Neural Networks 61 (2015) 1–9

Contents lists available at ScienceDirect

Neural Networks

journal homepage: www.elsevier.com/locate/neunet

New synchronization criteria for memristor-based networks: Adaptive control and feedback control schemes☆

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ARTICLE INFO

Article history:
Received 27 March 2014
Received in revised form 21 July 2014
Accepted 28 August 2014
Available online 8 September 2014

Keywords:
Memristor-based neural networks
Time delay
Adaptive control
State-feedback control
Synchronization

ABSTRACT

In this paper, we investigate synchronization for memristor-based neural networks with time-varying delay via an adaptive and feedback controller. Under the framework of Filippov's solution and differential inclusion theory, and by using the adaptive control technique and structuring a novel Lyapunov functional, an adaptive updated law was designed, and two synchronization criteria were derived for memristor-based neural networks with time-varying delay. By removing some of the basic literature assumptions, the derived synchronization criteria were found to be more general than those in existing literature. Finally, two simulation examples are provided to illustrate the effectiveness of the theoretical results.

1. Introduction

Memristors were proposed by Prof. Chua in his seminal paper Chua (1971). They share many properties and the same unit of measurement as resistors, but cannot be replaced by any of the other three circuit elements: resistor, capacitor, or inductor. As the fourth basic passive circuit element, they were not noticed by many researchers until the memristor prototype was manufactured by the Hewlett–Packard laboratory (Strukov, Snider, Stewart, & Williams, 2008; Tour & He, 2008). The main property of the memristor is that its memristance depends on the magnitude and polarity of the voltage and on how long the voltage has been applied. Hence, its memristance M can represent the functional relationship between charge and flux: $d\varphi = Mdq$, (see Fig. 1). Because of its memory function, the memristor has attracted increased attention. It can simulate the human brain quite realistically. It also has many potential applications, for example, it could increase the starting speed of a computer substantially and extend cell phone battery life by several months.

The mathematical model of memristor-based neural networks is a special case of a switched discontinuous system (Brown, 1994; Huang, Qu, & Li, 2005; Lou & Cui, 2008), whose switching rule depends on the network's state. However, this model has its own special features. The common method for dealing with the switched system (Hou, Zong, & Wu, 2011; Lian & Zhang, 2011; Zhang & Yu, 2009) is unsuitable for memristor-based neural networks. It is discontinuous on the right-hand side, and the synchronization study for a discontinuous right-hand side system is not easy. Recently, the dynamic behavior of memristor-based neural networks has become a popular topic (Wang, Li, Peng, Xiao, & Yang, 2014; Wu & Zeng, 2012, 2013; Zhang & Shen, 2013; Zhang, Shen, & Wang, 2013). Wu and Zeng (2012) studied closed-loop control problems of memristive systems, by designing optimal controllers. Some sufficient conditions in terms of linear matrix inequalities were obtained to ensure exponential stabilization of memristive cellular neural networks. By applying the drive–response concept, two different types of feedback controller were proposed to ensure exponential stability for the anti-synchronization error system in Wu and Zeng (2013). Chen, Zeng, and Jiang (2014) considered the model of fractional-order memristor-based neural networks (FMNN). They firstly proved the existence and uniqueness of its equilibrium point, then presented the sufficient criteria for global

☆ This work was jointly supported by the National Natural Science Foundation of China (NSFC) under Grants No. 61272530 and 11072059, and the Natural Science Foundation of Jiangsu Province of China under Grant No. BK2012741, and the "Fundamental Research Funds for the Central Universities", the JSPS Innovation Program under Grant CXLX13_075, and the Scientific Research Foundation of Graduate School of Southeast University YBJJ1407.

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http://dx.doi.org/10.1016/j.neunet.2014.08.015
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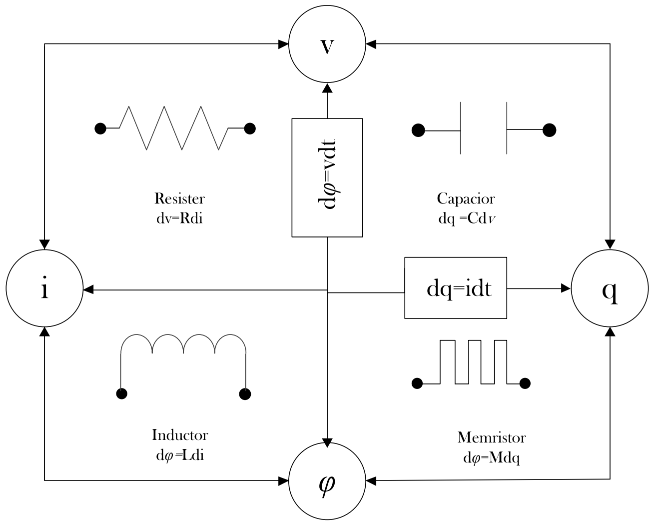

Fig. 1. Connection of four basic electrical elements (inspired by Strukov et al., 2008).

Mittag-Leffler stability and synchronization of the networks. Yang, Cao, and Yu (2014) discussed the problem of global exponential synchronization for a class of memristor-based Cohen–Grossberg neural networks with time-varying discrete delay and unbounded distributed delay. Through a nonlinear transformation, an alternative system of memristor-based Cohen–Grossberg neural networks was obtained. By designing a novel controller, corresponding synchronization criteria for memristor-based Cohen–Grossberg neural networks were given, the conditions established in the paper were improved upon, and the outcomes extended on those in existing papers.

The synchronization or anti-synchronization of memristor-based neural networks has received attention. In large-scale networks, they are unable to synchronize by themselves. Various effective control approaches and techniques have been proposed for synchronization. These include impulsive (Lu, Ho, Cao, & Kurths, 2011; Zhang & Sun, 2009), feedback (Cao & Wan, 2014; Rafikov & Balthazar, 2008), adaptive (Yang & Jiang, 2014; Zhou, Lu, & Lü, 2006), and intermittent (Huang, Li, Huang, & Han, 2013; Liu & Chen, 2011; Yang & Cao, 2009) control. The impulsive effects have been regarded as disturbances, the concept of average impulsive interval was used, and Lu et al. (2011) investigated the globally exponential synchronization of linearly coupled networks with impulsive disturbances. Authors have investigated drive–response fractional-order dynamic networks with uncertain parameters, by adopting an adaptive controller, which has a more general and simpler expression form. The adaptive laws of parameters were introduced by Yang and Jiang (2014). Adaptive controllers obtain effective results in actual applications. By designing suitable adaptive laws, adaptive controllers can adjust the coupling strength automatically. Furthermore, in the electronic implementation of memristor-based neural networks, time delays such as time-varying delays are inevitable because of the finite switching speed of the amplifiers, and they play an important role in the stability or synchronization of neural networks. They can result in network instability and should therefore be included (Cai & Huang, 2014; Cai, Huang, Guo, & Chen, 2012) in the mathematical model of memristor-based neural networks. To the best of our knowledge, no research exists on dealing with adaptive control for memristor-based neural networks with time-varying delay, despite its potential and practical importance.

Motivated by the aforementioned discussions, we deal with the synchronization of memristor-based neural networks with time delays using adaptive and feedback controllers, differential inclusion theory, and adaptive control techniques. By structuring novel Lyapunov functionals, an adaptive updated law is designed and new synchronization criteria for memristor-based neural network time-varying delays are proposed. Most previous work (Wang et al., 2014; Wu & Zeng, 2013; Zhang & Shen, 2013; Zhang et al., 2013) on the synchronization of memristor-based neural networks requires the basic assumption: $co\{\underline{a}_{ij},\bar{a}_{ij}\}f_j(x_j(t)) - co\{\underline{a}_{ij},\bar{a}_{ij}\}f_j(y_j(t)) \subseteq co\{\underline{a}_{ij},\bar{a}_{ij}\}(f_j(x_j(t)) - f_j(y_j(t)))$. This assumption is not always derived. The main contributions in this paper can be summarized as follows: (1) a novel adaptive control law is designed to study the synchronization of memristor-based neural networks; (2) the time-varying delay is considered and a new mathematical model of memristor-based neural networks is established, which more closely approximates the actual model; and (3) basic assumptions in existing references are removed and new sufficient conditions are obtained to ensure that the memristor-based neural networks with time delay reach synchronization. This result is easy to verify and extends results from previous work.

