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A mismatched membership function approach to sampled-data stabilization for T-S fuzzy systems with time-varying delayed signals

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ABSTRACT

This paper is concerned with the issue of sampled-data stabilization for T-S fuzzy systems with time-varying delays, where the mismatched membership function (MMF) approach is proposed. The superiority of the proposed method lies that delay interval is split into flexible terminals, and newly Lyapunov–Krasovskii function (LKF) is constructed. By employing the Wirtinger inequality and extended Jensen inequality, some sufficient conditions of the sampled-data control for T-S fuzzy systems are established, which are efficiently solved by using standard available numerical packages. Finally, the superiority of achieved result is illustrated by the truck-trailer model.

1. Introduction

In recent years, T-S fuzzy systems are introduced to describe the nonlinear systems in the form of IF-THEN rules, which have been proved to be efficient approaches because of its approximation properties and received extensively interest. It is noted that the nonlinear dynamic systems can be described by T-S fuzzy model and recast in linear matrix inequalities (LMIs), the issue of stability analysis for T-S fuzzy system has been extensively studied [1–6] and references therein.

The time delay is frequently encountered in the dynamical systems and it brings the instability and performance degradation of T-S fuzzy systems. Up to now, there exists two type of results subject to time delay: delay-independent stability [7] and delay-dependent ones [8–10]. It is noted that the delay-dependent research achievements make full use of the delay information, which obtain less conservative results and achieve lager upper bound of delay. Moreover, the T-S fuzzy system experienced are frequently time-varying delays. The fundamental objective of delay-dependent stability conditions are to accomplish the upper bounds of time-varying delays. Recently, the problem of stability analysis and control synthesis is the crucial feature in T-S fuzzy systems with time-varying delay, which has gained significance attention and many important results has been reported [11–20],[35–37]. It can be seen from [11–20] that, to achieve less conservative stability conditions, various techniques have been introduced for T-S fuzzy systems, to mention a few, the Jensen's inequality [13], free-weighting matrix approach [14,34], Leibniz–Newton method [15] and so on. However, these approaches suffer a common shortcoming, for example, with the increase of the number of subintervals, which leads to the increasing of complexity of computing.

On the other hand, in sampled-data control systems, it is important to choose a proper sampling interval when designing suitable controllers. Over the past few years, the stability analysis and controller synthesis issues of sampled-data control systems have attracted much attention because of its widely applications [21–28], and several fruitful results have been presented for the sampled-data control systems combined with LKFs. However, it should be pointed out that in the existing literature, the problem of sampled-data stabilization for T-S fuzzy systems with time-varying delays has not been considered in the literature, and remains as a topic for further investigation.

Motivated by the above discussion, in this paper, the issue of sampled-data stabilization for T-S fuzzy systems with time-varying delays is investigated. The main contributions of this paper are summarized as follows.

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(i) The novel LKF involving triple integrals and tunable parameter is proposed, which contains the full information of sampling pattern and flexible time-varying delay intervals, and a new set of sufficient criteria are developed.
(ii) The mismatched membership functions are introduced to eliminate the conservatism in a more reasonable way.
(iii) Some novel cross terms of time delay are introduced and the less conservative results are achieved.
(iv) The proposed fuzzy sampled-data controller is designed and truck-trailer system is employed to illustrate the superiority of proposed approach.

Notation: R^n denotes the $n-$dimensional Euclidean space. $X \geq 0$ (respectively,$X > 0$) means that the matrix X is a real symmetric positive semidefinite matrix the(respectively, positive definite). Superscripts -1 and T denote the inverse and transpose, respectively. The notation $*$ is used for the transposed element in symmetric positions. $Sym\{X\}$ is defined as $Sym\{X\} = X^T + X$. $diag\{\ldots\}$ denotes a diagonal matrix.

2. Preliminaries

Consider an T-S fuzzy system with time-varying delays. The ith rule of the T-S fuzzy system is expressed in the following IF-THEN form

Rule i:
If $\theta_1(t)$ is W_1^i and, …, and $\theta_g(t)$ is W_g^i,
Then

$$\dot{x}(t) = A_i x(t) + A_{di} x(t-\tau(t)) + B_i u(t),$$
$$x(t) = \phi(t), t \in [-\max(\tau_M, h), 0] \tag{1}$$

where $i = 1, 2, \ldots, r$, r is the number of If-Then rules; $x(t) \in \mathbb{R}^{n_x}$ and $u(t) \in \mathbb{R}^{n_u}$ stands for state and input vector, respectively; $W_j^i (i = 1, 2, \ldots, r; j = 1, 2, \ldots, g)$ represents the fuzzy sets; $\theta_j(t)(j = 1, 2, \ldots, g)$ means the premise variables. Denote $\theta(t) = [\theta_1(t), \ldots, \theta_g(t)]^T$, and $\theta(t)$ is independent on $x(t)$ nor $u(t)$. $\phi(t)$ is the initial condition of the system state; $\tau(t)$ and h denotes the state delay and the sample period, respectively, which satisfies the following condition:

$$0 \leq \tau_m \leq \tau(t) \leq \tau_M, \tau_d \leq \dot{\tau}(t) \leq \tau_D \tag{2}$$

and A_i, A_{di}, B_i $(i = 1, 2, \ldots, r)$ are constant matrices with compatible dimensions.

With the help the method of centroid for defuzzification, the T-S fuzzy system (1) can be inferred as

$$\dot{x}(t) = \sum_{i=1}^{r} \mu_i(\theta(t))[A_i x(t) + A_{di} x(t-\tau(t)) + B_i u(t)] \tag{3}$$

where $\mu_i(\theta(t)) = \frac{h_i(\theta(t))}{\sum_{i=1}^{r} h_i(\theta(t))}$, $h_i(\theta(t)) = \prod_{j=1}^{g} W_j^i(\theta_j(t))$ and $W_j^i(\theta_j(t))$ is the membership value of $\theta_j(t)$ in W_j^i. It is easily seen that $\forall i \in \{1, 2, \ldots, r\}$, $\mu_i(\theta(t))$ satisfies the following properties

$$\mu_i(\theta(t)) \geq 0, \sum_{i=1}^{r} \mu_i(\theta(t)) = 1$$

Similar to the T-S fuzzy system described in (1), the fuzzy sampled-data controller can be transformed into:

Rule I:
If $\theta_1(t_k)$ is W_1^i and, …, and $\theta_g(t_k)$ is W_g^i,
Then

$$u_i(t) = K_i x(t_k), \ t_k \leq t < t_{k+1} \tag{4}$$

where $K_i (i = 1, 2, \ldots, r)$ denotes the controller gains to be estimated; t_k is the sampling instant satisfies $0 < t_1 < t_2 < \cdots < t_k \ldots$; $x(t_k)$ means the state vector of subsystem at the instant t_k, which is a piecewise constant function.

The defuzzified output of controller (4) is deduced as

$$u(t) = \sum_{i=1}^{r} \mu_i(\theta(t_k)) K_i x(t_k), \ t_k \leq t < t_{k+1} \tag{5}$$

Denote the sampling interval $d(t) = t - t_k$ for $t_k \leq t < t_{k+1}$. It follows from (5) that $0 \leq d(t) < t_{k+1} - t_k \leq h$, where h is the known bound of sampling instants. Furthermore, it is easily to obtain that $\dot{d}(t) = 1, t \neq t_d$. The system (5) can be rewritten as

$$u(t) = \sum_{i=1}^{r} \mu_i(\theta(t_k)) K_i x(t - d(t)), t_k \leq t < t_{k+1} \tag{6}$$

Combining (3) with (6) yields the following closed-loop sampled-data fuzzy system (CLSDFS)

$$\dot{x}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i(\theta(t)) \mu_j(\theta(t_k))$$
$$\times [A_i x(t) + A_{di} x(t-\tau(t)) + B_i K_j x(t - d(t))] \tag{7}$$

Lemma 2.1. *[29] For any $t \in [t_k, t_{k+1})$, and symmetric matrices $Q_{ij} \in \mathbb{R}^{n \times n}$, the inequality $Q(x, x_k) = \sum_{i=1}^{r} \sum_{j=1}^{r} \omega_i(x(t)) \omega_j(x(t_k)) Q_{ij} < 0$ holds, if the functions $\omega_i(x(t))$ and $\omega_i(x(t_k))$ satisfying*

$$| \omega_i(x(t)) - \omega_i(x(t_k)) | \leq \delta_i, i \in \mathbb{S}$$

for given scalars $\delta_i > 0$ and matrices $R_{ij} > 0$, $N_{ij} > 0$, $U_{ij} = U_{ij}^T$, $U_{i(j+r)} = U_{(j+r)i}^T$, $\forall i, j \in \mathbb{S}$ such that

$$W_{ij} + W_{ji} \leq U_{ij} + U_{ji}$$

$$Q_{ij} - 2W_{ij} + \sum_{s=1}^{r} \delta_s (W_{is}^{+} + W_{sj}^{+}) \leq U_{i(j+r)} + U_{(j+r)i}$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ * & Y_{11} \end{bmatrix} < 0$$

where

$$W_{ij} = N_{ij} - R_{ij}, W_{ij}^{+} = R_{ij} + N_{ij}$$

$$Y_{11} = \begin{bmatrix} U_{11} & \ldots & U_{1r} \\ \vdots & \ddots & \vdots \\ U_{r1} & \ldots & U_{rr} \end{bmatrix}, \quad Y_{12} = \begin{bmatrix} U_{1(r+1)} & \ldots & U_{1(2r)} \\ \vdots & \ddots & \vdots \\ U_{r(r+1)} & \ldots & U_{r(2r)} \end{bmatrix}$$

Lemma 2.2. *[30] Let $a_l > 0 (l = 1, 2, \ldots, n) : \mathfrak{R}^m \mapsto \mathfrak{R}$, which belongs to an open subset $\mathbb{D}$ of $\mathfrak{R}^m$. One has*

$$\min_{(a_i | a_i > 0, \sum_i a_i = 1)} \sum_i \frac{1}{a_i} s_i(t) = \sum_i s_i(t) + \max_{b_{i,j(t)}} \sum_{i \neq j} b_{i,j(t)}$$

subject to

$$\{b_{i,j} : \mathcal{R}^m \mapsto \mathcal{R}, b_{j,i(t)} \triangleq b_{i,j(t)}, \begin{bmatrix} s_i(t) & b_{i,j(t)} \\ b_{j,i(t)} & s_j(t) \end{bmatrix} \geq 0\}$$

