PRML-HW25F05 神经网络

一、

1、反向传播算法：

①将输入向量x_n应用于网络，并使用加权和和激活值前向传播通过网络，以找到所有隐藏单元和输出单元的激活值。

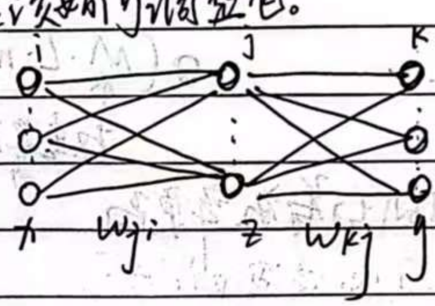
②使用误差方程δ_k评估所有输出单元的误差。

③使用反向传播算法反向传播δ，获得网络中每个隐藏单元的δ_j。

④使用链式法则计算所需导数。

2、反向传播算法的作用：利用链式法则，把输出误差从后往前传播，计算每个参数的梯度，从而知道应该如何调整它。

①前向传播：



$$a_j = \sum_i w_{ji} x_i$$

$$z_j = h(a_j)$$

$$a_k = \sum_j w_{kj} z_j$$

$$y_k = \sigma(a_k)$$ 损失函数 $L = \frac{1}{2}\|y_k - t_k\|^2$

②反向传播：

$$\delta_k = y_k - t_k$$

$$a_k = \sum_j h(a_j)\cdot w_{kj}$$

$$\delta_j = \frac{\partial L}{\partial a_j} = \sum_k \frac{\partial L}{\partial a_k}\cdot\frac{\partial a_k}{\partial a_j} = \sum_k \delta_k \cdot w_{kj}\cdot h'(a_j) = h'(a_j)\cdot\sum_k w_{kj}\delta_k$$

③参数更新：

$$\frac{\partial L}{\partial w_{ji}} = \delta_j x_i \qquad \frac{\partial L}{\partial w_{kj}} = \delta_k z_j$$

3. 链式法则: $\frac{d}{dx}[f(g(x))] = f'(g(x))\cdot g'(x)$

反向传播算法基于链式法则计算神经网络中每个权重的梯度。

4. 前向传播计算复杂度: $O\left(\sum_{l=1}^{L-1} n_l\cdot n_{l+1}\right) \approx O(L\cdot n^2)$

反向传播: $O\left(\sum_{l=1}^{L-1} n_l\cdot n_{l+1}\right) \approx O(L\cdot n^2)$

数值差分: $\frac{\partial L}{\partial w_{ij}} \approx \frac{L(w_{ij}+\varepsilon)-L(w_{ij}-\varepsilon)}{2\varepsilon}$

$O(\underset{\uparrow}{M}\cdot L\cdot n^2)$

权重参数个数

∴反向传播效率更高。

5. 不能，线性激活函数只能对输入进行线性变换，神经网络会退化为一个浅层的线性模型，无法捕捉数据中的非线性关系。

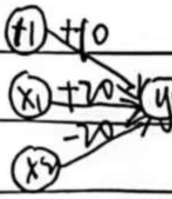
6. 动机: ① 线性模型存在维度灾难
② 随着人脑的发展，希望模拟人脑功能
③ 神经网络通过组合简单的神经元，可以表示更复杂的特征。

7. 异或 ⊕ XOR

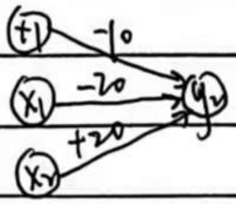
同或 ⊙ 或 ≡ XNOR

$x_1 \oplus x_2 = \underbrace{(x_1\wedge \neg x_2)}_{y_1} \vee \underbrace{(\neg x_1 \wedge x_2)}_{y_2}$

