Statistics of voids identified by DIVE in N-body simulations for different DE models

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nothingness of the universe: **cosmic voids**

Large scale structures of the universe

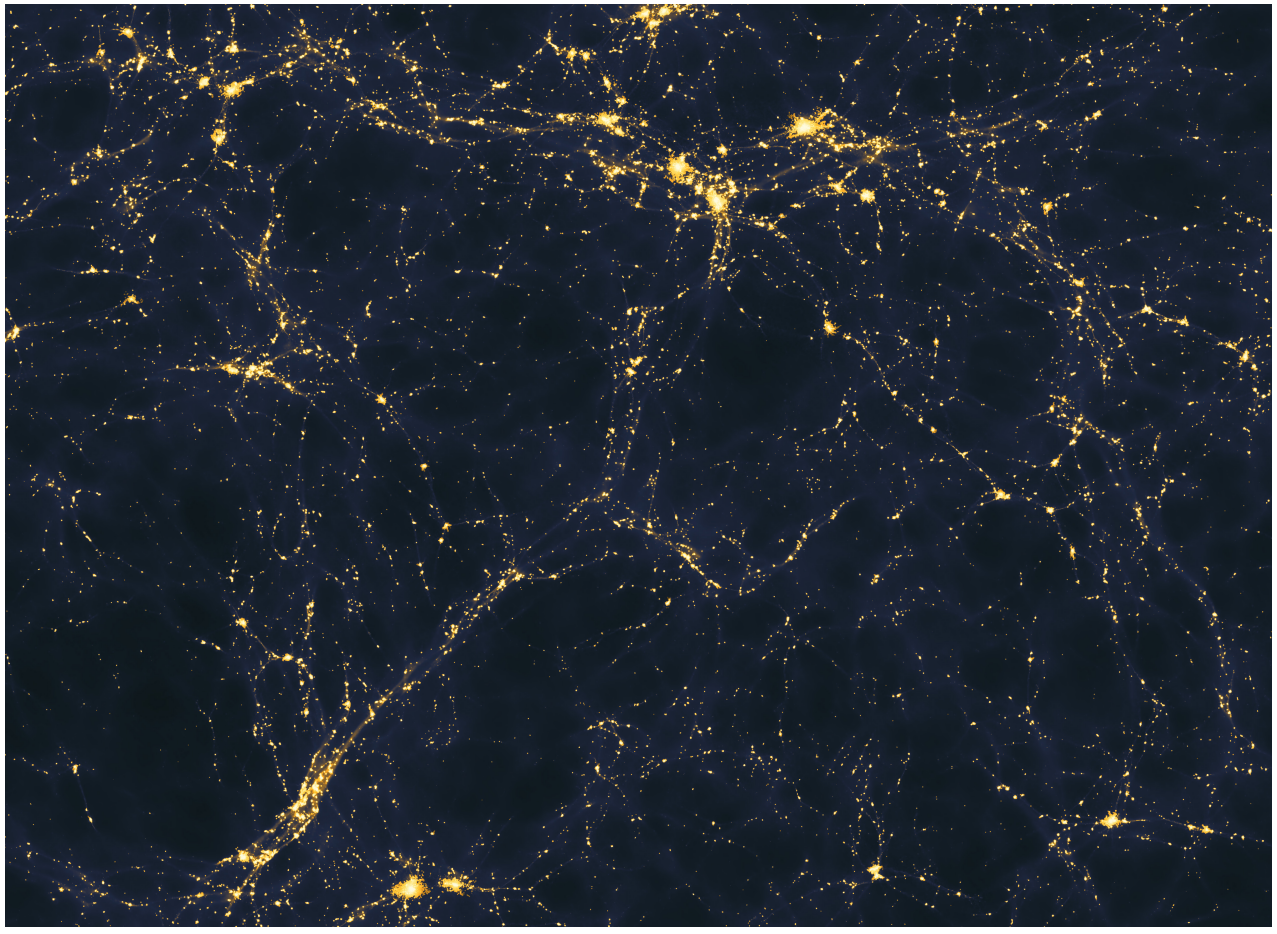

Figure: https://en.wikipedia.org/wiki/Observable_universe#Large-scale_structure

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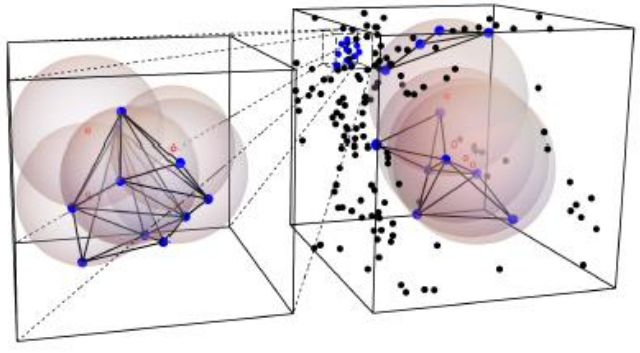

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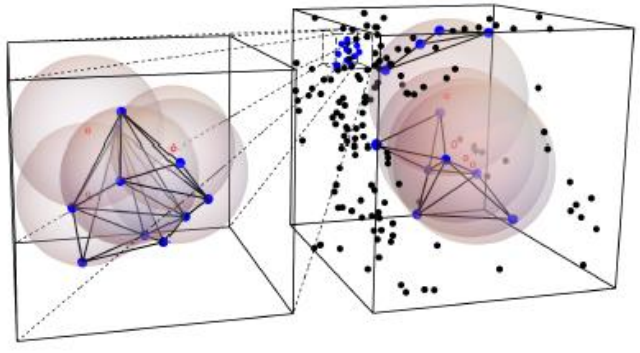

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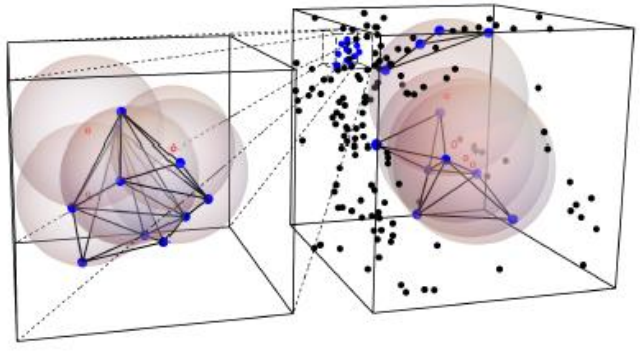
  - *All voids*



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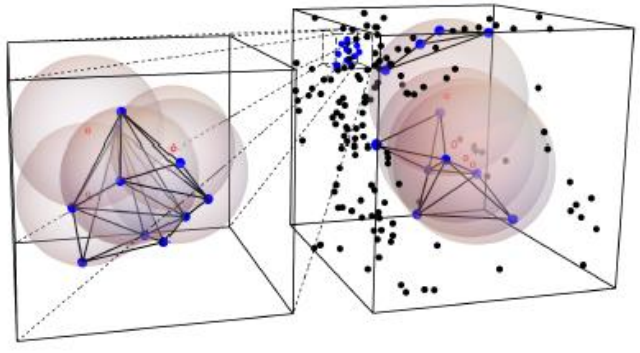

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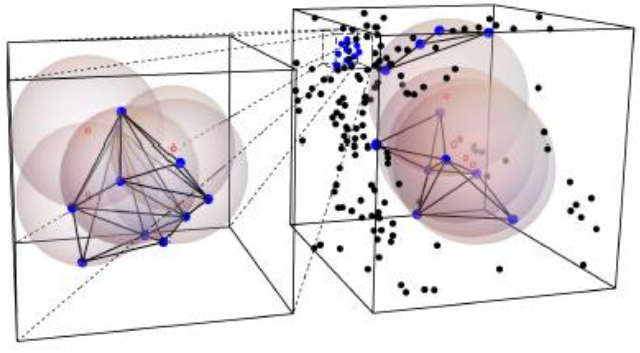

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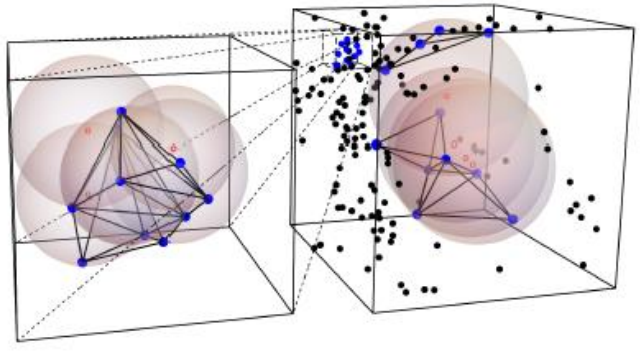

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    sampled from *all voids* by hierarchically removing overlapping voids

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Void statistics for ED in Lambda and R-P

