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Dynamic analysis of periodic solution for high-order discrete-time Cohen–Grossberg neural networks with time delays

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ABSTRACT

In this paper, we analyze the dynamic behavior of periodic solution for the high-order discrete-time Cohen–Grossberg neural networks (CGNNs) with time delays. First, the existence is studied based on the continuation theorem of coincidence degree theory and Young's inequality. And then, the criterion for the global exponential stability is given using Lyapunov method. Finally, simulation result shows the effectiveness of our proposed criterion.

1. Introduction

In the past ten years, the study on the dynamics of neural networks has attracted many researcher's attention (Cao, Feng, & Wang, 2008; Xiang & Cao, 2009). A new artificial neural network, CGNNs was first proposed in 1983. Its model is

$$x_i'(t) = -a_i(x_i(t))\left[b_i(x_i(t)) - \sum_{j=1}^{m} c_{ij}(t) f_j(x_j(t)) - \sum_{j=1}^{m} d_{ij}(t) f_j\left(x_j\left(t-\tau_{ij}(t)\right)\right) + I_i(t)\right],$$

$i = 1, 2, \ldots, m.$

Due to its wide application in parallel computation and signal processing, many researchers have studied the dynamics of CGNNs. And a large number of results have been made available (Cao & Song, 2006; Chen & Ruan, 2005; Cohen & Grossberg, 1983; Wang & Zou, 2002). However, most authors pay attention to low-order CGNNs (Huang, Chen, Huang, & Cao, 2007; Wang, Jiang, & Hu, 2014; Xiong & Cao, 2005). Because a low-order neural network has limitations in fault tolerance, convergence rate and storage capacity, it is needed to add high-order interaction to neural networks. The high-order neural network has higher fault tolerance, faster convergence rate and larger storage capacity than the low-order one. Some useful results on this topic have been presented (Chen, Zhao, & Fu, 2009; Chen, Zhao, & Ruan, 2007; Cheng et al., 2014; Nie & Cao, 2011; Qiu, 2010; Ren & Cao, 2007; Zhang, Qiu, & She, 2014).

The model of a high-order CGNNs with delay can be described by

$$x_i'(t) = -a_i(x_i(t))\left[b_i(x_i(t)) - \sum_{j=1}^{m} c_{ij}(t) f_j(x_j(t)) - \sum_{j=1}^{m}\sum_{k=1}^{m} d_{ijk}(t) f_j\left(x_j\left(t-\tau_{ij}(t)\right)\right) \times f_k(x_k(t-\tau_{ik}(t))) + I_i(t)\right], \quad (1)$$

where $i = 1, 2, \ldots, m$, $m \geq 2$ is the number of neurons in the networks, $x_i(t)$ denotes the state variable of the ith neuron at

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time t, $a_i(\cdot)$ represents an amplification function, and $b_i(\cdot)$ is an appropriately behaved function, $\tau_j(t)$ is time delay of jth neuron at time t, c_{ij} and d_{ijk} are the first-order and second-order connection weights of the neural networks, respectively. The activation function $f_j(x_j(t))$ denotes the output of the jth neuron at time t. $I_i(t)$ is the input from outside of the networks at time t.

To the best of our knowledge, there are few results about the properties of high-order discrete-time CGNNs until now. This is the motivation for the present study. In this paper, we discuss the existence and exponential stability of periodic solution for the following high-order discrete-time CGNNs:

$$\begin{aligned} x_i(n+1) = x_i(n) - a_i(x_i(n))\Bigg[& b_i(x_i(n)) - \sum_{j=1}^{m} c_{ij}(n) f_j(x_j(n)) \\ & - \sum_{j=1}^{m}\sum_{k=1}^{m} d_{ijk}(n) f_j(x_j(n-\tau_{ij}(n))) \\ & \times f_k(x_k(n-\tau_{ik}(n))) + I_i(n)\Bigg] \end{aligned} \tag{2}$$

where $c_{ij}(n)$, $d_{ijk}(n)$, $\tau_{ij}(n)$ and $\tau_{ik}(n)$ are ω-periodic non-negative integers. Initial conditions of (2) are

$$x_i(s) = \phi_i(s), \quad s \in [-\tau, 0],$$
$$\tau = \max_{0\le i,j\le m} \tau_{ij}, \qquad \tau_{ij} = \max_{0\le n\le\omega} \left\{|\tau_{ij}(n)|\right\}, \quad i = 1, 2, \ldots, m.$$
$$\overline{c_{ij}} = \max_{0\le n\le\omega} \left\{c_{ij}(n)\right\}, \qquad \overline{d_{ijk}} = \max_{0\le n\le\omega} \left\{d_{ijk}(n)\right\},$$
$$\bar{I}_i = \max_{0\le n\le\omega} \{I_i(n)\}.$$

We give some assumptions as follows.

- (H_1) $a_i(\mu), b_i(\mu) \in C(R, R), 0 < \underline{a}_i \le a_i(\mu) \le a_i, \underline{b}_i\mu \le b_i(\mu), 0 < \underline{a}_i\underline{b}_i < 1,\ i = 1, 2, \ldots, m.$
- (H_2) There exist non-negative constants N_i and e_i such that $f_i^2(x) \le N_i|x| + e_i, \forall x \in R,\ i = 1, 2, \ldots, m.$
- (H_3) There exists $M_j > 0$ such that $|f_j(x)| \le M_j$, $\left|f_j(x) - f_j(y)\right| \le L_j|x-y|, x \ne y,\ \forall x \in R,\ j = 1, 2, \ldots, m.$
- (H_4) There exists a non-negative constant L_i^a such that $|a_i(\mu) - a_i(\upsilon)| \le L_i^a|\mu - \upsilon|, \forall \mu, \upsilon \in R,\ i = 1, 2, \ldots, m.$
- (H_5) There exist a non-negative constant L_i^{ab} such that $(a_i(\mu)b_i(\mu) - a_i(\upsilon)b_i(\upsilon))(\mu - \upsilon) \ge 0, |a_i(\mu)b_i(\mu) - a_i(\upsilon)b_i(\upsilon)| \ge L_i^{ab}|\mu - \upsilon|, \forall \mu, \upsilon \in R.$

2. Existence of periodic solution

In this section, we present some definitions and lemmas before giving our main results.

