Signal Processing 140 (2017) 69–76

Contents lists available at ScienceDirect

Signal Processing

journal homepage: www.elsevier.com/locate/sigpro

Multi-target Bayes filter with the target detection

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ARTICLE INFO

Article history:
Received 12 October 2016
Revised 11 May 2017
Accepted 12 May 2017
Available online 13 May 2017

Keywords:
Target tracking
Multi-target Bayes filter
Target detection
Track initiation
State estimation

ABSTRACT

The probability hypothesis density (PHD) filter and marginal distribution Bayes (MDB) filter are two efficient Bayes approaches for multi-target tracking. However, these two filters fail to provide the state estimation of a target during its initial times due to the poor capability of the two filters on the target detection. To enhance the capability of the MDB filter on the target detection, we present a method for the target detection based on the rule-based track initiation technique, and develop a multi-target Bayes filter with the target detection by applying this target detection method to the MDB filter. Simulation results indicate that this filter has a stronger detecting and tracking capability of the target than the existing PHD and MDB filters.

1. Introduction

Multi-target tracking has attracted the attentions of researchers due to its wide applications in civil and military fields. This problem was discussed in many articles and a lot of tracking theories and algorithms of multiple targets were established in the past decades [1–31]. Traditional approaches for this problem use the measurements obtained by a sensor at several different times to detect new targets and establish new trajectories, and assign the measurement for each existing target to maintain its trajectory [1–4]. As the finite set statistics was established by Mahler, many novel Bayes tracking algorithms for multiple targets were developed in recent years. The remarkable achievements in this aspect include Mahler's probability hypothesis density (PHD) filter [5–6], Vo's Gaussian mixture PHD filter [7], particle PHD filter [8–10] and their extensions [11–25]. The optimal multi-target Bayes filter propagates the joint posterior density of the multi-target state through its prediction and update equations, which is generally intractable because of the set integrals in the prediction and update equations of the filter. This intractability is alleviated in Mahler's PHD filter because it propagates the first-order moment of the joint posterior density in the filter recursion [6]. However, the prediction and update equations of the PHD filter still involve multiple dimension integrals and are also intractable in general. Therefore, further approximation of the PHD filter is usually required [7,11]. Using a Gaussian mixture to represent the posterior intensity, Vo developed an implementation of the PHD filter for the linear Gaussian system, and extended this implementation to the nonlinear Gaussian system [7]. By applying the switching multiple models into Vo's Gaussian mixture PHD filter, Pasha proposed a Gaussian mixture PHD filter to track the maneuvering target that switches among several linear Gaussian models [12]. Adaptive target birth intensity techniques were also investigated independently in [13, 14] and [15] to avoid the requirement for exact knowledge of the birth target. To improve the stability of the target number estimation, the cardinalized PHD filter was developed in [16] to propagate the cardinality and moment in the filter recursion. In addition to the Gaussian mixture approach, the particle or sequential Monte Carlo approach is another approximation of the PHD filter [8–10]. The drawback of this approximation is that it requires a higher computational load than the Gaussian mixture approximation because a large number of particles need to be sampled in this approximation.

The main differences between the PHD filter and the traditional tracking approach are that the PHD filter avoids the data association which is used in the traditional approach to establish the new trajectories and maintain the existing trajectories [5–8] and that the PHD filter may provide the estimation of the target number. However, the PHD filter is prone to discard the information of a target if the target is missing from the measurement of a sensor. Besides, the PHD filter is not applicable to the closely spaced targets because it cannot distinguish distinct targets if these targets have a small separation [6]. To track the closely spaced targets more efficiently, Liu developed the marginal distribution Bayes (MDB) filter [27,28]. Unlike the PHD filter that propagates the first-order moment, the MDB filter transmits the marginal distribution and existence probability of each target in the filter's recursion. Based on the MDB filter, Liu also developed a sequential multi-

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http://dx.doi.org/10.1016/j.sigpro.2017.05.016

target MDB filter to track multiple targets [29]. The advantage of the sequential MDB filter is that it may handle the received measurements in real time and is more applicable to tracking the multiple targets than the PHD filter in case of low detection probability. By applying the switching multiple models and the adaptive estimating technique of turn rate to the sequential MDB filter, respectively, Liu proposed a sequential multiple target Bayes filter with jump Markov system models [30] and a multi-target Bayes filter with adaptive estimation of turn rate [31] to track the multiple maneuvering targets and the turn maneuvering targets, respectively. Similar to the PHD filter, Liu's MDB filters also avoid the data association that is usually used in the traditional approach of target tracking. The performance of these MDB filters have also been analyzed and examined through simulation experiments [27–31].

Available research results have demonstrated that Vo's PHD filter and Liu's MDB filter are efficient for multi-target tracking in the presence of clutter and noise [7,27,29] . However, these two filters fail to provide the state estimation of a target during its initial times due to the poor capability of these two filters on the target detection. The track initiation technique in the traditional tracking approaches [32] may provide an efficient solution for this problem. The objective of this technique is to detect a new target and initialize its track whenever the new target appears in the surveillance space. In [32], the two sequential track initiation techniques, namely, the rule-based technique and logic-based technique, are investigated. Based on the rule-based track initiation technique, an approach for the target detection by using the measurements obtained by the radar at three different scanning periods is presented in this paper. Applying the target detection approach to the MDB filter, we develop the MDB filter with the target detection to enhance the capability of the MDB filter on detecting multiple targets, which is also the main contribution of this paper. In terms of the OSPA distance [33], the tracking performance of the proposed filter is demonstrated by comparing it with Vo's Gaussian mixture PHD filter and Liu's MDB filter, which indicates that the proposed filter is best at detecting and tracking multiple targets among these filters. Please note that the target detection method presented in this paper may also be applied to the PHD filter to enhance its capability on detecting multiple targets because of the similarity between the MDB filter and PHD filter.

The rest of this paper is organized as follows: The target detection technique used in this paper is described in Section 2. The multi-target Bayes filter with the target detection is developed in Section 3. The performance of the developed filter is evaluated in Section 4. Conclusion is drawn in Section 5.

