HW6 Shu Bin

I looked at the solutions

1) $l(\mu_k, \Sigma_k) = \sum_k \sum_i r_{ik} \log \mathbb{P}(x_i | \theta_k) = -\frac{1}{2} \sum_i r_{ik} \left(\log |\Sigma_k| + (x_i - \mu_k)^T \Sigma_k^{-1} (x_i - \mu_k) \right)$

then $\frac{\partial l}{\partial \mu_k} = \sum_i r_{ik} \Sigma_k^{-1} (x_i - \mu_k) = 0$

@ optimality, $\sum_i r_{ik} x_i = \mu_k \sum_i r_{ik} \Rightarrow \mu_k = \frac{\sum r_{ik} x_i}{\sum_i r_{ik}} = \frac{\sum_i r_{ik} x_i}{r_k}$

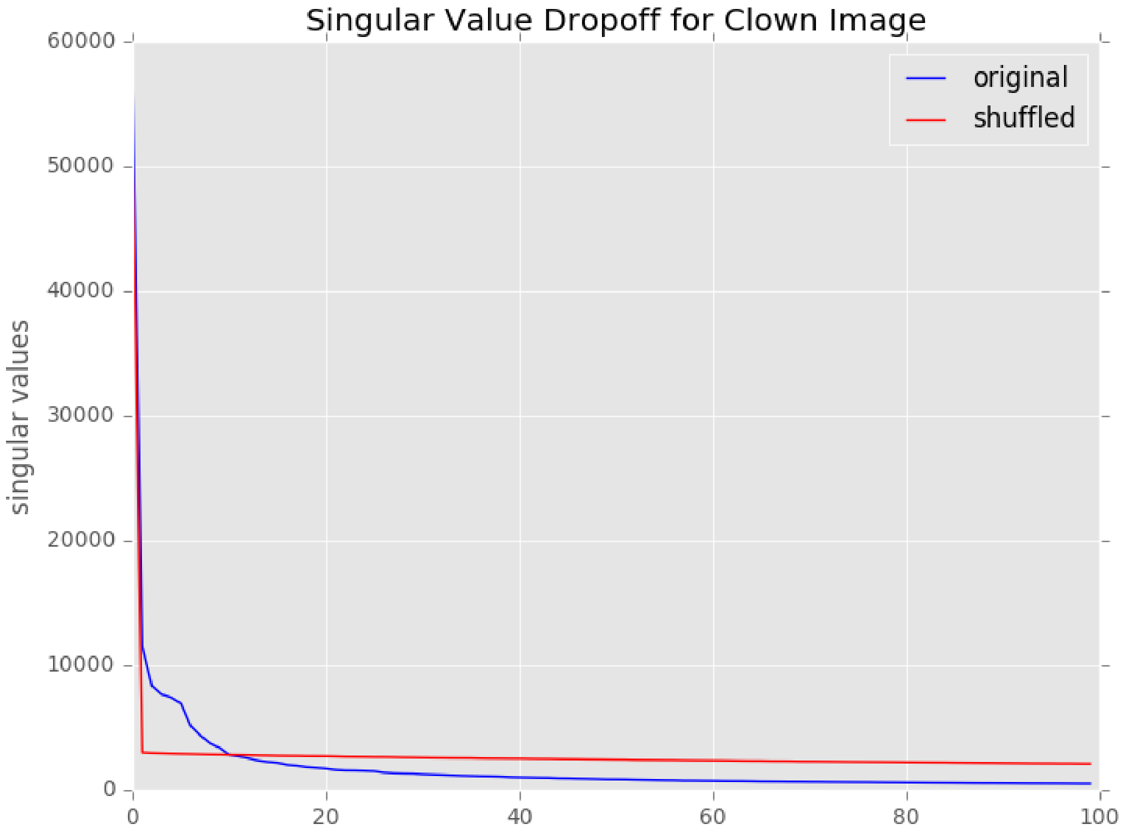
then $\frac{\partial l}{\partial \Sigma_k} = -\frac{1}{2} \sum_i r_{ik} \left(\Sigma_k^{-1} - \Sigma_k^{-1} (x_i - \mu_k)(x_i - \mu_k)^T \Sigma_k^{-1} \right) = 0$

thus the optimality condition is

$$\sum_i r_{ik} I = \left(\sum_i r_{ik} (x_i - \mu_k)(x_i - \mu_k)^T \right) \Sigma_k^{-1}$$

$$\sum_i r_{ik} I \Sigma_k = \sum_i r_{ik} (x_i - \mu_k)(x_i - \mu_k)^T$$

$$\Sigma_k = \frac{1}{r_k} \sum_i r_{ik} (x_i - \mu_k)(x_i - \mu_k)^T$$



Singular Value Dropoff for Clown Image
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10000
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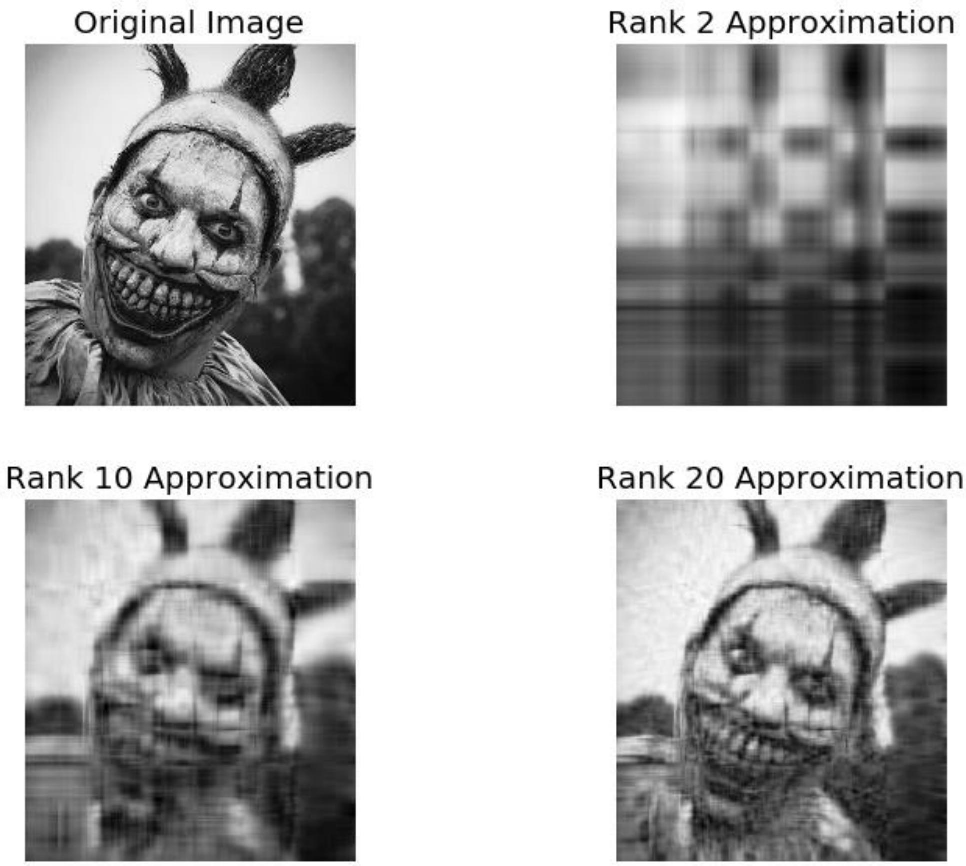
Original Image
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