Definition 1. Let $B = sE_n - A, s \in R, A \in R^{n\times n}$, and E_n be the identity matrix of size n. Suppose $s > 0$ and $A \ge 0$. If the spectral radius of A, $\rho(A)$, satisfies $\rho(A) \le s$, then B is a non-singular M-matrix. Here, $A \ge 0$ means that the items of A are non-negative.

From Definition 1, it is easy to get Lemma 1.

Lemma 1. *Let $A \ge 0$ be a real $n \times n$ matrix and $\rho(A) < 1$. Then, $E_n - A$ is a non-singular M-matrix.*

Lemma 2 (Zhang et al., 2014). *Let $A = (a_{ij})_{n\times n}$ with $a_{ij} \le 0,\ i,\ j = 1,\ 2, \ldots, m$, and $i \ne j$. The following statements are equivalent:*

1. *A is a non-singular M-matrix;*
2. *There exists a vector $\xi = [\xi_1, \xi_2, \ldots, \xi_n]^T > [0, 0, \ldots, 0]^T$ such that $A\xi > 0$;*
3. *$A^{-1} \ge 0$.*

Lemma 3 (Zhang et al., 2014). *Let Ω be an open bounded set in Banach space X, L : Dom $L \subset X \to X$ is a Fredholm operator with index zero, P : Dom $L \subset X \to \operatorname{Ker} L$, $Q : X \to X/\operatorname{Im} L$ are two projectors, and $N : \Omega \to X$ is L-compact on $\overline{\Omega}$. Moreover, we assume that the following conditions are satisfied:*

1. $Lx \ne \lambda Nx,\ \forall x \in \partial\Omega \cap \operatorname{Dom} L,\ \lambda \in (0, 1)$;
2. $QNx \ne 0,\ \forall x \in \partial\Omega \cap \operatorname{Ker} L$;
3. $\deg(QN, \Omega \cap \operatorname{Ker} L, 0) \ne 0$.

Then, the equation $Lx = Nx$ has at least one solution in $\overline{\Omega}$.

Remark 1. For the positive constants C and ε that satisfy $\frac{N_i}{C} \le \varepsilon$ and $\frac{C^2}{4} \ge e_i, i = 1, 2\cdots m$, we get

$$N_i\,|x| + e_i \le N_i\,|x| + \frac{C^2}{4} \le \left(\frac{N}{C}\,|x| + \frac{C}{2}\right)^2. \tag{3}$$

It follows from (H_2) that

$$f_i^2(x) \le N_i|x| + e_i. \tag{4}$$

Combining (3) and (4) yields

$$f_i^2(x) \le \left(\frac{N}{C}\,|x| + \frac{C}{2}\right)^2. \tag{5}$$

As a result, it is easy to get $|f_i(x)| \le \frac{N_i}{C}|x| + \frac{C}{2},\ \forall x \in R$. This conclusion will be used in the proof of the following Theorem 1.

Theorem 1. *Assume that (H_1) and (H_2) hold and that $E_m - H$ is a non-singular M-matrix, where $H = \begin{bmatrix} 0 & H_2 \\ H_1 & 0 \end{bmatrix}$, $H_i = \frac{a_i}{\underline{a}_i\underline{b}_i}\sum_{j=1}^{m}\sum_{k=1}^{m}\frac{\overline{d_{ijk}}}{2}\left(N_j + N_k\right)$, then the system (2) has at least one ω-periodic solution.*

Proof. Define $l_m = \{x = \{x = \{x(n)\} : x(n) \in R^m, n \in Z\}\}$. Let $l^\omega \in l_m$ denote the subspace of all ω-periodic sequences equipped with the norm $\|\cdot\|$, i.e.,

$$\|x\| = \left\|(x_1, \ldots, x_m)^T\right\| = \max_{k\in I_\omega}\sum_{i=1}^{m}|x_i(k)|, \quad x \in l^\omega,$$

where $I_\omega = \{0, 1, \ldots, \omega - 1\}$. Then l^ω is a Banach space.

Let $l_0^\omega = \left\{y \in \{y(n)\} \in l^\omega : \sum_{n=0}^{\omega-1} y(n) = 0\right\}$, $l_c^\omega = \{y = \{y(n)\} \in l^\omega : y(n) = c \in R^m, n \in Z\}$. Then l_0^ω and l_c^ω are both closed linear subspaces of l^ω, and $l^\omega = l_0^\omega \oplus l_c^\omega$, $\dim l_c^\omega = m$. Take $X = Y = l^\omega$, let

$$\begin{aligned} Nx_i(n) = -a_i\left(x_i(n)\right)\Bigg[& b_i\left(x_i(n)\right) - \sum_{j=1}^{m} c_{ij}(n) f_j\left(x_j(n)\right) \\ & - \sum_{j=1}^{m}\sum_{k=1}^{m} d_{ijk}(n) f_j\left(x_j\left(n-\tau_{ij}(n)\right)\right) \\ & \times f_k\left(x_k\left(n-\tau_{ik}(n)\right)\right) + I_i(n)\Bigg], \end{aligned}$$

and $(Lx)(n) = x(n+1) - x(n)$ for $x \in X, n \in Z$.

Then L is a bounded linear operator with $\operatorname{Ker} L = l_c^\omega$, $\operatorname{Im} L = l_0^\omega$. It follows that L is a Fredholm mapping of index zero. We define

$$Px = \frac{1}{\omega}\sum_{n=0}^{\omega-1} x(n), \quad x \in X, \quad \text{and}$$

$$Qy = \frac{1}{\omega}\sum_{n=0}^{\omega-1} y(n), \quad y \in Y.$$

It is easy to show that P and Q are continuous projectors that satisfied $\operatorname{Im} P = \operatorname{Ker} L$ and $\operatorname{Im} L = \operatorname{Ker} Q = \operatorname{Im} I - Q$, where I is the identity mapping. What is more, the generalized inverse (of L) $K_P : \operatorname{Im} L \to \operatorname{Ker} P \cap \operatorname{Dom} L$ exists. Obviously, QN and $K_p(I-Q)N$ are continuous. Since X is a finite-dimensional Banach space, we cannot difficultly get that $K_P(I-Q)N(\overline{\Omega})$ is compact for any bounded set $\Omega \subset X$. Moreover, because $QN(\overline{\Omega})$ is bounded, N is an L-compact on $\overline{\Omega}$ with any open bounded set $\Omega \subset X$.

