Sampling Signals with Finite Rate of Innovation

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Abstract

- This paper proposes a method to uniformly sample non-bandlimited signal at the rate of innovation using an appropriate kernel and perfectly reconstructed.
- We define the rate of innovation as the degrees of freedom per unit time.
- We identify the innovative part of a signal, such as time instants and weights of Diracs using an annihilating filter.

Applications and Importance

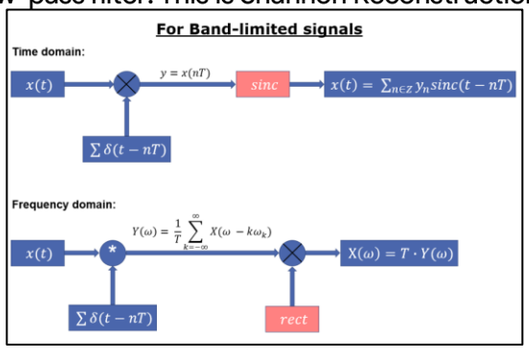
- We establish a connection between sampling theory and spectral analysis & error correction coding.
- We can apply this method to several bandlimited signals, and non bandlimited signals as well.

Miscellaneous Discussion

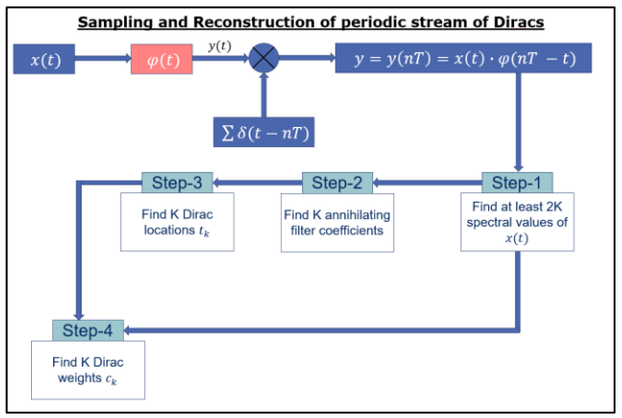
- In practice, noise will be present, which can be dealt with by using oversampling and solving the various systems involved using the singular value decomposition.
- We also can compare our idea with the classic Shannon bandwidth, which finds the dimension of subspace per unit time, and allows us to represent the signals of interest.
- We can use our methods in ultrawide band communication as well.

Method Used

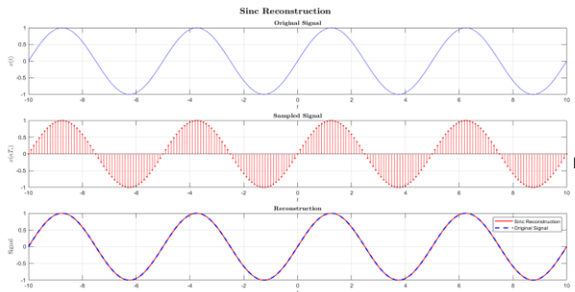
- In the case of bandlimited signals, we sample them using a stream of diracs. This sampled signal is a sufficient characterisation of the original signal as we can perfectly reconstruct it using the samples by passing it through an ideal sinc low-pass filter. This is Shannon Reconstruction.



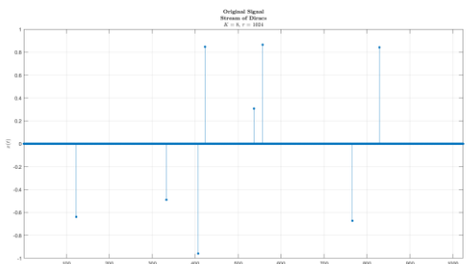
- In the case of non-bandlimited signals, we work with those with a finite rate of innovation.
- For a stream of diracs, we pass it through a sinc sampling kernel. The obtained samples are a sufficient characterization of the original signal.
- We make use of the annihilating filter method to find the location of the diracs and their weights so that we can perfectly reconstruct the signal.



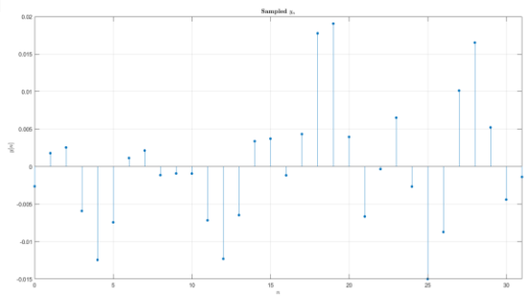
Results and MATLAB plots



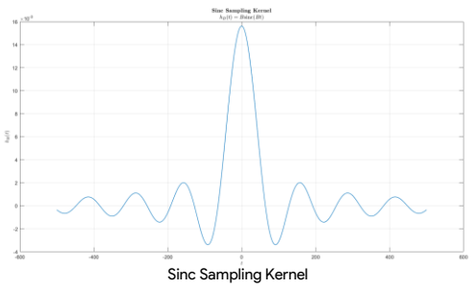
Reconstruction of bandlimited signal using sinc interpolation



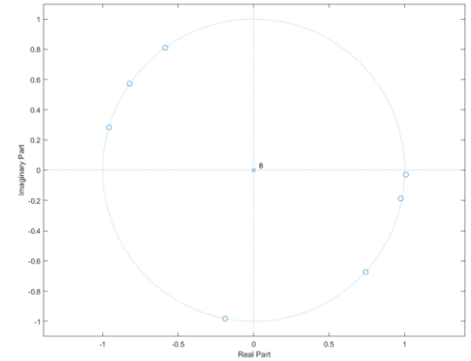
Stream of Diracs
$K = 8, \tau = 1024$



Sampled signal



Sinc Sampling Kernel



Roots of the Annihilating Filter

Conclusion

Since, many cases like the non-uniform splines, derivative of diracs, and piecewise polynomials can be boiled down to the case of stream of diracs, we can use this method for those cases as well.

References

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[3] ‘Tutorial on Sparse Sampling: Theory, Algorithms and Applications, October 2008’, Pier Luigi Dragotti, Martin Vetterli, Thierry Blu, IEEE.
[4] ‘Sampling curves with finite rate of innovation’, Hanjie Pan, Thierry Blu, Pier-Luigi Dragotti.