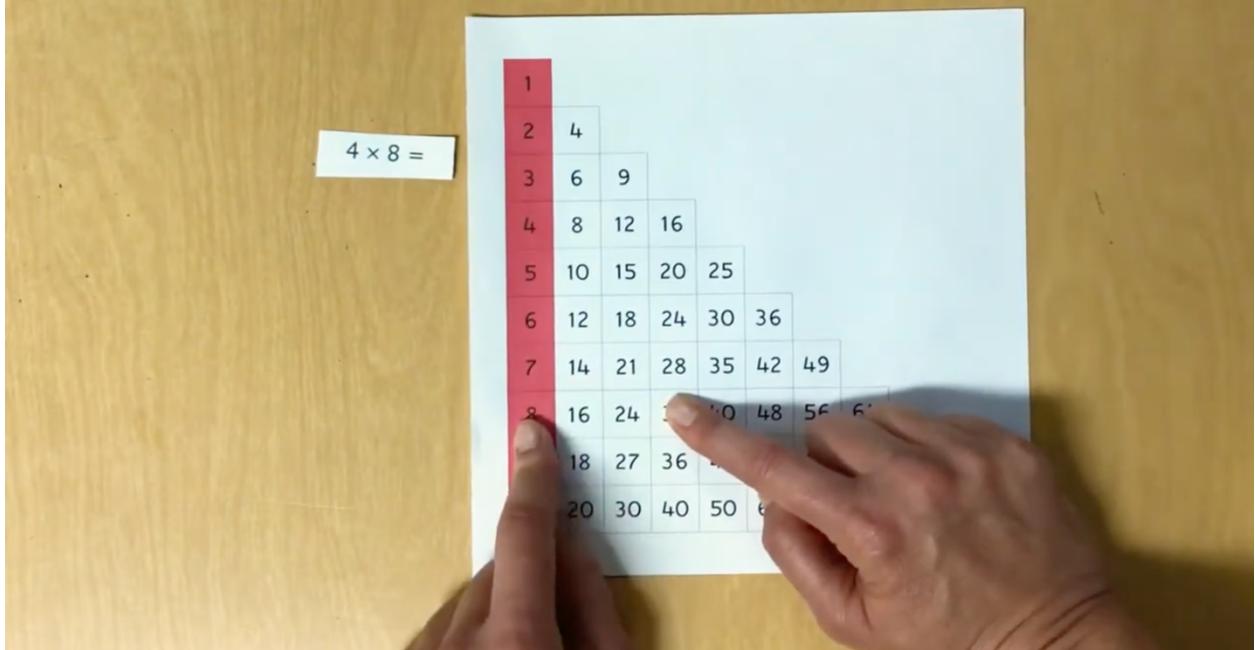
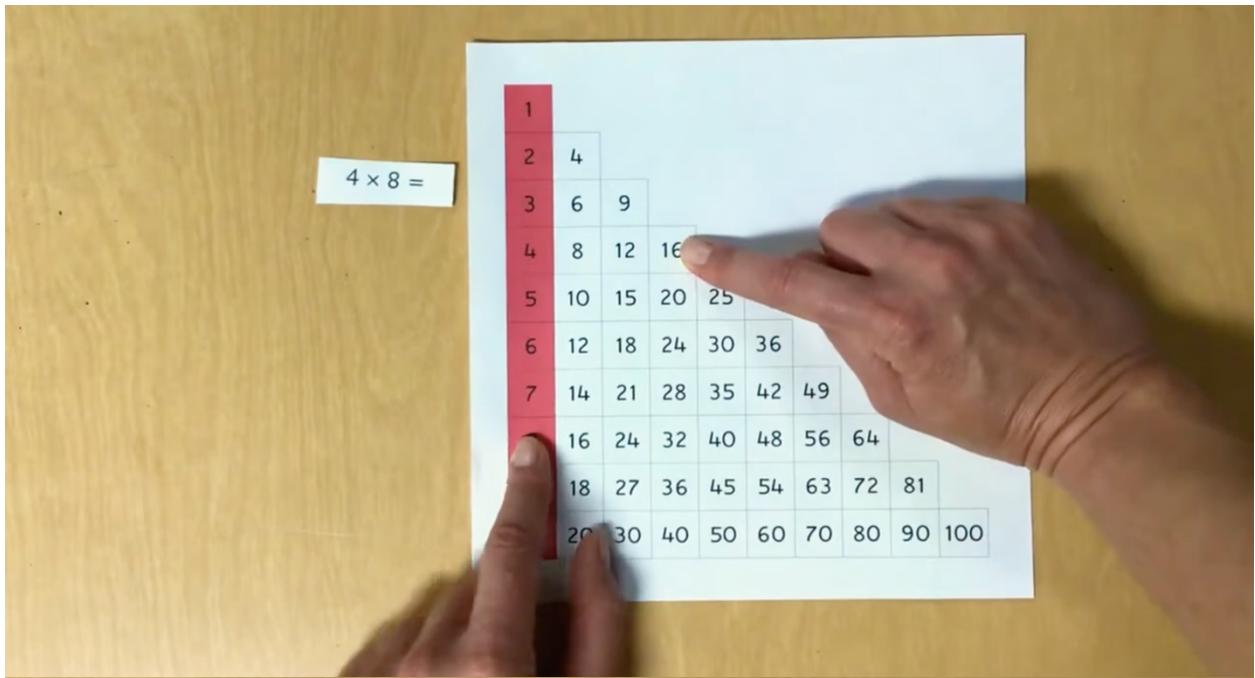
✓ **Support:** Use bead bars alongside charts for struggling learners.