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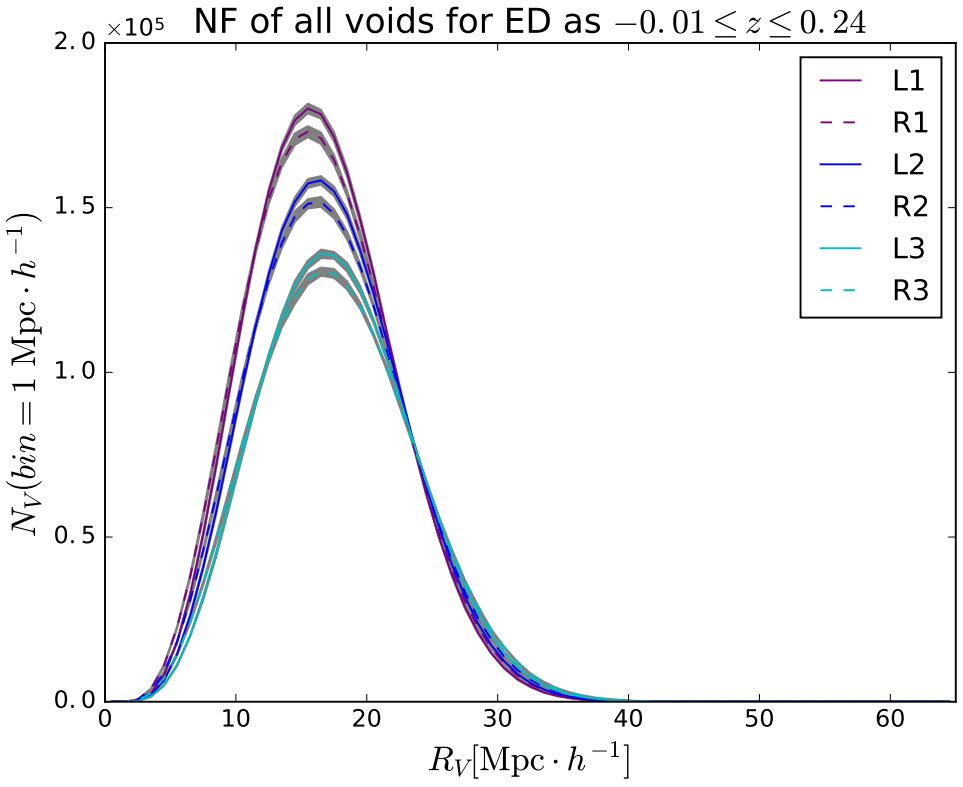

Figure: L1, L2, L3, R1, R2 and R3 correspond to Lambda with $z = 0$, Lambda with $z = 0.11$, Lambda with $z = 0.24$, R-P with $z = -0.01$, R-P with $z = 0.11$, and R-P with $z = 0.25$, respectively.

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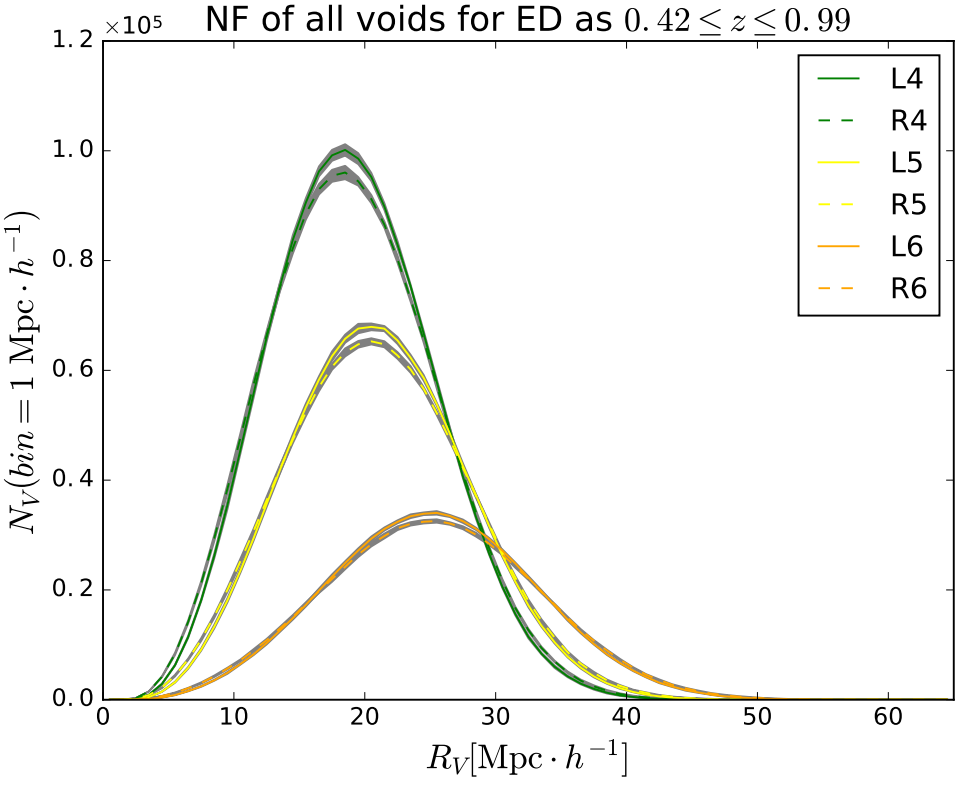

Figure: L4, L5, L6, R4, R5 and R6 correspond to Lambda with $z = 0.43$, Lambda with $z = 0.66$, Lambda with $z = 0.99$, R-P with $z = 0.42$, R-P with $z = 0.66$, and R-P with $z = 0.99$, respectively.

Void statistics for ED in Lambda and R-P

Difference between NFs of *all voids* as $-0.01 \leq z \leq 0.99$

Void statistics for ED in Lambda and R-P
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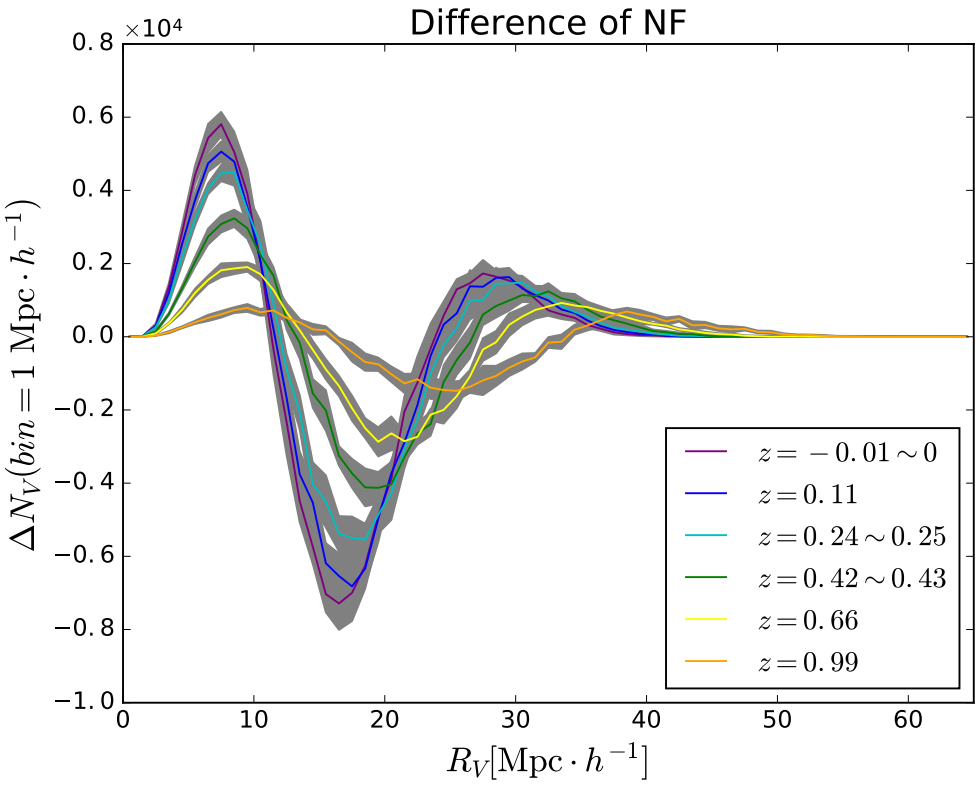

Figure: The peaks and dip are shown significant enough to overcome the uncertainties and may serve as potential DE model-degeneracy-breakers from the perspective of (DT) void statistics for Lambda and R-P.