Lemma 2.3. *[31] Denote x is a differentiable function: $[\alpha, \beta] \to \mathcal{R}^n$. For symmetric matrices $\mathcal{R} > 0$ and $\mathcal{N}_i \in \mathcal{R}^{4n \times n}$, the following inequality holds:*

$$-\int_{\alpha}^{\beta} \dot{x}^T(s) R \dot{x}(s) ds \leq \vartheta^T \Omega \vartheta,$$

where

$$\Omega = (\beta - \alpha)\Big(\mathcal{N}_1 \mathcal{R}^{-1} \mathcal{N}_1^T + \frac{1}{3} \mathcal{N}_2 \mathcal{R}^{-1} \mathcal{N}_2^T + \frac{1}{5} \mathcal{N}_3 \mathcal{R}^{-1} \mathcal{N}_3^T\Big)$$
$$+ Sym\{\mathcal{N}_1 \Xi_1 + \mathcal{N}_2 \Xi_2 + \mathcal{N}_3 \Xi_3\}$$

$$\Xi_1 = e_1 - e_2 \quad \Xi_2 = e_1 + e_2 - 2e_3 \quad \Xi_3 = e_1 - e_2 - 6e_3 + 6e_4$$

$$\vartheta = \begin{bmatrix} x^T(\beta) & x^T(\alpha) & \frac{1}{\beta-\alpha}\int_\alpha^\beta x^T(s)ds \\ & \frac{2}{(\beta-\alpha)^2}\int_\alpha^\beta\int_\alpha^s x^T(u)duds]^T \end{bmatrix}.$$

Lemma 2.4. [32] *If there exist matrices $\mathcal{Z}_{i,j} \in \mathcal{R}^{3n\times 3n}(i,j=1,2,3) > 0$, one has*

$$-\int_{t-\gamma(t)}^t \dot{z}^T(s)\mathcal{Z}_{33}\dot{z}(s)ds \le \int_{t-\gamma(t)}^t \bar{w}^T(t)\bar{\Xi}\bar{w}(t)ds,$$

where $\bar{w}(t) = \begin{bmatrix} z^T(t) & z^T(t-\gamma(t)) & \dot{z}^T(s)\end{bmatrix}$ *and* $\bar{\Xi} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ * & Z_{22} & Z_{23} \\ * & * & 0 \end{bmatrix}$

Lemma 2.5. [33] *For given scalars m and n with $m<n$, matrix $\mathcal{Y} > 0$, for any differentiable function x in $[m,n] \to \mathcal{R}^n$:*

$$\frac{(n-m)^2}{2}\int_m^n\int_\theta^n \dot{x}^\mathsf{T}(s)\mathcal{Y}\dot{x}(s)dsd\theta \ge \left(\int_m^n\int_\theta^n \dot{x}(s)dsd\theta\right)^\mathsf{T}\mathcal{Y}\left(\int_m^n\int_\theta^n \dot{x}(s)dsd\theta\right) + 2\Theta_\vartheta^\mathsf{T}\mathcal{Y}\Theta_\vartheta$$

where $\Theta_\vartheta = -\int_m^n\int_\theta^n \dot{x}(s)dsd\theta + \frac{3}{n-m}\int_m^n\int_\theta^n\int_v^n \dot{x}(v)dvdsd\theta$.

Lemma 2.6. [33] *For given matrix $Y>0$ and vector $\omega(s)$, one has*

$$\int_{t_1}^{t_2}\omega^T(s)Y\omega(s)ds \le (t_2-t_1)\xi^T(t)F^TY^{-1}F\xi(t) + 2\xi^T(t)F^T\int_{t_1}^{t_2}\omega(s)ds$$

where vector $\xi(t)$ denotes the vector and matrix F means matrix with compatible dimension.

3. Main results

The notations are defined as

$$\zeta(t) = \Big[x^\mathsf{T}(t) \quad x^T(t-\tau(t)) \quad x^\mathsf{T}(t-\tau_m(t)) \quad x^T(t-\tau_M(t)) \quad x^T(t-d(t)) \quad \dot{x}^T(t) \quad \frac{1}{\tau(t)}\int_{t-\tau(t)}^t x^T(s)ds \times \frac{1}{\tau_M(t)-\tau(t)}\int_{t-\tau_M(t)}^{t-\tau(t)} x^T(s)ds \quad \frac{1}{\tau(t)-\tau_m(t)} \times \int_{t-\tau(t)}^{t-\tau_m(t)} x^T(s)ds \quad \frac{1}{(\tau(t))^2}\int_{-\tau(t)}^0\int_{t+\theta}^t x^T dsd\theta \times \frac{1}{(\tau(t)-\tau_m(t))^2}\int_{-\tau(t)}^{-\tau_m(t)}\int_{t+\theta}^t x^T(s)dsd\theta \times \frac{1}{(\tau_M(t)-\tau(t))^2}\int_{-\tau_M(t)}^{-\tau(t)}\int_{t+\theta}^t x^T(s)dsd\theta\Big]^T$$

$$\chi_1(t) = \Big[x^T(t) \quad x^T(t-\tau(t)) \quad \frac{1}{\tau(t)}\int_{t-\tau(t)}^t x^T(s)ds \quad \frac{2}{(\tau(t))^2}\int_{-\tau(t)}^0\int_{t+\theta}^t x^T(s)dsd\theta\Big]^T$$

$$\chi_2(t) = \Big[x^T(t-\tau(t)) \quad x^T(t-\tau_M(t)) \quad \frac{1}{\tau_M(t)-\tau(t)}\int_{t-\tau_M(t)}^{t-\tau(t)} x^T(s)ds \quad \frac{2}{(\tau_M(t)-\tau(t))^2}\int_{-\tau_M(t)}^{-\tau(t)}\int_{t+\theta}^t x^T(s)dsd\theta\Big]^T$$

$$\chi_3(t) = \Big[x^T(t-\tau_m(t)) \quad x^T(t-\tau(t)) \quad \frac{1}{\tau(t)-\tau_m(t)}\int_{t-\tau(t)}^{t-\tau_m(t)} x^T(s)ds \quad \frac{2}{(\tau(t)-\tau_m(t))^2}\int_{-\tau(t)}^{-\tau_m(t)}\int_{t+\theta}^t x^T(s)dsd\theta\Big]^T$$

$$\chi_4(t) = \Big[x^T(t-\tau(t)) \quad x^T(t-\tau_M(t)) \quad \frac{1}{\tau_M(t)-\tau(t)}\int_{t-\tau_M(t)}^{t-\tau(t)} x^T(s)ds \quad \frac{2}{(\tau_M(t)-\tau(t))^2}\int_{-\tau_M(t)}^{-\tau(t)}\int_{t+\theta}^t x^T(s)dsd\theta\Big]^T$$

Theorem 3.1. *For given scalar $h>0$, $\tau_m \le \tau_M$, if there exist symmetric matrices $P>0$, $Q_s>0$ $(s=1,2,\ldots,4)$ $R_1>0$, $R_2>0$, $S_1>0$, $S_2>0$, $W>0$, $X_l>0$ $(l=1,2,3)$ and matrices $\mathcal{M}_l$, $\mathcal{N}_l$, $\mathcal{K}_l$, $\mathcal{L}_l$ $(l=1,2,3)$, $\mathcal{I}_{mn}$, $\mathcal{J}_{mn}$, $\mathcal{K}_{mn}$, $\mathcal{L}_{mn}(m=1,2;n=1,2,3)$, $\tilde{W}_{1ij}>0$, $\tilde{U}_{1ij}>0$, $\tilde{Y}_{1ij}$, such that*

$$\begin{bmatrix} R_1 & Y_1 \\ * & R_1 \end{bmatrix} \ge 0 \tag{8}$$

$$\begin{bmatrix} R_2 & Y_2 \\ * & R_2 \end{bmatrix} \ge 0 \tag{9}$$

$$\tilde{W}_{lij} + \tilde{W}_{lji} \le \tilde{U}_{lij} + \tilde{U}_{lji}, l=1,2,\ldots,8, i,j \in \mathbb{S} \tag{10}$$

$$\Pi_{lij} - 2\tilde{W}_{lij} + \sum_{s=1}^r \delta_s(\tilde{W}_{lis}^+ + \tilde{W}_{lsj}^+) \le \tilde{U}_{li(j+r)} + \tilde{U}_{l(j+r)i}, \quad l=1,2,\ldots,8, i,j \in \mathbb{S} \tag{11}$$

$$\begin{bmatrix} \tilde{Y}_{l11} & \tilde{Y}_{l12} \\ * & \tilde{Y}_{l11} \end{bmatrix} < 0, l=1,2,\ldots,8 \tag{12}$$

where

$\tilde{W}_{lij} = \tilde{N}_{lij} - \tilde{R}_{lij}, \tilde{W}_{lij}^+ = \tilde{R}_{lij} + \tilde{N}_{lij}, l=1,2,\ldots,8$

$$\tilde{Y}_{l11} = \begin{bmatrix} \tilde{U}_{l11} & \cdots & \tilde{U}_{l1r} \\ \vdots & \ddots & \vdots \\ \tilde{U}_{lr1} & \cdots & \tilde{U}_{lrr} \end{bmatrix}, l=1,2,\ldots,8$$

$$\tilde{Y}_{l12} = \begin{bmatrix} \tilde{U}_{l1(r+1)} & \cdots & \tilde{U}_{l1(2r)} \\ \vdots & \ddots & \vdots \\ \tilde{U}_{lr(r+1)} & \cdots & \tilde{U}_{lr(2r)} \end{bmatrix}, l=1,2,\ldots,8$$