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Conclusion:

Finger Charts solidify **math fact fluency** through visual-motor repetition. Over time, children rely less on the chart and recall facts automatically.





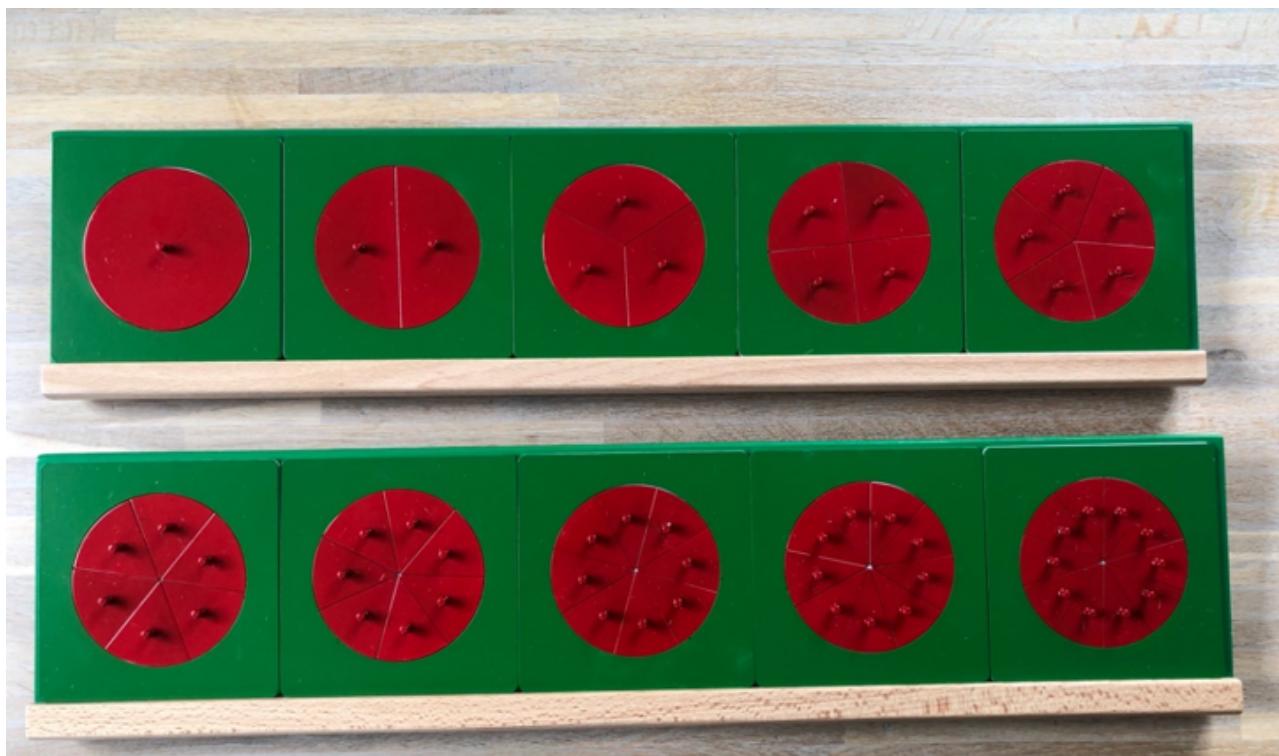
$$4 \times 8 =$$

1				
2	4			
3	6	9		
4	8	12	16	
5	10	15	20	25
6	12	18	24	30
7	14	21	28	35
8	16	24	32	40
9		36	45	54
0	40	50		