In Section 2, the model formulation and some preliminaries are presented. In Section 3, adaptive synchronization criteria for memristor-based neural networks are obtained. In Section 4, synchronization criteria for memristor-based neural networks are derived by feedback control. Two numerical examples are given to demonstrate the validity of the proposed results in Section 5. Some conclusions are made in Section 6.

Notation: $\mathbb{R}$ denotes the set of real numbers, $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, $\mathbb{R}^{m\times n}$ denotes the set of all $m \times n$ real matrices. For $\tau > 0$, $C([-\tau,0];\mathbb{R}^n)$ denotes the family of continuous functions φ from $[-\tau,0]$ to $\mathbb{R}^n$ with the norm $\|\varphi\| = \sup_{-\tau\le s\le 0}\max_{1\le i\le n}|\varphi_i(s)|$. The solutions of memristor-based networks are considered in Filippov's sense, and $[\cdot,\cdot]$ represents the interval. $co(Q)$ denotes the closure of the convex hull of Q. If not stated explicitly, matrices are assumed to have compatible dimensions for algebraic operations.

2. Model description and preliminaries

We consider the following memristor-based neural networks with time-varying delay:

$$\dot{x}_i(t) = -c_i(x_i(t)) + \sum_{j=1}^{n} a_{ij}(x_i(t))f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij}(x_i(t)) \tag{1}$$
$$g_j(x_j(t-\tau(t))) + I_i, \quad t \ge 0,\ i = 1,2,\ldots,n,$$

where $x_i(t)$ is the voltage of the capacitor $\mathbf{C}_i$; $c_i(x_i(t))$ are appropriately behaved functions; $\tau(t)$ is the time-varying delay that satisfies differentiability and $0 \le \tau(t) \le \tau$, $\dot{\tau}(t) \le \sigma < 1$ where τ and σ are nonnegative constants; $f_j(\cdot)$ and $g_j(\cdot)$ are feedback functions; I_i is the external input; and

$$a_{ij}(x_i(t)) = \frac{\mathbf{W}_{ij}}{\mathbf{C}_i} \times \mathrm{sgin}_{ij}, \qquad b_{ij}(x_i(t)) = \frac{\mathbf{M}_{ij}}{\mathbf{C}_i} \times \mathrm{sgin}_{ij},$$

$$\mathrm{sgin}_{ij} = \begin{cases} 1, & i \ne j \\ -1, & i = j, \end{cases}$$

in which $\mathbf{W}_{ij}$ and $\mathbf{M}_{ij}$ denote the memductances of resistors $\mathbf{R}_{ij}$ and $\mathbf{F}_{ij}$, respectively. $\mathbf{R}_{ij}$ represents the resistors between the feedback function $f_i(x_i(t))$ and $x_i(t)$. $\mathbf{F}_{ij}$ represents the resistors between the feedback function $g_i(x_i(t-\tau(t)))$ and $x_i(t)$. According to the memristor features and the current–voltage characteristics, $a_{ij}(x_i(t))$ and $b_{ij}(x_i(t))$ are memristor-based connection weights that satisfy the following conditions:

$$a_{ij}(x_i(t)) = \begin{cases} \acute{a}_{ij}, & \mathrm{sgin}_{ij}\dfrac{df_j(x_j(t))}{dt} - \dfrac{dx_i(t)}{dt} \le 0, \\ \grave{a}_{ij}, & \mathrm{sgin}_{ij}\dfrac{df_j(x_j(t))}{dt} - \dfrac{dx_i(t)}{dt} > 0, \end{cases}$$
$$b_{ij}(x_i(t)) = \begin{cases} \acute{b}_{ij}, & \mathrm{sgin}_{ij}\dfrac{df_j(x_j(t))}{dt} - \dfrac{dx_i(t)}{dt} \le 0, \\ \grave{b}_{ij}, & \mathrm{sgin}_{ij}\dfrac{df_j(x_j(t))}{dt} - \dfrac{dx_i(t)}{dt} > 0, \end{cases} \tag{2}$$

for $i, j = 1, 2, \ldots, n$, where $\acute{a}_{ij}$, $\grave{a}_{ij}$, $\acute{b}_{ij}$, $\grave{b}_{ij}$, are constants. The initial value associated with system (1) is $\phi(s) = (\phi_1(s), \phi_2(s), \ldots, \phi_n(s))^T \in \mathcal{C}([-\tau, 0]; \mathbb{R}^n)$.

Remark 1. Memristor-based recurrent networks can be seen as a special case of switched networks where the switching rule depends on the network state, but its analysis method is different from that of general switched systems because of its characteristics. From a mathematical perspective, memristor-based differential equations obey Bernoulli's nonlinear differential equations.

We have made the following assumptions:

($\mathcal{H}1$): Behaved function $c_i(x)$ satisfies $\dot{c}_i(x) \ge \beta_i > 0$, and $c_i(0) = 0, i = 1, 2, \ldots, n$.

($\mathcal{H}2$): For any two different $x,\ y \in \mathbb{R}$,

$$|f_j(x) - f_j(y)| \le h_j|x - y|, \qquad |g_j(x) - g_j(y)| \le k_j|x - y|,$$

where $h_j, k_j (j = 1, 2, \ldots, n)$ are positive constants.

($\mathcal{H}3$): For any $x \in \mathbb{R}$, there exist positive constants M_j, N_j, such that

$$|f_j(x)| \le M_j, \qquad |g_j(x)| \le N_j.$$

From Eq. (2), memristor-based recurrent networks are a discontinuous switched system. In this case, the solutions of (1) are considered in Filippov's sense. In the following, we introduce some definitions on the set-valued map and define the Filippov solution.

Definition 2.1 (*Aubin & Frankowska, 2009*)**.** Let $E \subseteq \mathbb{R}^n$, $x \mapsto F(x)$ is called a set-valued map from $E \hookrightarrow \mathbb{R}^n$, if for each point x of a set $E \subseteq \mathbb{R}^n$, there corresponds a nonempty set $F(x) \subseteq \mathbb{R}^n$.

Definition 2.2 (*Aubin & Frankowska, 2009*)**.** A set-valued map F with nonempty values is said to be upper semi-continuous at $x_0 \in E \subseteq \mathbb{R}^n$, if for any open set N containing $F(x_0)$, there exists a neighborhood M of x_0 such that $F(M) \subseteq N$. $F(x)$ is said to have a closed (convex, compact) image if, for each $x \in E$, $F(x)$ is closed (convex, compact).

Definition 2.3 (*Filippov, 1960*)**.** For differential system $\frac{dx}{dt} = f(t, x)$, where $f(t, x)$ is discontinuous in x. The set-valued map of $f(t, x)$ is defined as:

$$F(t, x) = \bigcap_{\delta>0} \bigcap_{\mu(N)=0} co[f(B(x, \delta) \setminus N)],$$

where $B(x, \delta) = \{y : \|y - x\| \le \delta\}$ is the ball of center x and radius δ; intersection is taken over all sets N of measure zero and over all $\delta > 0$; and $\mu(N)$ is the Lebesgue measure of set N.