$$\begin{cases} \Pi_{1ij} = \Pi_{ij}(\tau(t),\dot{\tau}(t),d(t)) \mid_{\tau(t)=\tau_m,\dot{\tau}(t)=\tau_d,d(t)=0} \\ \Pi_{2ij} = \Pi_{ij}(\tau(t),\dot{\tau}(t),d(t)) \mid_{\tau(t)=\tau_m,\dot{\tau}(t)=\tau_d,d(t)=h} \\ \Pi_{3ij} = \Pi_{ij}(\tau(t),\dot{\tau}(t),d(t)) \mid_{\tau(t)=\tau_m,\dot{\tau}(t)=\tau_D,d(t)=0} \\ \Pi_{4ij} = \Pi_{ij}(\tau(t),\dot{\tau}(t),d(t)) \mid_{\tau(t)=\tau_m,\dot{\tau}(t)=\tau_D,d(t)=h} \\ \Pi_{5ij} = \Pi_{ij}(\tau(t),\dot{\tau}(t),d(t)) \mid_{\tau(t)=\tau_M,\dot{\tau}(t)=\tau_d,d(t)=0} \\ \Pi_{6ij} = \Pi_{ij}(\tau(t),\dot{\tau}(t),d(t)) \mid_{\tau(t)=\tau_M,\dot{\tau}(t)=\tau_d,d(t)=h} \\ \Pi_{7ij} = \Pi_{ij}(\tau(t),\dot{\tau}(t),d(t)) \mid_{\tau(t)=\tau_M,\dot{\tau}(t)=\tau_D,d(t)=0} \\ \Pi_{8ij} = \Pi_{ij}(\tau(t),\dot{\tau}(t),d(t)) \mid_{\tau(t)=\tau_M,\dot{\tau}(t)=\tau_D,d(t)=h} \end{cases}$$

$$\Pi_{lij}(\tau(t),\dot{\tau}(t),d(t)) = \begin{bmatrix} \Sigma_{ij} & \Lambda & \sqrt{d(t)}Y \\ * & -\Delta & 0 \\ * & * & -X_1 \end{bmatrix},$$

$$
\begin{aligned}
&\Sigma_{ij}(\tau(t),\dot{\tau}(t),d(t))\\
&\quad= e_6Pe_1^T+e_1Pe_6^T+\Omega_1(\tau(t),\dot{\tau}(t),d(t))\\
&\qquad+\Omega_2(\tau(t),\dot{\tau}(t),d(t))+\Omega_3(\tau(t),\dot{\tau}(t),d(t))\\
&\qquad+\tau_M(t)e_6S_1e_6^T+(\tau_M(t)-\tau_m(t))e_6S_2e_6^T\\
&\qquad+Sym\{\mathcal{M}_1\Xi_1+\mathcal{M}_2\Xi_2+\mathcal{M}_3\Xi_3\}\\
&\qquad+Sym\{\mathcal{N}_1\Xi_1+\mathcal{N}_2\Xi_2+\mathcal{N}_3\Xi_3\}\\
&\qquad+(1-\alpha\dot{\tau}(t))Sym\{\mathcal{G}_1\Xi_1+\mathcal{G}_2\Xi_2+\mathcal{G}_3\Xi_3\}\\
&\qquad+(1-\alpha\dot{\tau}(t))Sym\{\mathcal{H}_1\Xi_1+\mathcal{H}_2\Xi_2+\mathcal{H}_3\Xi_3\}\\
&\qquad+\Big[e_1(\tau(t)\mathcal{I}_{11}+\mathcal{I}_{13}^T+\mathcal{I}_{13})e_1^T+2e_1(\tau(t)\mathcal{I}_{12}-\mathcal{I}_{13}+\mathcal{I}_{23}^T)e_2^T\\
&\qquad+e_2(\tau(t)\mathcal{I}_{22}-\mathcal{I}_{23}-\mathcal{I}_{23}^T)e_2^T\Big]\\
&\qquad+\Big[e_2((\tau_M(t)-\tau(t))\mathcal{J}_{11}+\mathcal{J}_{13}^T+\mathcal{J}_{13})e_2^T\\
&\qquad+2e_2((\tau_M(t)-\tau(t))\mathcal{J}_{12}-\mathcal{J}_{13}+\mathcal{J}_{23}^T)e_4^T\\
&\qquad+e_4((\tau_M(t)-\tau(t))\mathcal{J}_{22}-\mathcal{J}_{23}-\mathcal{J}_{23}^T)e_4^T\Big]\\
&\qquad+(1-\alpha\dot{\tau}(t))\Big[e_3((\tau(t)-\tau_m(t))\mathcal{K}_{11}+\mathcal{K}_{13}^T+\mathcal{K}_{13})e_3^T\\
&\qquad+2e_3((\tau(t)-\tau_m(t))\mathcal{K}_{12}-\mathcal{K}_{13}+\mathcal{K}_{23}^T)e_2^T\\
&\qquad+e_2((\tau(t)-\tau_m(t))\mathcal{K}_{22}-\mathcal{K}_{23}-\mathcal{K}_{23}^T)e_2^T\Big]\\
&\qquad+(1-\alpha\dot{\tau}(t))\Big[e_2((\tau_M(t)-\tau(t))\mathcal{L}_{11}+\mathcal{L}_{13}^T+\mathcal{L}_{13})e_2^T\\
&\qquad+2e_2((\tau_M(t)-\tau(t))\mathcal{L}_{12}-\mathcal{L}_{13}+\mathcal{L}_{23}^T)e_4^T\\
&\qquad+e_4((\tau_M(t)-\tau(t))\mathcal{L}_{22}-\mathcal{L}_{23}-\mathcal{L}_{23}^T)e_4^T\Big]\\
&\qquad+(h-d(t))e_6X_1e_6^T+2Y(e_1-e_5)+((d_k-2d(t))e_5X_2e_5^T\\
&\qquad+h^2e_6X_3e_6^T-\frac{\pi^2}{4}(e_1-e_5)X_3(e_1-e_5)^T\\
&\qquad+2(e_1M_1^T+e_6M_2^T)(e_6^{\mathsf T}-A_ie_1^{\mathsf T}+A_{di}e_2^{\mathsf T})\\
&\qquad+2e_1B_iG_je_5^{\mathsf T}+2e_6\epsilon B_iG_je_5^{\mathsf T},\ M_2=\epsilon M_1
\end{aligned}
$$

$$\Lambda=[\Lambda_1\quad\Lambda_2\quad\Lambda_3\quad\Lambda_4]\quad\Delta=\mathrm{diag}\{\Delta_1\quad\Delta_2\quad\Delta_3\quad\Delta_4\}$$

$$
\begin{aligned}
\Lambda_1&=[\sqrt{\tau(t)}\Phi_1\mathcal{M}_1\quad\sqrt{\tau(t)}\Phi_1\mathcal{M}_2\quad\sqrt{\tau(t)}\Phi_1\mathcal{M}_3]\\
\Lambda_2&=[\sqrt{\tau_M(t)-\tau(t)}\Phi_1\mathcal{N}_1\quad\sqrt{\tau_M(t)-\tau(t)}\Phi_1\mathcal{N}_2\\
&\qquad\sqrt{\tau_M(t)-\tau(t)}\Phi_1\mathcal{N}_3]\\
\Lambda_3&=[\sqrt{(1-\alpha\dot{\tau}(t))(\tau(t)-\tau_m(t))}\Phi_1\mathcal{L}_1\\
&\qquad\sqrt{(1-\alpha\dot{\tau}(t))(\tau(t)-\tau_m(t))}\Phi_1\mathcal{L}_2\\
&\qquad\sqrt{(1-\alpha\dot{\tau}(t))(\tau(t)-\tau_m(t))}\Phi_1\mathcal{L}_3]
\end{aligned}
$$

$$
\begin{aligned}
\Lambda_4&=[\sqrt{(1-\alpha\dot{\tau}(t))(\tau_M(t)-\tau(t))}\Phi_1\mathcal{K}_1\\
&\qquad\sqrt{(1-\alpha\dot{\tau}(t))(\tau_M(t)-\tau(t))}\Phi_1\mathcal{K}_2\\
&\qquad\sqrt{(1-\alpha\dot{\tau}(t))(\tau_M(t)-\tau(t))}\Phi_1\mathcal{K}_3]
\end{aligned}
$$

$$
\begin{aligned}
&\Delta_i=\mathrm{diag}\{R_1\quad 3R_1\quad 5R_1\}\quad(i=1,2,\ldots,4)\\
&\Omega_1(\tau(t),\dot{\tau}(t),d(t))=e_1(Q_1+Q_2+Q_4)e_1^T-(1-\dot{\tau}(t))e_2Q_2e_2^T\\
&\qquad\qquad+(1-\alpha\dot{\tau}(t))[e_3(Q_3-Q_4)e_3^T-e_4Q_3e_4^T]
\end{aligned}
$$

$$
\begin{aligned}
&\Omega_2(\tau(t),\dot{\tau}(t),d(t))\\
&\quad=e_1[\tau_M(t)R_1+(\tau_M(t)-\tau_m(t))R_2]e_1^T\\
&\qquad-\frac{1-\alpha\dot{\tau}(t)}{\tau_M(t)}[\tau(t)e_7\ (\tau_M(t)-\tau(t))e_8]\\
&\qquad\times\begin{bmatrix}R_1 & Y_1\\ * & R_1\end{bmatrix}[\tau(t)e_7\ (\tau_M(t)-\tau(t))e_8]^T\\
&\qquad-\frac{1-\alpha\dot{\tau}(t)}{(1-\alpha)(\tau_M-\tau_m)}\\
&\qquad\times[(\tau(t)-\tau_m(t))e_9\ (\tau_M(t)-\tau(t))e_8]\begin{bmatrix}R_2 & Y_2\\ * & R_2\end{bmatrix}\\
&\qquad\times[(\tau(t)-\tau_m(t))e_9\ (\tau_M(t)-\tau(t))e_8]^T
\end{aligned}
$$

$$\Omega_3(\tau(t),\dot{\tau}(t),d(t))=\frac{(\tau_M(t)-\tau_m(t))^2}{2}e_6We_6^T+\Theta_1+\Theta_2$$

$$
\begin{aligned}
\phi_1&=\begin{bmatrix}e_1 & e_2 & e_7 & 2e_{10}\end{bmatrix}\phi_2=\begin{bmatrix}e_2 & e_4 & e_8 & 2e_{12}\end{bmatrix}\\
\phi_3&=\begin{bmatrix}e_3 & e_2 & e_9 & 2e_{11}\end{bmatrix}\phi_4=\begin{bmatrix}e_2 & e_4 & e_8 & 2e_{12}\end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
\Theta_1&=(1-\alpha\dot{\tau}(t))\Big[-2(e_1-e_8)W(e_1-e_8)^T\\
&\qquad-4\Big(-\frac{1}{2}e_1-e_8+3e_{12}\Big)W\Big(-\frac{1}{2}e_1-e_8+3e_{12}\Big)^T\Big]\\
\Theta_2&=(1-\alpha\dot{\tau}(t))\Big[-2(e_1-e_9)W(e_1-e_9)^T\\
&\qquad-4\Big(-\frac{1}{2}e_1-e_9+3e_{11}\Big)W\Big(-\frac{1}{2}e_1-e_9+3e_{11}\Big)^T\Big]
\end{aligned}
$$

$$\tau_m(t)=\alpha\tau(t)+(1-\alpha)\tau_m,\ \tau_M(t)=\alpha\tau(t)+(1-\alpha)\tau_M$$