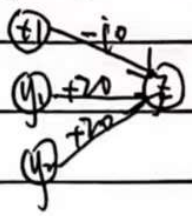
$x_1\wedge\neg x_2$



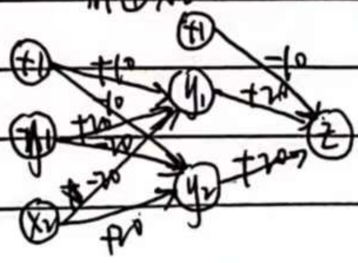
$\neg x_1\wedge x_2$



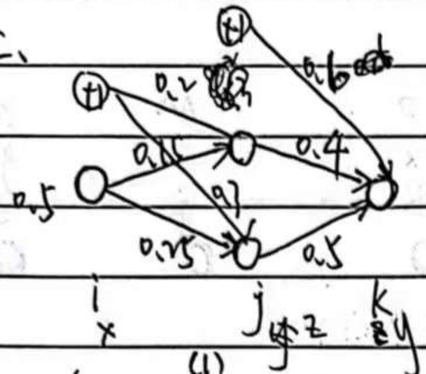
$y_1\vee y_2$



$x_1\oplus x_2$



二、



$$h(x) = \frac{1}{1+e^{-x}}$$

1. $a_{11}^{(1)} = 0.15 \times 0.5 + 0.2 = 0.275$

$a_{12}^{(1)} = 0.25 \times 0.5 + 0.3 = 0.425$

$z\cancel{y}_1 = h(a_{11}) = \frac{1}{1+e^{-0.275}} = 0.568$

$z\cancel{y}_2 = h(a_{12}) = \frac{1}{1+e^{-0.425}} = 0.606$

$a_{11}^{(2)} = 0.568 \times 0.4 + 0.606 \times 0.5 + 0.6 = 1.13$

$y = h(a_{11}^{(2)}) = \frac{1}{1+e^{-1.13}} = 0.75$

2. $\cancel{\delta_k = t_k - y_k = 1 - 0.75 - 1 = -0.25}$ $\cancel{y_k - t_k}$ $\cancel{\delta_k = (t_k}$

$\delta_k = (y-t) \cdot \sigma'(z)$

$\cancel{= (0.75-1) \times \sigma(z)(1-\sigma(z))}$

$\sigma'(z) = \sigma(z)(1-\sigma(z))$ $\cancel{0.75 \times (1-0.75}$

$= 0.75 \times (1-0.75) = 0.19$

$\delta_k = \cancel{(1-0.75)}\ (0.75-1) \times 0.19 = 0.046$

3. 求 $\frac{\partial E}{\partial w_{jk}}$

$\frac{\partial E}{\partial w_{kj}} = \delta_k z_j \cancel{= -0.25 \times z_j}$

$\frac{\partial E}{\partial w_{21}} = \delta_k z_1 = 0.046 \times 0.568 = 0.026$

$\frac{\partial E}{\partial w_{22}} = \delta_k z_2 = 0.046 \times 0.606 = 0.028$

4. $w_{21} = 0.4 - 0.1 \times 0.026 = 0.397$

$w_{22} = 0.5 - 0.1 \times 0.028 = 0.497$

三、1. $w^{(1)}: (D+1)\times \cancel{(M+1)}\ M$

$w^{(2)}: (M+1)\times K$

2.

(2,1) $\cancel{x}$ $(3,2)^T$ (2,3) (~~m~~ 3,1)

$$a^{(1)} = w^{(1)} x$$

$$z = \sigma(a^{(1)})$$

$$a^{(2)} = \begin{bmatrix} 1 \\ \sigma(a^{(1)}) \end{bmatrix}$$

$$y = \cancel{\sigma\left(w^{(2)}\begin{bmatrix} 1 \\ \sigma(a^{(1)}) \end{bmatrix}\right)}\ \sigma\left(w^{(2)}\begin{bmatrix} 1 \\ \sigma(a^{(1)}) \end{bmatrix}\right)$$

3. $$\frac{\partial y_k}{\partial w_{md}} = \frac{\partial y_k}{\partial w_{km}} \cdot \frac{\partial w_{km}}{\partial w_{md}} = z_m$$

$$y_k = z_m w_{km},\ \cancel{w_{km}}$$

$$= \sigma(x_D \cdot w_{MD}) \cdot w_{KM}$$

$$\sigma'(a) = \sigma(a)(1-\sigma(a))$$

$$\frac{\partial y_k}{\partial w_{md}} = \frac{\partial y_k}{\partial a_k^{(2)}} \cdot \frac{\partial a_k^{(2)}}{\partial z_m} \cdot \frac{\partial z_m}{\partial a_m^{(1)}} \cdot \frac{\partial a_m^{(1)}}{\partial w_{md}}$$

$$y_k(1-y_k) \quad \cancel{w_{kj}}\ w_{km} \quad z_m(1-z_m) \quad x_d$$

$$\therefore \frac{\partial y_k}{\partial w_{md}} = y_k(1-y_k) \cdot w_{km} \cdot z_m(1-z_m) \cdot x_d$$

四、1、 $W_{out} = \frac{32-3}{1} + 1 = 30 \Rightarrow 30 \times 30$

2、 $K_{eff} = 5 + (5-1) \times (2-1) = 9 \Rightarrow 64 \times 64$

3、参数共享、局部感知、平移不变性

五、1、神经网络：neural networks 2、过拟合 overfitting
3、正则化：regularization
4、随机梯度下降：stochastic gradient desent
5、逻辑回归：logistic regression.
6、线性回归：linear regression
7、广义线性模型：generalized linear regression
8、均方误差：mean squared error
9、平均绝对误差：mean absolute error