## 6.1 Fraction Insets

(Ages 5.5-6.5 | Prerequisites: Division with Racks & Tubes, Strong Number Sense 1-10)



## A. Materials

- **Metal fraction insets** (1-10 denominators, red)
- **Fraction puzzle frames** (green)
- **Label cards** (e.g.,  $1/2$ ,  $1/3$ )
- **Small paper circles** for drawing
- **Tray with compartments**

## B. Direct Purposes

1. Introduce **fractions as equal parts**
2. Develop **sensory memory** of fractional relationships
3. Prepare for **operations with fractions**

## C. Presentation

### 1. Naming Fractions (1-5 denominators)

1. Remove  $1/2$  piece: "*This is one half.*"
2. Trace circumference: "*One of two equal parts.*"
3. Repeat with  $1/3$ ,  $1/4$

### 2. Three-Period Lesson

**Period 1:** "*This is one third. This is one fourth.*"

**Period 2:** "*Show me one fifth. Where is one half?*"

**Period 3:** "*What is this?*" (Point to  $1/4$ )

### 3. Drawing Fractions

1. Child traces inset on paper
2. Colors one part: "*I colored one third.*"

## D. Control of Error

✓ **Visual:**

- Pieces fit only in correct frames
- No gaps when insets are complete

✓ **Tactile:**

- Irregular pieces feel unbalanced

## E. Extensions

### 1. "Find the Whole"

- Give child 1/4 piece: "*What fraction completes me?*"