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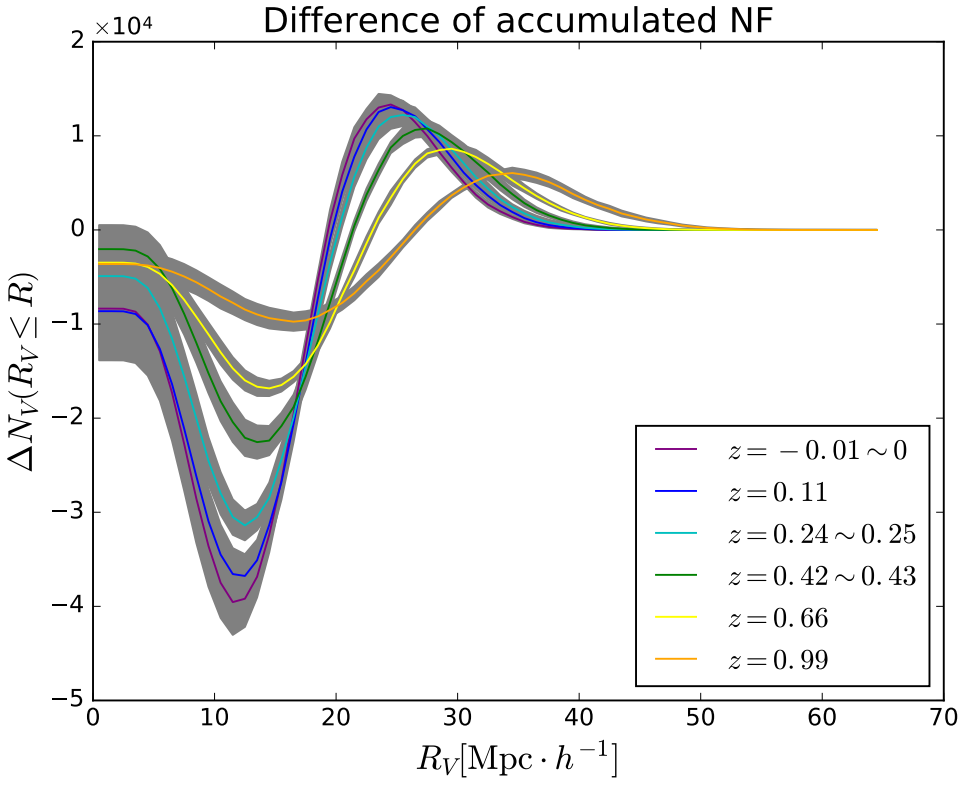

Figure: There are one dip and one peak in the curve of difference between ANFs, which are also significant enough to overcome the uncertainties.

Evolution with respect to the redshift z

Total number of *all voids* $N_V(0 \leq R)$ versus tracer number

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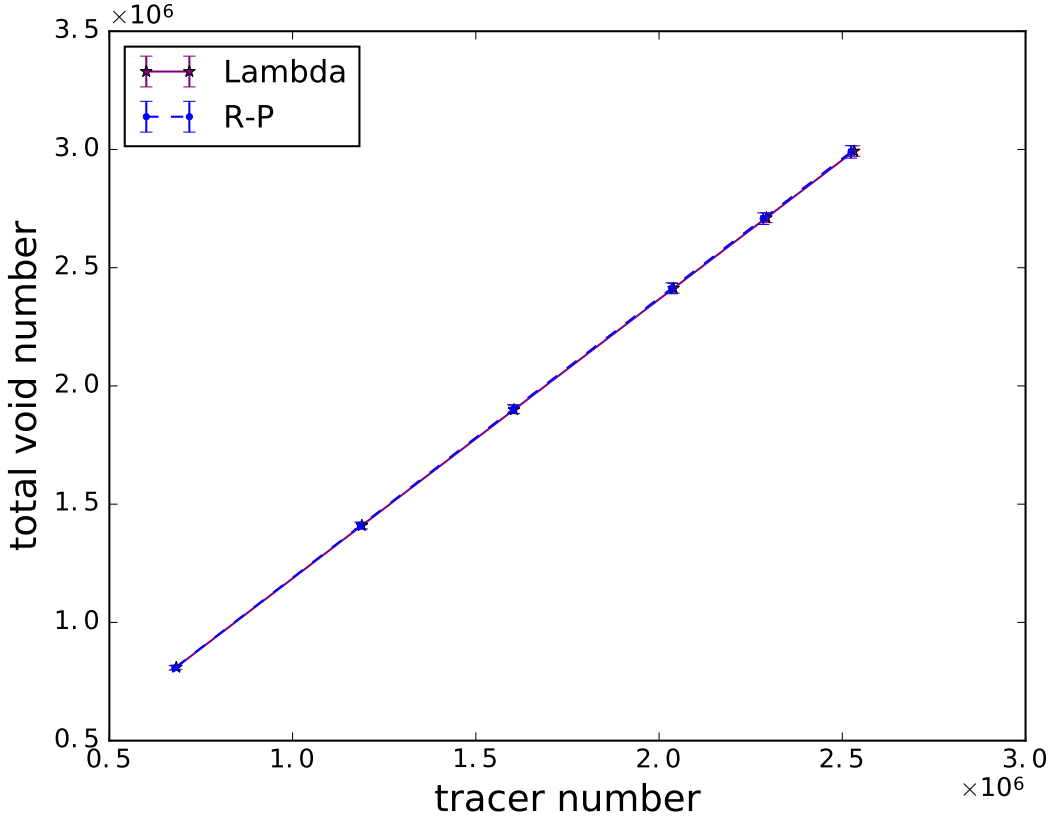

Figure: The curves of Lambda and R-P coincide with each other perfectly. Besides, a perfect linear relation is shown.

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- Some parameters of void statistics are likely to be DE model-independent.
- There is a perfect linear relation between the total number of *all voids* and the number of tracers.

A rough interpretation
From the perspective of tracer clustering

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- **For *all voids* in R-P compared with those in Lambda under the condition of ED, there are more voids of small radius and large radius, while less voids of medium radius.**

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- **For *all voids* in R-P compared with those in Lambda under the condition of ED, there are more voids of small radius and large radius, while less voids of medium radius.**
- Roughly speaking, we can further divide the tracers into clusters as components of the walls and filaments in LSS.

A rough interpretation
Exemplar spatial distributions of tracers and voids

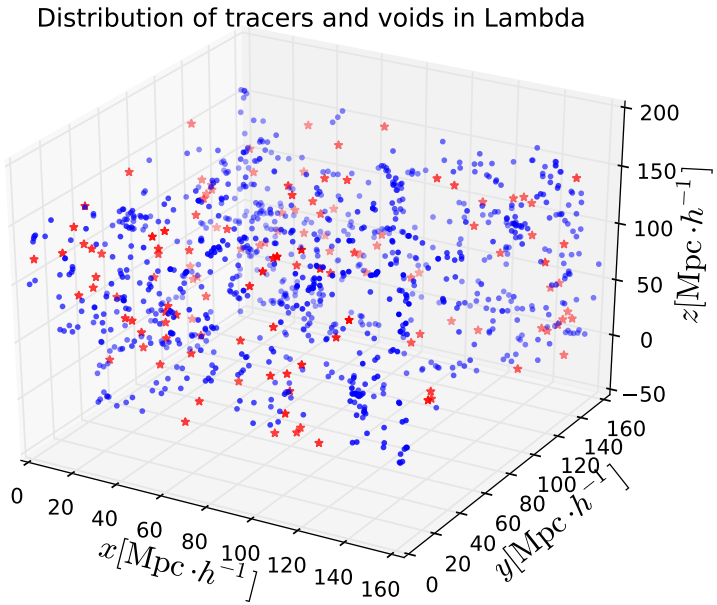

Figure: Tracers: red asterisks, Voids: blue solid points

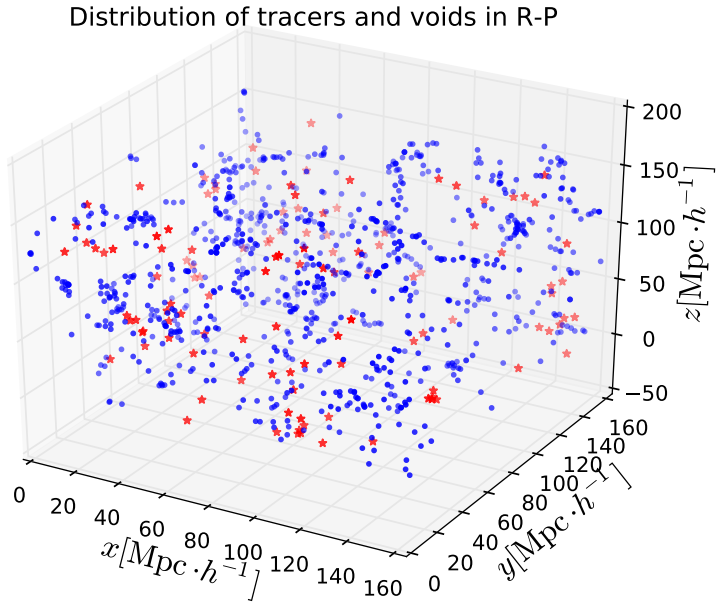

Figure: The clustering of tracers (and voids) for R-P is slightly more compact.