Denote $x_i(\xi) = \min_{n\in I_\omega} x_i(n), \xi \in I_\omega, i = 1, 2, \dots, m$. From Remark 1 and $Lx = \lambda Nx$, $\lambda \in (0, 1)$, we have

$$\begin{aligned}
x_i(\xi) &\le x_i(n+1)\\
&= x_i(n) - a_i\left(x_i(n)\right)\Bigg[b_i\left(x_i(n)\right) - \sum_{j=1}^m c_{ij}(n) f_j\left(x_j(n)\right)\\
&\quad - \sum_{j=1}^m\sum_{k=1}^m d_{ijk}(n) f_j\left(x_j\left(n-\tau_{ij}(n)\right)\right)\\
&\quad \times f_k\left(x_k\left(n-\tau_{ik}(n)\right)\right) + I_i(n)\Bigg]\\
&\le \left(1-\lambda \underline{a_i}\,\underline{b_i}\right)|x_i(n)| + \lambda a_i \sum_{j=1}^m |c_{ij}(n) f_j\left(x_j(n)\right)|\\
&\quad + \lambda a_i \sum_{j=1}^m\sum_{k=1}^m d_{ijk}\\
&\quad \times \left|f_j\left(x_j\left(n-\tau_{ij}(n)\right)\right) f_k\left(x_k\left(n-\tau_{ik}(n)\right)\right)\right| + \lambda a_i I_i(n)\\
&\le \left(1-\lambda \underline{a_i}\,\underline{b_i}\right)|x_i(n)| + \lambda a_i \sum_{j=1}^m \overline{c_{ij}}\left(\frac{N_j}{C}|x_j(n)| + \frac{C}{2}\right)\\
&\quad + \lambda a_i \sum_{j=1}^m\sum_{k=1}^m d_{ijk}\frac{1}{2}\left(f_j^2\left(x_j\left(n-\tau_{ij}(n)\right)\right)\right.\\
&\quad \left. + f_k^2\left(x_k\left(n-\tau_{ik}(n)\right)\right)\right) + \lambda a_i \overline{I_i}\\
&\le \left(1-\lambda \underline{a_i}\,\underline{b_i}\right)|x_i(n)| + \lambda a_i \sum_{j=1}^m \frac{\overline{c_{ij}} N_j}{C}|x_j(n)|\\
&\quad + \lambda a_i \sum_{j=1}^m \frac{\overline{c_{ij}} C}{2}\\
&\quad + \frac{\lambda a_i}{2}\sum_{j=1}^m\sum_{k=1}^m \overline{d_{ijk}}\left(N_j|x_j(n-\tau_{ij}(n))| + N_k|x_k(n-\tau_{ik}(n))|\right)\\
&\quad + \frac{\lambda a_i}{2}\sum_{j=1}^m\sum_{k=1}^m \overline{d_{ijk}}\left(e_j+e_k\right) + \lambda a_i \overline{I_i}.
\end{aligned}$$

It follows that

$$\begin{aligned}
&\max|x_i(n)| \le \left(1-\lambda \underline{a_i}\,\underline{b_i}\right)\max|x_i(n)|\\
&\quad + \lambda a_i \sum_{j=1}^m \overline{c_{ij}}\varepsilon \max|x_j(n)| + \lambda a_i \sum_{j=1}^m \frac{\overline{c_{ij}} C}{2}\\
&\quad + \lambda a_i \sum_{j=1}^m\sum_{k=1}^m \frac{\overline{d_{ijk}}}{2}\left(N_j+N_k\right)\max|x_i(n)|\\
&\quad + \frac{\lambda a_i}{2}\sum_{j=1}^m\sum_{k=1}^m \overline{d_{ijk}}\left(e_j+e_k\right) + \lambda a_i \overline{I_i}\\
&\le \left(1-\lambda \underline{a_i}\,\underline{b_i}\right)\max|x_i(n)| + \lambda \underline{a_i}\,\underline{b_i}\left[\frac{a_i}{\underline{a_i}\,\underline{b_i}}\sum_{j=1}^m \overline{c_{ij}}\varepsilon\right.\\
&\quad \left. + \frac{a_i}{\underline{a_i}\,\underline{b_i}}\sum_{j=1}^m\sum_{k=1}^m \frac{\overline{d_{ijk}}}{2}(N_j+N_k)\right]\max|x_i(n)| + \lambda \underline{a_i}\,\underline{b_i}\delta_i
\end{aligned}\tag{6}$$

where $\delta_i = \frac{a_i}{\underline{a_i}\,\underline{b_i}}\left(\sum_{j=1}^m \frac{\overline{c_{ij}}}{2}C + \frac{1}{2}\sum_{j=1}^m\sum_{k=1}^m \overline{d_{ijk}}\left(e_j+e_k\right) + \overline{I_i}\right)$, $\varepsilon = \max_{1\le j\le m}\frac{N_j}{C}$.

From Lemma 2 and the fact that $E_m - H$ is a non-singular M-matrix, we know that there exists a vector $\eta = [\eta_1, \dots, \eta_n]^T > 0$ such that $(E_m - H)\eta > 0$. So, there exists a small $\varepsilon > 0$ such that $(E_m - H)\eta > \varepsilon B\eta$, that is $(E_m - (H + \varepsilon B))\,\eta > 0$. It follows from Lemma 2 that $E_m - (H + \varepsilon B)$ is a non-singular M-matrix. Set $F = [\delta_1, \dots, \delta_m]$,. It follows from (6) that

$$E_m - (H+\varepsilon B)[|x_1(n)|, \dots, |x_m(n)|]^T \le F,$$

$$B = \frac{a_i}{\underline{a_i}\,\underline{b_i}}\sum_{j=1}^m \overline{c_{ij}}, \qquad H = \frac{a_i}{\underline{a_i}\,\underline{b_i}}\sum_{j=1}^m\sum_{k=1}^m \frac{\overline{d_{ijk}}}{2}(N_j+N_k).$$

Since $E_m - (H + \varepsilon B)$ is a non-singular M-matrix, we obtain

$$\xi = [\xi_1, \dots, \xi_m]^T = (E_m - (H+\varepsilon B))^{-1}F + [1, \dots, 1]^T > 0.$$

This implies that $[|x_1(n)|, \dots, |x_2(n)|]^T < \xi$, that is, $|x_i(n)| < \xi$, $i = 1, 2, \dots, m$. Set $d = \sum_{i=1}^m \xi_i$. So, d is independent of λ and $\|x\| = \sum_{i=1}^m |x_i(n)| < d$. Consequently, we take an open bounded subset to be $\Omega = \{x(n)|x(n) \in X, \|x\| < d\}$. Then, it is easy to get $Lx \ne \lambda Nx$, $\forall x \in \partial\Omega \cap \operatorname{Dom} L$, $\lambda \in (0, 1)$, This proves that the first condition of Lemma 3 is satisfied.