### 2. Equivalent Fractions

- Compare 1/2 circle to two 1/4 pieces

### 3. Real-World Fractions

- Cut fruit into halves/quarters

## F. Teacher Notes

### Key Language

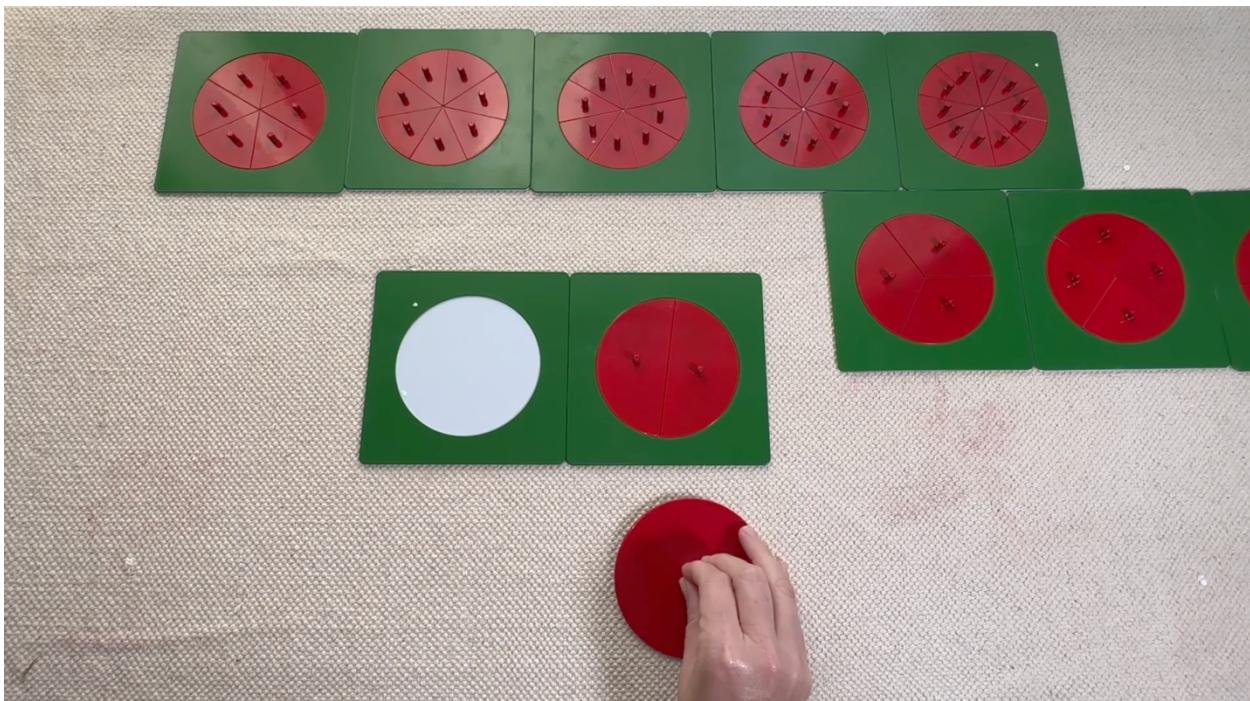
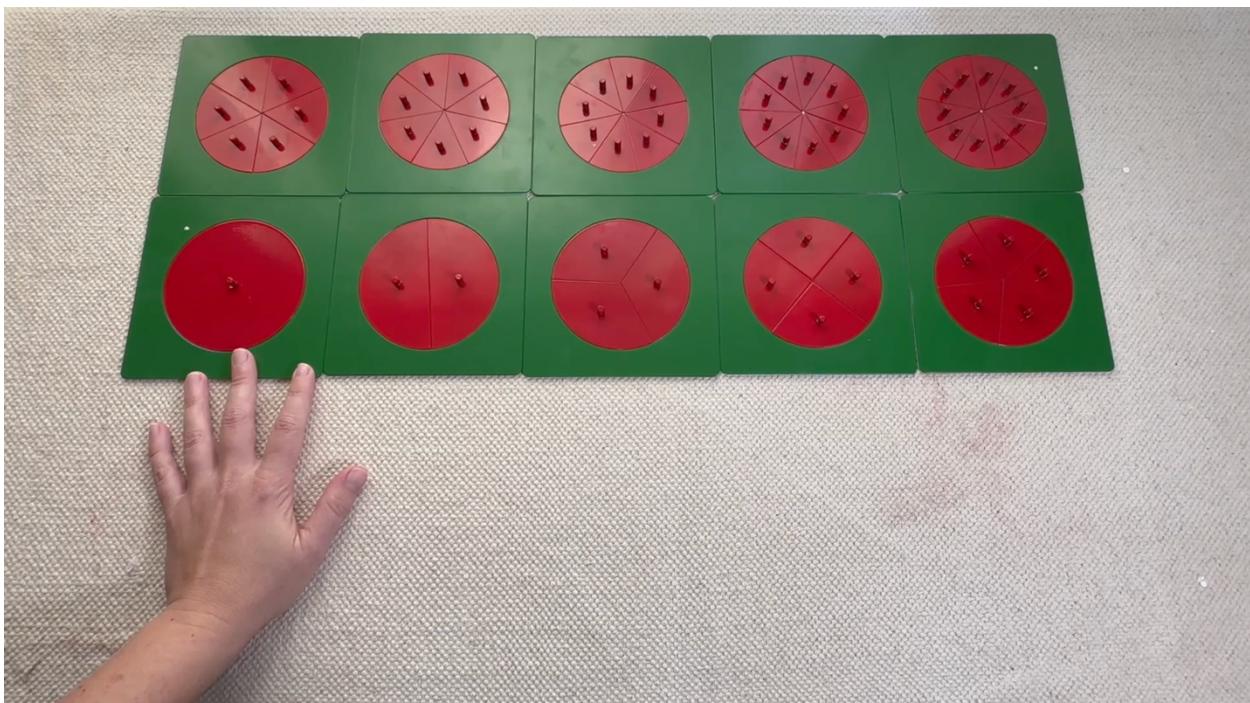
- Always use "*equal parts*"
- "*The denominator tells how many total parts.*"

