MATH189 HW1

Shu Bin

1) $E(\vec{y}) = E(A\vec{x}+\vec{b}) = \int_S (A\vec{x}+\vec{b})p(\vec{x})d\vec{x}$

$= \int_S A\vec{x}p(\vec{x}) + \vec{b}p(\vec{x})d\vec{x}$

$= \int_S A\vec{x}p(\vec{x}) + \int_S \vec{b}p(\vec{x})d\vec{x}$

$= A\int_S \vec{x}p(\vec{x}) + \vec{b}\int_S p(\vec{x})d\vec{x}$

$= AE(\vec{x}) + \vec{b}$

$Cov(\vec{y}) = Cov(A\vec{x}+\vec{b}) = E[(A\vec{x}+\vec{b} - E(A\vec{x}+\vec{b}))(A\vec{x}+\vec{b} - E(A\vec{x}+\vec{b}))^T]$

$= E[(A\vec{x}+\vec{b} - AE(\vec{x}) - \vec{b})(A\vec{x}+\vec{b} - AE(\vec{x}) - \vec{b})^T]$

$= E[(A\vec{x} - AE(\vec{x}))(A\vec{x} - AE(\vec{x}))^T]$

$= E[(A(\vec{x} - E(\vec{x}))(A(\vec{x} - E(\vec{x}))^T]$

$= E[A(\vec{x} - E(\vec{x}))(\vec{x} - E(\vec{x}))^T A^T]$

$= AE[(\vec{x} - E(\vec{x}))(\vec{x} - E(\vec{x}))^T]A^T$

$= A\,Cov(\vec{x})A^T = A\Sigma A^T$

2) a) let $X = \begin{pmatrix} 1 & 0 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{pmatrix}$ $\vec{y} = \begin{pmatrix} 1 \\ 3 \\ 6 \\ 8 \end{pmatrix}$

$\vec{x}^T X = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 3 & 4 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} 4 & 9 \\ 9 & 29 \end{pmatrix}$ $X^T\vec{y} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 3 & 4 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \\ 6 \\ 8 \end{pmatrix} = \begin{pmatrix} 18 \\ 56 \end{pmatrix}$

$\hat{\theta}_0 = \dfrac{\det\begin{pmatrix} 18 & 9 \\ 56 & 29 \end{pmatrix}}{\det\begin{pmatrix} 4 & 9 \\ 9 & 29 \end{pmatrix}} = \dfrac{18}{35}$ $\hat{\theta}_1 = \dfrac{\det\begin{pmatrix} 4 & 18 \\ 9 & 56 \end{pmatrix}}{\det\begin{pmatrix} 4 & 9 \\ 9 & 29 \end{pmatrix}} = \dfrac{62}{35}$