A Filippov solution of system (1) with initial condition $x(0) = x_0$ is absolutely continuous on any subinterval $t \in [t_1, t_2]$ of $[0, T]$, which satisfies $x(0) = x_0$, and the differential inclusion:

$$\frac{dx}{dt} \in F(t, x). \quad \text{for a.a. } t \in [0, T].$$

By applying the theory of differential inclusion, the memristor-based networks (1) can be written as follows:

$$\dot{x}_i(t) \in -c_i(x_i(t)) + \sum_{j=1}^{n} co(\underline{a}_{ij}, \overline{a}_{ij})f_j(x_j(t)) + \sum_{j=1}^{n} co(\underline{b}_{ij}, \overline{b}_{ij})g_j(x_j(t - \tau(t))) + I_i,$$
$$\text{for a.e. } t \ge 0,\ 1, 2, \ldots, n, \tag{3}$$

where $\underline{a}_{ij} = \min\{\acute{a}_{ij}, \grave{a}_{ij}\}$, $\overline{a}_{ij} = \max\{\acute{a}_{ij}, \grave{a}_{ij}\}$, $\underline{b}_{ij} = \min\{\acute{b}_{ij}, \grave{b}_{ij}\}$, $\overline{b}_{ij} = \max\{\acute{b}_{ij}, \grave{b}_{ij}\}$, $a^+_{ij} = \max\{|\underline{a}_{ij}|, |\overline{a}_{ij}|\}$, $b^+_{ij} = \max\{|\underline{b}_{ij}|, |\overline{b}_{ij}|\}$, and

$$co(\underline{a}_{ij}, \overline{a}_{ij}) = \begin{cases} \acute{a}_{ij}, & \mathrm{sgin}_{ij}\dfrac{df_j(x_j(t))}{dt} - \dfrac{dx_i(t)}{dt} < 0, \\ \left[\underline{a}_{ij}, \overline{a}_{ij}\right], & \mathrm{sgin}_{ij}\dfrac{df_j(x_j(t))}{dt} - \dfrac{dx_i(t)}{dt} = 0, \\ \grave{a}_{ij}, & \mathrm{sgin}_{ij}\dfrac{df_j(x_j(t))}{dt} - \dfrac{dx_i(t)}{dt} > 0, \end{cases}$$
$$co(\underline{b}_{ij}, \overline{b}_{ij}) = \begin{cases} \acute{b}_{ij}, & \mathrm{sgin}_{ij}\dfrac{df_j(x_j(t))}{dt} - \dfrac{dx_i(t)}{dt} < 0, \\ \left[\underline{b}_{ij}, \overline{b}_{ij}\right], & \mathrm{sgin}_{ij}\dfrac{df_j(x_j(t))}{dt} - \dfrac{dx_i(t)}{dt} = 0, \\ \grave{b}_{ij}, & \mathrm{sgin}_{ij}\dfrac{df_j(x_j(t))}{dt} - \dfrac{dx_i(t)}{dt} > 0 \end{cases}$$

or equivalently, there exist $\gamma_{ij}(t) \in co(\underline{a}_{ij}, \overline{a}_{ij})$ and $\delta_{ij}(t) \in co(\underline{b}_{ij}, \overline{b}_{ij})$, such that

$$\dot{x}_i(t) = -c_i(x_i(t)) + \sum_{j=1}^{n} \gamma_{ij}(t)f_j(x_j(t)) + \sum_{j=1}^{n} \delta_{ij}(t) g_j(x_j(t - \tau(t))) + I_i, \quad t \ge 0. \tag{4}$$

We consider the drive–response synchronization and the response system of (1) can be described as follows:

$$\dot{y}_i(t) = -c_i(y_i(t)) + \sum_{j=1}^{n} a_{ij}(y_i(t))f_j(y_j(t)) + \sum_{j=1}^{n} b_{ij}(y_i(t)) g_j(y_j(t - \tau(t))) + I_i + u_i(t), \quad t \ge 0,\ i = 1, 2, \ldots, n, \tag{5}$$

where $u_i(t)$ is the adaptive or feedback controller to be designed for reaching synchronization of the drive–response system. Similar to the above discussion, the parameters in system (5) can also be defined as:

$$a_{ij}(y_i(t)) = \begin{cases} \acute{a}_{ij}, & \mathrm{sgin}_{ij}\dfrac{df_j(y_j(t))}{dt} - \dfrac{dy_i(t)}{dt} \le 0, \\ \grave{a}_{ij}, & \mathrm{sgin}_{ij}\dfrac{df_j(y_j(t))}{dt} - \dfrac{dy_i(t)}{dt} > 0, \end{cases}$$
$$b_{ij}(y_i(t)) = \begin{cases} \acute{b}_{ij}, & \mathrm{sgin}_{ij}\dfrac{df_j(y_j(t))}{dt} - \dfrac{dy_i(t)}{dt} \le 0, \\ \grave{b}_{ij}, & \mathrm{sgin}_{ij}\dfrac{df_j(y_j(t))}{dt} - \dfrac{dy_i(t)}{dt} > 0. \end{cases}$$

Similar to the analysis of (3) and (4), the response system can be rewritten as:

$$\dot{y}_i(t) \in -c_i(y_i(t)) + \sum_{j=1}^{n} co(\underline{a}_{ij}, \overline{a}_{ij})f_j(y_j(t)) + \sum_{j=1}^{n} co(\underline{b}_{ij}, \overline{b}_{ij})g_j(y_j(t - \tau(t))) + I_i + u_i(t),$$
$$\text{for a.e. } t \ge 0,\ 1, 2, \ldots, n, \tag{6}$$

where

$$co(\underline{a}_{ij}, \overline{a}_{ij}) = \begin{cases} \acute{a}_{ij}, & \text{sgin}_{ij}\dfrac{df_j(y_j(t))}{dt} - \dfrac{dy_i(t)}{dt} < 0, \\ [\underline{a}_{ij}, \overline{a}_{ij}], & \text{sgin}_{ij}\dfrac{df_j(y_j(t))}{dt} - \dfrac{dy_i(t)}{dt} = 0, \\ \grave{a}_{ij}, & \text{sgin}_{ij}\dfrac{df_j(y_j(t))}{dt} - \dfrac{dy_i(t)}{dt} > 0, \end{cases}$$

$$co(\underline{b}_{ij}, \overline{b}_{ij}) = \begin{cases} \acute{b}_{ij}, & \text{sgin}_{ij}\dfrac{df_j(y_j(t))}{dt} - \dfrac{dy_i(t)}{dt} < 0, \\ [\underline{b}_{ij}, \overline{b}_{ij}], & \text{sgin}_{ij}\dfrac{df_j(y_j(t))}{dt} - \dfrac{dy_i(t)}{dt} = 0, \\ \grave{b}_{ij}, & \text{sgin}_{ij}\dfrac{df_j(y_j(t))}{dt} - \dfrac{dy_i(t)}{dt} > 0. \end{cases}$$

or equivalently, there exist $\bar{\gamma}_{ij}(t) \in co(\underline{a}_{ij}, \overline{a}_{ij})$ and $\bar{\delta}_{ij}(t) \in co(\underline{b}_{ij}, \overline{b}_{ij})$, such that:

$$\dot{y}_i(t) = -c_i(y_i(t)) + \sum_{j=1}^{n} \bar{\gamma}_{ij}(t) f_j(y_j(t)) + \sum_{j=1}^{n} \bar{\delta}_{ij}(t) \quad (7)$$
$$g_j(y_j(t-\tau(t))) + I_i + u_i(t), \quad t \geq 0.$$

The initial value associated with system (5) is $\varphi(s) = (\varphi_1(s), \varphi_2(s), \ldots, \varphi_n(s))^T \in \mathcal{C}([-\tau, 0]; \mathbb{R}^n)$.

To obtain the main results in this paper, the following definition and lemmas are introduced.

Definition 2.4. The response system (5) is said to be globally exponentially synchronized to drive system (1) if there exist positive scalars $\alpha > 0$ and $\beta > 0$ such that:

$$|y_i(t) - x_i(t)| \leq \beta \|\phi - \varphi\| e^{-\alpha(t-t_0)}, \quad t \geq t_0.$$

Lemma 2.1 (*Chain Rule Clarke, 1983*). *If $V(t) : \mathbb{R}^n \to \mathbb{R}$ is C-regular and $x(t)$ is absolutely continuous on any compact subinterval of $[0, +\infty)$, then $x(t)$ and $V(x(t)) : [0, +\infty) \to \mathbb{R}$ are differentiable for a.a. $t \in [0, +\infty)$ and*

$$\dot{V}(x(t)) = \nu(t)\dot{x}(t), \quad \forall \nu(t) \in \partial V(x(t)),$$

where $\partial V(x(t))$ is the Clarke generalized gradient of V at $x(t)$.

