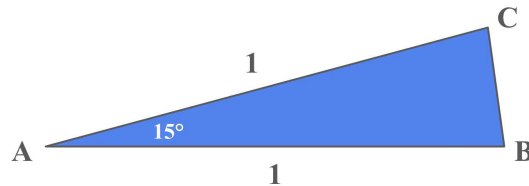


Riddler Classic Solution
January 7, 2022
Can You Trek The Triangle? | FiveThirtyEight

Amare the ant is traveling within Triangle ABC, as shown below. Angle A measures 15 degrees, and sides AB and AC both have length 1.

Amare starts at point B and wants to ultimately arrive on side AC. However, the queen of his colony has asked him to make several stops along the way. Specifically, his path must:

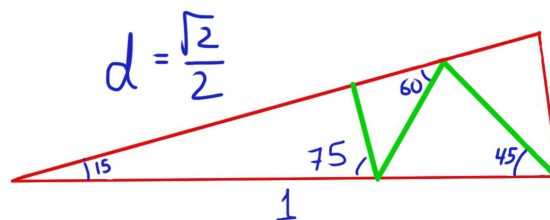


- Start at point B.
- Second, touch a point — any point — on side AC.
- Third, touch a point — any point — back on side AB.
- Finally, proceed to a point — any point — on side AC (not necessarily the same point he touched earlier).

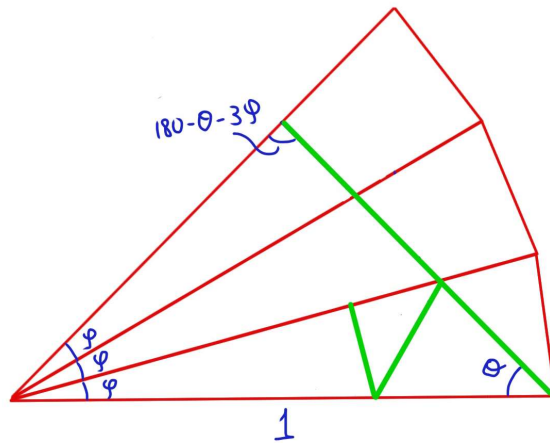
What is the shortest distance Amare can travel to complete the queen's desired path?

The shortest possible distance is $0.7071 = \frac{\sqrt{2}}{2}$.

The path he must follow is shown below.



The easiest way to see this is the shortest path is to stack three copies of the triangle on top of each other. That makes it easy to compute the distance travelled as a function of the initial angle of departure from AB.

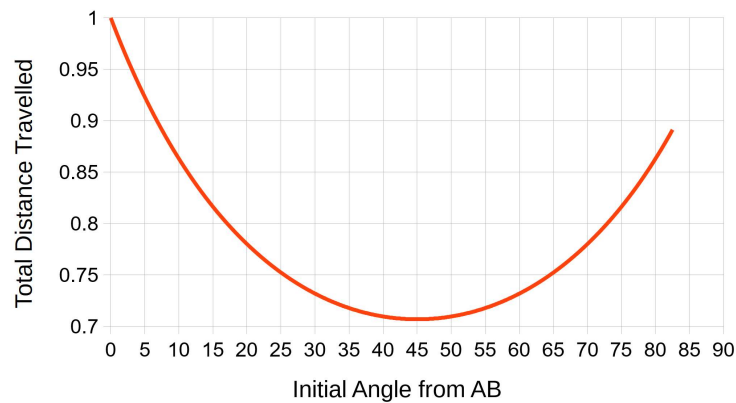


I use the law of sines to calculate the distance travelled as a function of θ , the initial angle of departure.

$$\frac{d}{\sin 3\phi} = \frac{1}{\sin(180 - \theta - 3\phi)}$$

$$d = \frac{\sin 3\phi}{\sin(180 - \theta - 3\phi)}$$

For $\phi = 15^\circ$, I find numerically that the distance is minimum at $\theta = 45^\circ$, as shown below.



I use simple geometry to find the other two angles of departure. The main geometric facts I used are that the internal angles of a triangle sum to 180° and that the opposite angles of two intersecting lines are equal.