Then, the CLSDFS (7) is asymptotically stable. Furthermore, the fuzzy sampled-data controller is given by

$$K_j=G_jM_1^{-\mathsf T}$$

Proof. Construct the following LKF:

$$V(t)=\sum_{i=1}^{7}V_i(t)\tag{13}$$

where

$$V_1(t)=x^T(t)Px(t)$$

$$
\begin{aligned}
V_2(t)&=\int_{t-d(t)}^{t}x^T(s)Q_1x(s)ds+\int_{t-\tau(t)}^{t}x^T(s)Q_2x(s)ds\\
&\quad+\int_{t-\tau_M(t)}^{t-\tau_m(t)}x^T(s)Q_3x(s)ds+\int_{t-\tau_m(t)}^{t}x^T(s)Q_4x(s)ds
\end{aligned}
$$

$$
\begin{aligned}
V_3(t)&=\int_{-\tau_M(t)}^{0}\int_{t+\theta}^{t}x^T(s)R_1x(s)dsd\theta\\
&\quad+\int_{-\tau_M(t)}^{-\tau_m(t)}\int_{t+\theta}^{t}x^T(s)R_2x(s)dsd\theta
\end{aligned}
$$

$$
\begin{aligned}
V_4(t)&=\int_{-\tau_M(t)}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)S_1\dot{x}(s)dsd\theta\\
&\quad+\int_{-\tau_M(t)}^{-\tau_m(t)}\int_{t+\theta}^{t}\dot{x}^T(s)S_2\dot{x}(s)dsd\theta
\end{aligned}
$$

$$V_5(t)=\int_{-\tau_M(t)}^{-\tau_m(t)}\int_{\theta}^{0}\int_{t+\lambda}^{t}\dot{x}^T(s)W\dot{x}(s)dsd\lambda d\theta$$

$$
\begin{aligned}
V_6(t)&=(h-t+t_k)\int_{t_k}^{t}\dot{x}^T(s)X_1\dot{x}(s)ds\\
&\quad+(t_{k+1}-t)(t-t_k)x^T(t_k)X_2x(t_k)
\end{aligned}
$$

$$V_7(t) = h^2 \int_{t_k}^{t} \dot{x}^T(s) X_3 \dot{x}(s) ds - \frac{\pi^2}{4} \int_{t_k}^{t} (x(s) - x(t_k))^T X_3 (x(s) - x(t_k)) ds$$

Let $\mathfrak{L}(\cdot)$ be the weak-infinitesimal operator of $\{x(t),\ t \geq 0\}$. we then obtain

$$\mathfrak{L}V(t) = \sum_{i=1}^{7} \mathfrak{L}V_i(t) \tag{14}$$

where

$$\mathfrak{L}V_1(t) = 2x^T(t)P\dot{x}(t) = \zeta^T(t)(e_6Pe_1^T + e_1Pe_6^T)\zeta(t) \tag{15}$$

$$\begin{aligned}
\mathfrak{L}V_2(t) &= x^T(t)Q_1x(t) - (1 - \dot{d}(t))x^T(t - d(t))Q_1x(t - d(t)) \\
&\quad + x^T(t)Q_2x(t) - (1 - \dot{\tau}(t))x^T(t - \tau(t))Q_2x(t - \tau(t)) \\
&\quad + (1 - \alpha\dot{\tau}(t))x^T(t - \tau_m(t))Q_3x(t - \tau_m(t)) \\
&\quad - (1 - \alpha\dot{\tau}(t))x^T(t - \tau_M(t))Q_3x(t - \tau_M(t)) \\
&\quad + x^T(t)Q_4x(t) - (1 - \alpha\dot{\tau}(t))x^T(t \\
&\quad - \tau_m(t))Q_4x(t - \tau_m(t)) \\
&= x^T(t)Q_1x(t) + x^T(t)Q_2x(t) \\
&\quad - (1 - \dot{\tau}(t))x^T(t - \tau(t))Q_2x(t - \tau(t)) \\
&\quad + (1 - \alpha\dot{\tau}(t))x^T(t - \tau_m(t))Q_3x(t - \tau_m(t)) \\
&\quad - (1 - \alpha\dot{\tau}(t))x^T(t - \tau_M(t))Q_3x(t - \tau_M(t)) \\
&\quad + x^T(t)Q_4x(t) - (1 - \alpha\dot{\tau}(t))x^T(t \\
&\quad - \tau_m(t))Q_4x(t - \tau_m(t)) \\
&= \zeta^T(t)\Omega_1(\tau(t), \dot{\tau}(t), d(t))\zeta(t)
\end{aligned} \tag{16}$$

$$\begin{aligned}
\mathfrak{L}V_3(t) &= \tau_M(t)x^T(t)R_1x(t) + (\tau_M(t) - \tau_m(t))x^T(t)R_2x(t) \\
&\quad - (1 - \alpha\dot{\tau}(t)) \int_{t-\tau_M(t)}^{t} x^T(s)R_1x(s)ds \\
&\quad - (1 - \alpha\dot{\tau}(t)) \int_{t-\tau_M(t)}^{t-\tau_m(t)} x^T(s)R_2x(s)ds
\end{aligned} \tag{17}$$

By using Lemma 2.2, we have

$$\begin{aligned}
&- \int_{t-\tau_M(t)}^{t} x^T(s)R_1x(s)ds \\
&= - \int_{t-\tau(t)}^{t} x^T(s)R_1x(s)ds - \int_{t-\tau_M(t)}^{t-\tau(t)} x^T(s)R_1x(s)ds \\
&\leq -\frac{1}{\tau_M(t)} \begin{bmatrix} \int_{t-\tau(t)}^{t} x(s)ds \\ \int_{t-\tau_M(t)}^{t-\tau(t)} x(s)ds \end{bmatrix}^T \begin{bmatrix} R_1 & Y_1 \\ * & R_1 \end{bmatrix} \begin{bmatrix} \int_{t-\tau(t)}^{t} x(s)ds \\ \int_{t-\tau_M(t)}^{t-\tau(t)} x(s)ds \end{bmatrix}
\end{aligned} \tag{18}$$

Similarly, one has

$$\begin{aligned}
- \int_{t-\tau_M(t)}^{t-\tau_m(t)} x^T(s)R_2x(s)ds &= - \int_{t-\tau(t)}^{t-\tau_m(t)} x^T(s)R_2x(s)ds \\
&\quad - \int_{t-\tau_M(t)}^{t-\tau(t)} x^T(s)R_2x(s)ds \\
&\leq -\frac{1}{\tau_M(t) - \tau_m(t)} \begin{bmatrix} \int_{t-\tau(t)}^{t-\tau_m(t)} x(s)ds \\ \int_{t-\tau_M(t)}^{t-\tau(t)} x(s)ds \end{bmatrix}^T \\
&\quad \times \begin{bmatrix} R_2 & Y_2 \\ * & R_2 \end{bmatrix} \begin{bmatrix} \int_{t-\tau(t)}^{t-\tau_m(t)} x(s)ds \\ \int_{t-\tau_M(t)}^{t-\tau(t)} x(s)ds \end{bmatrix}
\end{aligned} \tag{19}$$

$$\begin{aligned}
\mathfrak{L}V_3(t) &= \tau_M(t)x^T(t)R_1x(t) + (\tau_M(t) - \tau_m(t))x^T(t)R_2x(t) \\
&\quad - (1 - \alpha\dot{\tau}(t)) \int_{t-\tau_M(t)}^{t} x^T(s)R_1x(s)ds \\
&\quad - (1 - \alpha\dot{\tau}(t)) \int_{t-\tau_M(t)}^{t-\tau_m(t)} x^T(s)R_2x(s)ds \\
&\leq \tau_M(t)x^T(t)R_1x(t) + (\tau_M(t) - \tau_m(t))x^T(t)R_2x(t) \\
&\quad - \frac{(1 - \alpha\dot{\tau}(t))}{\tau_M(t)} \begin{bmatrix} \int_{t-\tau(t)}^{t} x(s)ds \\ \int_{t-\tau_M(t)}^{t-\tau(t)} x(s)ds \end{bmatrix}^T \begin{bmatrix} R_1 & Y_1 \\ * & R_1 \end{bmatrix} \\
&\quad \times \begin{bmatrix} \int_{t-\tau(t)}^{t} x(s)ds \\ \int_{t-\tau_M(t)}^{t-\tau(t)} x(s)ds \end{bmatrix} - \frac{(1 - \alpha\dot{\tau}(t))}{\tau_M(t) - \tau_m(t)} \begin{bmatrix} \int_{t-\tau(t)}^{t-\tau_m(t)} x(s)ds \\ \int_{t-\tau_M(t)}^{t-\tau(t)} x(s)ds \end{bmatrix}^T \\
&\quad \times \begin{bmatrix} R_2 & Y_2 \\ * & R_2 \end{bmatrix} \begin{bmatrix} \int_{t-\tau(t)}^{t-\tau_m(t)} x(s)ds \\ \int_{t-\tau_M(t)}^{t-\tau(t)} x(s)ds \end{bmatrix} \\
&= \tau_M(t)x^T(t)R_1x(t) + (\tau_M(t) - \tau_m(t))x^T(t)R_2x(t) \\
&\quad - \frac{(1 - \alpha\dot{\tau}(t))}{\tau_M(t)} \begin{bmatrix} \int_{t-\tau(t)}^{t} x(s)ds \\ \int_{t-\tau_M(t)}^{t-\tau(t)} x(s)ds \end{bmatrix}^T \begin{bmatrix} R_1 & Y_1 \\ * & R_1 \end{bmatrix} \\
&\quad \times \begin{bmatrix} \int_{t-\tau(t)}^{t} x(s)ds \\ \int_{t-\tau_M(t)}^{t-\tau(t)} x(s)ds \end{bmatrix} \\
&\quad - \frac{(1 - \alpha\dot{\tau}(t))}{(1 - \alpha)(\tau_M - \tau_m)} \begin{bmatrix} \int_{t-\tau(t)}^{t-\tau_m(t)} x(s)ds \\ \int_{t-\tau_M(t)}^{t-\tau(t)} x(s)ds \end{bmatrix}^T \begin{bmatrix} R_2 & Y_2 \\ * & R_2 \end{bmatrix} \\
&\quad \times \begin{bmatrix} \int_{t-\tau(t)}^{t-\tau_m(t)} x(s)ds \\ \int_{t-\tau_M(t)}^{t-\tau(t)} x(s)ds \end{bmatrix} \\
&= \zeta^T(t)\Omega_2(\tau(t), \dot{\tau}(t), d(t))\zeta(t)
\end{aligned} \tag{20}$$