Lemma 2.2 (*Zhao & Zhang, 2011*). *For $-\infty < a < b \leq +\infty$, let $\psi_i(t) \in \mathcal{C}([a, b]; \mathbb{R})$, $(i = 1, 2, \ldots, n)$ satisfy the following integral delay inequality:*

$$\begin{cases} \psi_i(t) \leq e^{-\check{\chi}_i(t-a)}\psi_i(a) + \displaystyle\int_a^t e^{-\check{\chi}_i(t-s)} \Bigg[\sum_{j=1}^{n} \tilde{\chi}_{ij}\psi_j(s) \\ \quad + \displaystyle\sum_{j=1}^{n} \hat{\chi}_{ij}\psi_j(s-\tau(s))\Bigg] ds, \quad t \in [a, b), \\ \psi_i(a+s) = \sigma_i(s) \quad s \in [-\tau, 0], \end{cases} \quad (8)$$

where $\check{\chi}_i$, $\tilde{\chi}_{ij}$, and $\hat{\chi}_{ij}$, $(i, j = 1, 2, \ldots, n)$ are positive constants. Assume that

$$-\check{\chi}_i + \sum_{j=1}^{n} (\tilde{\chi}_{ij} + \hat{\chi}_{ij}) < 0, \quad \text{and}$$

$$\psi_i(t) \leq M\|\sigma\|^2, \quad t \in [a-\tau, a], \ i = 1, 2, \ldots, n$$

then

$$\psi_i(t) \leq M\|\sigma\|^2, \quad t \in (a, b),$$

where $M > 0$ is a positive constant.

3. Adaptive control for memristor-based neural network

In this section, we consider the adaptive synchronization of memristor-based neural networks with time-varying delay. Based on the invariant principle of functional differential equations and adaptive control techniques, a simple criterion for synchronization is proposed, which is different from the literature. We define the error between the drive and the response systems as $e_i(t) = y_i(t) - x_i(t)$. From (4) and (7), we have:

$$\begin{aligned} \dot{e}_i(t) = & -\tilde{c}_i(e_i(t)) + \sum_{j=1}^{n} \gamma_{ij}(t)\tilde{f}_j(e_j(t)) \\ & + \sum_{j=1}^{n} \delta_{ij}(t)\tilde{g}_j(e_j(t-\tau(t))) \\ & + \sum_{j=1}^{n} (\bar{\gamma}_{ij}(t) - \gamma_{ij}(t)) f_j(y_j(t)) \\ & + \sum_{j=1}^{n} (\bar{\delta}_{ij}(t) - \delta_{ij}(t)) g_j(y_j(t-\tau(t))) \\ & + \epsilon_i e_i(t) - \eta_i\, signe_i(t), \end{aligned} \quad (9)$$

where $\tilde{c}_i(e_i(t)) = c_i(e_i(t) + x_i(t)) - c_i(x_i(t))$, $\tilde{f}_i(e_i(t)) = f_i(e_i(t) + x_i(t)) - f_i(x_i(t))$, and $\tilde{g}_i(e_i(t-\tau(t))) = g_i(e_i(t-\tau(t)) + x_i(t-\tau(t))) - g_i(x_i(t-\tau(t)))$. We chose the adaptive controller as $u_i(t) = \epsilon_i e_i(t) - \eta_i\, signe_i(t)$, where ϵ_i is the updated coupling strength, and η_i is the constant control gain.

Theorem 1. *Under assumptions $(\mathcal{H}_1)$–$(\mathcal{H}_3)$, the response system (5) will synchronize globally with the drive system (1) when the coupling strength ϵ_i is updated by the following law:*

$$\dot{\epsilon}_i = -\alpha_i e_i^2(t).$$

And $\eta_i > \sum_{j=1}^{n} |\acute{a}_{ij} - \grave{a}_{ij}| M_j + \sum_{j=1}^{n} |\acute{b}_{ij} - \grave{b}_{ij}| N_j$, where $\alpha_i > 0$, $i = 1, 2, \ldots, n$ are arbitrary positive constants.

Proof. Consider the following Lyapunov functional:

$$\begin{aligned} V_i(t) = & \frac{1}{2}e_i^2(t) + \frac{1}{2\alpha_i}(\epsilon_i + l)^2 \\ & + \frac{1}{2(1-\sigma)} \sum_{j=1}^{n} \int_{t-\tau(t)}^{t} \tilde{g}_j^2(e_j(s))ds, \end{aligned} \quad (10)$$

where l is a constant to be determined. From Lemma 2.1, we have:

$$\begin{aligned} \dot{V}_i(t) = & e_i(t)\dot{e}_i(t) - (\epsilon_i + l)e_i^2(t) + \frac{1}{2(1-\sigma)} \sum_{j=1}^{n} \tilde{g}_j^2(e_j(t)) \\ & - \frac{1-\dot{\tau}(t)}{2(1-\sigma)} \sum_{j=1}^{n} \tilde{g}_j^2(e_j(t-\tau(t))) \\ = & e_i(t)\Bigg[-\tilde{c}_i(e_i(t)) + \sum_{j=1}^{n} \gamma_{ij}(t)\tilde{f}_j(e_j(t)) \\ & + \sum_{j=1}^{n} \delta_{ij}(t)\tilde{g}_j(e_j(t-\tau(t))) \\ & + \sum_{j=1}^{n} (\bar{\gamma}_{ij}(t) - \gamma_{ij}(t)) f_j(y_j(t)) \\ & + \sum_{j=1}^{n} (\bar{\delta}_{ij}(t) - \delta_{ij}(t)) g_j(y_j(t-\tau(t))) \end{aligned}$$

$$
\begin{aligned}
&\left. + \epsilon_i e_i(t) - \eta_i\, signe_i(t) \right] - (\epsilon_i + l) e_i^2(t) \\
&+ \frac{1}{2(1-\sigma)} \sum_{j=1}^{n} \tilde{g}_j^2(e_j(t)) \\
&- \frac{1-\dot{\tau}(t)}{2(1-\sigma)} \sum_{j=1}^{n} \tilde{g}_j^2(e_j(t-\tau(t))) \\
\le\; & -e_i(t)\tilde{c}_i(e_i(t)) + \sum_{j=1}^{n} |e_i(t)|\,|\gamma_{ij}(t)\|\tilde{f}_j(e_j(t))| \\
&+ \sum_{j=1}^{n} |e_i(t)\|\delta_{ij}(t)|\,|\tilde{g}_j(e_j(t-\tau(t)))| \\
&+ \sum_{j=1}^{n} |e_i(t)|\,|(\bar{\gamma}_{ij}(t) - \gamma_{ij}(t))\|f_j(y_j(t))| \\
&+ \sum_{j=1}^{n} |e_i(t)|\,|(\bar{\delta}_{ij}(t) - \delta_{ij}(t))\|g_j(y_j(t-\tau(t)))| \\
&+ \epsilon_i e_i^2(t) - \eta_i |e_i(t)| - (\epsilon_i + l) e_i^2(t) \\
&+ \frac{1}{2(1-\sigma)} \sum_{j=1}^{n} \tilde{g}_j^2(e_j(t)) \\
&- \frac{1-\dot{\tau}(t)}{2(1-\sigma)} \sum_{j=1}^{n} \tilde{g}_j^2(e_j(t-\tau(t))). \qquad (11)
\end{aligned}
$$

From the assumptions $(\mathcal{H}_1)$–$(\mathcal{H}_3)$, we have

$$
\begin{aligned}
&-(1-\dot{\tau}(t))/2(1-\sigma) \le -\frac{1}{2}, \\
&|e_i(t)\|\gamma_{ij}(t)\|\tilde{f}_j(e_j(t))| \le \frac{(a_{ij}^{+})^2}{2} e_i^2(t) + \frac{h}{2} e_j^2(t), \\
&|e_i(t)\|\delta_{ij}(t)\|\tilde{g}_j(e_j(t-\tau(t)))| \\
&\quad \le \frac{(b_{ij}^{+})^2}{2} e_i^2(t) + \frac{1}{2}\tilde{g}_j^2(e_j(t-\tau(t))), \qquad (12) \\
&|e_i(t)\|(\bar{\gamma}_{ij}(t) - \gamma_{ij}(t))\|f_j(y_j(t))| \le |\acute{a}_{ij} - \grave{a}_{ij}| M_j |e_i(t)|, \\
&|e_i(t)\|(\bar{\delta}_{ij}(t) - \delta_{ij}(t))\|g_j(y_j(t-\tau(t)))| \le |\acute{b}_{ij} - \grave{b}_{ij}| N_j |e_i(t)|, \\
&\tilde{g}_j^2(e_j(t)) \le k e_j^2(t),
\end{aligned}
$$

where $h = \max\{h_i^2 | i = 1, 2, \ldots, n\}$ and $k = \max\{k_i^2 | i = 1, 2, \ldots, n\}$.