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- More compact clusters of tracers lead to even smaller voids of small radius ($\lesssim 10\mathrm{Mpc}\cdot h^{-1}$), and wider separations among clusters even larger voids of large radius ($\gtrsim 30\mathrm{Mpc}\cdot h^{-1}$).

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- Disadvantages of DT voids
  - We are not able to construct a simple (one-to-one) relation between DT voids and low-density regions in LSS.
  - It remains an unsettled issue how the properties of tracers (e.g. luminosity of galaxies, mass (cut) of DM haloes) affect DT void statistics.

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Thanks for your attention.

I can show you some recent results, if there is still time left...

Recent results: comparison with SDSS DR12 data

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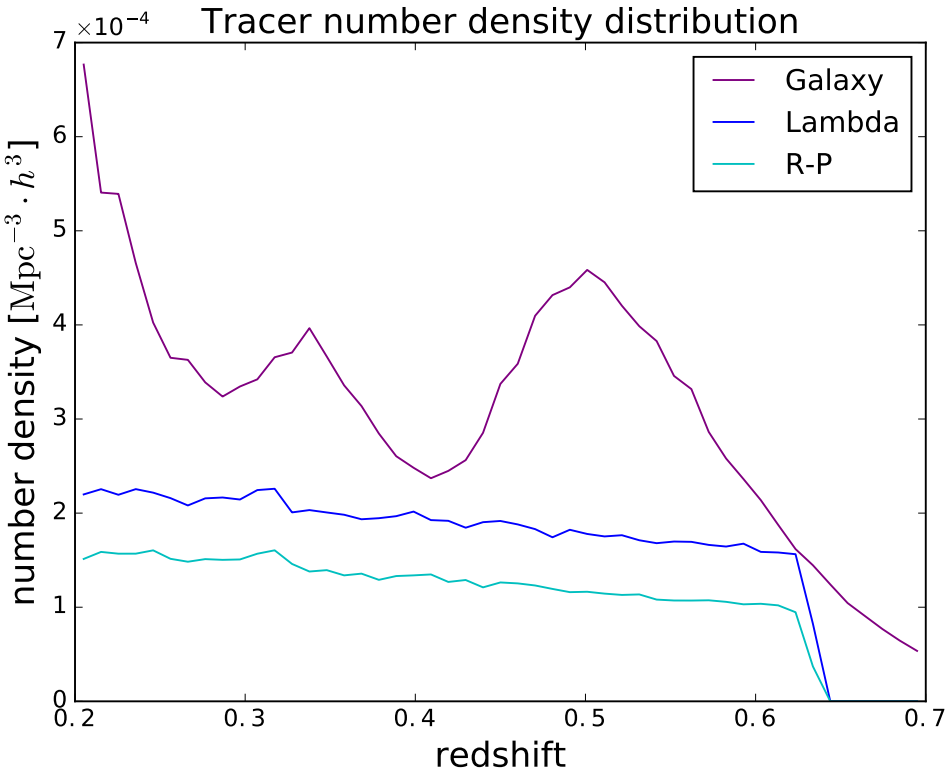
- Quantities: number function (NF), difference of number function (DNF), accumulated number function (ANF), difference of accumulated number function (DANF)

Recent results

- Modification of the tracer number density distribution with respect to the redshift



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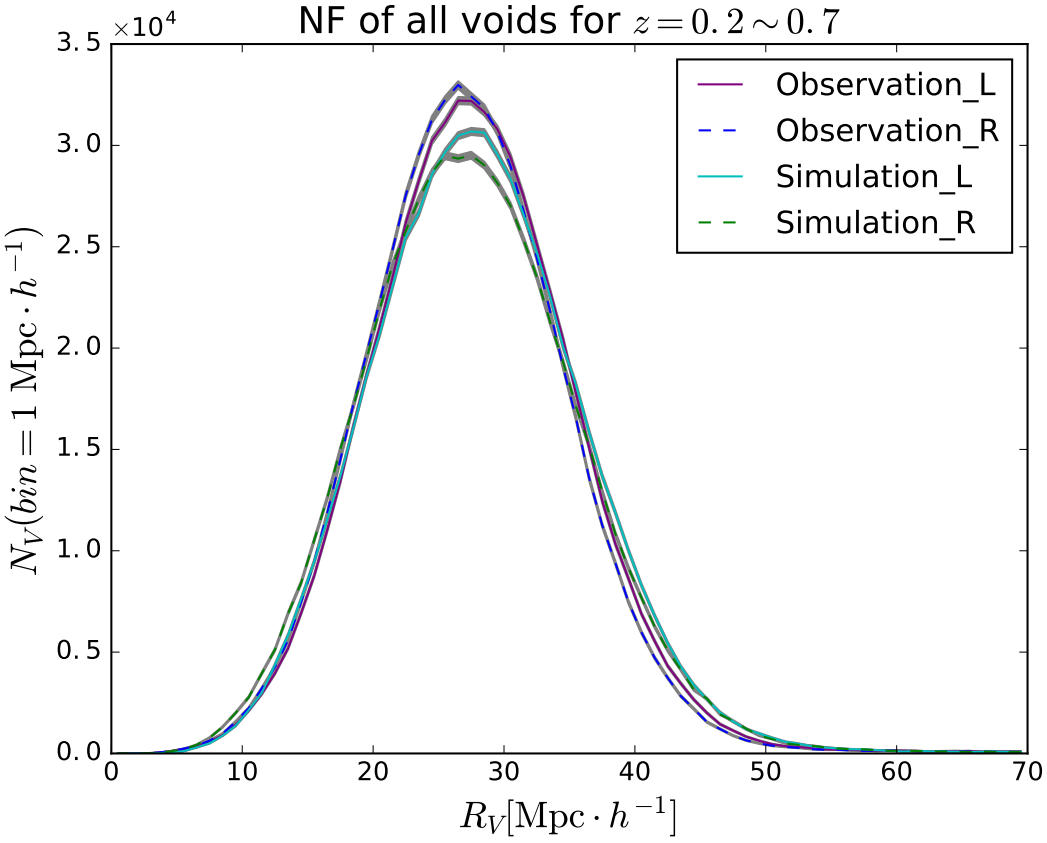


Figure: The uncertainties calculated from Poisson fluctuations (i.e. $\delta N = \sqrt{N}$) are almost negligible.