Second, if $x(n) \in \partial\Omega \cap \operatorname{Ker} L$, then $x(n) = \alpha = [\alpha_1, \dots, \alpha_m]^T$ is a constant vector in R^m, $\|x\| = \|\alpha\| = \sum_{i=1}^m |\alpha_i| = d$. We set

$$\widehat{a_i}(n) = \frac{1}{\omega}\sum_{n=0}^{\omega-1} a_i(n), \qquad \widehat{b_i}(n) = \frac{1}{\omega}\sum_{n=0}^{\omega-1} b_i(n),$$

$$\widehat{c_{ij}} = \frac{1}{\omega}\sum_{n=0}^{\omega-1} c_{ij}(n),$$

$$\widehat{d_{ijk}} = \frac{1}{\omega}\sum_{n=0}^{\omega-1} d_{ijk}(n), \qquad \widehat{I_i} = \frac{1}{\omega}\sum_{n=0}^{\omega-1} I_i(n).$$

Then

$$\begin{aligned}
QNx_i(n) &= -\widehat{a_i}\left(x_i(n)\right)\Bigg[\widehat{b_i}\left(x_i(n)\right) - \sum_{j=1}^m \widehat{c_{ij}}(n) f_j(\alpha_j)\\
&\quad - \sum_{j=1}^m\sum_{k=1}^m \widehat{d_{ijk}}(n) f_j(\alpha_j) f_k(\alpha_k) + \widehat{I_i}(n)\Bigg].
\end{aligned}$$

We claim that $QNx(n) \ne 0$, $\forall x(n) \in \partial\Omega \cap \operatorname{Ker} L$. If it is not true, then

$$QNx_i(n) = QN\alpha_i = 0, \quad i = 1, 2, \dots, m.$$

It follows that

$$\begin{aligned}
&-\widehat{a_i}\left(x_i(n)\right)\Bigg[\widehat{b_i}\left(x_i(n)\right) - \sum_{j=1}^m \widehat{c_{ij}}(n) f_j(\alpha_j)\\
&\quad - \sum_{j=1}^m\sum_{k=1}^m \widehat{d_{ijk}}(n) f_j(\alpha_j) f_k(\alpha_k) + \widehat{I_i}(n)\Bigg] = 0.
\end{aligned}$$

Following the same deduction procedure shown above yields

$$\begin{aligned}
&\max|\alpha_i|\\
&\quad < \frac{a_i}{\underline{a_i}\,\underline{b_i}}\sum_{j=1}^m\left[\overline{c_{ij}}\varepsilon + \frac{a_i}{\underline{a_i}\,\underline{b_i}}\sum_{k=1}^m \frac{\overline{d_{ijk}}}{2}\left(N_j+N_k\right)\right]\max|\alpha_j| + \delta_i.
\end{aligned}$$

It implies that $(E-(H+\varepsilon B))[\alpha_1,\alpha_2,\ldots,\alpha_m]^T \le F$. Thus, we have $|\alpha_i| < \xi_i$, $\|\alpha\| = \sum_{i=1}^m |\alpha_i| < \sum_{i=1}^m \xi_i = d$ which contradicts $\|x\| = \|\alpha\| = \sum_{i=1}^m |\alpha_i| = d$. That gives $QNx(n) \neq 0, \forall x(n) \in \partial\Omega \cap \operatorname{Ker} L$. So, we verify the second condition of Lemma 3.

Next, we define a continuous function

$$S : \Omega \times [0,1] \to X \quad S(x,\mu) = [S(x_1,\mu),\ldots,S(x_m,\mu)],$$

where

$$S(x_i,\mu) = -\mu\,\widehat{a_i}(x_i(n))\,\widehat{b_i}(x_i(n)) + (1-\mu)QNx_i(n),\quad i=1,2,\ldots,m.$$

If $S(x,\mu)=0$, $\forall x(n) \in \partial\Omega \cap \operatorname{Ker} L$, then $x(n) = \alpha = [\alpha_1,\ldots,\alpha_m]^T$ is a constant vector in R^m with $\|\alpha\| = d$, and

$$\begin{aligned} S(x_i,\mu) &= S(a_i,\mu) \\ &= -\mu\,\widehat{a_i}(x_i(n))\,\widehat{b_i}(x_i(n)) + (1-\mu)\Bigg\{-\widehat{a_i}(x_i(n))\Bigg[\widehat{b_i}(x_i(n)) \\ &\quad - \sum_{j=1}^m \widehat{c_{ij}} f_j(a_j) - \sum_{j=1}^m\sum_{k=1}^m \widehat{d_{ijk}} f_j(a_j) f_k(a_k) + \widehat{I_i}\Bigg]\Bigg\} = 0 \end{aligned}$$

for $i=1,2,\ldots,m$. It follows that

$$\begin{aligned} &\widehat{a_i}(x_i(n))\,\widehat{b_i}(x_i(n)) = (1-\mu)\,\widehat{a_i}(x_i(n)) \\ &\quad\times\left[\sum_{j=1}^m \widehat{c_{ij}}(n) f_j(\alpha_j) + \sum_{j=1}^m\sum_{k=1}^m \widehat{d_{ijk}} f_j(\alpha_j) f_k(\alpha_k) + \widehat{I_i}(n)\right] \\ &\underline{a_i}\,\underline{b_i}\,|a_i| \le (1-\mu)a_i\left[\sum_{j=1}^m \overline{c_{ij}} f_j(\alpha_j) + \sum_{j=1}^m\sum_{k=1}^m \overline{d_{ijk}} f_j(\alpha_j) f_k(\alpha_k) + \overline{I_i}\right] \\ &\max|a_i| \le \frac{(1-\mu)a_i}{\underline{a_i}\,\underline{b_i}}\left[\sum_{j=1}^m \overline{c_{ij}} f_j(\alpha_j) + \sum_{j=1}^m\sum_{k=1}^m \overline{d_{ijk}} f_j(\alpha_j) f_k(\alpha_k) + \overline{I_i}\right] \\ &\le \frac{a_i}{\underline{a_i}\,\underline{b_i}}\left[\sum_{j=1}^m \overline{c_{ij}} f_j(\alpha_j) + \sum_{j=1}^m\sum_{k=1}^m \overline{d_{ijk}} f_j(\alpha_j) f_k(\alpha_k) + \overline{I_i}\right] \\ &\le \frac{a_i}{\underline{a_i}\,\underline{b_i}}\left[\sum_{j=1}^m \overline{c_{ij}}\varepsilon + \sum_{j=1}^m\sum_{k=1}^m \frac{\overline{d_{ijk}}}{2}(N_j+N_k)\right]\max|\alpha_i| + \delta_i. \end{aligned}$$