2. Target detection method

Track initiation techniques have been investigated in [32]. Based on the rule-based track initiation technique, we will present the target detection approach to detect the new target and estimate its initial state vectors by using three measurement sets that are obtained by the radar at three different scanning periods. The radar measures the range and azimuth of the target, and its measurement in polar coordinates is denoted by

$$\boldsymbol{z}_{r,\theta} = \begin{bmatrix} r \\ \theta \end{bmatrix} = \begin{bmatrix} \sqrt{x^2+y^2} \\ \arctan\dfrac{y}{x} \end{bmatrix} + \begin{bmatrix} v_r \\ v_\theta \end{bmatrix} \tag{1}$$

where (x, y) denotes the position of the target, and v_r and v_θ are the measurement noises with standard deviations σ_r and σ_θ, respectively. Converting this measurement in polar coordinates into a measurement in Cartesian coordinates [34], we have

$$\boldsymbol{z} = \begin{bmatrix} r\cos\theta \\ r\sin\theta \end{bmatrix} \tag{2}$$

The error covariance of converted measurement $\boldsymbol{z}$ is given by

$$\boldsymbol{R}_{xy} = \begin{bmatrix} \sigma_x^2 & \sigma_{xy}^2 \\ \sigma_{yx}^2 & \sigma_y^2 \end{bmatrix} \tag{3}$$

where

$$\sigma_x^2 = \sigma_r^2\cos^2\theta + r^2\sigma_\theta^2\sin^2\theta \tag{4}$$

$$\sigma_y^2 = \sigma_r^2\sin^2\theta + r^2\sigma_\theta^2\cos^2\theta \tag{5}$$

$$\sigma_{xy}^2 = \sigma_{yx}^2 = \sin\theta\cos\theta\left(\sigma_\theta^2 - r^2\sigma_\theta^2\right) \tag{6}$$

Let $\boldsymbol{Z}_{k-2} = (\boldsymbol{z}_{1,k-2}, \boldsymbol{z}_{2,k-2}, \cdots, \boldsymbol{z}_{M_{k-2},k-2})$, $\boldsymbol{Z}_{k-1} = (\boldsymbol{z}_{1,k-1}, \boldsymbol{z}_{2,k-1}, \cdots, \boldsymbol{z}_{M_{k-1},k-1})$ and $\boldsymbol{Z}_k = (\boldsymbol{z}_{1,k}, \boldsymbol{z}_{2,k}, \cdots, \boldsymbol{z}_{M_k,k})$ denote three measurement sets obtained at scanning periods $k-2$, $k-1$ and k, respectively, let t_{k-2}, t_{k-1} and t_k denote the times of these three scanning periods and let M_{k-2}, M_{k-1} and M_k denote the number of measurements at these three scanning periods. Each measurement in three measurement sets is the converted measurement given by Eq. (2). We pick a measurement from each measurement set. The picked measurements from the three sets are denoted by $\boldsymbol{z}_{e,k-2}$, $\boldsymbol{z}_{f,k-1}$ and $\boldsymbol{z}_{g,k}$, respectively, where $e = 1, 2, \cdots, M_{k-2}$, $f = 1, 2, \cdots, M_{k-1}$ and $g = 1, 2, \cdots, M_k$. We then test whether the picked three measurements satisfy the following three requirements that are given based on the rule-based track initiation technique in [32].

(1) Velocity requirement: the moving velocity of the target is greater than or equal to $v_{\min}$ and is less than or equal to $v_{\max}$, namely,

$$v_{\min} \le \frac{\left\|\boldsymbol{z}_{f,k-1} - \boldsymbol{z}_{e,k-2}\right\|_2}{t_{k-1} - t_{k-2}} \le v_{\max}, \tag{7}$$

$$v_{\min} \le \frac{\left\|\boldsymbol{z}_{g,k} - \boldsymbol{z}_{f,k-1}\right\|_2}{t_k - t_{k-1}} \le v_{\max}, \tag{8}$$

where $\|\cdot\|_2$ denotes the 2-norm of a vector, and $v_{\min}$ and $v_{\max}$ are minimal and maximal velocities, respectively.

(2) Acceleration requirement: the acceleration of the target is less than or equal to $a_{\max}$, namely,

$$\left|\frac{\left\|\boldsymbol{z}_{g,k} - \boldsymbol{z}_{f,k-1}\right\|_2}{t_k - t_{k-1}} - \frac{\left\|\boldsymbol{z}_{f,k-1} - \boldsymbol{z}_{e,k-2}\right\|_2}{t_{k-1} - t_{k-2}}\right| \Big/ (t_k - t_{k-1}) \le a_{\max}, \tag{9}$$

where $|\cdot|$ denotes the absolute value of a number and $a_{\max}$ is the maximal acceleration.

(3) Angle requirement: the cosine of the included angle between vectors $\boldsymbol{z}_{f,k-1} - \boldsymbol{z}_{e,k-2}$ and $\boldsymbol{z}_{g,k} - \boldsymbol{z}_{f,k-1}$ is greater than or equal to minimal value $c_{\min}$, namely,

$$\frac{\left(\boldsymbol{z}_{g,k} - \boldsymbol{z}_{f,k-1}, \boldsymbol{z}_{f,k-1} - \boldsymbol{z}_{e,k-2}\right)}{\left\|\boldsymbol{z}_{g,k} - \boldsymbol{z}_{f,k-1}\right\|_2 \times \left\|\boldsymbol{z}_{f,k-1} - \boldsymbol{z}_{e,k-2}\right\|_2} \ge c_{\min}, \tag{10}$$

where $(\cdot, \cdot)$ denotes the dot product of two vectors.

We confirm that a target is detected if measurements $\boldsymbol{z}_{e,k-2}$, $\boldsymbol{z}_{f,k-1}$ and $\boldsymbol{z}_{g,k}$ satisfy the above three requirements, and then estimate the state vectors of the detected target at times t_{k-2}, t_{k-1} and

t_k by using the least square technique to establish its track. Let

$$\boldsymbol{C} = \begin{bmatrix} 1 & t_{k-2}-t_k & 0 & 0 \\ 0 & 0 & 1 & t_{k-2}-t_k \\ 1 & t_{k-1}-t_k & 0 & 0 \\ 0 & 0 & 1 & t_{k-1}-t_k \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \tag{11}$$

$$\boldsymbol{Y} = \begin{bmatrix} \boldsymbol{z}_{e,k-2} \\ \boldsymbol{z}_{f,k-1} \\ \boldsymbol{z}_{g,k} \end{bmatrix}, \tag{12}$$

$$\boldsymbol{R} = \begin{bmatrix} \boldsymbol{R}_{xy}^1 & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{R}_{xy}^2 & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{R}_{xy}^3 \end{bmatrix}, \tag{13}$$