Recent results

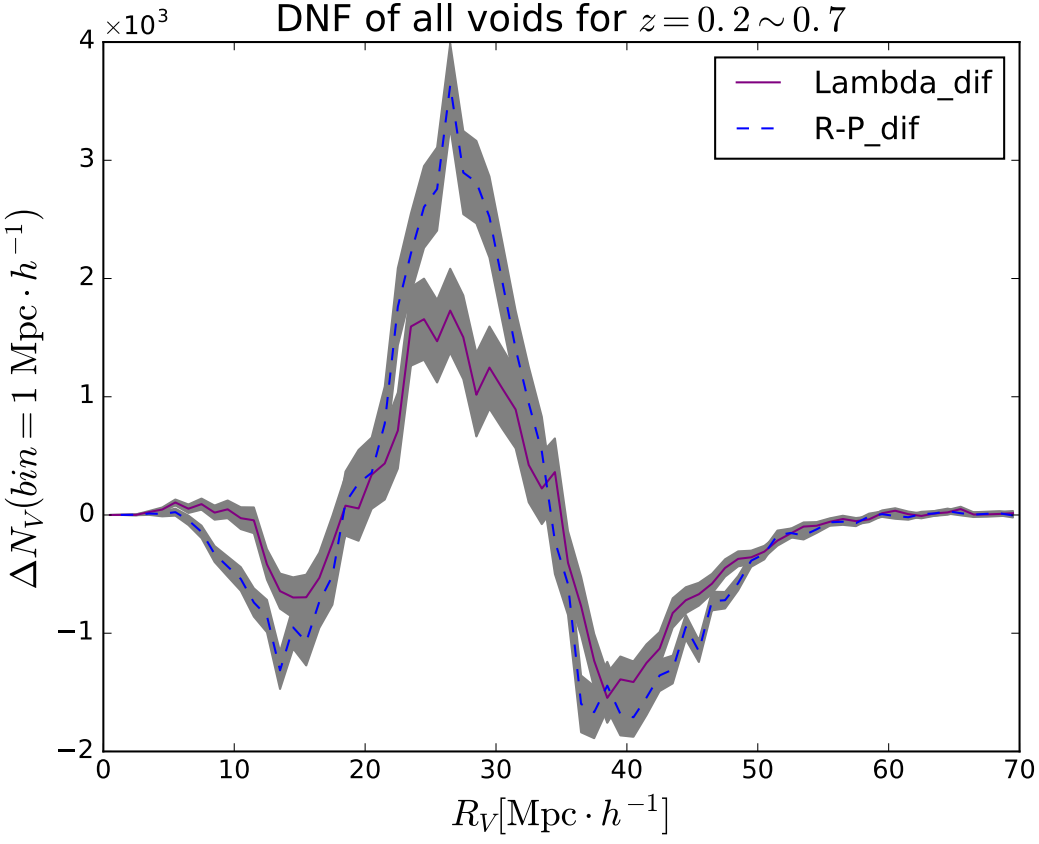
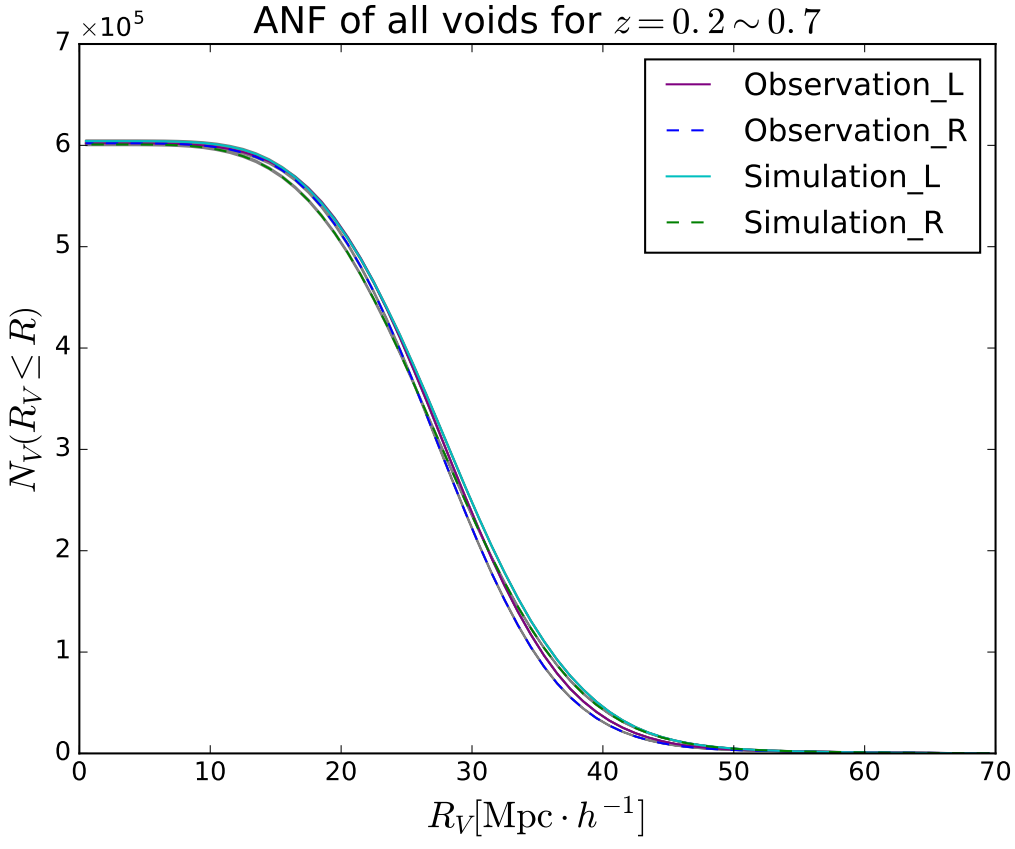


Figure: Here we estimate the uncertainties in the most conservative way, and they still do not affect the conclusion.

Recent results



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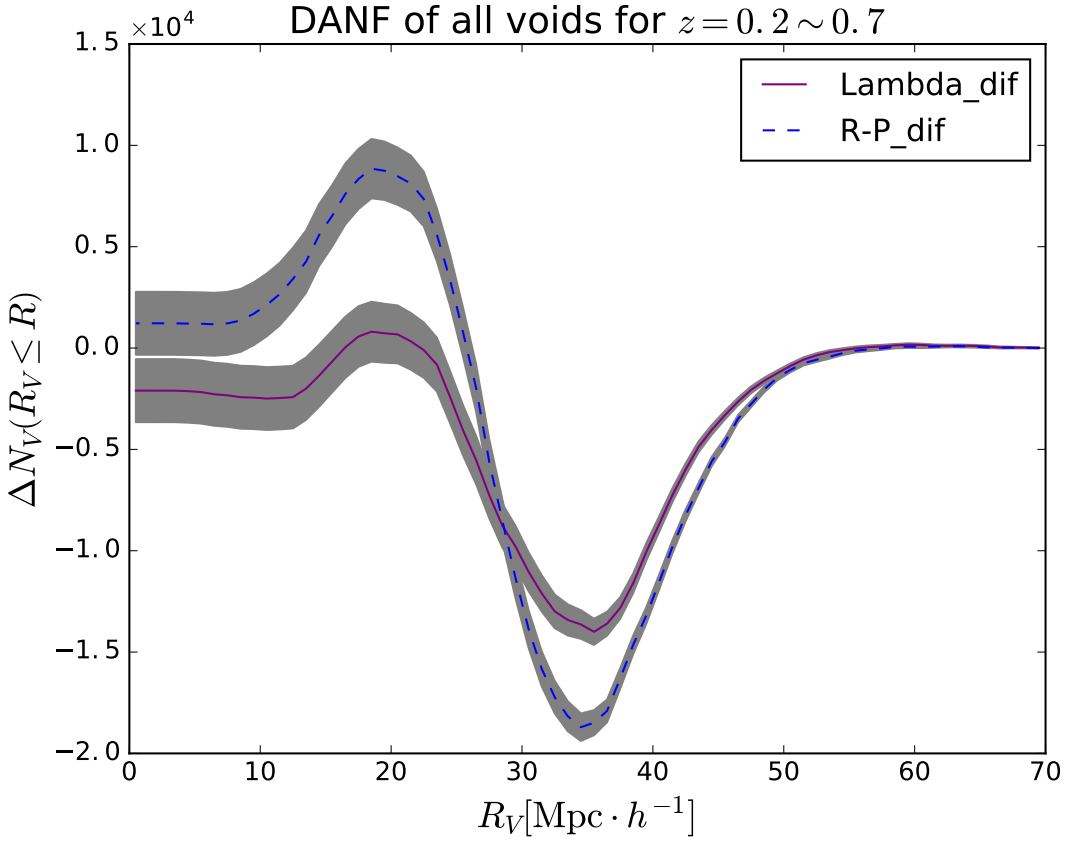

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- Problems
  - consistency of galaxies and DM haloes as tracers of DT voids
  - sampling of tracers
  - evaluation of uncertainties

References

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Liang Y., Zhao C., Chuang C.-H., Kitaura F.-S., Tao C., 2015, preprint, (arXiv:1511.04391)

Rasera Y., Alimi J.-M., Courtin J., Roy F., Corasaniti P.-S., Füzfa A., Boucher V., 2010, in Alimi J.-M., Fuözfa A., eds, American Institute of Physics Conference Series Vol. 1241, American Institute of Physics Conference Series. pp 1134–1139 (arXiv:1002.4950), doi:10.1063/1.3462610

Zhao C., Tao C., Liang Y., Kitaura F.-S., Chuang C.-H., 2016, MNRAS,

Evolution with respect to the redshift z

Peak position R_p versus the redshift z

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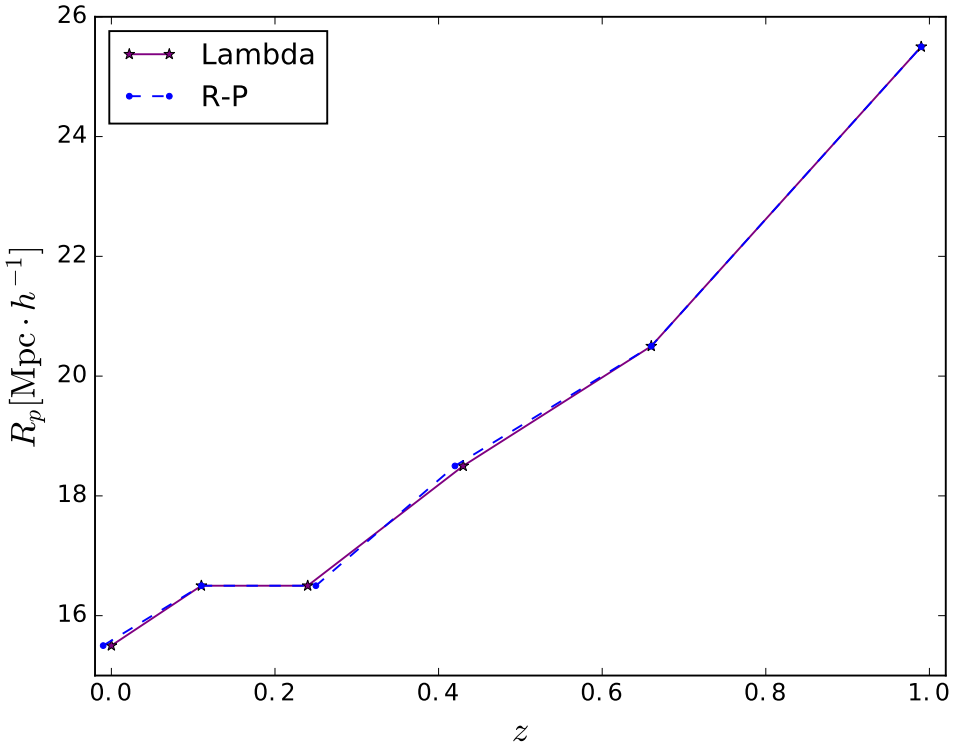

Figure: The curves of the two DE models almost completely coincide with each other. R_p increases as z increases, where a 'platform' occurs as $0.11 \leq z \leq 0.25$.