In the same manner, we obtain $\|\alpha\| < d$. So, we can conclude that $S(x,\mu) \neq 0$ for $\mu \in [0,1]$ and every $x \in \partial\Omega \cap \operatorname{Ker} L$. Combining the homotopy invariance theorem gives

$$\begin{aligned} \deg(QN, \Omega \cap \operatorname{Ker} L, 0) &= \deg(S(\cdot,0), \Omega \cap \operatorname{Ker} L, 0) \\ &= \deg(S(\cdot,1), \Omega \cap \operatorname{Ker} L, 0) \neq 0. \end{aligned}$$

As a result, the third condition in Lemma 3 is satisfied.

From Lemma 3, we know that $Lx(n) = Nx(n)$ has at least one solution in $\overline{\Omega}$. That is, the system has at least one ω-periodic solution. The proof is completed.

Remark 2. In previous work (Chen et al., 2009, 2007; Cheng et al., 2014; Ren & Cao, 2007), the condition $|f_j(x)| \le M_j$ is needed when proving the existence of periodic solution for the high-order CGNNs. However, we remove this limitation by using Young's inequality in this paper. This is the main advantage of our obtained results over the previous work.

3. Uniqueness and exponential stability

In this section, we use the Lyapunov theory to study the uniqueness and exponential stability of the system (2).

Theorem 2. *Assume that* $(H_1), (H_3), (H_4), (H_5)$ *are satisfied, and* $L_i^{ab} < 1$,

$$\begin{aligned} &L_i^{ab} - L_i^a \sum_{j=1}^m \overline{c_{ij}} M_j - L_i^a \sum_{j=1}^m\sum_{k=1}^m \overline{d_{ijk}} M_j M_k - L_i^a \overline{I_i} \\ &\quad - L_i \sum_{j=1}^m a_j \overline{c_{ij}} - L_i \sum_{j=1}^m\sum_{k=1}^m a_j\left(\overline{d_{jik}} + \overline{d_{jki}}\right) M_k > 0, \\ &\quad i = 1,2,\ldots,m \end{aligned} \tag{7}$$

then the system (2) has exactly one ω-periodic solution $x^*(n) = \left[x_1^*(n), x_2^*(n),\ldots,x_m^*(n)\right]^T$. Moreover, it is globally exponentially stable, that is, $x^*(n)$ satisfies the following inequality for any solution of (2) $x(n) = [x_1(n), x_2(n),\ldots,x_m(n)]^T$:

$$\sum_{i=1}^m |x_i(n) - x_i^*(n)| \le \upsilon\left[\frac{1}{\xi}\right]^n \sum_{i=1}^m \left[\sup_{s\in[-\tau,0]} \left|\varphi_i(s) - x_i^*(n)\right|\right], \tag{8}$$

where υ, $\xi > 1$ are constants, and $\varphi(s) = [\varphi_1(s), \varphi_2(s),\ldots,\varphi_m(s)]^T$ is the initial condition of $x(n)$.

Proof. If (H_3) holds, (H_2) is satisfied with $N_i = 0$. It is easy to get $H = 0$ in Theorem 1. Moreover, Definition 1 gives that $E_m - H$ is a non-singular M-matrix. So, Theorem 1 tells us that the system (2) exists at least one ω-periodic solution $x^*(n) = \left[x_1^*(n), x_2^*(n),\ldots,x_m^*(n)\right]^T$.

Let $u_i(n) = x_i(n) - x_i^*(n)$, $i = 1,2\ldots m$. The system (2) changes to

$$\begin{aligned} u_i(n+1) &= u_i(n) - \alpha_i(u_i(n)) + \beta_i(u_i(n)) + \overline{\beta_i}(u_i(n)) \\ &\quad + \gamma_i(u_i(n)) + \overline{\gamma_i}(u_i(n)) - \theta_i(u_i(n)), \end{aligned}$$

where

$$\begin{aligned} \alpha_i(u_i(n)) &= a_i\left(u_i(n) + x_i^*(n)\right) b_i\left(u_i(n) + x_i^*(n)\right) \\ &\quad - a_i\left(x_i^*(n)\right) b_i\left(x_i^*(n)\right), \\ \beta_i(u_i(n)) &= a_i\left(u_i(n) + x_i^*(n)\right) \sum_{j=1}^m c_{ij}(n) \\ &\quad \times \left[f_j\left(u_j(n) + x_j^*(n)\right) - f_j\left(x_j^*(n)\right)\right], \\ \overline{\beta_i}(u_i(n)) &= a_i\left(u_i(n) + x_i^*(n)\right) \sum_{j=1}^m\sum_{k=1}^m d_{ijk}(n) \\ &\quad \times \Big[f_j\left(u_j\left(n - \tau_{ij}(n)\right) + x_j^*\left(n - \tau_{ij}(n)\right)\right) \\ &\quad \times f_k\left(u_k\left(n - \tau_{ik}(n)\right) + x_k^*\left(n - \tau_{ik}(n)\right)\right) \\ &\quad - f_j\left(x_j^*\left(n - \tau_{ij}(n)\right)\right) f_k\left(x_k^*\left(n - \tau_{ik}(n)\right)\right)\Big], \\ \gamma_i(u_i(n)) &= \left[a_i\left(u_i(n) + x_i^*(n)\right) - a_i\left(x_i^*(n)\right)\right] \\ &\quad \times \sum_{j=1}^m c_{ij}(n) f_j\left(x_j^*(n)\right), \\ \overline{\gamma_i}(u_i(n)) &= \left[a_i\left(u_i(n) + x_i^*(n)\right) - a_i\left(x_i^*(n)\right)\right] \\ &\quad \times \sum_{j=1}^m\sum_{k=1}^m d_{ijk}(n) * f_j\left(x_j^*\left(n - \tau_{ij}(n)\right)\right) \\ &\quad \times f_k\left(x_k^*\left(n - \tau_{ik}(n)\right)\right), \\ \theta_i(u_i(n)) &= \left[a_i\left(u_i(n) + x_i^*(n)\right) - a_i\left(x_i^*(n)\right)\right] I_i(n). \end{aligned}$$