where $\boldsymbol{R}_{xy}^1$,$\boldsymbol{R}_{xy}^2$ and $\boldsymbol{R}_{xy}^3$ are the error covariance matrices of converted measurements $\boldsymbol{z}_{e,k-2}$, $\boldsymbol{z}_{f,k-1}$ and $\boldsymbol{z}_{g,k}$, respectively, and are obtained by using Eq. (3). Using $\boldsymbol{\xi} = [x \quad \dot{x} \quad y \quad \dot{y}]^T$ to denote the state vector of a target where (x, y) is its position vector and $(\dot{x}, \dot{y})$ is its velocity vector, the state estimation and covariance matrix of the detected target at time t_k are given by

$$\boldsymbol{\xi}_{\gamma,k}^i = (\boldsymbol{C}^{\mathrm{T}}\boldsymbol{C})^{-1}\boldsymbol{C}^{\mathrm{T}}\boldsymbol{Y}, \tag{14}$$

$$\boldsymbol{P}_{\gamma,k}^i = (\boldsymbol{C}^{\mathrm{T}}\boldsymbol{C})^{-1}\boldsymbol{C}^{\mathrm{T}}\boldsymbol{R}C(\boldsymbol{C}^{\mathrm{T}}\boldsymbol{C})^{-1}. \tag{15}$$

The state estimation of the detected target at time t_{k-1} are given by

$$\boldsymbol{\xi}_{\gamma,k-1}^i = (\boldsymbol{C}_1^{\mathrm{T}}\boldsymbol{C}_1)^{-1}\boldsymbol{C}_1^{\mathrm{T}}\boldsymbol{Y}, \tag{16}$$

where

$$\boldsymbol{C}_1 = \begin{bmatrix} 1 & t_{k-2}-t_{k-1} & 0 & 0 \\ 0 & 0 & 1 & t_{k-2}-t_{k-1} \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & t_k-t_{k-1} & 0 & 0 \\ 0 & 0 & 1 & t_k-t_{k-1} \end{bmatrix}. \tag{17}$$

Similarly, the state estimation of the detected target at time t_{k-2} are given by

$$\boldsymbol{\xi}_{\gamma,k-2}^i = (\boldsymbol{C}_2^{\mathrm{T}}\boldsymbol{C}_2)^{-1}\boldsymbol{C}_2^{\mathrm{T}}\boldsymbol{Y}, \tag{18}$$

where

$$\boldsymbol{C}_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & t_{k-1}-t_{k-2} & 0 & 0 \\ 0 & 0 & 1 & t_{k-1}-t_{k-2} \\ 1 & t_k-t_{k-2} & 0 & 0 \\ 0 & 0 & 1 & t_k-t_{k-2} \end{bmatrix}. \tag{19}$$

After the state estimations of a detected target are obtained, three measurements which are used for estimating its state vectors have to be abandoned from sets $\boldsymbol{Z}_{k-2}$, $\boldsymbol{Z}_{k-1}$ and $\boldsymbol{Z}_k$, respectively.