Evolution with respect to the redshift z

Relative abundances versus the scale factor a

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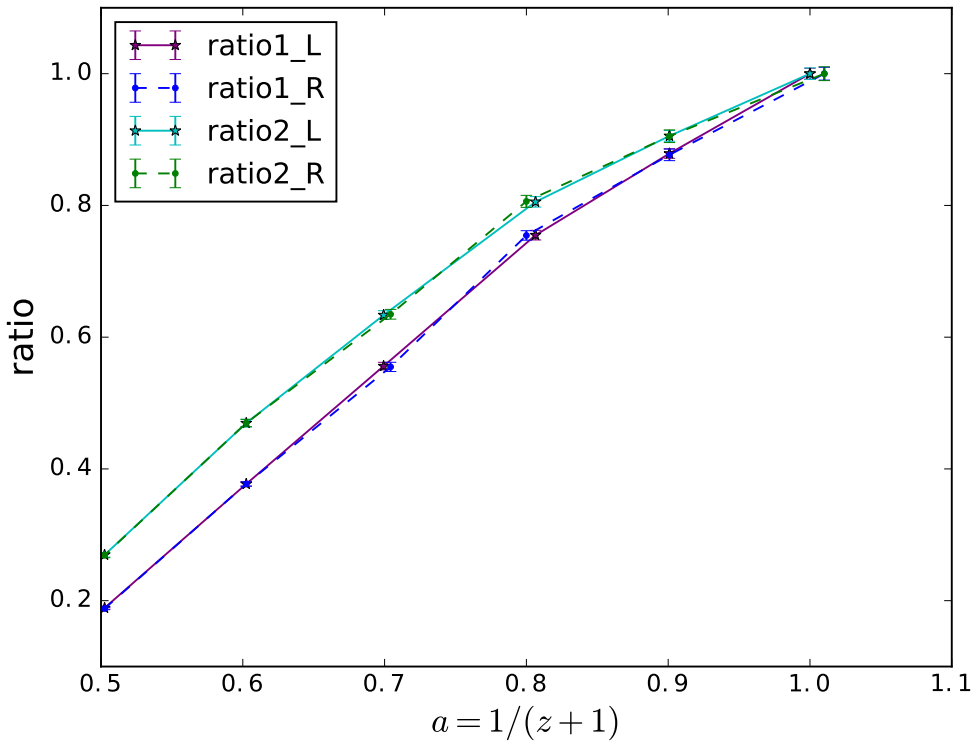

Figure: ratio 1 is the ratio of the number of *all voids* at the peak of NF with certain z to that with $z = 0$ (Lambda) or $z = -0.01$ (R-P), ratio 2 is the ratio of the total number of *all voids* for certain z to that for $z = 0 \sim -0.01$.

Evolution with respect to the redshift z
Shape of the NF curve

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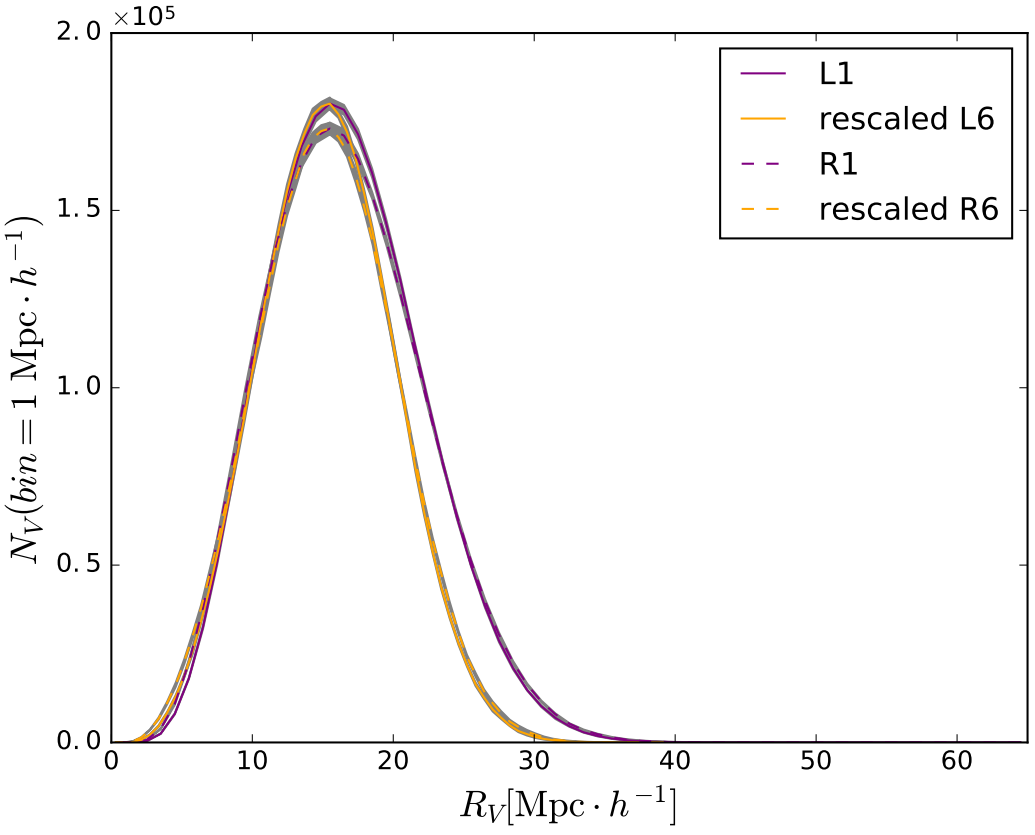

Figure: NF at $z = z_0 = 0$ (Lambda) or $z = z_0 = -0.01$ (R-P) and the rescaled NF at $z = 0.99$ of *all voids* for the two DE models.

Trial results from Group 1 simulations

NF of *all voids* for ED at the snapshot $z = 0 \sim -0.01$

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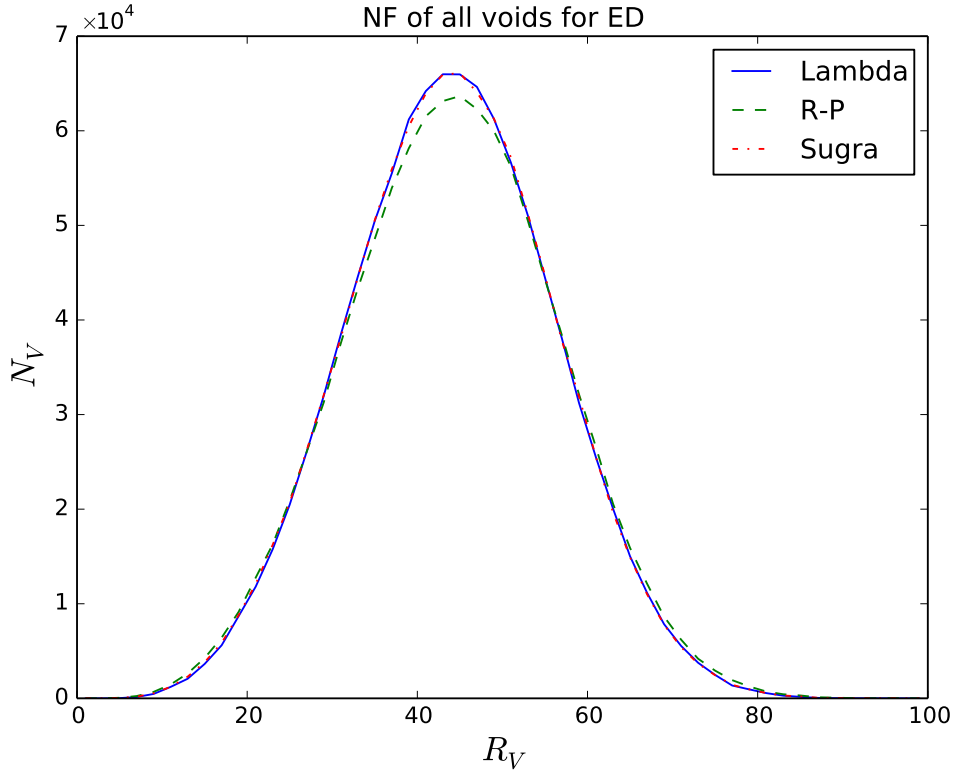