$$\begin{aligned}
\mathfrak{L}V_4(t) &= \tau_M(t)\dot{x}^T(t)S_1\dot{x}(t) + (\tau_M(t) - \tau_m(t))\dot{x}^T(t)S_2\dot{x}(t) \\
&\quad - (1 - \alpha\dot{\tau}(t)) \int_{t-\tau_M(t)}^{t} \dot{x}^T(s)S_1\dot{x}(s)ds \\
&\quad - (1 - \alpha\dot{\tau}(t)) \int_{t-\tau_M(t)}^{t-\tau_m(t)} \dot{x}^T(s)S_2\dot{x}(s)ds \\
&= \tau_M(t)\dot{x}^T(t)S_1\dot{x}(t) + (\tau_M(t) - \tau_m(t))\dot{x}^T(t)S_2\dot{x}(t) \\
&\quad - \int_{t-\tau(t)}^{t} \dot{x}^T(s)S_1\dot{x}(s)ds - \int_{t-\tau_M(t)}^{t-\tau(t)} \dot{x}^T(s)S_1\dot{x}(s)ds \\
&\quad - (1 - \alpha\dot{\tau}(t)) \int_{t-\tau(t)}^{t-\tau_m(t)} \dot{x}^T(s)S_2\dot{x}(s)ds \\
&\quad - (1 - \alpha\dot{\tau}(t)) \int_{t-\tau_M(t)}^{t-\tau(t)} \dot{x}^T(s)S_2\dot{x}(s)ds \\
&\leq \tau_M\dot{x}^T(t)S_1\dot{x}(t) + (\tau_M - \tau_m)\dot{x}^T(t)S_2\dot{x}(t) \\
&\quad - \int_{t-\tau(t)}^{t} \dot{x}^T(s)\{S_1 - \mathcal{I}_{33}\}\dot{x}(s)ds \\
&\quad - \int_{t-\tau(t)}^{t} \dot{x}^T(s)\mathcal{I}_{33}\dot{x}(s)ds \\
&\quad - \int_{t-\tau_M(t)}^{t-\tau(t)} \dot{x}^T(s)\{S_1 - \mathcal{J}_{33}\}\dot{x}(s)ds \\
&\quad - \int_{t-\tau_M(t)}^{t-\tau(t)} \dot{x}^T(s)\mathcal{J}_{33}\dot{x}(s)ds \\
&\quad - (1 - \alpha\dot{\tau}(t)) \int_{t-\tau(t)}^{t-\tau_m(t)} \dot{x}^T(s)\{S_2 - \mathcal{K}_{33}\}\dot{x}(s)ds \\
&\quad - (1 - \alpha\dot{\tau}(t)) \int_{t-\tau(t)}^{t-\tau_m(t)} \dot{x}^T(s)\mathcal{K}_{33}\dot{x}(s)ds
\end{aligned}$$

$$-(1-\alpha\dot{\tau}(t))\int_{t-\tau_M(t)}^{t-\tau(t)}\dot{x}^T(s)\{S_2-\mathcal{L}_{33}\}\dot{x}(s)ds$$
$$-(1-\alpha\dot{\tau}(t))\int_{t-\tau_M(t)}^{t-\tau(t)}\dot{x}^T(s)\mathcal{L}_{33}\dot{x}(s)ds \quad (21)$$

Let $\Upsilon_1=S_1-\mathcal{I}_{33}\geq 0$, $\Upsilon_2=S_1-\mathcal{J}_{33}\geq 0$, $\Upsilon_3=S_2-\mathcal{K}_{33}\geq 0$, and $\Upsilon_4=S_2-\mathcal{L}_{33}\geq 0$, it follows from Lemma 2.3 that

$$\begin{aligned}
&-\int_{t-\tau(t)}^{t}\dot{x}^T(s)\Upsilon_1\dot{x}(s)ds\\
&\leq \chi_1^{\mathsf T}(t)\Big[\tau(t)\mathcal{M}_1\Upsilon_1^{-1}\mathcal{M}_1^{\mathsf T}+\frac{\tau(t)}{3}\mathcal{M}_2\Upsilon_1^{-1}\mathcal{M}_2^{\mathsf T}\\
&+\frac{\tau(t)}{5}\mathcal{M}_3\Upsilon_1^{-1}\mathcal{M}_3^{\mathsf T}+Sym\{\mathcal{M}_1\Xi_1+\mathcal{M}_2\Xi_2+\mathcal{M}_3\Xi_3\}\Big]\chi_1(t)\\
&=\zeta^T(t)\Big\{\phi_1\Big[\tau(t)\mathcal{M}_1\Upsilon_1^{-1}\mathcal{M}_1^{\mathsf T}+\frac{\tau(t)}{3}\mathcal{M}_2\Upsilon_1^{-1}\mathcal{M}_2^{\mathsf T}\\
&+\frac{\tau(t)}{5}\mathcal{M}_3\Upsilon_1^{-1}\mathcal{M}_3^{\mathsf T}\Big]^T\phi_1^T+Sym\{\mathcal{M}_1\Xi_1+\mathcal{M}_2\Xi_2+\mathcal{M}_3\Xi_3\}\Big\}\zeta(t)
\end{aligned}\quad (22)$$

$$\begin{aligned}
&\int_{t-\tau_M(t)}^{t-\tau(t)}\dot{x}^T(s)\Upsilon_2\dot{x}(s)ds\\
&\leq \chi_2^{\mathsf T}(t)\Big[(\tau_M(t)-\tau(t))\mathcal{N}_1\Upsilon_2^{-1}\mathcal{N}_1^{\mathsf T}+\frac{(\tau_M(t)-\tau(t))}{3}\mathcal{N}_2\Upsilon_2^{-1}\mathcal{N}_2^{\mathsf T}\\
&+\frac{(\tau_M(t)-\tau(t))}{5}\mathcal{N}_3\Upsilon_2^{-1}\mathcal{N}_3^{\mathsf T}+Sym\{\mathcal{N}_1\Xi_1+\mathcal{N}_2\Xi_2+\mathcal{N}_3\Xi_3\}\Big]\chi_2(t)\\
&=\zeta^T(t)\Big\{\phi_2\Big[(\tau_M(t)-\tau(t))\mathcal{N}_1\Upsilon_2^{-1}\mathcal{N}_1^{\mathsf T}\\
&+\frac{(\tau_M(t)-\tau(t))}{3}\mathcal{N}_2\Upsilon_2^{-1}\mathcal{N}_2^{\mathsf T}+\frac{(\tau_M(t)-\tau(t))}{5}\mathcal{N}_3\Upsilon_2^{-1}\mathcal{N}_3^{\mathsf T}\Big]^T\phi_2^T\\
&+Sym\{\mathcal{N}_1\Xi_1+\mathcal{N}_2\Xi_2+\mathcal{N}_3\Xi_3\}\Big\}\zeta(t)\\
&-(1-\alpha\dot{\tau}(t))\int_{t-\tau(t)}^{t-\tau_m(t)}\dot{x}^T(s)\Upsilon_3\dot{x}(s)ds\\
&\leq(1-\alpha\dot{\tau}(t))\chi_3^{\mathsf T}(t)\Big[(\tau(t)-\tau_m(t))\mathcal{G}_1\Upsilon_3^{-1}\mathcal{G}_1^{\mathsf T}\\
&+\frac{(\tau(t)-\tau_m(t))}{3}\mathcal{G}_2\Upsilon_3^{-1}\mathcal{G}_2^{\mathsf T}+\frac{(\tau(t)-\tau_m(t))}{5}\mathcal{G}_3\Upsilon_3^{-1}\mathcal{G}_3^{\mathsf T}\\
&+Sym\{\mathcal{G}_1\Xi_1+\mathcal{G}_2\Xi_2+\mathcal{G}_3\Xi_3\}\Big]\chi_3(t)
\end{aligned}\quad (23)$$

$$\begin{aligned}
&=(1-\alpha\dot{\tau}(t))\zeta^T(t)\Big\{\phi_3\Big[(\tau(t)-\tau_m(t))\mathcal{G}_1\Upsilon_3^{-1}\mathcal{G}_1^{\mathsf T}\\
&+\frac{(\tau(t)-\tau_m(t))}{3}\mathcal{G}_2\Upsilon_3^{-1}\mathcal{G}_2^{\mathsf T}+\frac{(\tau(t)-\tau_m(t))}{5}\mathcal{G}_3\Upsilon_3^{-1}\mathcal{G}_3^{\mathsf T}\Big]^T\phi_3^T\\
&+Sym\{\mathcal{G}_1\Xi_1+\mathcal{G}_2\Xi_2+\mathcal{G}_3\Xi_3\}\Big\}\zeta(t)
\end{aligned}\quad (24)$$

$$\begin{aligned}
&-(1-\alpha\dot{\tau}(t))\int_{t-\tau_M(t)}^{t-\tau(t)}\dot{x}^T(s)\Upsilon_4\dot{x}(s)ds\\
&\leq(1-\alpha\dot{\tau}(t))\chi_4^{\mathsf T}(t)\Big[(\tau_M(t)-\tau(t))\mathcal{H}_1\Upsilon_4^{-1}\mathcal{H}_1^{\mathsf T}\\
&+\frac{(\tau_M(t)-\tau(t))}{3}\mathcal{H}_2\Upsilon_4^{-1}\mathcal{H}_2^{\mathsf T}+\frac{(\tau_M(t)-\tau(t))}{5}\mathcal{H}_3\Upsilon_4^{-1}\mathcal{H}_3^{\mathsf T}\\
&+Sym\{\mathcal{H}_1\Xi_1+\mathcal{H}_2\Xi_2+\mathcal{H}_3\Xi_3\}\Big]\chi_4(t)\\
&=(1-\alpha\dot{\tau}(t))\zeta^T(t)\Big\{\phi_4\Big[(\tau_M(t)-\tau(t))\mathcal{H}_1\Upsilon_4^{-1}\mathcal{H}_1^{\mathsf T}\\
&+\frac{(\tau_M(t)-\tau(t))}{3}\mathcal{H}_2\Upsilon_4^{-1}\mathcal{H}_2^{\mathsf T}+\frac{(\tau_M(t)-\tau(t))}{5}\mathcal{H}_3\Upsilon_4^{-1}\mathcal{H}_3^{\mathsf T}\Big]^T\phi_4^T\\
&+Sym\{\mathcal{H}_1\Xi_1+\mathcal{H}_2\Xi_2+\mathcal{H}_3\Xi_3\}\Big\}\zeta(t)
\end{aligned}\quad (25)$$