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given {z_{e,k-2}}_{e=1}^{M_{k-2}}, {z_{f,k-1}}_{f=1}^{M_{k-1}}, {z_{g,k}}_{g=1}^{M_k}, t_{k-2}, t_{k-1}, t_k, σ_r and σ_θ
set m_e := 0 for e = 1,···,M_{k-2}.
set n_f := 0 for f = 1,···,M_{k-1}.
set l_g := 0 for g = 1,···,M_k.
i := 0.
for  e = 1,···,M_{k-2}
   for  f = 1,···,M_{k-1}
      for  g = 1,···,M_k
         if m_e = 0 and n_f = 0 and l_g = 0
            if z_{e,k-2}, z_{f,k-1} and z_{g,k} satisfy Eqs. (7),(8),(9) and (10)
               i := i+1.
               m_e := 1, n_f := 1, l_g := 1.
               using Eq. (14) to calculate ξ^i_{γ,k} .
               using Eq. (15) to calculate P^i_{γ,k}.
               using Eq. (16) to calculate ξ^i_{γ,k-1}.
               using Eq. (18) to calculate ξ^i_{γ,k-2}.
            end
         end
      end
   end
end
delete measurements with m_e = 1 or n_f = 1 or l_g = 1 from
{z_{e,k-2}}_{e=1}^{M_{k-2}}, {z_{f,k-1}}_{f=1}^{M_{k-1}} and {z_{g,k}}_{g=1}^{M_k}, respectively.
N := i.
output {ξ^i_{γ,k-2}, ξ^i_{γ,k-1}, ξ^i_{γ,k}, P^i_{γ,k}}_{i=1}^N .
```

Fig. 1. Pseudo-code for the target detection.

By repeating the aforementioned method, we may detect individual targets from measurement sets $\boldsymbol{Z}_{k-2}$, $\boldsymbol{Z}_{k-1}$ and $\boldsymbol{Z}_k$, and estimate their state vectors at times t_{k-2}, t_{k-1} and t_k and covariance matrix at time t_k.

Fig. 1 shows the pseudo-Code for the target detection where N denotes the number of the detected targets at time t_k. The advantage of the target detection method is that it may provide the state estimations of individual detected targets at times t_{k-2}, t_{k-1} and t_k, which is extremely important for tracking these detected targets at subsequent times.

3. Mult-target Bayes filter with the target detection

We have presented the multi-target Bayes filter in [27] and [29]. In this section, we apply the target detection method in Section 2 to the multi-target Bayes filter in [29] to design a multi-target Bayes filter with the target detection. The novel filter is described as follows:

Step 1: Prediction

In this step, we use marginal distribution $N(\boldsymbol{x}_{i,k-1}; \boldsymbol{\xi}_{i,k-1}, \boldsymbol{P}_{i,k-1})$ and existence probability $\rho_{i,k-1}$ of distinct targets at time step $k-1$ to predict their marginal distribution and existence probability at time step k where $i = 1, 2, \cdots, N_{k-1}$ and N_{k-1} denotes the number of targets at time step $k-1$. The predicted marginal distribution and existence probability are as follows:

$$N(\boldsymbol{x}_{i,k}; \boldsymbol{\xi}_{i,k|k-1}, \boldsymbol{P}_{i,k|k-1}) \quad i = 1, 2, \cdots, N_{k-1} \tag{20}$$

$$\rho_{i,k|k-1} = p_{S,k}\rho_{i,k-1}, \; i = 1, 2, \cdots, N_{k-1} \tag{21}$$

where $p_{S,\,k}$ is survival probability, and $\boldsymbol{\xi}_{i,k|k-1}$ and $\boldsymbol{P}_{i,k|k-1}$ are given by

$$\boldsymbol{\xi}_{i,k|k-1} = \boldsymbol{\Phi}_{k-1}\boldsymbol{\xi}_{i,k-1} \tag{22}$$

$$\boldsymbol{P}_{i,k|k-1} = \boldsymbol{Q}_{k-1} + \boldsymbol{\Phi}_{k-1}\boldsymbol{P}_{i,k-1}\boldsymbol{\Phi}_{k-1}^{\mathrm{T}} \tag{23}$$

where $\boldsymbol{\Phi}_{k-1}$ and $\boldsymbol{Q}_{k-1}$ denotes the state transition matrix and covariance matrix, respectively.

Step 2: Classification of measurements

In this step, we split measurement set $\boldsymbol{Z}_k = (\boldsymbol{z}_{1,k}, \cdots, \boldsymbol{z}_{M_k,k})$ at time step k, where M_k denotes the number of measurements, into two categories according to the following procedures:

We first use converted measurement $\boldsymbol{z}_{j,\,k}$ where $j = 1, 2, \cdots, M_k$ to compute the probability ρ_j^c as

$$\rho_j^c = \frac{\lambda_{c,k}}{\lambda_{c,k} + p_{D,k}\sum_{i=1}^{N_{k-1}} \rho_{i,k|k-1} N(\boldsymbol{z}_{j,k}; \boldsymbol{H}_k\boldsymbol{\xi}_{i,k|k-1}, \boldsymbol{H}_k\boldsymbol{P}_{i,k|k-1}\boldsymbol{H}_k^{\mathrm{T}} + \boldsymbol{R}_{j,k})} \tag{24}$$

where $\lambda_{c,\,k}$, $p_{D,\,k}$, $\boldsymbol{H}_k$ and $\boldsymbol{R}_{j,\,k}$ denote the clutter rate, detection probability, observation matrix and error covariance matrix of converted measurement $\boldsymbol{z}_{j,\,k}$, respectively. We then classify converted measurement $\boldsymbol{z}_{j,\,k}$ into the first category if $\rho_j^c \le 0.5$, which indicates $\boldsymbol{z}_{j,\,k}$ is originated from an existing target. Otherwise, we classify $\boldsymbol{z}_{j,\,k}$ into the second category, which suggests $\boldsymbol{z}_{j,\,k}$ is a clutter-originated measurement or a measurement originated from a new target.