Figure: The peaks of NFs for Lambda and Sugra are very close to each other and slightly higher than that for R-P. ($[R_V] = \mathrm{Mpc} \cdot h^{-1}$, $bin = 2\mathrm{Mpc} \cdot h^{-1}$)

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NF of *all voids* for ED at the snapshot $z = 0.66 \sim 0.65$

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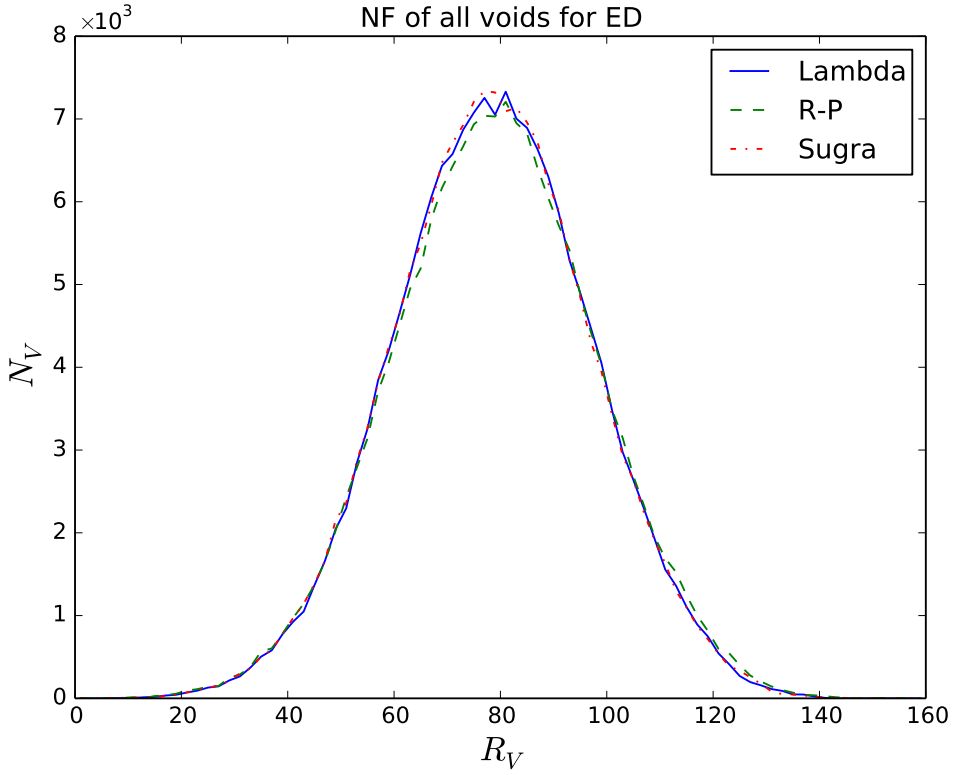

Figure: The results are similar, however, more unreliable than those for $z = 0 \sim -0.01$ due to the smaller tracer number. ($[R_V] = \mathrm{Mpc} \cdot h^{-1}$, $bin = 2\mathrm{Mpc} \cdot h^{-1}$)

Trial results from Group 1 simulations

NF of *disjoint voids* for ED at the snapshot $z = 0 \sim -0.01$

Trial results from Group 1 simulations

NF of *disjoint voids* for ED at the snapshot $z = 0 \sim -0.01$

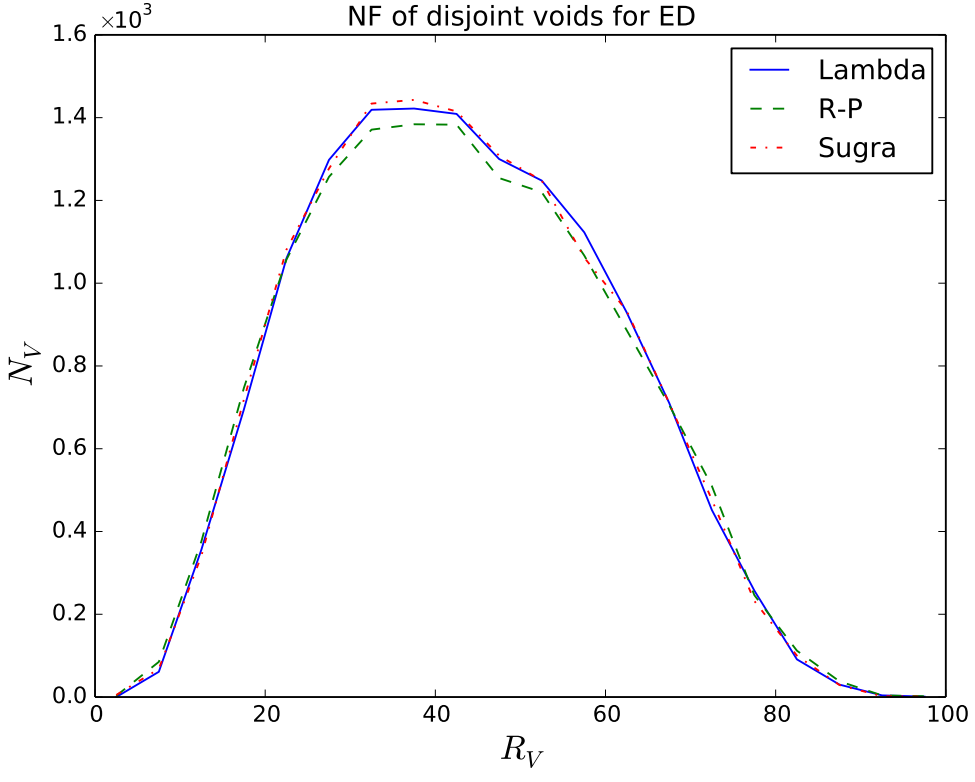

Figure: NFs of *disjoint voids* are not as symmetric as those of *all voids*, which is a general feature. ($[R_V] = \mathrm{Mpc} \cdot h^{-1}$, $bin = 5\mathrm{Mpc} \cdot h^{-1}$)

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NF of *disjoint voids* for ED at the snapshot $z = 0.66 \sim 0.65$

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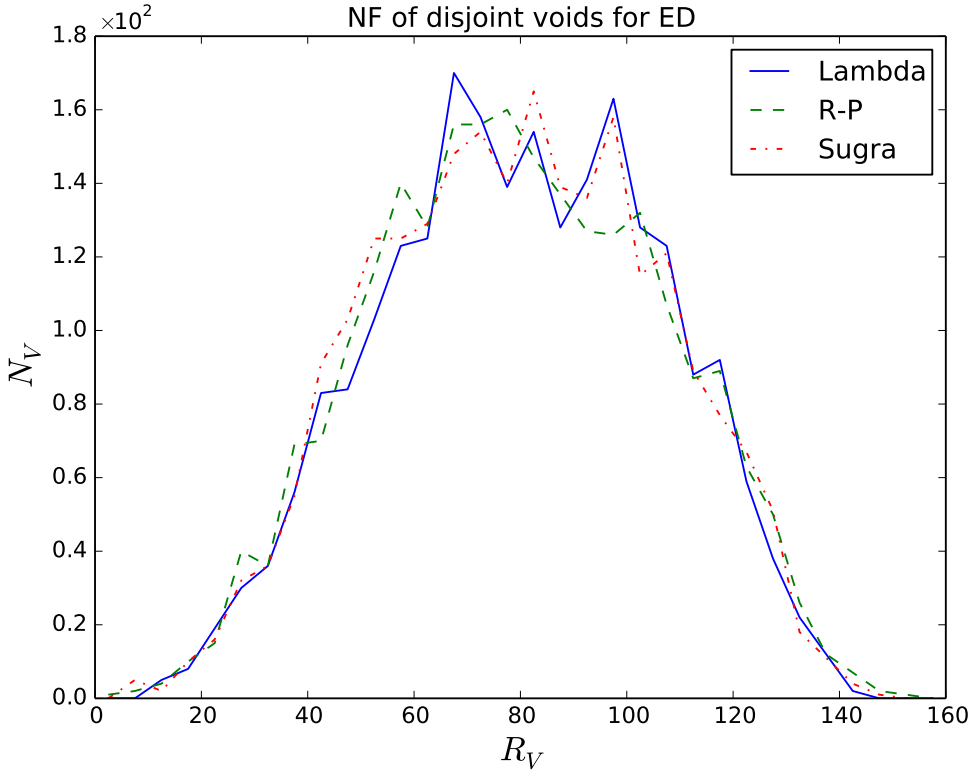

Figure: The fluctuations indicate that the numbers of *disjoint voids* are too small to do statistics for high redshifts. ($[R_V] = \mathrm{Mpc} \cdot h^{-1}$, $bin = 5\mathrm{Mpc} \cdot h^{-1}$)