Let

$$F(\mu) = 1-\mu+L_i^{ab}\mu-L_i^a\mu\sum_{j=1}^m\overline{c_{ij}}M_j - L_i^a\mu\sum_{j=1}^m\sum_{k=1}^m\overline{d_{ijk}}M_jM_k - L_i^a\mu\overline{I_i} - L_i\mu\sum_{j=1}^m a_j\overline{c_{ij}} - L_i\sum_{j=1}^m\sum_{k=1}^m a_j\left(\overline{d_{jik}}+\overline{d_{jki}}\right)M_k\mu^{\tau_{ji}+1},$$
$$i=1,2,\ldots,m.\ \mu\in[1,\infty).$$

So,

$$F(1) = L_i^{ab}-L_i^a\sum_{j=1}^m\overline{c_{ij}}M_j-L_i^a\sum_{j=1}^m\sum_{k=1}^m\overline{d_{ijk}}M_jM_k - L_i^a\overline{I_i}-L_i\sum_{j=1}^m a_j\overline{c_{ij}} - L_i\sum_{j=1}^m\sum_{k=1}^m a_j\left(\overline{d_{jik}}+\overline{d_{jki}}\right)M_k > 0.$$

Since F_i is continuous on $[1,\infty)$, there exists $\xi>1$, such that

$$F(\xi) = 1-\xi+L_i^{ab}\xi-L_i^a\xi\sum_{j=1}^m\overline{c_{ij}}M_j - L_i^a\xi\sum_{j=1}^m\sum_{k=1}^m\overline{d_{ijk}}M_jM_k - L_i^a\xi\overline{I_i}-L_i\xi\sum_{j=1}^m a_j\overline{c_{ij}} - L_i\sum_{j=1}^m\sum_{k=1}^m a_j\left(\overline{d_{jik}}+\overline{d_{jki}}\right)M_k\xi^{\tau_{ji}+1}\ge 0,$$
$$i=1,2,\ldots,m. \tag{9}$$

Set $z_i(n)=\xi^n|u_i(n)|$, then

$$\begin{aligned}
z_i(n+1) &= \xi^{n+1}|u_i(n+1)| \\
&\le \xi^{n+1}[|u_i(n)-\alpha_i(u_i(n))|+|\beta_i(u_i(n))| + |\overline{\beta_i}(u_i(n))|+|\gamma_i(u_i(n))|+|\overline{\gamma_i}(u_i(n))|+|\theta_i(u_i(n))|] \\
&\le \xi^{n+1}\Bigg\{\left(1-L_i^{ab}\right)|u_i(n)|+a_i\sum_{j=1}^m\overline{c_{ij}}L_j|u_j(n)| \\
&\quad + a_i\sum_{k=1}^m\sum_{j=1}^m\left(\overline{d_{ikj}}+\overline{d_{ijk}}\right)M_kL_jf_j\left|u_j\left(n-\tau_{ij}(n)\right)\right| \\
&\quad + L_i^a\sum_{j=1}^m\overline{c_{ij}}M_j|u_i(n)| \\
&\quad + L_i^a\sum_{j=1}^m\sum_{k=1}^m\overline{d_{ijk}}M_jM_k|u_i(n)|+L_i^a\overline{I_i}|u_i(n)|\Bigg\} \\
&\le \xi^{n+1}\Bigg\{1-L_i^{ab}+L_i^a\sum_{j=1}^m\overline{c_{ij}}M_j \\
&\quad + L_i^a\sum_{j=1}^m\sum_{k=1}^m\overline{d_{ijk}}M_jM_k+L_i^a\overline{I_i}+L_i\sum_{j=1}^m\overline{c_{ji}}a_j\Bigg\}|u_i(n)| \\
&\quad + a_i\sum_{k=1}^m\sum_{j=1}^m\left(\overline{d_{jik}}+\overline{d_{jki}}\right)M_kL_jf_j\left|u_j\left(n-\tau_{ij}(n)\right)\right| \\
&\le \xi\Bigg\{1-L_i^{ab}+L_i^a\sum_{j=1}^m\overline{c_{ij}}M_j \\
&\quad + L_i^a\sum_{j=1}^m\sum_{k=1}^m\overline{d_{ijk}}M_jM_k+L_i^a\overline{I_i}+L_i\sum_{j=1}^m\overline{c_{ji}}a_j\Bigg\}|z_i(n)| \\
&\quad + a_i\sum_{k=1}^m\sum_{j=1}^m\left(\overline{d_{jik}}+\overline{d_{jki}}\right)M_kL_j\xi^{\tau_{ij}+1}|z_j(n-\tau_{ij}(n))|.
\end{aligned}$$

Let $V(n)=\sum_{i=1}^m\left\{z_i(n)+a_i\sum_{k=1}^m\sum_{j=1}^m\left(\overline{d_{jik}}+\overline{d_{jki}}\right)M_kL_j\xi^{\tau_{ij}+1}\sum_{s=n-\tau_{ij}}^{n-1}z_j(s)\right\}$. It gives