Addressing each measurement in measurement set $\boldsymbol{Z}_k = (\boldsymbol{z}_{1,k}, \cdots, \boldsymbol{z}_{M_k,k})$ according to the aforementioned method, we may split measurement set $\boldsymbol{Z}_k$ into two categories. The measurements in the first category and second category are denoted by sets $\boldsymbol{Z}_k^m = (\boldsymbol{z}_{1,k}^m, \cdots, \boldsymbol{z}_{M_{1,k},k}^m)$ and $\boldsymbol{Z}_k^c = (\boldsymbol{z}_{1,k}^c, \cdots, \boldsymbol{z}_{M_{2,k},k}^c)$, respectively, where $M_{1,\,k}$ and $M_{2,\,k}$ denote the measured numbers of the first category and second category at time step k, respectively, and $M_{1,k} + M_{2,k} = M_k$.

Step 3: Update and state extraction of existing targets

Using each measurement in measurement set $\boldsymbol{Z}_k^m = (\boldsymbol{z}_{1,k}^m, \cdots, \boldsymbol{z}_{M_{1,k},k}^m)$ to update each predicted distribution in Eq. (20) according to Bayes rule, we obtain the updated distribution and existence probability as follows:

$$N(\boldsymbol{x}_{i,k}; \boldsymbol{\xi}_{i,k}^j, \boldsymbol{P}_{i,k}^j),\ i = 1, 2, \cdots, N_{k-1},\ j = 1, 2,\ \cdots, M_{1,k} \tag{25}$$

$$\rho_{i,k}^j = \frac{p_{D,k}\rho_{i,k|k-1}N(\boldsymbol{z}_{j,k}^m; \boldsymbol{H}_k\boldsymbol{\xi}_{i,k|k-1}, \boldsymbol{H}_k\boldsymbol{P}_{i,k|k-1}\boldsymbol{H}_k^{\mathrm{T}} + \boldsymbol{R}_{j,k}^m)}{\lambda_{c,k} + p_{D,k}\sum_{i=1}^{N_{k-1}} \rho_{i,k|k-1}N(\boldsymbol{z}_{j,k}^m; \boldsymbol{H}_k\boldsymbol{\xi}_{i,k|k-1}, \boldsymbol{H}_k\boldsymbol{P}_{i,k|k-1}\boldsymbol{H}_k^{\mathrm{T}} + \boldsymbol{R}_{j,k}^m)}, \quad i = 1, 2, \cdots, N_{k-1},\ j = 1, 2,\ \cdots, M_{1,k} \tag{26}$$

where $\boldsymbol{R}_{j,k}^m$ is the error covariance matrix of converted measurement $\boldsymbol{z}_{j,k}^m$, and $\boldsymbol{\xi}_{i,k}^j$ and $\boldsymbol{P}_{i,k}^j$ are given by

$$\boldsymbol{\xi}_{i,k}^j = \boldsymbol{\xi}_{i,k|k-1} + \boldsymbol{A}_{i,k}^j \cdot (\boldsymbol{z}_{j,k}^m - \boldsymbol{H}_k\boldsymbol{\xi}_{i,k|k-1}) \tag{27}$$

$$\boldsymbol{P}_{i,k}^j = (\boldsymbol{I} - \boldsymbol{A}_{i,k}^j \cdot \boldsymbol{H}_k)\boldsymbol{P}_{i,k|k-1} \tag{28}$$

$$\boldsymbol{A}_{i,k}^j = \boldsymbol{P}_{i,k|k-1}\boldsymbol{H}_k^{\mathrm{T}}[\boldsymbol{H}_k\boldsymbol{P}_{i,k|k-1}\boldsymbol{H}_k^{\mathrm{T}} + \boldsymbol{R}_{j,k}^m]^{-1} \tag{29}$$

Let $\rho_{i,k}^{M_{1,k}+1} = \rho_{i,k|k-1}$, $\boldsymbol{\xi}_{i,k}^{M_{1,k}+1} = \boldsymbol{\xi}_{i,k|k-1}$ and $\boldsymbol{P}_{i,k}^{M_{1,k}+1} = \boldsymbol{P}_{i,k|k-1}$. We find the index with maximum existence probability from set$\{\rho_{i,k}^j; j = 1, 2, \cdots, M_{1,k}+1\}$, namely,

$$q = \arg\max_{j\in\{1,\cdots,M_{1,k}+1\}} \{\rho_{i,k}^j\} \tag{30}$$

The updated distribution and existence of target i at time step k are given by

$$N(\boldsymbol{x}_{i,k}; \boldsymbol{\xi}_{i,k}, \boldsymbol{P}_{i,k}) = N(\boldsymbol{x}_{i,k}; \boldsymbol{\xi}_{i,k}^q, \boldsymbol{P}_{i,k}^q),\ i = 1, 2, \cdots, N_{k-1}, \tag{31}$$

$$\rho_{i,k} = \rho_{i,k}^q,\ i = 1, 2, \cdots, N_{k-1}, \tag{32}$$

We then eliminate the targets whose existence probability $\rho_{i,\,k}$ is less than threshold τ and select the state vectors of the targets whose existence probability $\rho_{i,\,k}$ is greater than 0.5 as the output of this filter at time step k.

Step 4: Target detection

In this step, we use measurement sets $\boldsymbol{Z}_{k-2}^c = (\boldsymbol{z}_{1,k-2}^c, \cdots, \boldsymbol{z}_{M_{2,k-2},k-2}^c)$ at time step $k-2$, $\boldsymbol{Z}_{k-1}^c = (\boldsymbol{z}_{1,k-1}^c, \cdots, \boldsymbol{z}_{M_{2,k-1},k-1}^c)$ at time step $k-1$ and $\boldsymbol{Z}_k^c = (\boldsymbol{z}_{1,k}^c, \cdots, \boldsymbol{z}_{M_{2,k},k}^c)$ at time step k to detect individual targets.

We select a measurement from each set; denote the selected measurements by $\boldsymbol{z}_{e,k-2}^c$, $\boldsymbol{z}_{f,k-1}^c$ and $\boldsymbol{z}_{g,k}^c$, respectively, where $e = 1, 2, \cdots, M_{2,k-2}$, $f = 1, 2, \cdots, M_{2,k-1}$ and $g = 1, 2, \cdots, M_{2,k}$; replace measurements $\boldsymbol{z}_{e,k-2}$, $\boldsymbol{z}_{f,k-1}$ and $\boldsymbol{z}_{g,k}$ in Eqs. (7), (8), (9) and (10) with $\boldsymbol{z}_{e,k-2}^c$, $\boldsymbol{z}_{f,k-1}^c$ and $\boldsymbol{z}_{g,k}^c$, respectively, and test whether the selected three measurements satisfy the requirements in Eqs. (7), (8), (9) and (10).