According to Lemma 2.4,

$$\begin{aligned}
&-\int_{t-\tau(t)}^{t}\dot{x}^T(s)\mathcal{I}_{33}\dot{x}(s)ds\\
&\leq\int_{t-\tau(t)}^{t}\begin{bmatrix}x(t)\\x(t-\tau(t))\\\dot{x}(s)\end{bmatrix}^T\begin{bmatrix}\mathcal{I}_{11}&\mathcal{I}_{12}&\mathcal{I}_{13}\\ *&\mathcal{I}_{22}&\mathcal{I}_{23}\\ *&*&0\end{bmatrix}\begin{bmatrix}x(t)\\x(t-\tau(t))\\\dot{x}(s)\end{bmatrix}ds\\
&=\zeta^T(t)\big[e_1(\tau(t)\mathcal{I}_{11}+\mathcal{I}_{13}^T+\mathcal{I}_{13})e_1^T+2e_1(\tau(t)\mathcal{I}_{12}-\mathcal{I}_{13}\\
&+\mathcal{I}_{23}^T)e_2^T+e_2(\tau(t)\mathcal{I}_{22}-\mathcal{I}_{23}-\mathcal{I}_{23}^T)e_2^T\big]\zeta(t)
\end{aligned}\quad (26)$$

Similarly, one has

$$\begin{aligned}
&-\int_{t-\tau_M(t)}^{t-\tau(t)}\dot{x}^T(s)\mathcal{J}_{33}\dot{x}(s)ds\\
&\leq\int_{t-\tau_M(t)}^{t-\tau(t)}\begin{bmatrix}x(t-\tau(t))\\x(t-\tau_M(t))\\\dot{x}(s)\end{bmatrix}^T\begin{bmatrix}\mathcal{J}_{11}&\mathcal{J}_{12}&\mathcal{J}_{13}\\ *&\mathcal{J}_{22}&\mathcal{J}_{23}\\ *&*&0\end{bmatrix}\begin{bmatrix}x(t-\tau(t))\\x(t-\tau_M(t))\\\dot{x}(s)\end{bmatrix}ds\\
&=\zeta^T(t)\big[e_2((\tau_M(t)-\tau(t))\mathcal{J}_{11}+\mathcal{J}_{13}^T+\mathcal{J}_{13})e_2^T+2e_2((\tau_M(t)\\
&-\tau(t))\mathcal{J}_{12}-\mathcal{J}_{13}+\mathcal{J}_{23}^T)e_4^T\\
&+e_4((\tau_M(t)-\tau(t))D_{22}-\mathcal{J}_{23}-\mathcal{J}_{23}^T)e_4^T\big]\zeta(t)
\end{aligned}\quad (27)$$

$$-(1-\alpha\dot{\tau}(t))\int_{t-\tau(t)}^{t-\tau_m(t)}\dot{x}^T(s)\mathcal{K}_{33}\dot{x}(s)ds \quad (28)$$

$$\begin{aligned}
&\leq(1-\alpha\dot{\tau}(t))\int_{t-\tau(t)}^{t-\tau_m(t)}\begin{bmatrix}x(t-\tau_m(t))\\x(t-\tau(t))\\\dot{x}(s)\end{bmatrix}^T\begin{bmatrix}\mathcal{K}_{11}&\mathcal{K}_{12}&\mathcal{K}_{13}\\ *&\mathcal{K}_{22}&\mathcal{K}_{23}\\ *&*&0\end{bmatrix}\\
&\times\begin{bmatrix}x(t-\tau_m(t))\\x(t-\tau(t))\\\dot{x}(s)\end{bmatrix}ds\\
&=(1-\alpha\dot{\tau}(t))\zeta^T(t)\big[e_3((\tau(t)-\tau_m(t))\mathcal{K}_{11}+\mathcal{K}_{13}^T+\mathcal{K}_{13})e_3^T\\
&+2e_3((\tau(t)-\tau_m(t))\mathcal{K}_{12}-\mathcal{K}_{13}+\mathcal{K}_{23}^T)e_2^T\\
&+e_2((\tau(t)-\tau_m(t))\mathcal{K}_{22}-\mathcal{K}_{23}-\mathcal{K}_{23}^T)e_2^T\big]\zeta(t)
\end{aligned}\quad (28)$$

$$-(1-\alpha\dot{\tau}(t))\int_{t-\tau_M(t)}^{t-\tau(t)}\dot{x}^T(s)\mathcal{L}_{33}\dot{x}(s)ds \quad (29)$$

$$\begin{aligned}
&\leq(1-\alpha\dot{\tau}(t))\int_{t-\tau_M(t)}^{t-\tau(t)}\begin{bmatrix}x(t-\tau(t))\\x(t-\tau_M(t))\\\dot{x}(s)\end{bmatrix}^T\begin{bmatrix}\mathcal{L}_{11}&\mathcal{L}_{12}&\mathcal{L}_{13}\\ *&\mathcal{L}_{22}&\mathcal{L}_{23}\\ *&*&0\end{bmatrix}\\
&\times\begin{bmatrix}x(t-\tau(t))\\x(t-\tau_M(t))\\\dot{x}(s)\end{bmatrix}ds\\
&=(1-\alpha\dot{\tau}(t))\zeta^T(t)\big[e_2((\tau_M(t)-\tau(t))\mathcal{L}_{11}+\mathcal{L}_{13}^T+\mathcal{L}_{13})e_2^T\\
&+2e_2((\tau_M(t)-\tau(t))\mathcal{L}_{12}-\mathcal{L}_{13}+\mathcal{L}_{23}^T)e_4^T\\
&+e_4((\tau_M(t)-\tau(t))\mathcal{L}_{22}-\mathcal{L}_{23}-\mathcal{L}_{23}^T)e_4^T\big]\zeta(t)
\end{aligned}\quad (29)$$

By using Lemma 2.5, we get

$$\begin{aligned}
\mathfrak{L}V_5(t)&=\frac{(\tau_M(t)-\tau_m(t))^2}{2}\dot{x}^T(t)W\dot{x}(t)\\
&-(1-\alpha\dot{\tau}(t))\int_{-\tau_M(t)}^{-\tau_m(t)}\int_{t+\theta}^{t}\dot{x}^T(s)W\dot{x}(s)dsd\theta\\
&=\frac{(\tau_M(t)-\tau_m(t))^2}{2}\dot{x}^T(t)W\dot{x}(t)
\end{aligned}$$

$$
\begin{aligned}
&-(1-\alpha\dot{\tau}(t))\int_{-\tau_M(t)}^{-\tau(t)}\int_{t+\theta}^{t}\dot{x}^T(s)W\dot{x}(s)dsd\theta \qquad (30)\\
&-(1-\alpha\dot{\tau}(t))\int_{-\tau(t)}^{-\tau_m(t)}\int_{t+\theta}^{t}\dot{x}^T(s)W\dot{x}(s)dsd\theta\\
\le\ &\frac{(\tau_M(t)-\tau_m(t))^2}{2}\dot{x}^T(t)W\dot{x}(t)\\
&-(1-\alpha\dot{\tau}(t))\Big\{\frac{2}{(\tau_M(t)-\tau(t))^2}[((\tau_M(t)-\tau(t)))x(t)\\
&-\int_{t-\tau_M(t)}^{t-\tau(t)}x(s)ds\Big]^T W\Big[(\tau_M(t)-\tau(t))-\int_{t-\tau_M(t)}^{t-\tau(t)}x(s)ds\Big]\\
&-\frac{4}{(\tau_M(t)-\tau(t))^2}\Big[-\frac{\tau_M(t)-\tau(t)}{2}x(t)-\int_{t-\tau_M(t)}^{t-\tau_m(t)}x(s)ds\\
&+\frac{3}{\tau_M(t)-\tau(t)}\int_{-\tau_M(t)}^{-\tau_m(t)}\int_{t+\theta}^{t}x(s)dsd\theta\Big]^T\\
&W\Big[-\frac{\tau_M(t)-\tau(t)}{2}x(t)-\int_{t-\tau_M(t)}^{t-\tau_m(t)}x(s)ds\\
&+\frac{3}{\tau_M(t)-\tau(t)}\int_{-\tau_M(t)}^{-\tau_m(t)}\int_{t+\theta}^{t}x(s)dsd\theta\Big]\\
&-\frac{2}{(\tau(t)-\tau_m(t))^2}[(\tau(t)-\tau_m(t))x(t)\\
&-\int_{t-\tau(t)}^{t-\tau_m(t)}x(s)ds\Big]^T W[(\tau(t)-\tau_m(t))x(t)\\
&-\int_{t-\tau(t)}^{t-\tau_m(t)}x(s)ds\Big]\\
&-\frac{4}{(\tau(t)-\tau_m(t))^2}\Big[-\frac{(\tau(t)-\tau_m(t))}{2}x(t)\\
&-\int_{t-\tau(t)}^{t-\tau_m(t)}x(s)ds \qquad (30)\\
&+\frac{3}{\tau(t)-\tau_m(t)}\int_{-\tau(t)}^{-\tau_m(t)}\int_{t+\theta}^{t}x(s)dsd\theta\Big]^T\\
&W\Big[-\frac{(\tau(t)-\tau_m(t))}{2}x(t)-\int_{t-\tau(t)}^{t-\tau_m(t)}x(s)ds\\
&+\frac{3}{\tau(t)-\tau_m(t)}\int_{-\tau(t)}^{-\tau_m(t)}\int_{t+\theta}^{t}x(s)dsd\theta\Big]\Big\}\\
=\ &\frac{(\tau_M(t)-\tau_m(t))^2}{2}\dot{x}^T(t)W\dot{x}(t)-(1-\alpha\dot{\tau}(t))\\
&\Big\{2\Big[x(t)-\frac{1}{\tau_M(t)-\tau(t)}\int_{t-\tau_M(t)}^{t-\tau(t)}x(s)ds\Big]^T\\
&\times W\Big[x(t)-\frac{1}{\tau_M(t)-\tau(t)}\int_{t-\tau_M(t)}^{t-\tau(t)}x(s)ds\Big]\\
&-4\Big[-\frac{1}{2}x(t)-\frac{1}{\tau_M(t)-\tau(t)}\int_{t-\tau_M(t)}^{t-\tau(t)}x(s)ds\\
&+\frac{3}{(\tau_M(t)-\tau(t))^2}\int_{-\tau_M(t)}^{-\tau_m(t)}\int_{t+\theta}^{t}x(s)dsd\theta\Big]^T\\
&W\Big[-\frac{1}{2}x(t)-\frac{1}{\tau_M(t)-\tau(t)}\int_{t-\tau_M(t)}^{t-\tau(t)}x(s)ds\\
&+\frac{3}{(\tau_M(t)-\tau(t))^2}\int_{-\tau_M(t)}^{-\tau_m(t)}\int_{t+\theta}^{t}x(s)dsd\theta\Big]\\
&-2\Big[x(t)-\frac{1}{\tau(t)-\tau_m(t)}\int_{t-\tau(t)}^{t-\tau_m(t)}x(s)ds\Big]^T\\
&\times W\Big[x(t)-\frac{1}{\tau(t)-\tau_m(t)}\int_{t-\tau(t)}^{t-\tau_m(t)}x(s)ds\Big]\\
&-4\Big[-\frac{1}{2}x(t)-\frac{1}{\tau(t)-\tau_m(t)}\int_{t-\tau(t)}^{t-\tau_m(t)}x(s)ds\\
&+\frac{3}{(\tau(t)-\tau_m(t))^2}\int_{-\tau(t)}^{-\tau_m(t)}\int_{t+\theta}^{t}x(s)dsd\theta\Big]^T\\
&W\Big[-\frac{1}{2}x(t)-\frac{1}{\tau(t)-\tau_m(t)}\int_{t-\tau(t)}^{t-\tau_m(t)}x(s)ds\\
&+\frac{3}{(\tau(t)-\tau_m(t))^2}\int_{-\tau(t)}^{-\tau_m(t)}\int_{t+\theta}^{t}x(s)dsd\theta\Big]\Big\}\\
=\ &\zeta^T(t)\Omega_3(\tau(t),\dot{\tau}(t),d(t))\zeta(t) \qquad (30)
\end{aligned}
$$