Substituting (12) into (11) yields:

$$
\begin{aligned}
\dot{V}_i(t) \le\; & -\beta_i e_i^2 + \sum_{j=1}^{n} |\acute{a}_{ij} - \grave{a}_{ij}| M_j |e_i(t)| \\
&+ \sum_{j=1}^{n} |\acute{b}_{ij} - \grave{b}_{ij}| N_j |e_i(t)| + \sum_{j=1}^{n} \frac{(a_{ij}^{+})^2}{2} e_i(t)^2 \\
&+ \sum_{j=1}^{n} \frac{h}{2} e_j(t)^2 + \sum_{j=1}^{n} \frac{(b_{ij}^{+})^2}{2} e_i(t)^2 - \eta_i |e_i(t)| \\
&- l e_i(t)^2 + \frac{1}{2(1-\sigma)} \sum_{j=1}^{n} k e_j^2(t). \qquad (13)
\end{aligned}
$$

Let

$$
\eta_i > \sum_{j=1}^{n} |\acute{a}_{ij} - \grave{a}_{ij}| M_j + \sum_{j=1}^{n} |\acute{b}_{ij} - \grave{b}_{ij}| N_j,
$$

$$
\begin{aligned}
l =\; & -\min_{1\le i\le n}\{\beta_i\} + \max_{1\le i\le n}\left\{ \sum_{j=1}^{n} \frac{(a_{ij}^{+})^2 + (b_{ij}^{+})^2}{2} \right\} \\
&+ \frac{h}{2} + \frac{k}{2(1-\sigma)} + 1.
\end{aligned}
$$

From (13), we obtain $\dot{V}_i(t) < -e_i(t)^2$. $\dot{V}_i(t) = 0$ if and only if $e_i(t) = 0$, $i = 1, 2, \ldots, n$. The set $E_i = \{(e_i(t), \epsilon_i)^T : e_i(t) = 0, \epsilon_i = \epsilon_{i0}\}$ is the largest invariant set contained in set $\{\dot{V}_i(t) = 0\}$, according to the invariant principle, the trajectories of error system (9) converge asymptotically to the set E_i, i.e. $e_i(t) \to 0$ when $t \to 0$. The response system (5) will synchronize globally with the drive system (1) under the adaptive controller.

4. Feedback control for memristor-based neural network

In this section, by utilizing some inequality techniques and Lemma 2.2, several sufficient conditions are obtained to ensure exponential synchronization of memristor-based neural networks with feedback control. We choose the following feedback control strategy:

$$
u_i(t) = -p_i e_i(t) - \eta_i\, signe_i(t), \qquad (14)
$$

where p_i and η_i are control gains to be determined.

Theorem 2. *Under assumptions $(\mathcal{H}_1)$–$(\mathcal{H}_3)$, if there exists a constant $\alpha > 0$, such that the following inequalities hold:*

(a) $\eta_i > \sum_{j=1}^{n} |\acute{a}_{ij} - \grave{a}_{ij}| M_j + \sum_{j=1}^{n} |\acute{b}_{ij} - \grave{b}_{ij}| N_j$;

(b) $-(p_i + \beta_i - \alpha) + \sum_{j=1}^{n} (a_{ij}^{+} h_j + b_{ij}^{+} k_j e^{\alpha\tau}) < 0$;

then the response system (5) will synchronize globally exponentially with the drive system (1) under feedback controller (14).

Proof. Define a Lyapunov functional by:

$$
V_i = e^{\alpha t}|e_i(t)|, \quad i = 1, 2, \ldots, n.
$$

In view of Lemma 2.1, evaluating the time derivative of V_i along the trajectory of (9) gives

$$
\begin{aligned}
\dot{V}_i(t) &= \alpha e^{\alpha t}|e_i(t)| + e^{\alpha t}\, signe_i(t)\dot{e}_i(t) \\
&= \alpha e^{\alpha t}|e_i(t)| + e^{\alpha t} signe_i(t) \Bigg[-\tilde{c}_i(e_i(t)) \\
&\quad + \sum_{j=1}^{n} \gamma_{ij}(t)\tilde{f}_j(e_j(t)) + \sum_{j=1}^{n} \delta_{ij}(t)\tilde{g}_j(e_j(t-\tau(t))) \\
&\quad + \sum_{j=1}^{n} (\bar{\gamma}_{ij}(t) - \gamma_{ij}(t)) f_j(y_j(t)) \\
&\quad + \sum_{j=1}^{n} (\bar{\delta}_{ij}(t) - \delta_{ij}(t)) g_j(y_j(t-\tau(t))) \\
&\quad + \epsilon_i e_i(t) - \eta_i\, signe_i(t) \Bigg].
\end{aligned}
$$

From assumption $(\mathcal{H}2)$, we have

$$
\begin{aligned}
&\tilde{f}_j(e_j(t)) \le h_j e_j(t), \\
&\tilde{g}_j(e_j(t-\tau(t))) \le k_j e_j(t-\tau(t)). \qquad (15)
\end{aligned}
$$

Combining (15) and assumption ($\mathscr{H}3$), the condition (a) in Theorem 2 yields:

$$\begin{aligned}
\dot{V}_i(t) &\le \alpha e^{\alpha t}|e_i(t)| + e^{\alpha t}\Bigg[-\beta_i|e_i(t)| + \sum_{j=1}^{n} a_{ij}^{+}h_j|e_j(t)| \\
&\quad + \sum_{j=1}^{n} b_{ij}^{+}k_j|e_j(t-\tau(t))| + \sum_{j=1}^{n}|\acute{a}_{ij}-\grave{a}_{ij}|M_j \\
&\quad + \sum_{j=1}^{n}|\acute{b}_{ij}-\grave{b}_{ij}|N_j - p_i|e_i(t)| - \eta_i\Bigg] \\
&\le \alpha e^{\alpha t}|e_i(t)| + e^{\alpha t}(-\beta_i - p_i)|e_i(t)| \\
&\quad + \sum_{j=1}^{n} a_{ij}^{+}h_j e^{\alpha t}|e_j(t)| \\
&\quad + \sum_{j=1}^{n} b_{ij}^{+}k_j e^{\alpha\tau}e^{\alpha(t-\tau(t))}|e_j(t-\tau(t))| \\
&\le (\alpha - \beta_i - p_i)V_i(t) + \sum_{j=1}^{n} a_{ij}^{+}h_jV_j(t) \\
&\quad + \sum_{j=1}^{n} b_{ij}^{+}k_j e^{\alpha\tau}V_j(t-\tau(t)).
\end{aligned}$$

Let $-\breve{\chi}_i = \alpha - \beta_i - p_i$, $\tilde{\chi}_{ij} = a_{ij}^{+}h_j$, $\hat{\chi}_{ij} = b_{ij}^{+}k_je^{\alpha\tau}$ then:

$$\dot{V}_i(t) \le -\breve{\chi}_iV_i(t) + \sum_{j=1}^{n}\tilde{\chi}_{ij}V_j(t) + \sum_{j=1}^{n}\hat{\chi}_{ij}V_j(t-\tau(t)),$$

that is to say,

$$\begin{cases}
V_i(t) \le e^{-\breve{\chi}_i(t)}V_i(0) + \displaystyle\int_0^t e^{-\breve{\chi}_i(t-s)}\Bigg[\sum_{j=1}^{n}\tilde{\chi}_{ij}V_j(s) \\
\quad + \displaystyle\sum_{j=1}^{n}\hat{\chi}_{ij}V_j(s-\tau(s))\Bigg]ds, \\
\quad t \in [0,+\infty),\ i = 1,2,\ldots,n.
\end{cases} \tag{16}$$

Because

$$|e_i(t)| \le \|\varphi - \phi\| \le \|\varphi - \phi\|e^{-\alpha t}, \quad t \le 0,\ i = 1,2,\ldots,n$$

then

$$V_i(t) \le \|\varphi - \phi\|, \quad t \le 0.$$

By combining (16) with condition (b), from Lemma 2.2, it follows that:

$$V_i(t) \le \|\varphi - \phi\|, \quad t \ge 0.$$

Therefore

$$|e_i(t)| \le \|\varphi - \phi\|e^{-\alpha t}, \quad t \ge 0.$$

Based on the above analysis, one always has $|e_i(t)| \le \|\varphi-\phi\|e^{-\alpha t}$, for $t \ge 0$. If conditions (a) and (b) in Theorem 2 hold, from Definition 2.4, the response system (5) will synchronize globally exponentially with the drive system (1) under the feedback controller.