If measurements $\boldsymbol{z}_{e,k-2}^c$, $\boldsymbol{z}_{f,k-1}^c$ and $\boldsymbol{z}_{g,k}^c$ satisfy the requirements in Eqs. (7), (8), (9) and (10). we confirm that a target is detected. Replacing measurements $\boldsymbol{z}_{e,k-2}$, $\boldsymbol{z}_{f,k-1}$ and $\boldsymbol{z}_{g,k}$ in Eq. (12) with $\boldsymbol{z}_{e,k-2}^c$, $\boldsymbol{z}_{f,k-1}^c$ and $\boldsymbol{z}_{g,k}^c$, respectively, and using the error covariance matrices of converted measurements $\boldsymbol{z}_{e,k-2}^c$, $\boldsymbol{z}_{f,k-1}^c$ and $\boldsymbol{z}_{g,k}^c$ as covariance matrices $\boldsymbol{R}_{xy}^1$, $\boldsymbol{R}_{xy}^2$ and $\boldsymbol{R}_{xy}^3$ in Eq. (13), respectively, we obtain state vector $\boldsymbol{\xi}_{\gamma,k}^i$ and error covariance $\boldsymbol{P}_{\gamma,k}^i$ by using Eqs. (14) and (15), respectively. The marginal distribution of the detected target is given by

$$N(\boldsymbol{x}_{i,k}; \boldsymbol{\xi}_{\gamma,k}^i, \boldsymbol{P}_{\gamma,k}^i) \tag{33}$$

The state estimations $\boldsymbol{\xi}_{\gamma,k-1}^i$ and $\boldsymbol{\xi}_{\gamma,k-2}^i$ of the detected target at time steps $k-1$ and $k-2$ are given by Eqs (16) and (18), respectively. At the same time, we designate a known ρ_γ as the existence probability of the detected target at time step k, namely,

$$\rho_{\gamma,k}^i = \rho_\gamma \tag{34}$$

After the state estimations of the detected target are obtained, we discard the three measurements which are used for estimating the states of the detected target from sets $\boldsymbol{Z}_{k-2}^c$, $\boldsymbol{Z}_{k-1}^c$ and $\boldsymbol{Z}_k^c$, respectively.

By repeating the aforementioned method, we may detect individual targets and obtain their marginal distributions at time step k and their state estimations at time steps $k-1$ and $k-2$.

Step 5: State extraction of detected targets

We export the state estimations of each detected target at time steps k, $k-1$ and $k-2$ to supply the output of this filter at time steps k, $k-1$ and $k-2$, respectively. At the same time, we add the marginal distributions and existence probabilities of the detected targets at time step k to those of the existing targets at time step k to form the marginal distributions and existence probabilities of individual targets at time step k, and transmit them, along with measurement sets $\boldsymbol{Z}_{k-1}^c$ and $\boldsymbol{Z}_k^c$, to the next time step. The pseudo-Code for the multi-target Bayes filter with the target detection is given in Fig. 2.

The advantages of the multi-target Bayes filter with the target detection over the PHD and MDB filters are as follows: (1) It may provide the initial state estimations of the detected target, which is extremely important for tracking the target at subsequent times. (2) It propagates less Gaussian terms to the next time step because

given $\left\{\xi_{i,k-1}, \boldsymbol{P}_{i,k-1}, \rho_{i,k-1}\right\}_{i=1}^{N_{k-1}}, \left\{z_{e,k-2}^{c}\right\}_{e=1}^{M_{2,k-2}}, \left\{z_{f,k-1}^{c}\right\}_{f=1}^{M_{2,k-1}}, \left\{z_{j,k}\right\}_{j=1}^{M_k}$,
$t_{k-2}, t_{k-1}, t_k, \sigma_r$ and σ_θ .
Step1: prediction.
if $N_{k-1} > 0$
  Step2: Classification of measurements.
  Step3: Update and state extraction of existing targets.
else
  $\boldsymbol{Z}_k^c := \left\{z_{j,k}\right\}_{j=1}^{M_k}$.
end
Step 4: Target detection.
Step 5: State extraction of detected targets.

Fig. 2. Pseudo-code for the proposed multi-target Bayes filter.

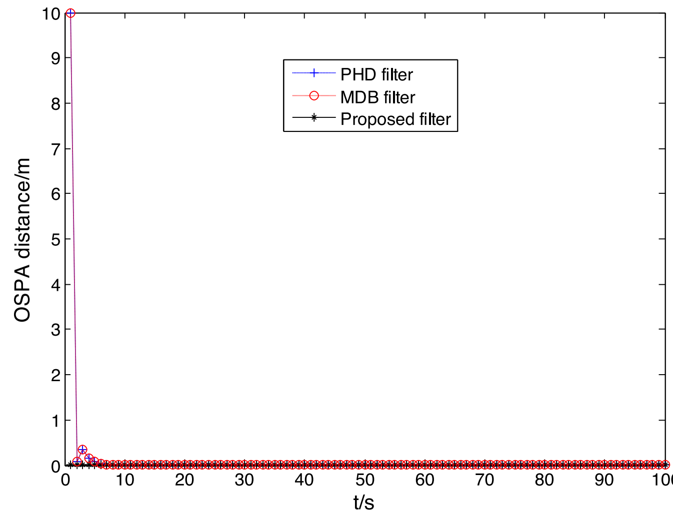

Fig. 3. The experimental result in Example 1.

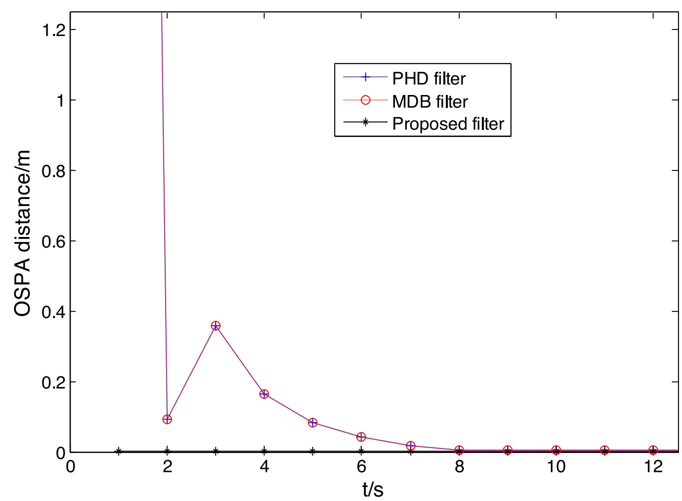

Fig. 4. A detail view of the experimental result in Example 1.

only the Gaussian terms relevant to the existing targets and detected targets are propagated to the next time step.

***Remark*:** The initial position, velocity and error covariance of the detected targets or newly appearing targets provided in the developed filter are accurate, whereas the initial velocity of newly appearing targets is usually assumed to be zero and their initial error covariance is generally chosen based on the maximal target velocity in the PHD filter and MDB filter [14]. In this case, the inaccuracy in the initial state and error covariance of new targets becomes larger, which may lead to a failure of target detection or track initialization [14].

4. Simulation results