Applying Lemma 2.6, we have

$$
\begin{aligned}
\mathfrak{L}V_6(t) =\ & (h-d(t))\dot{x}^T(t)X_1\dot{x}(t)-\int_{t-d(t)}^{t}\dot{x}^T(s)X_1\dot{x}(s)ds\\
&+(d_k-2d(t))x^T(t-d(t))X_2x(t-d(t))\\
\le\ & (h-d(t))\dot{x}^T(t)X_1\dot{x}(t)+d(t)\zeta^T(t)YX_1^{-1}F\zeta(t)\\
&+2\zeta^T(t)Y\int_{t-d(t)}^{t}\dot{x}(s)ds\\
&+((d-2d(t))x^T(t-d(t))X_2x(t-d(t))\\
=\ & \zeta^T(t)(h-d(t))e_6X_1e_6^T\zeta(t)+d(t)\zeta^T(t)YX_1^{-1}F\zeta(t)\\
&+2\zeta^T(t)Y(e_1-e_5)\zeta(t)+((d-2d(t))\zeta^T(t)e_5X_2e_5^T\zeta(t)
\end{aligned} \tag{31}
$$

$$
\begin{aligned}
\mathfrak{L}V_7(t) =\ & h^2\dot{x}^T(t)X_3\dot{x}(t)-\frac{\pi^2}{4}(x(t)-x(t_k))^TX_3(x(t)-x(t_k))\\
=\ & h^2\dot{x}^T(t)X_3\dot{x}(t)-\frac{\pi^2}{4}(x(t)\\
&-x(t-d(t)))^TX_3(x(t)-x(t-d(t)))\\
=\ & h^2\zeta^Te_6X_3e_6^T\zeta(t)-\frac{\pi^2}{4}\zeta^T(t)(e_1-e_5)X_3(e_1-e_5)^T\zeta(t)
\end{aligned} \tag{32}
$$

By introducing the relaxation matrix M_1 and M_2 with compatible dimension, one has the following zero equation

$$
\begin{aligned}
&(x^TM_1^T+\dot{x}^T(t)M_2^T)(\dot{x}(t)-A^ix(t)+A_{di}x(t-\tau(t))\\
&+B_iK_jx(t-d(t)))=0
\end{aligned} \tag{33}
$$

Combing (13)–(33) gives

$$
\mathfrak{L}V(t)\le\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(\theta(t))\mu_j(\theta_k)\zeta^T\overline{\Pi}_{lij}(\tau(t),\dot{\tau}(t),d(t))\zeta(t) \tag{34}
$$

where

$$
\begin{aligned}
\overline{\Pi}_{lij}(\tau(t),\dot{\tau}(t),d(t)) =\ & \Sigma_{lij}(\tau(t),\dot{\tau}(t),d(t))\\
&+\Lambda\Delta^{-1}\Lambda^{\intercal}+d(t)YX_1^{-1}Y^{\intercal}
\end{aligned}
$$

If the LMIs (8)–(12) are feasible, it follows from Lemma 2.1 that

$$
\mathfrak{L}V(t)<0. \tag{35}
$$

which implies the CLSDFS (7) is asymptotically stable. This completes the proof. □

Remark 1. Newly mode-dependent LKF is proposed with the aid of the tunable parameter, which makes full use of flexible delay intervals, and some novel stability criteria can be achieved.

Remark 2. Notice that in this paper, the mismatched membership functions (MMFs) $\omega_i(z(t))$ and $\omega_i(z(t_k))$ are introduced to obtain the less conservative results. It should be pointed out that, MMFs in Lemma 2.1 are introduced to tackle with the fuzzy systems with grades of membership. The relationship of $\omega_i(z(t))$ and $\omega_i(z(t_k))$ are neglected in the existing literature [2,3,13,14].

Remark 3. Inspired by Wang et al. [35], a new LKF has been proposed with the aid of a variable parameter α in the proof of Theorem 3.1. Different from the existing results in [9,14,15], the maximum and minimum bound of time delay is estimated as $\tau_M(t) = \alpha\tau(t) + (1-\alpha)\tau_M$ and $\tau_m(t) = \alpha\tau(t) + (1-\alpha)\tau_m$. Some novel cross terms $1-\alpha\dot{\tau}(t)$, $(1-\alpha)(\tau_M - \tau_m)$, $\tau_M(t) - \tau(t)$ and $\tau(t) - \tau_m(t)$ are introduced to gives less conservative criteria.

Remark 4. The time delay $\tau(t)$ satisfies $\dot{\tau}(t) \le \tau_D$, compared with [14,17], the constraint $\tau_D < 1$ is eliminated in the time-derivative of the proposed LKF, that is, the established stability condition in this paper is less conservative than those presented in [14,17].

Remark 5. It is worth pointing out that Lemma 2.3 reveals the relationship among mode-dependent terms $\int_{t-\tau_M(t)}^{t-\tau_m(t)} \dot{x}^{\mathsf{T}}(s)S_2\dot{x}(s)ds$, $x(t-\tau_M(t))$, $x(t-\tau_m(t))$, $\int_{t-\tau_M(t)}^{t-\tau(t)} x(s)ds$, $\int_{t-\tau(t)}^{t-\tau_m(t)} x(s)ds$, $\int_{-\tau(t)}^{-\tau_m(t)}\int_{t+\theta}^{t} x(s)dsd\theta$ and $\int_{-\tau_M(t)}^{-\tau(t)}\int_{t+\theta}^{t} x(s)dsd\theta$, which may induces less conservative results.

Corollary 3.1. *For a given scalar $h > 0$, if there exist symmetric matrices $P > 0$, $X_l > 0$ $(l = 1,2,3)$ and matrices Y_1, Y_2, $\mathcal{M}_l$, $\mathcal{N}_l$, $\mathcal{K}_l$, $\mathcal{L}_l$ $(l = 1,2,3)$, $\mathcal{I}_{mn}$, $\mathcal{J}_{mn}$, $\mathcal{K}_{mn}$, $\mathcal{L}_{mn}(m = 1,2; n = 1,2,3)$, $\tilde{W}_{1ij} > 0$, $\tilde{U}_{1ij} > 0$, $\tilde{Y}_{1ij}$, such that*

$$\begin{bmatrix} R_1 & Y_1 \\ * & R_1 \end{bmatrix} \ge 0 \tag{36}$$

$$\begin{bmatrix} R_2 & Y_2 \\ * & R_2 \end{bmatrix} \ge 0 \tag{37}$$

$$\bar{W}_{lij} + \bar{W}_{lji} \le \bar{U}_{lij} + \bar{U}_{lji}, l = 1,2, i, j \in \mathbb{S} \tag{38}$$

$$\begin{aligned}&\Pi_{lij} - 2\bar{W}_{lij} + \sum_{s=1}^{r} \delta_s(\bar{W}_{lis}^{+} + \bar{W}_{lsj}^{+}) \\ &\quad \le \bar{W}_{li(j+r)} + \bar{W}_{l(j+r)i}, l = 1,2, i, j \in \mathbb{S}\end{aligned} \tag{39}$$

$$\begin{bmatrix} \bar{Y}_{l11} & \bar{Y}_{l12} \\ * & \bar{Y}_{l11} \end{bmatrix} < 0, l = 1,2 \tag{40}$$

where

$$\bar{W}_{lij} = \bar{N}_{lij} - \bar{R}_{lij}, \bar{W}_{lij}^{+} = \bar{R}_{lij} + \bar{N}_{lij}, l = 1,2$$

$$\bar{Y}_{l11} = \begin{bmatrix} \bar{U}_{l11} & \cdots & \bar{U}_{l1r} \\ \vdots & \ddots & \vdots \\ \bar{U}_{lr1} & \cdots & \bar{U}_{lrr} \end{bmatrix} (l = 1,2),$$