Remark 2. In the literature (Wang et al., 2014; Wu & Zeng, 2013; Zhang & Shen, 2013; Zhang et al., 2013), authors investigated the synchronization or anti-synchronization control of memristor-based neural networks under the following basic assumption:

$$\begin{aligned}
&co\{\underline{a}_{ij},\overline{a}_{ij}\}f_j(x_j(t)) - co\{\underline{a}_{ij},\overline{a}_{ij}\}f_j(y_j(t)) \\
&\quad \subseteq co\{\underline{a}_{ij},\overline{a}_{ij}\}(f_j(x_j(t)) - f_j(y_j(t))).
\end{aligned}$$

Unfortunately, this assumption cannot always hold (for a more detailed explanation, see Yang et al. (2014)). By removing the basic assumption, we use a new analysis method to study the synchronization control of memristor-based neural networks, with proposed criteria that are easy to verify.

Remark 3. Memristor-based neural networks has been an important and challenging topic in recent years. A suitable research method is still being established for these networks. Wu, Zhang, and Ding (2013) divided the proof into four steps with the proposed synchronization criterion depending on the switching parameter T_i, which could result in less conservatism.

Remark 4. In this paper, we consider the synchronization of memristor-based neural networks, which can be seen as a discontinuous right-hand side system. Differential inclusion theory is a useful tool to deal with such a system. Cai et al. (2012) and Cai and Huang (2014) adopted a differential inclusion method to study the dynamics of a class of time-varying delayed neural networks with discontinuous activation function and memristor-based BMA neural networks with time-varying delay, respectively. The former studied the existence, uniqueness and global exponential stability of a periodic solution for the neural networks (Cai et al., 2012). By using the matrix measure approach and generalized Halanay inequalities, the latter investigated global dissipativity and a positive periodic solution for memristor-based BMA neural networks with time-varying delay (Cai & Huang, 2014). These provide new ideas and methods for memristor-based neural networks with time-varying delay. It is necessary to study the dissipativity and passivity of memristor-based neural networks by using the matrix measure method and the Halanay inequality technique.

5. Numerical examples

In this section, two illustrative examples are given to check the synchronization criteria obtained in Theorems 1 and 2.

Example 1. Consider two-order memristor-based recurrent neural networks as follows:

$$\begin{cases}
\dot{x}_1(t) = -c_1(x_1(t)) + a_{11}(x_1(t))f_1(x_1(t)) \\
\quad + a_{12}(x_1(t))f_2(x_2(t)) + b_{11}(x_1(t))g_1(x_1(t-\tau(t))) \\
\quad + b_{12}(x_1(t))g_2(x_2(t-\tau(t))), \\
\dot{x}_2(t) = -c_2(x_2(t)) + a_{21}(x_1(t))f_1(x_1(t)) \\
\quad + a_{22}(x_2(t))f_2(x_2(t)) + b_{21}(x_2(t))g_1(x_1(t-\tau(t))) \\
\quad + b_{22}(x_2(t))g_2(x_2(t-\tau(t))),
\end{cases} \tag{17}$$

where

$$a_{11}(x_1(t)) = \begin{cases} -0.5, & -\dfrac{df_1(x_1(t))}{dt} - \dfrac{dx_1(t)}{dt} \le 0, \\ 0.5, & -\dfrac{df_1(x_1(t))}{dt} - \dfrac{dx_1(t)}{dt} > 0, \end{cases}$$

$$a_{12}(x_1(t)) = \begin{cases} 3, & \dfrac{df_2(x_2(t))}{dt} - \dfrac{dx_1(t)}{dt} \le 0, \\ 4, & \dfrac{df_2(x_1(t))}{dt} - \dfrac{dx_1(t)}{dt} > 0, \end{cases}$$

$$a_{21}(x_2(t)) = \begin{cases} 5, & \dfrac{df_1(x_1(t))}{dt} - \dfrac{dx_2(t)}{dt} \le 0, \\ 6, & \dfrac{df_1(x_1(t))}{dt} - \dfrac{dx_2(t)}{dt} > 0, \end{cases}$$

$$a_{22}(x_2(t)) = \begin{cases} -0.5, & -\dfrac{df_2(x_2(t))}{dt} - \dfrac{dx_2(t)}{dt} \le 0, \\ 0.5, & -\dfrac{df_2(x_2(t))}{dt} - \dfrac{dx_2(t)}{dt} > 0 \end{cases}$$

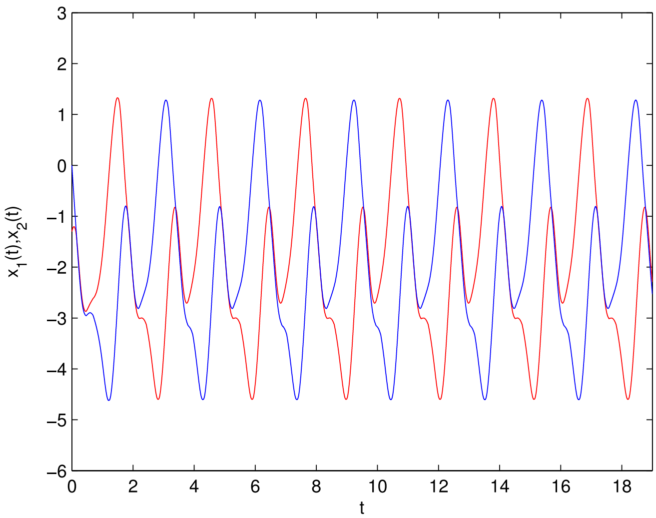

Fig. 2. State trajectories of variable $x_1(t), x_2(t)$.

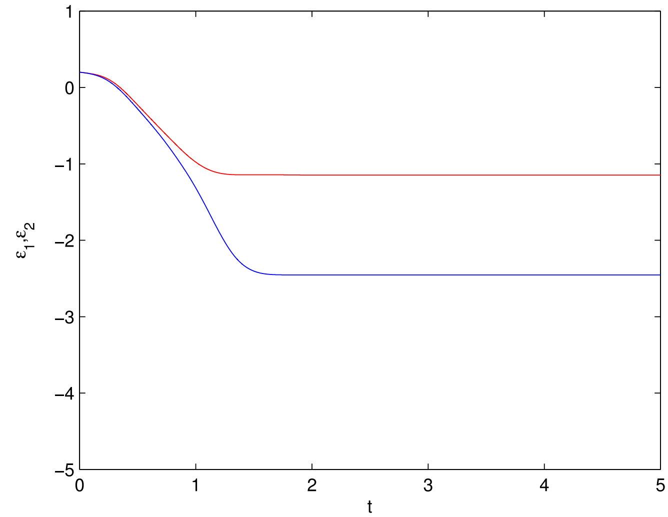

Fig. 4. Trajectories of coupling strength ϵ_1, ϵ_2.

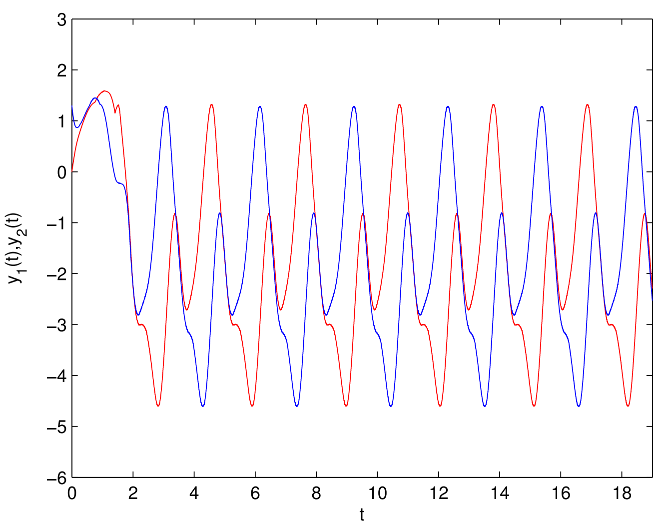

Fig. 3. State trajectories of variable $y_1(t), y_2(t)$.