We use three examples to reveal the performance of the MDB filter with the target detection. The first is to show the performance of this filter for two targets with the constant velocity in the absence of missed detection, clutter and noise. The second is to reveal the tracking performance of this filter for eight targets with the constant velocity in the presence of missed detection, clutter and noise. The third is to demonstrate the performance of this filter for eight targets with the constant acceleration in the presence of missed detection, clutter and noise. In the experiments, we use Vo's PHD filter and Liu's MDB filter as the comparing objects, and select OSPA distance [33] with parameters $c = 10$ m and $p = 2$ as the measure. The state transition matrix $\boldsymbol{\Phi}_{k-1}$ in Eqs. (22) and (23), covariance matrix $\boldsymbol{Q}_{k-1}$ in Eq. (23), observation matrix $\boldsymbol{H}_k$ in Eqs. (24), (26), (27), (28) and (29) are identical to matrices $\boldsymbol{F}_{k-1}$, $\boldsymbol{Q}_{k-1}$, $\boldsymbol{H}_k$ in [29], respectively, where σ_v^2 denotes the variance of process noise. The converted measurement in Eq. (2) and converted error covariance in Eq. (3) are used in the proposed filter, Vo's PHD filter and Liu's MDB filter. The parameters $v_{\min}$, $v_{\max}$, $a_{\max}$ and $c_{\min}$ for the target detection in the proposed filter are set to $v_{\min} = 10\ \mathrm{ms}^{-1}$, $v_{\max} = 70\ \mathrm{ms}^{-1}$, $a_{\max} = 25\ \mathrm{ms}^{-2}$ and $c_{\min} = 0.7071$.

Example 1. Two targets are considered in this example. Target 1 with initial position at (−950 m,−850 m) moves at constant velocity ($\dot{x} = 19$ m/s and $\dot{y} = 18$ m/s) from $t = 1$ s to $t = 100$ s. Target 2 moves at constant velocity ($\dot{x} = 18$ m/s and $\dot{y} = -17$ m/s) from $t = 1$ s to $t = 100$ s, and its initial position is at (−950 m, 850 m). We use parameters $\Delta t_k = t_k - t_{k-1} = 1$ s, $\sigma_v = 0\ \mathrm{m/s}^2$, $\sigma_r = 0$ m, $\sigma_\theta = 0$ rad, $p_{S,k} = 1.0$, $\lambda_{c,k} = 0\ \mathrm{m}^{-2}$ and $p_{D,k} = 1.0$ to generate the measurements of the radar with scanning period $T = 1$ s for 100 scanning periods, set parameters of the proposed filter to $\Delta t_k = 1$ s $\sigma_v = 1$ m/s, $p_{S,k} = 0.6$, $\lambda_{c,k} = 2.5 \times 10^{-11}\ \mathrm{m}^{-2}$, $p_{D,k} = 1.0$, $\tau = 0.001$, $\sigma_r = 3$ m, $\sigma_\theta = 0.001$ rad and $\rho_\gamma = 0.9$, and address the measurements by the proposed filter, Vo's PHD filter and Liu's MDB filter and MDB filter, respectively. Figs. 3 and 4 show the experimental result and a detail view of this result, respectively.

The proposed filter uses the measurements from three consecutive scanning periods to detect the targets and estimate their initial states. Due to this fact, although the two targets appear at $t = 1$ s in this example, the proposed filter detects the two targets at $t = 3$ s and obtains their initial state estimations at this time. Figs. 3 and 4 reveal that sufficiently small OSPA distances appear at the plot of the proposed filter at $t = 1$ s, $t = 2$ s and $t = 3$ sbecause the proposed filter may provide the accurate state estimations of the two targets at $t = 1$ s, $t = 2$ s and $t = 3$ s when they are detected at $t = 3$ s, whereas Vo's PHD filter and Liu's MDB filter fail to provide the state estimations of the two targets at $t = 1$ sbecause of their poor capacity on the target detection. Fig. 4 also reveals that the PHD and MDB filters have larger OSPA distances at $t = 2$ s, $t = 3$ s and several subsequent times than the proposed filter. The reason for this phenomenon is that the proposed filter provides much more accurate state estimations than the PHD and MDB filters at these times because it has a strong detecting capability of the target.

Example 2. In this example, eight targets with different initial velocities appear from different initial positions at different times.

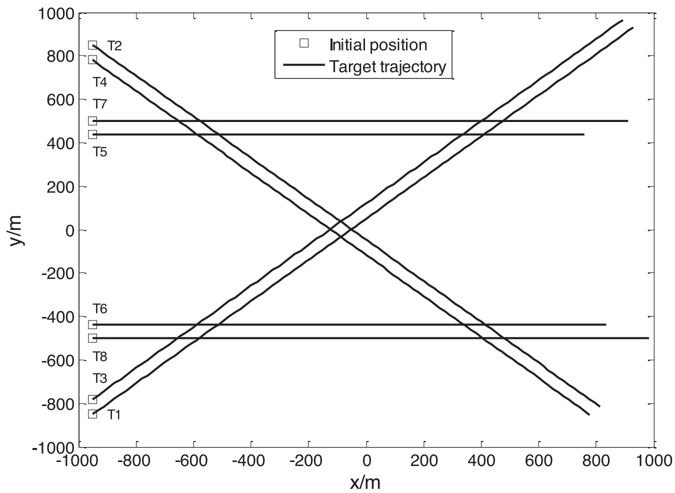

Fig. 5. Moving trajectories of eight targets.

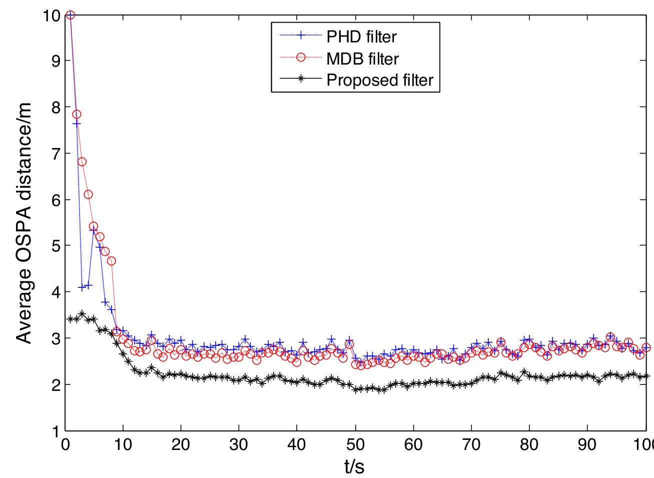

Fig. 7. Average OSPA distances in Example 2.

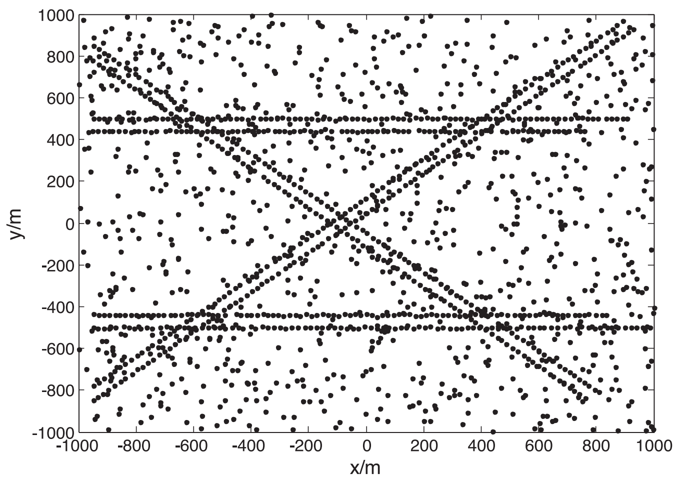

Fig. 6. Converted measurements.

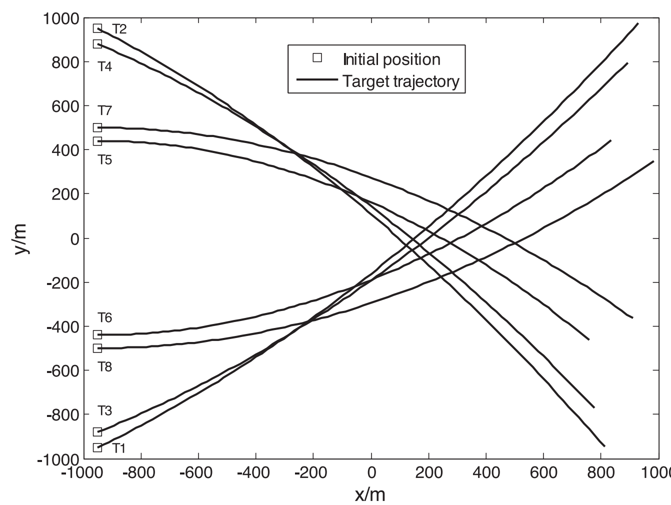

Fig. 8. Moving trajectories of the targets in Example 3.

Targets 1, 2, 3, 4, 5, 6, 7 and 8 appear at $t = 1$ s, $t = 2$ s, $t = 3$ s, $t = 4$ s, $t = 5$ s, $t = 6$ s $t = 7$ s and $t = 8$ s, respectively, and continue to exist before $t = 100$ s. The measurements with missed detection, clutter and noise are generated by using parameters $\Delta t_k = 1$ s, $\sigma_v = 0$ m/s^2, $\sigma_r = 3$ m, $\sigma_\theta = 0.001$ rad, $p_{S,k} = 1.0$, $\lambda_{c,k} = 2.5 \times 10^{-6}$ m^{-2} and $p_{D,k} = 0.98$. The moving trajectories of eight targets and converted measurements are shown in Figs. 5 and 6, respectively.