$$\bar{Y}_{l12} = \begin{bmatrix} \bar{U}_{l1(r+1)} & \cdots & \bar{U}_{l1(2r)} \\ \vdots & \ddots & \vdots \\ \bar{U}_{lr(r+1)} & \cdots & \bar{U}_{lr(2r)} \end{bmatrix} (l = 1,2)$$

$$\begin{cases} \bar{\Pi}_{1ij} = \Pi_{ij}(\tau(t), \dot{\tau}(t), d(t)) \mid_{d(t)=0} \\ \bar{\Pi}_{2ij} = \Pi_{ij}(\tau(t), \dot{\tau}(t), d(t)) \mid_{d(t)=h} \end{cases}$$

$$\bar{\Pi}_{lij}(d(t)) = \begin{bmatrix} \bar{\Sigma}_{ij} & \sqrt{d(t)}Y \\ * & -X_1 \end{bmatrix}$$

$$\begin{aligned}\bar{\Sigma}_{ij}(d(t)) = {} & e_6Pe_1^T + e_1Pe_6^T + e_1Q_1e_1^{\mathsf{T}} \\ & + (h - d(t))e_6X_1e_6^T + 2Y(e_1 - e_5) \\ & + ((d_k - 2d(t))e_5X_2e_5^T + h^2e_6X_3e_6^T \\ & - \frac{\pi^2}{4}(e_1 - e_5)X_3(e_1 - e_5)^T \\ & + (e_1M_1^T + e_6M_2^T)(e_6^{\mathsf{T}} - A_ie_1^{\mathsf{T}} + A_{di}e_2^{\mathsf{T}} + B_iK_je_5^{\mathsf{T}}),\end{aligned}$$

4. Illustrative example

Example 1. In this example, a truck-trailer system (TTS) is considered, which is an extremely common industrial process subject to sampled-data control

$$\begin{cases} \dot{x}_1(t) = -a\frac{v\bar{t}}{Lt_0}x_1(t) - (1-a)\frac{v\bar{t}}{Lt_0}x_1(t - t_d) + \frac{v\bar{t}}{lt_0}u(t) \\ \dot{x}_2(t) = a\frac{v\bar{t}}{Lt_0}x_1(t) + (1-a)\frac{v\bar{t}}{Lt_0}x_1(t - t_d) \\ \dot{x}_3(t) = \frac{v\bar{t}}{Lt_0}\sin(x_2(t) + a(v\bar{t}/2L)x_1(t) \\ \quad +(1-a)(v\bar{t}/2L)x_1(t - t_d)) \end{cases}$$

where $x(t) = [x_1(t)x_2(t)x_3(t)]^{\mathsf{T}}$, $l = 2.8$, $L = 5.5$, $v = -1.0$, $\bar{t} = 2.0$, $t_0 = 0.5$. $x_1(t) \in [-\pi/2, \pi/2]$, $\dot{x}_1(t) \in [-3,3]$, $x_2(t) \in [-\pi/2, \pi/2]$, $\dot{x}_2(t) \in [-2,2]$, $[x_1(0)\ x_2(0)\ x_3(0)] = [0.5\ \ -0.2\ \ 1]$. The above nonlinear TTS can be modeled as T-S fuzzy system by imposing:

Plant rule 1:

IF $\theta(t) = x_2(t) + a(v\bar{t}/2L)x_1(t) + (1-a)(v\bar{t}/2L)x_1(t - t_d)$ is about 0,

THEN

$$\dot{x}(t) = A_1x(t) + A_{1d}x(t - t_d) + B_1u(t),$$

Plant rule 2:

IF $\theta(t) = x_2(t) + a(v\bar{t}/2L)x_1(t) + (1-a)(v\bar{t}/2L)x_1(t - t_d)$ is about π or $-\pi$,

THEN

$$\dot{x}(t) = A_2x(t) + A_{2d}x(t - t_d) + B_2u(t)$$

where

$$A_1 = \begin{bmatrix} -\frac{v\bar{t}}{Lt_0} & 0 & 0 \\ \frac{v\bar{t}}{Lt_0} & 0 & 0 \\ \frac{v^2\bar{t}^{-2}}{2Lt_0} & \frac{v\bar{t}}{t_0} & 0 \end{bmatrix}, \quad A_{d1} = \begin{bmatrix} -(1-a)\frac{v\bar{t}}{Lt_0} & 0 & 0 \\ (1-a)\frac{v\bar{t}}{Lt_0} & 0 & 0 \\ (1-a)\frac{v^2\bar{t}^{-2}}{2Lt_0} & 0 & 0 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} \frac{v\bar{t}}{lt} \\ 0 \\ 0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -\frac{v\bar{t}}{Lt_0} & 0 & 0 \\ \frac{v\bar{t}}{Lt_0} & 0 & 0 \\ \frac{dv^2\bar{t}^{-2}}{2Lt_0} & \frac{dv\bar{t}}{t_0} & 0 \end{bmatrix}, \quad A_{d2} = \begin{bmatrix} -(1-a)\frac{v\bar{t}}{Lt_0} & 0 & 0 \\ (1-a)\frac{v\bar{t}}{Lt_0} & 0 & 0 \\ (1-a)\frac{dv^2\bar{t}^{-2}}{2Lt_0} & 0 & 0 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} \frac{v\bar{t}}{lt} \\ 0 \\ 0 \end{bmatrix},$$

Table 1
The maximum bound of $h_{\max}$.

Method	$\tau_M = 0.5$	$\tau_M = 2$
[11]	0.374	0.251
Theorem 3.1	0.389	0.260

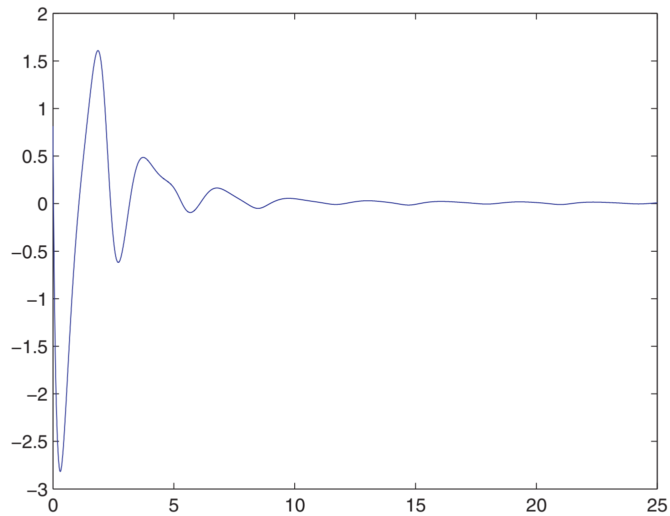

Fig. 1. State response of x_1.

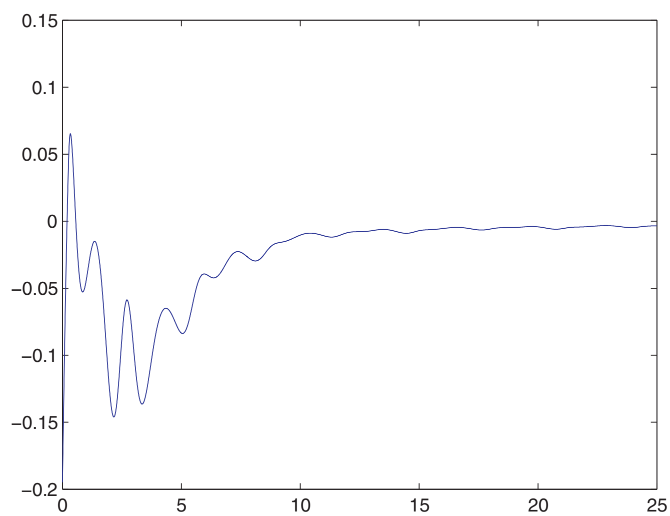

Fig. 2. State response of x_2.

setting $d = 0.1t_0/\pi$ and limiting function

$$\lambda_1(\theta(t)) = \left(1 - \frac{1}{1+\exp(-3(\theta(t)-0.5\pi))}\right) \times \left(\frac{1}{1+\exp(-3(\theta(t)+0.5\pi))}\right)$$

$$\lambda_2(\theta(t)) = 1 - \lambda_1(\theta(t)).$$

We suppose that time delay $\tau(t)$ satisfying $\tau_d = \tau_D = 0$, Table 1 depicts the maximum bound $h_{\max}$ with different τ_M. It is clear that the proposed method can provide larger upper bounds than the method in [11], which means that our result can achieves better performance and less conservativeness than the existing literature.

In the simulation, state trajectory for each state of TTS are given in Figs. 1–3. As shown in Figs. 1–3, it is clear that with the aid of sampled-data control, the closed-loop TTS is asymptotically stable.

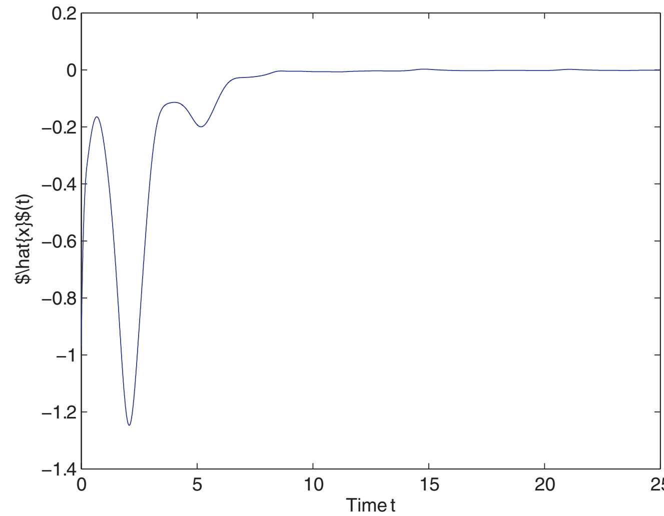

Fig. 3. State response of x_3.

5. Conclusions

In this paper, the problem of sampled-data stabilization for T-S fuzzy systems with time-varying delays has been studied. The mismatched membership functions are introduced to eliminate the conservatism in a more reasonable way, and the less conservative results are achieved. Both free-matrix-based and novel LKF involving triple integrals is employed to reduce the conservativeness of proposed method. Finally, a TTS is employed to verify the effectiveness of the proposed methodology. It is worth noting that in the future, it is of significance to investigate the sampled-data fuzzy control for multi-agent systems.

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