thus $\vec{y} = \frac{18}{35} + \frac{62}{35}\vec{x}$

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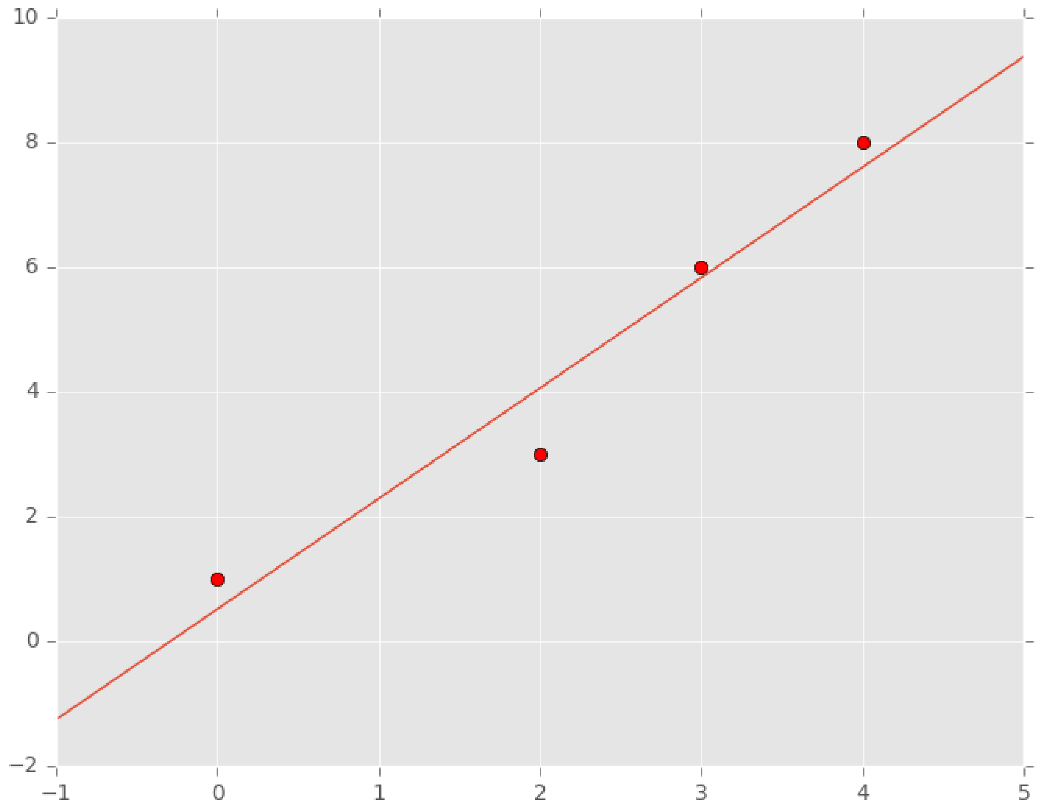
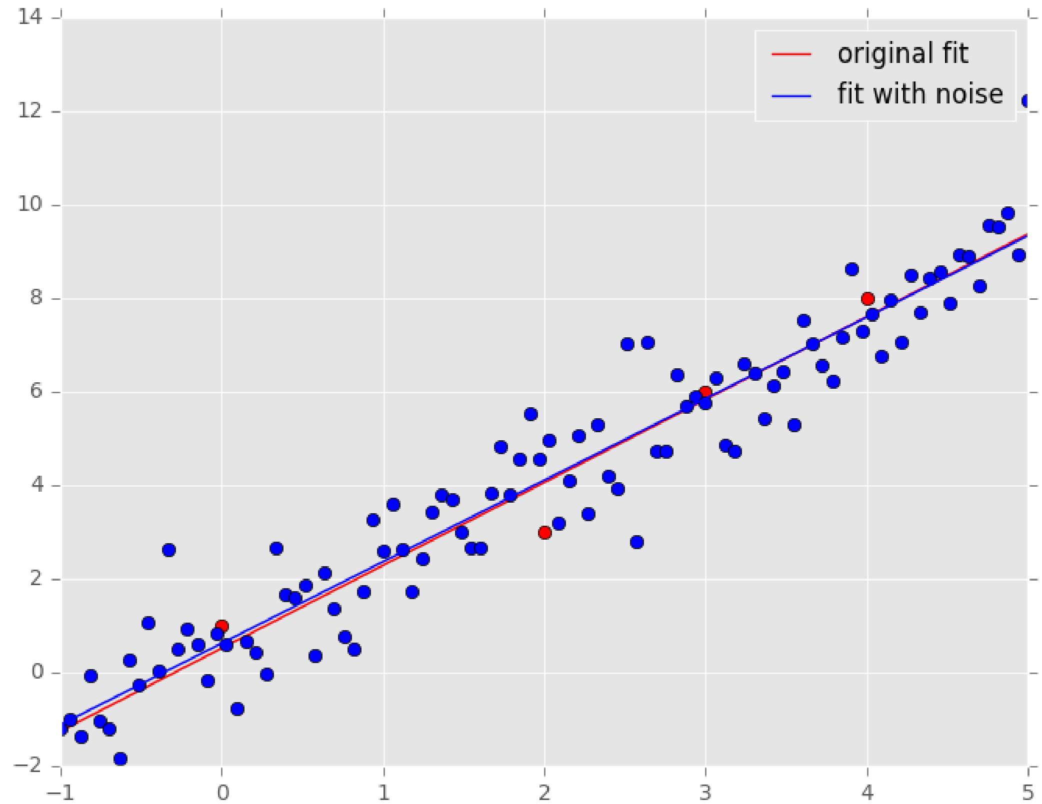
b) $\hat{\theta} = (x^T x)^{-1} x^T y = \left[\begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 3 & 4 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{pmatrix} \right]^{-1} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 3 & 4 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \\ 6 \\ 8 \end{pmatrix} = \begin{pmatrix} \frac{18}{35} \\ \frac{62}{35} \end{pmatrix}$



10
8
6
4
2
0
−2
−1
0
1
2
3
4
5


original fit
fit with noise
14
12
10
8
6
4
2
0
−2
−1
0
1
2
3
4
5