Setting parameters of the proposed filter to $\Delta t_k = 1$ s $\sigma_v = 1$ m/s,$p_{S,k} = 0.6$, $\lambda_{c,k} = 2.5 \times 10^{-6}$ m^{-2}, $p_{D,k} = 0.98$, $\tau = 0.001$, $\sigma_r = 3$ m, $\sigma_\theta = 0.001$ rad and $\rho_\gamma = 0.9$, we use Vo's PHD filter, Liu's MDB filter and proposed filter to handle the measurements, respectively. Fig. 7 shows the average OSPA distance for 120 trials and also reveals that the proposed filter performs better than Vo's PHD filter or Liu's MDB filter because its average OSPA distance is the least at each time. This performance advantage of the proposed filter appears more obvious from $t = 1$ s to 8 s because it has a strong detecting capability of the target. Fig. 7 also shows that the proposed filter may track the detected targets better than Vo's PHD filter or Liu's MDB filter at the subsequent times (from $t = 9$ s to $t = 100$ s) because it provides more accurate initial state estimations of the detected targets.

Example 3. In this example, eight targets with different initial velocities and accelerations appear in the scene at $t = 1$ s, $t = 2$ s, $t = 3$ s, $t = 4$ s, $t = 5$ s, $t = 6$ s $t = 7$ s and $t = 8$ s, respectively, and continue to exist in the scene before $t = 100$ s. Moving trajectories of the targets and converted measurements are shown in Figs. 8 and 9, respectively.

To handle the converted measurements, identical parameters to Example 2 are also used to set the proposed filter in this example. We use Vo's PHD filter, Liu's MDB filter and proposed filter to handle the converted measurements for 120 Monte Carlo runs, and show the experimental result in Fig. 10. Based on Fig. 10, similar conclusion to Example 2 is also drawn. The proposed filter is the best among these three filters due to its strong detecting capability of the target. In addition, the result in Fig. 10 suggests that the filter with the motion model of constant velocity may be applied for tracking the targets that move at a constant acceleration.

Computational complexity: The proposed filter is originated from Liu's MDB filter. The difference between them is that Liu's MDB filter uses the measurements at current time to generate the new marginal distributions, whereas the developed method first splits current measurements into two categories and then use the second category of measurements from three consecutive times to detect individual targets and estimate their initial states. Due to the similarity between the proposed filter and MDB filter, its computational complexity is identical to that of Liu's MDB filter or Vo's PHD filter. The computational complexity of the proposed filter is

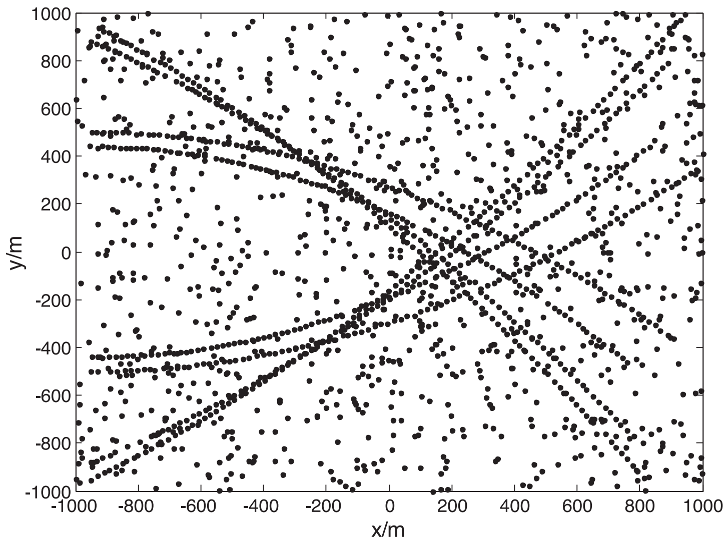

Fig. 9. Converted measurements in Example 3.

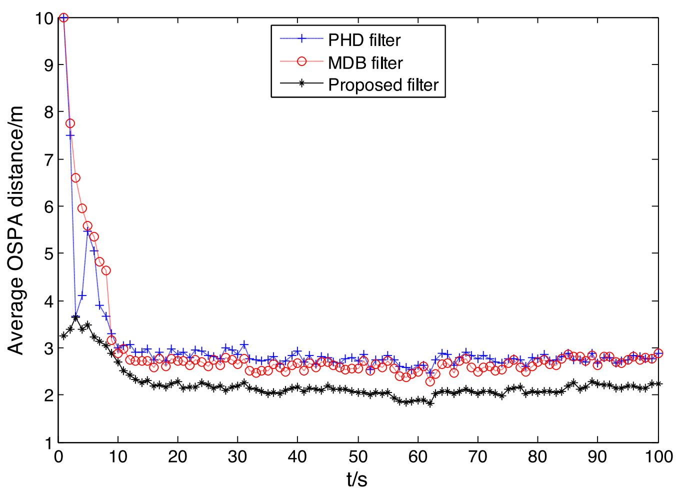

Fig. 10. Average OSPA distances in Example 3.

Table 1
Average performing time (s).

Filter	PHD filter	MDB filter	Proposed filter
Example 2	7.8422	8.7151	0.8132
Example 3	7.7861	8.5247	0.8126

also $O(M_k \times N_{k-1})$ where M_k is the number of measurements at time step k and N_{k-1} is the number of Gaussian terms or marginal distributions propagated from time step $k-1$ to time step k. Despite the identical computational complexity, The proposed filter requires less computational load than Vo's PHD filter or Liu's MDB filter due to the fact that N_{k-1} in the proposed filter approximates the number of real targets in the scene because only the marginal distributions and existence probabilities of the existing targets and detected targets are propagated to the next time step in the filter recursion, whereas Vo's PHD filter or Liu's MDB filter may propagate many Gaussian terms irrelevant to real targets, along with the Gaussian terms relevant to real targets, to the next time step. This performance advantage of the proposed filter is validated by the average performing times in Examples 2 and 3. Based on Table 1, the proposed filter requires much less performing time than Vo's PHD filter or Liu's MDB filter.

5. Conclusions

In this research, we present a target detection method based on the rule-based track initiation technique. This approach uses the measurements obtained at three consecutive times to detect individual targets and estimate their initial states. Applying this target detection method to the MDB filter, we develop a multi-target Bayes filter with the target detection. Its performance is tested by comparing it with Vo's PHD filter and Liu's MDB filter, which indicates that this filter has a stronger capability on detecting and tracking multiple targets than Vo's PHD filter or Liu's MDB filter. The extension of the applicable scope of the proposed filter and further improvement on its tracking performance are potential research topics for the multi-target Bayes filter with the target detection in the future.

Acknowledgments

This work was supported by the National Natural Science Foundation of China (61271107, 61301074) and Shenzhen Basic Research Project (JCYJ20140418095735618).

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