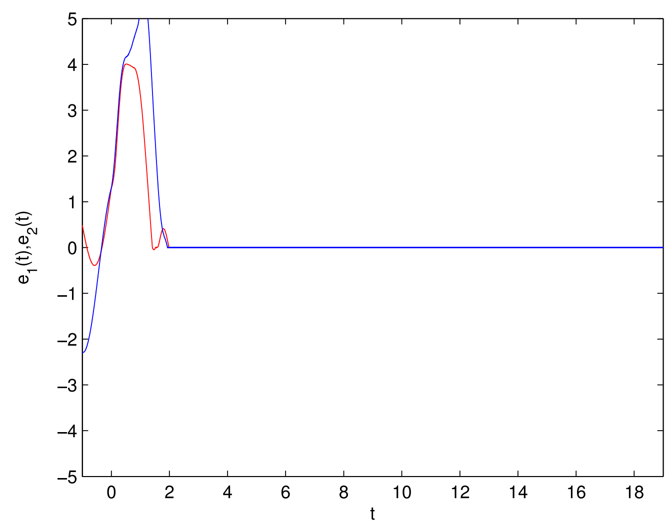

Fig. 5. State trajectories of variable $e_1(t), e_2(t)$.

$$b_{11}(x_1(t)) = \begin{cases} 0.1, & -\dfrac{df_1(x_1(t))}{dt} - \dfrac{dx_1(t)}{dt} \le 0, \\ -0.1, & -\dfrac{df_1(x_1(t))}{dt} - \dfrac{dx_1(t)}{dt} > 0, \end{cases}$$

$$b_{12}(x_1(t)) = \begin{cases} 5, & \dfrac{df_2(x_2(t))}{dt} - \dfrac{dx_1(t)}{dt} \le 0, \\ 6, & \dfrac{df_2(x_2(t))}{dt} - \dfrac{dx_1(t)}{dt} > 0, \end{cases}$$

$$b_{21}(x_2(t)) = \begin{cases} 3 & \dfrac{df_1(x_1(t))}{dt} - \dfrac{dx_2(t)}{dt} \le 0, \\ 4, & \dfrac{df_1(x_1(t))}{dt} - \dfrac{dx_2(t)}{dt} > 0, \end{cases}$$

$$b_{22}(x_2(t)) = \begin{cases} -0.1, & -\dfrac{df_2(x_2(t))}{dt} - \dfrac{dx_2(t)}{dt} \le 0, \\ 0.1, & -\dfrac{df_2(x_2(t))}{dt} - \dfrac{dx_2(t)}{dt} > 0, \end{cases}$$

and behaved functions $c_i(x_i(t)) = x_i(t)$, activation functions $f_i(x_i) = g_i(x_i) = \sin(x_i)$, $i = 1, 2$, and $\tau(t) = 0.1$. We choose $\alpha_1 = \alpha_2 = 0.1$ and the control gains $\eta_1 = 3.5$ and $\eta_2 = 3.9$. It is obvious that the condition in Theorem 1 holds. According to Theorem 1, we can conclude that the system (17) will synchronize with its response system under an adaptive controller.

For the numerical simulations, we choose the coupling strength ϵ_i updated by $\dot{\epsilon}_i = -0.1 * e_i^2(t)$, and take the initial conditions as $\epsilon_1(0) = \epsilon(0) = 0.2$, $(\phi_1(t), \phi_2(t)) = (-1.3 * \text{cos}t, -1.1 * \sin(2 * t))$, $(\varphi_1(t), \varphi_2(t)) = (1.5 * \sin(3 * t), 1.3 * \cos(3 * t))$, $\forall t \in [-1, 0)$. Fig. 2 shows the state trajectories of variables $x_1(t)$ and $x_2(t)$. Fig. 3 shows the state trajectories of variables $y_1(t)$ and $y_2(t)$. The trajectories of coupling strength are shown in Fig. 4. Fig. 5 shows the driver system (17) achieve synchronization under the update law of the coupling strength, and the connection weight coefficients $a_{11}(x_1(t))$ and $a_{11}(y_1(t))$ can be seen in Figs. 6 and 7. The numerical simulations verify the effectiveness of Theorem 1.

Example 2. Consider two-order memristor-based recurrent neural networks under state feedback control:

$$\begin{cases} \dot{x}_1(t) = -c_1(x_1(t)) + a_{11}(x_1(t))f_1(x_1(t)) \\ \quad + a_{12}(x_1(t))f_2(x_2(t)) + b_{11}(x_1(t))g_1(x_1(t - \tau(t))) \\ \quad + b_{12}(x_1(t))g_2(x_2(t - \tau(t))) + I_1, \\ \dot{x}_2(t) = -c_2(x_2(t)) + a_{21}(x_1(t))f_1(x_1(t)) \\ \quad + a_{22}(x_2(t))f_2(x_2(t)) + b_{21}(x_2(t))g_1(x_1(t - \tau(t))) \\ \quad + b_{22}(x_2(t))g_2(x_2(t - \tau(t))) + I_2, \end{cases} \tag{18}$$

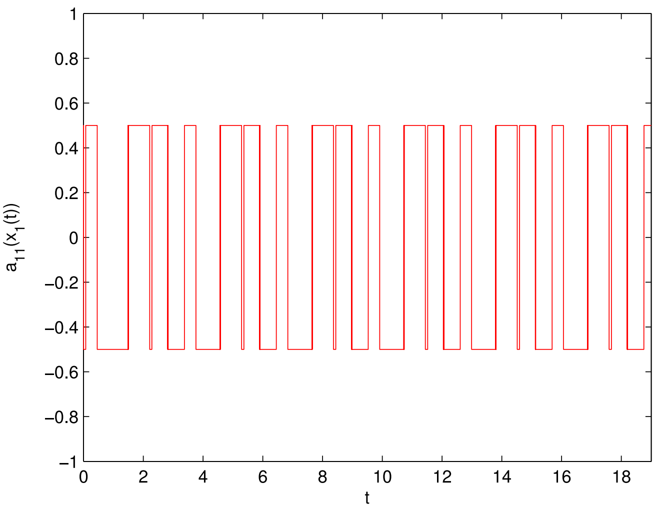

Fig. 6. Trajectories of connection weight coefficient $a_{11}(x_1(t))$.

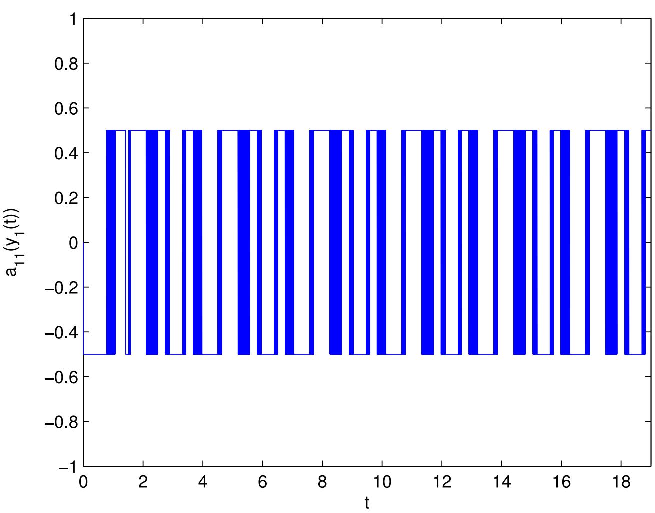

Fig. 7. Trajectories of connection weight coefficient $a_{11}(y_1(t))$.

where

$$a_{11}(x_1(t)) = \begin{cases} 1+\pi/4, & -\dfrac{df_1(x_1(t))}{dt} - \dfrac{dx_1(t)}{dt} \le 0, \\ 0.9+\pi/4, & -\dfrac{df_1(x_1(t))}{dt} - \dfrac{dx_1(t)}{dt} > 0, \end{cases}$$

$$a_{12}(x_1(t)) = \begin{cases} 20, & \dfrac{df_2(x_2(t))}{dt} - \dfrac{dx_1(t)}{dt} \le 0, \\ 19.5, & \dfrac{df_2(x_2(t))}{dt} - \dfrac{dx_1(t)}{dt} > 0, \end{cases}$$

$$a_{21}(x_2(t)) = \begin{cases} 0.09, & \dfrac{df_1(x_1(t))}{dt} - \dfrac{dx_2(t)}{dt} \le 0, \\ 0.1, & \dfrac{df_1(x_1(t))}{dt} - \dfrac{dx_2(t)}{dt} > 0, \end{cases}$$

$$a_{22}(x_2(t)) = \begin{cases} 0.9+\pi/4, & -\dfrac{df_2(x_2(t))}{dt} - \dfrac{dx_2(t)}{dt} \le 0, \\ 1+\pi/4, & -\dfrac{df_2(x_2(t))}{dt} - \dfrac{dx_2(t)}{dt} > 0, \end{cases}$$

$$b_{11}(x_1(t)) = \begin{cases} -\sqrt{2}-0.1, & -\dfrac{df_1(x_1(t))}{dt} - \dfrac{dx_1(t)}{dt} \le 0, \\ -\sqrt{2}, & -\dfrac{df_1(x_1(t))}{dt} - \dfrac{dx_1(t)}{dt} > 0, \end{cases}$$

