Squid Game Riddler
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[Can You Survive Squid Game Riddler? | FiveThirtyEight]

For each player, I calculate the probability that the player will lose the game on the nth tile. For the first player, this is trivial. The probability of losing on the first tile is ½. On the second tile is ¼. On the third 1/8, etc.

$$P(\textit{player } 1 \textit{ will lose on tile } n) = \frac{1}{2^n}$$

Player 2 cannot lose on tile 1. Whatever the outcome of player 1's first jump, player 2 knows where to jump on tile 1.

Play 2 must guess on the second tile only if player 1 lost on the first tile. So:

$$P(\textit{player } 2 \textit{ will lose on tile } 2) = \frac{1}{2} \cdot P(\textit{player } 1 \textit{ will lose on tile } 1) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

There are two possible ways for player 2 to lose on tile 3:

1. If player 1 lost on tile 2, then player 2 must guess incorrectly on tile 3.
2. If player 1 lost on tile 1, then player 2 must guess correctly on tile 2 then incorrectly on tile 3.

$$P(\textit{player } 2 \textit{ will lose on tile } 3) = \frac{1}{2} \cdot P(\textit{player } 1 \textit{ will lose on tile } 2) + \frac{1}{2} \cdot \frac{1}{2} \cdot P(\textit{player } 1 \textit{ will lose on tile } 1)$$

$$= \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$$

There are three possible ways for player 2 to lose on tile 4:

1. If player 1 lost on tile 3, then player 2 must guess incorrectly on tile 4.
2. If player 1 lost on tile 2, then player 2 must guess correctly on tile 3, then incorrectly on tile 4.
3. If player 1 lost on tile 1, then player 2 must guess correctly on tiles 2 and 3, then incorrectly on tile 4.

There are four possible ways for player 2 to lose on tile 5:

1. If player 1 lost on tile 4, then player 2 must guess incorrectly on tile 5.
2. If player 1 lost on tile 3, then player 2 must guess correctly on tile 4, then incorrectly on tile 5.
3. If player 1 lost on tile 2, then player 2 must guess correctly on tile 3 and 4, then incorrectly on tile 5.
4. If player 1 lost on tile 1, then player 2 must guess correctly on tiles 2, 3 and 4, then incorrectly on tile 5.

I can start to see a pattern. Player X knows how to jump for the first X-1 tiles, so can never lose for tiles up to X-1. For higher tiles, the probability player X will lose on tile n is given by this sum:

$$P(X,n)=\sum_{i=1}^{n-1} P\frac{(X-1,n-i)}{2^i}$$

Since I know the values of P(1,n), I can use this formula to calculate all the values P(X,n).

For each player, I can find the probability that player will win by adding the probabilities of losing on each tile and subtracting from 1.

These probabilities are shown in column 2 of the following table:

Player	**Probability of winning**	**Probability of being first winner**	**Number of winners**	**Expectation**
1	3.8147E-06	3.8147E-06	16	0.00006
2	7.2479E-05	6.8665E-05	15	0.00103
3	6.5613E-04	5.8365E-04	14	0.00817
4	3.7689E-03	3.1128E-03	13	0.04047
5	1.5442E-02	1.1673E-02	12	0.14008
6	4.8126E-02	3.2684E-02	11	0.35953
7	1.1894E-01	7.0816E-02	10	0.70816
8	2.4034E-01	1.2140E-01	9	1.09259
9	4.0726E-01	1.6692E-01	8	1.33539
10	5.9274E-01	1.8547E-01	7	1.29829
11	7.5966E-01	1.6692E-01	6	1.00154
12	8.8106E-01	1.2140E-01	5	0.60699
13	9.5187E-01	7.0816E-02	4	0.28326
14	9.8456E-01	3.2684E-02	3	0.09805
15	9.9623E-01	1.1673E-02	2	0.02335
16	9.9934E-01	3.1128E-03	1	0.00311
No winner		6.5613E-04	0	0.00000
Sum		**1**		**7.00008**

To find the expected number of winners, I must find the probability of each possible number of winners. For there to be 16 winners, player 1 must win. For 15 winners, player 2 must win and player 1 must lose. For 14 winners, player 3 must win and player 2 must lose, etc.

So for player X, the probability that 17-X players will win is the probability that player X wins minus the probability that player X-1 wins. These values are given in the third column of the table above. The probability there is no winner is the probability that player 16 loses. This is given on the second to last line in the table. Notice that the probabilities for 0 winners to 16 winners add to 1, as they must.

To find the expected number of winners, I simply multiply each possible number of winners by the probability of that number of winners and add the results. This is shown in column 5 of the table.

The sum of the expectations is 7.000, so I expect that on average 7 players will win the game.