$$\begin{aligned}
\Delta V(n) &= V(n+1)-V(n) \\
&= \sum_{i=1}^m\Bigg\{z_i(n+1)+a_i\sum_{k=1}^m\sum_{j=1}^m\left(\overline{d_{jik}}+\overline{d_{jki}}\right)M_kL_j\xi^{\tau_{ij}+1} \\
&\quad\times\sum_{s=n-\tau_{ij}+1}^{n}z_j(s)-z_i(n) \\
&\quad - a_i\sum_{k=1}^m\sum_{j=1}^m\left(\overline{d_{jik}}+\overline{d_{jki}}\right)M_kL_j\xi^{\tau_{ij}+1}\sum_{s=n-\tau_{ij}}^{n-1}z_j(s)\Bigg\} \\
&\le \sum_{i=1}^m\Bigg\{\xi\Bigg\{1-L_i^{ab}+L_i^a\sum_{j=1}^m\overline{c_{ij}}M_j \\
&\quad + L_i^a\sum_{j=1}^m\sum_{k=1}^m\overline{d_{ijk}}M_jM_k+L_i^a\overline{I_i}+L_i\sum_{j=1}^m\overline{c_{ji}}a_j\Bigg\}|z_i(n)| \\
&\quad + a_i\sum_{k=1}^m\sum_{j=1}^m\left(\overline{d_{jik}}+\overline{d_{jki}}\right)M_kL_j\xi^{\tau_{ij}+1}z_j(n)-z_i(n)\Bigg\} \\
&= -\sum_{i=1}^m\Bigg[1-\xi+L_i^{ab}\xi-L_i^a\xi\sum_{j=1}^m\overline{c_{ij}}M_j \\
&\quad - L_i^a\xi\sum_{j=1}^m\sum_{k=1}^m\overline{d_{ijk}}M_jM_k-L_i^a\xi\overline{I_i}-L_i\xi\sum_{j=1}^m a_j\overline{c_{ij}} \\
&\quad - L_i\sum_{j=1}^m\sum_{k=1}^m a_j\left(\overline{d_{jik}}+\overline{d_{jki}}\right)M_k\xi^{\tau_{ji}+1}\Bigg]z_i(n)\le 0.
\end{aligned}$$

So, $V(n)\le V(0)$ and

$$\begin{aligned}
\sum_{i=1}^m z_i(n) &\le \sum_{i=1}^m\Bigg\{z_i(0)+a_i\sum_{k=1}^m\sum_{j=1}^m\left(\overline{d_{jik}}+\overline{d_{jki}}\right)M_kL_j\xi^{\tau_{ij}+1} \\
&\quad\times\sum_{s=-\tau_{ij}}^{-1}z_j(s)\Bigg\} \\
&\le \sum_{i=1}^m\left\{1+L_i\sum_{k=1}^m\sum_{j=1}^m\left(\overline{d_{jik}}+\overline{d_{jki}}\right)M_ka_j\xi^{\tau_{ij}+1}\tau_{ji}\right\} \\
&\quad\times\sup_{s\in[-\tau,0]}z_j(s).
\end{aligned}$$

We easily get the conclusion that

$$\sum_{i=1}^m|u_i(n)|\le v\left[\frac{1}{\xi}\right]^n\sum_{i=1}^m\left[\sup_{s\in[-\tau,0]}|\varphi_i(s)-x_i^*(n)|\right],$$

where $v=\max_{1\le i\le m}\left\{1+L_i\sum_{k=1}^m\sum_{j=1}^m\left(\overline{d_{jik}}+\overline{d_{jki}}\right)M_ka_j\xi^{\tau_{ij}+1}\tau_{ji}\right\}$. The proof is completed.

4. Illustrative example

In this section, an example is presented to demonstrate the validity of our theoretical results. Let us consider a model of this system with $m = 2$ as

$$\begin{cases} \triangle x_1(n) = -a_1\left(x_1(n)\right)\Bigg[b_1\left(x_1(n)\right) - \sum_{j=1}^{2} c_{1j}(n) f_j\left(x_j(n)\right) \\ \quad - \sum_{j=1}^{2}\sum_{k=1}^{2} d_{1jk}(n) f_j\left(x_j\left(n-\tau_{1j}(n)\right)\right) \\ \quad \times f_k\left(x_k\left(n-\tau_{1k}(n)\right)\right) + I_1(n)\Bigg] \\ \triangle x_2(n) = -a_2\left(x_2(n)\right)\Bigg[b_2\left(x_2(n)\right) - \sum_{j=1}^{2} c_{2j}(n) f_j\left(x_j(n)\right) \\ \quad - \sum_{j=1}^{2}\sum_{k=1}^{2} d_{2jk}(n) f_j\left(x_j\left(n-\tau_{2j}(n)\right)\right) \\ \quad \times f_k\left(x_k\left(n-\tau_{2k}(n)\right)\right) + I_2(n)\Bigg] \end{cases} \tag{10}$$

where $\triangle x_1 = x_1(n+1) - x_1(n)$, $\triangle x_2 = x_2(n+1) - x_2(n)$, and

$$\begin{cases} a_1\left(x_1(n)\right) = 1/2, \quad a_2\left(x_2(n)\right) = 1/2, \\ b_1(x) = 3/4(e^x + x), \quad b_2(x) = 1/2(e^x + x), \\ c_{11}(n) = \frac{1}{16}\sin\frac{n\pi}{2}, \quad c_{21}(n) = \frac{1}{14}\cos\frac{n\pi}{2}, \\ c_{12}(n) = c_{22}(n) = 0, \\ I_1(n) = I_2(n) = 1/20, \\ d_{112}(n) = \frac{1}{12}\sin\frac{n\pi}{2}, \quad d_{211}(n) = \frac{1}{16}\cos\frac{n\pi}{2}, \\ d_{111}(n) = d_{121}(n) = d_{122}(n) \\ \quad = d_{212}(n) = d_{221}(n) = d_{222}(n) = 0, \\ f_1(x) = \sin x, f_2(x) = \cos x. \end{cases} \tag{11}$$

Thus, the functions $a_i\left(x_i(n)\right), b_i\left(x_i(n)\right)$ satisfy the assumption (H_1) with $\underline{a}_1 = 1/2$, $a_1 = 1/2$, $\underline{b}_1 = 9/8$, $\underline{b}_2 = 3/4$, the assumption (H_4) with $L_1^a = 1$, $L_2^a = 1$, and the assumption (H_5) with $L_1^{ab} = 0.37$, $L_2^{ab} = 0.25$. Meanwhile, $c_{ij}(n)$, $d_{ij}(n)$ are 4-periodic functions. In addition, it is obvious to get that $f_1(x), f_2(x)$ satisfy the assumption (H_3) with $M_1 = 1$, $M_2 = 1$ and $L_1 = 1$, $L_2 = 1$. In addition, it is not difficult to verify that the condition (4) in Theorem 2 is satisfied. From Theorem 2, we get that the system has a unique 4-periodic solution which is globally exponentially stable. To verify the above conclusion, we arbitrarily choose the initial conditions of (10) to be

$$\begin{cases} \phi(-2) = [\phi_1(-2), \phi_2(-2)]^T = [-0.7, -0.7]^T, \\ \phi(-1) = [\phi_1(-1), \phi_2(-1)]^T = [-0.62, -0.65]^T, \\ \phi(0) = [\phi_1(0), \phi_2(0)]^T = [-0.63, -0.6]^T. \end{cases} \tag{12}$$