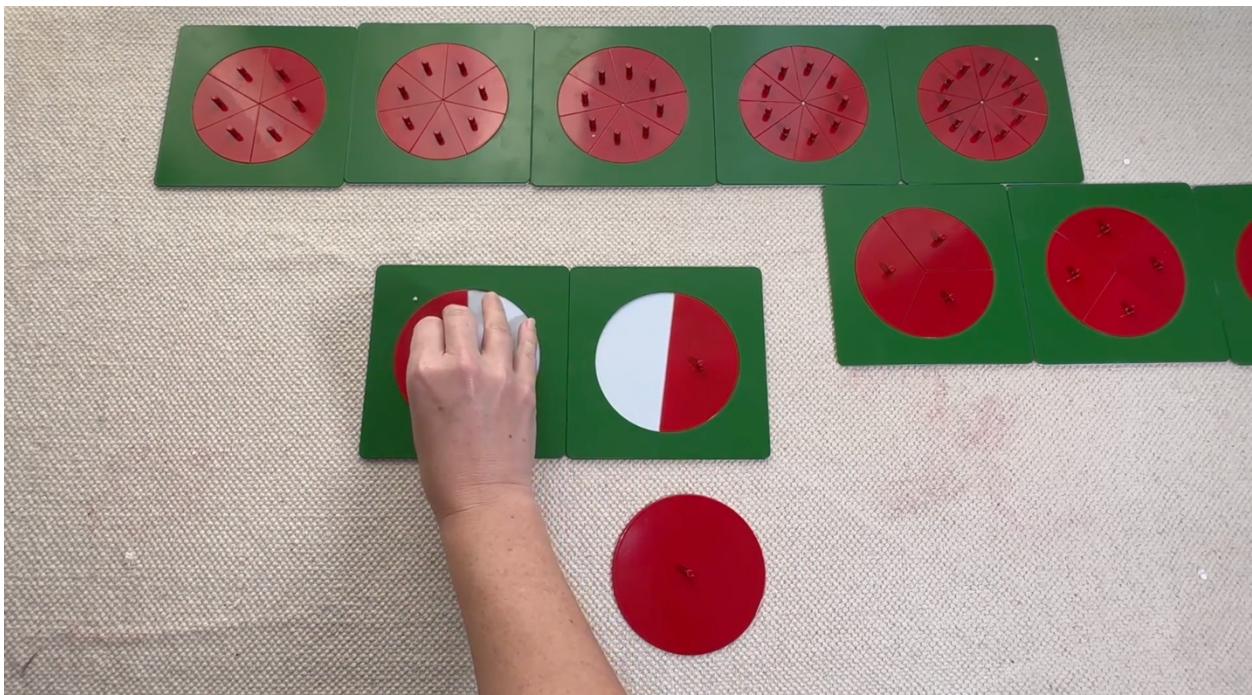
### Observation Signs

- Correctly orients irregular pieces (thirds, fifths)
- Spontaneously compares sizes

### Quote:

*"The child's hand discovers fractions before their mind computes them."*





## 6.2 Fraction Skittles

(Ages 5.5-6.5 | Prerequisites: Fraction Insets, Addition Strip Board)



### A. Materials

- **Four wooden skittles** (whole, halves, thirds, fourths) with removable segments
- **Tray** with compartments for segments
- **Fraction labels** (1, 1/2, 1/3, 1/4)
- **Small container** for "remainders"

### B. Direct Purposes

1. Introduce **fractional operations** (addition/subtraction)
2. Develop **concept of equivalency**
3. Prepare for **abstract fraction work**

### C. Presentation

#### 1. Adding Like Fractions (e.g., $1/4 + 2/4$ )

1. Assemble whole skittle: "*This is 1 whole.*"
2. Remove two 1/4 pieces: "*Here's one quarter... and two quarters.*"
3. Combine on table: "*One quarter plus two quarters makes three quarters.*"

## 2. Three-Period Lesson

**Period 1:** "*This is one third. Two thirds make this much.*"

**Period 2:** "*Show me one half plus one half.*"

**Period 3:** "*How much is three fourths minus one fourth?*"

## 3. Discovering Equivalents

1. Compare two 1/4 pieces to one 1/2:  
*"Two quarters equal one half."*

### D. Control of Error

✓ **Visual:**

- Segments perfectly reconstruct skittles
- Mismatches create obvious gaps

✓ **Kinesthetic:**

- Extra segments indicate incorrect operations

### E. Extensions

#### 1. "Fraction Stories"

- \*"*You ate 1/3 of pizza, then another 1/3. How much total?\**"

#### 2. Recording Work

- Draw equations:  $1/2 + 1/2 = 1$

### 3. Advanced Equivalents

- How many thirds equal a whole?

#### F. Teacher Notes

##### Key Language

- *"Same denominators stay together"*
- *"How many parts make a whole?"*

##### Observation Signs

- Combines fractions systematically
- Notices equivalencies without prompting

##### Next Steps

- Leads to **Decimal Fraction Board**
- Prepares for **Money Lessons** (decimal fractions)

#### G. Common Errors

Issue	Solution
Adds denominators	<i>"The bottom number names the parts – we don't change it"</i>
Forces mismatched pieces	<i>"Do these pieces fit together smoothly?"</i>

##### Quote:

*"The skittles prove that fractions aren't scary – just broken-up wholes waiting to be reunited."*

### **Conclusion for a Montessori Math Album (Ages 3-6):**

*"As we come to the end of our Montessori math journey, remember that every number, shape, and pattern you've explored is a step toward understanding the beautiful order of our world. Math is not just about counting—it's about discovering, problem-solving, and seeing the connections all around us. Keep working with your hands, asking questions, and finding joy in learning. Whether you're arranging the Number Rods, exploring the Golden Beads, or simply noticing numbers in nature, you are building a strong foundation for a lifetime of curiosity and confidence. Happy exploring, little mathematicians!"*

This conclusion reinforces the Montessori philosophy of hands-on, joyful learning while celebrating the child's progress.