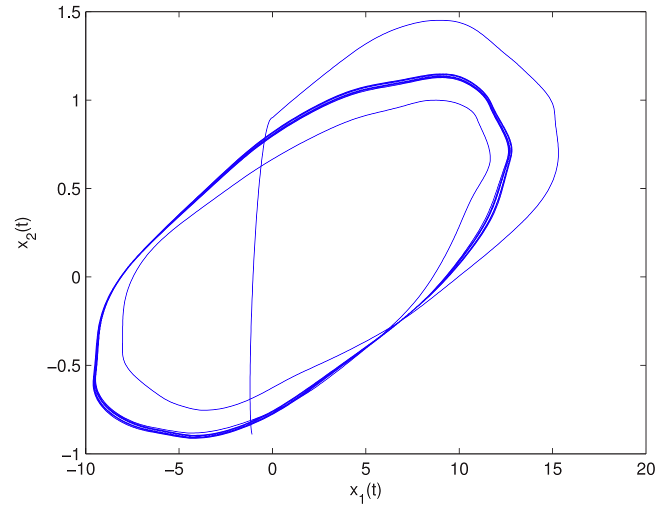

Fig. 8. Phase trajectories of memristor-based network model (18).

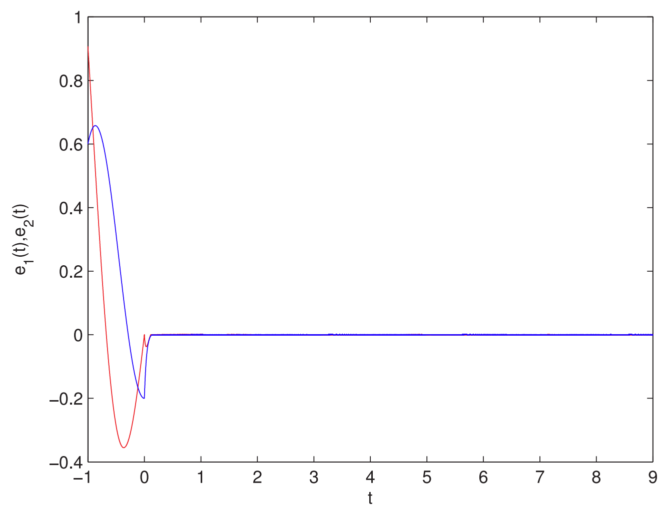

Fig. 9. Error state trajectories of variable $e_1(t), e_2(t)$.

$$b_{12}(x_1(t)) = \begin{cases} 0.1, & \dfrac{df_2(x_2(t))}{dt} - \dfrac{dx_1(t)}{dt} \le 0, \\ 0.09, & \dfrac{df_2(x_2(t))}{dt} - \dfrac{dx_1(t)}{dt} > 0, \end{cases}$$

$$b_{21}(x_2(t)) = \begin{cases} 0.09 & \dfrac{df_1(x_1(t))}{dt} - \dfrac{dx_2(t)}{dt} \le 0, \\ 0.1, & \dfrac{df_1(x_1(t))}{dt} - \dfrac{dx_2(t)}{dt} > 0, \end{cases}$$

$$b_{22}(x_2(t)) = \begin{cases} -\sqrt{2}-0.1, & -\dfrac{df_2(x_2(t))}{dt} - \dfrac{dx_2(t)}{dt} \le 0, \\ -\sqrt{2}, & -\dfrac{df_2(x_2(t))}{dt} - \dfrac{dx_2(t)}{dt} > 0, \end{cases}$$

and behaved functions $c_i(x_i(t)) = x_i(t)$; activation functions $f_i(x_i) = g_i(x_i) = \sin(x_i)$; $i = 1, 2$; $\tau(t) = 1$; and $I_1 = I_2 = 0.1$. We choose the control gain $p_1 = 23, p_2 = 25$, and $\eta_1 = \eta_2 = 1$. It is obvious that conditions (a)–(b) in Theorem 2 hold. According to Theorem 2, we can conclude that system (18) will synchronize to the response system under state feedback control.

For the numerical simulations, we take all initial conditions $(\phi_1(t), \phi_2(t)) = (1.2*\sin(2*t), 0.9*\cos(3*t)), (\varphi_1(t), \varphi_2(t)) = (1.3*\sin(3*t), 0.7*\cos(2*t)), \forall t \in [-1, 0)$. Fig. 8 shows the phase trajectories of the memristor-based network model (18). Fig. 9 shows the driver system (18) achieve synchronization

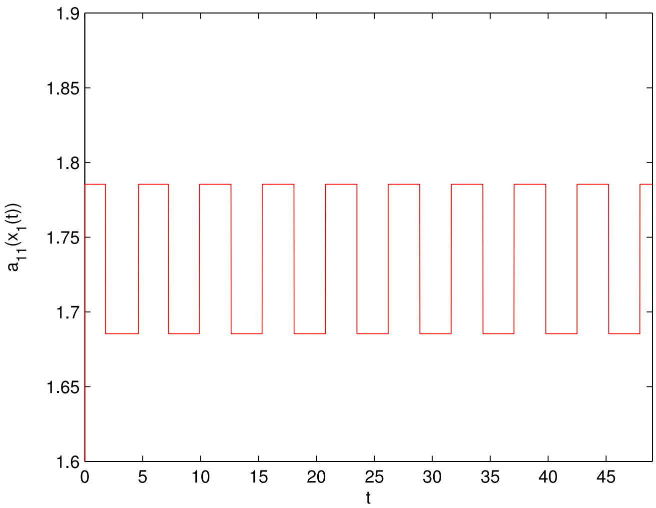

Fig. 10. Trajectories of connection weight coefficient $a_{11}(x_1(t))$.

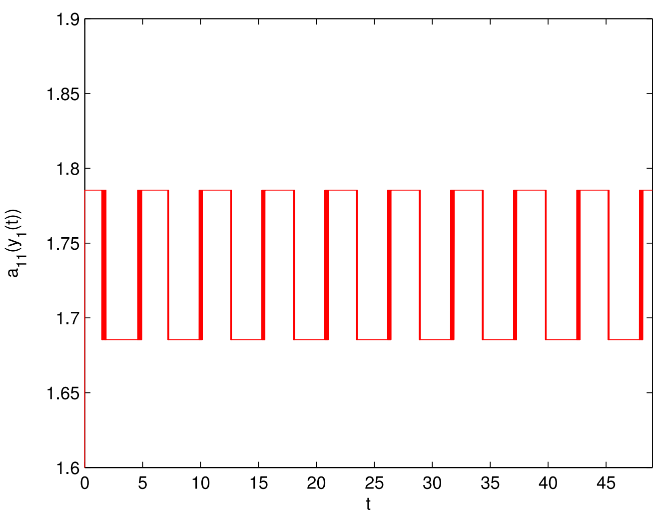

Fig. 11. Trajectories of connection weight coefficient $a_{11}(y_1(t))$.

under feedback control, and connection weight coefficients a_{11} change as the state $x_1(t)$ and $y_1(t)$ are shown in Figs. 10 and 11, respectively. The numerical simulations verify the effectiveness of Theorem 2.

6. Conclusions

We have considered the synchronization control of memristor-based neural networks with time-varying delay. Adaptive and feedback controllers have been taken into consideration. Synchronization criteria for memristor-based neural networks have been established by using the designed adaptive law, which is easy to verify. By simulation, we have found that the time trajectories of memristor-based neural networks depend significantly on their initial value. The analysis method in this paper may result in a new perspective on memristor-based neural networks. Several interesting problems exist for these networks. Some novel mathematics should still be improved upon. In our future work, we will consider introducing the semi-tensor product method to study memristor-based neural networks.

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