Simulation results show that the solution of (10) with (12) asymptotically converge to a 4-periodic sequence $x^{\#}(n) = \left[x_1^{\#}(n), x_2^{\#}(n)\right]^T$, the periodic of $x^{\#}(n)$ are

$x_1^{\#}(n): -0.6235, -0.5786, -0.5969, -0.6418;$

$x_2^{\#}(n): -0.6344, -0.6213, -0.6253, -0.6361.$

We claim that $x^{\#}(n)$ is the only one 4-periodic solution of (10). (See Fig. 1.)

In order to show the global stability of the periodic solution $x^{\#}(n)$, we arbitrarily choose two initial conditions for system (10) as

$$\begin{cases} \bar{\phi}(-2) = [\bar{\phi}_1(-2), \bar{\phi}_2(-2)]^T = [-0.5, -0.7]^T, \\ \bar{\phi}(-1) = [\bar{\phi}_1(-1), \bar{\phi}_2(-1)]^T = [-0.8, -0.9]^T, \\ \bar{\phi}(0) = [\bar{\phi}_1(0), \bar{\phi}_2(0)]^T = [-0.7, -0.8]^T. \end{cases} \tag{13}$$

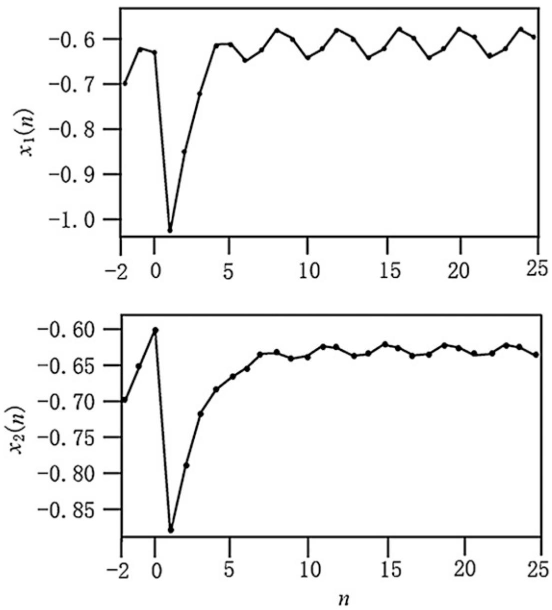

Fig. 1. Simulation results for (10) with initial condition (12).

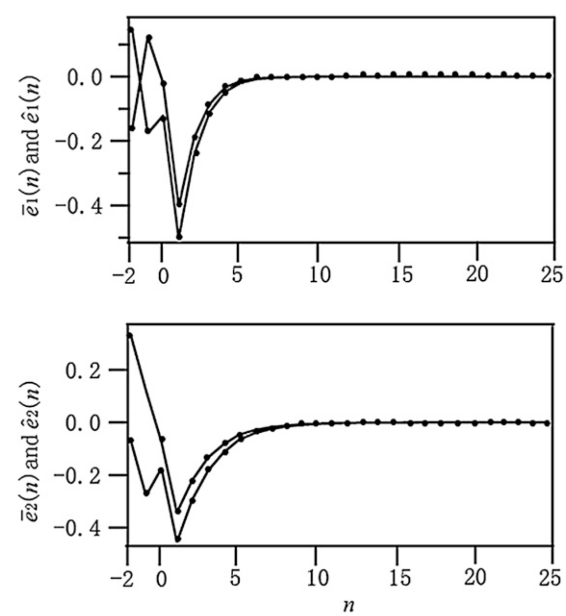

Fig. 2. Simulation results for $\bar{e}(n)$ and $\hat{e}(n)$ with from -2 to 25.

$$\begin{cases} \hat{\phi}(-2) = [\hat{\phi}_1(-2), \hat{\phi}_2(-2)]^T = [-0.7, -0.7]^T, \\ \hat{\phi}(-1) = [\hat{\phi}_1(-1), \hat{\phi}_2(-1)]^T = [-0.62, -0.65]^T, \\ \hat{\phi}(0) = [\hat{\phi}_1(0), \hat{\phi}_2(0)]^T = [-0.63, -0.6]^T. \end{cases} \tag{14}$$

We denote

$$\bar{e}(n) = [\bar{e}_1(n), \bar{e}_2(n)]^T \triangleq \bar{x}(n) - x^{\#}(n),$$
$$\hat{e}(n) = \left[\hat{e}_1(n), \hat{e}_2(n)\right]^T \triangleq \hat{x}(n) - x^{\#}(n),$$

where $\bar{x}(n) = [\bar{x}_1(n), \bar{x}_2(n)]^T$ is the solution of (10) with (13), and $\hat{x}(n) = \left[\hat{x}_1(n), \hat{x}_2(n)\right]^T$ is the solution of (10) with (14). The simulation results for the error states $\bar{e}(n)$ and $\hat{e}(n)$ are given in Fig. 2. Note that $\bar{e}(n)$ and $\hat{e}(n)$ quickly converge to zero. So, the periodic solution $x^{\#}(n)$ is globally stable.

5. Conclusions

In this paper, we analyze the dynamic behavior of periodic solution for the high-order discrete-time CGNNs with time delays. By using the continuation theorem and Young's inequality, the criterion for the existence of periodic solution has been derived. Secondly, a Lyapunov function is constructed to solve global exponential stability. Finally, a numerical example was presented to show the effectiveness of our theorems. The proposed criteria in this paper are easy to verify. In addition, the methods presented in this paper are easy to extend to other high-order discrete-time neural networks. In the future, we will further study the finite-time stabilization of high-order discrete-time CGNNs with time delays (Liu, Ho, Yu, & Cao, 2014; Liu, Jiang, & Cao, 2013; Liu, Park, Jiang, & Cao, 2014), and the dynamics of stochastic CGNNs with delays (Fu & Li, 2011; Li & Fu, 2011).

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