Trial results from Group 1 simulations

NF of *all voids* for EV at the snapshot $z = 0 \sim -0.01$

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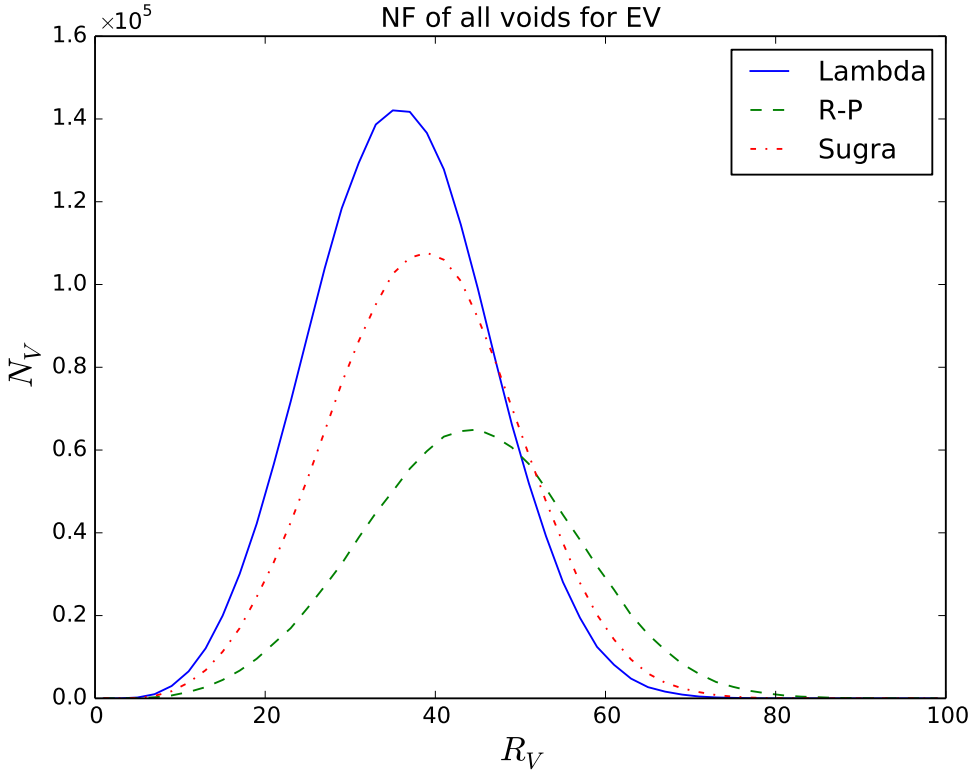

Figure: Different DE models have different peak positions (R_ps) due to different tracer numbers: $R_p(Lambda) < R_p(Sugra) < R_p(R-P)$. ($[R_V] = \mathrm{Mpc} \cdot h^{-1}$, $bin = 2\mathrm{Mpc} \cdot h^{-1}$)

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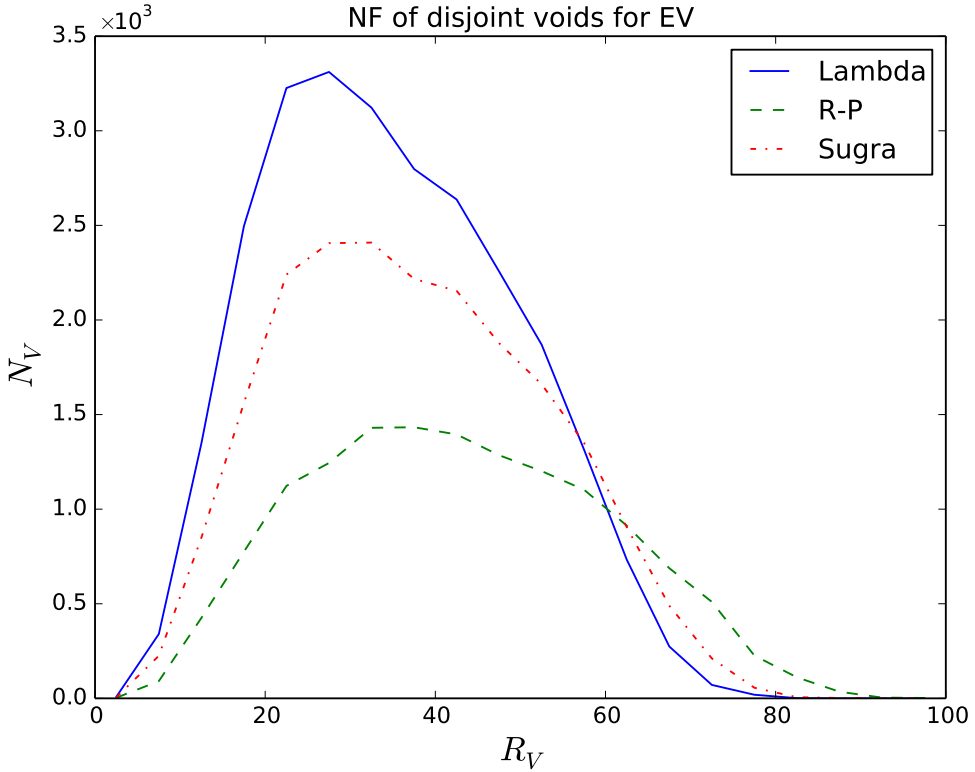

Figure: Similar to the case of *all voids*, the differences among different models are significant, and $R_p(Lambda) < R_p(Sugra) < R_p(R-P)$. ($[R_V] = \mathrm{Mpc} \cdot h^{-1}$, $bin = 5\mathrm{Mpc} \cdot h^{-1}$)

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ANF of *all voids* for EV as $z = 0 \sim -0.01$

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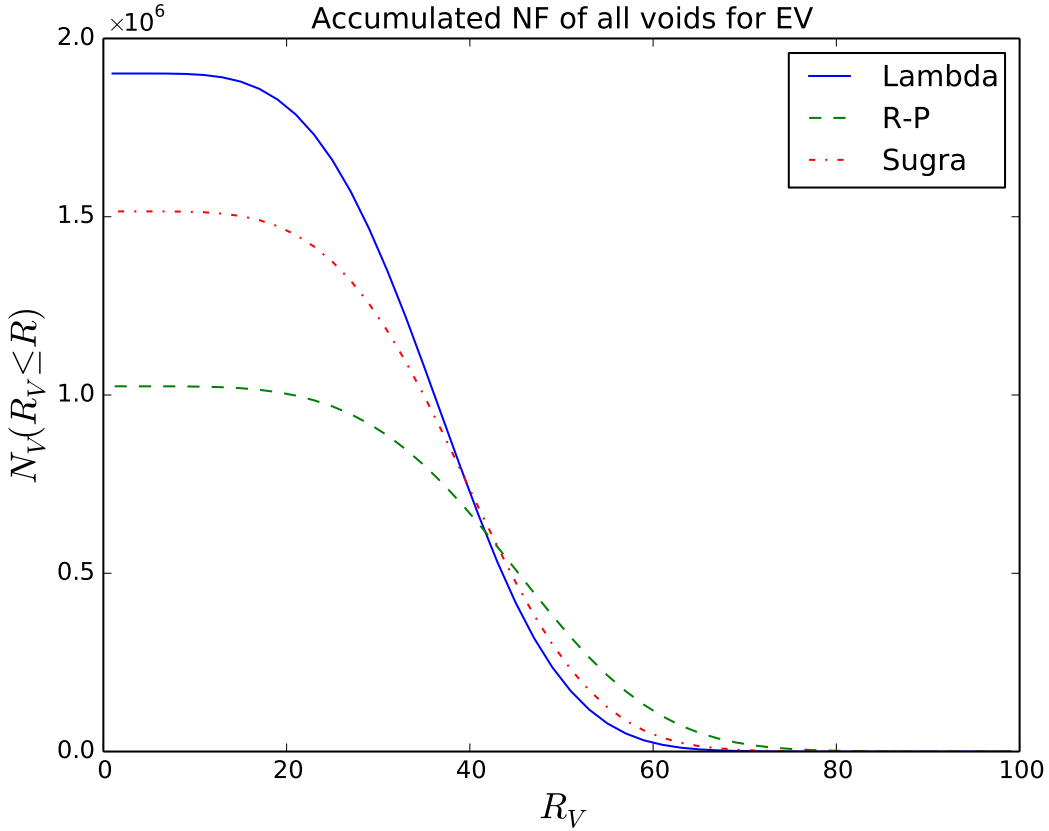

Figure: The curves with different tracer numbers intersect in a region $R_V = 40 \sim 45 \mathrm{Mpc} \cdot \mathrm{h}^{-1}$ rather than at a point. ($[R_V] = \mathrm{Mpc} \cdot h^{-1}$)

Trial results from Group 1 simulations

ANF of *disjoint voids* for EV as $z = 0 \sim -0.01$

Trial results from Group 1 simulations
ANF of *disjoint voids* for EV as $z = 0 \sim -0.01$

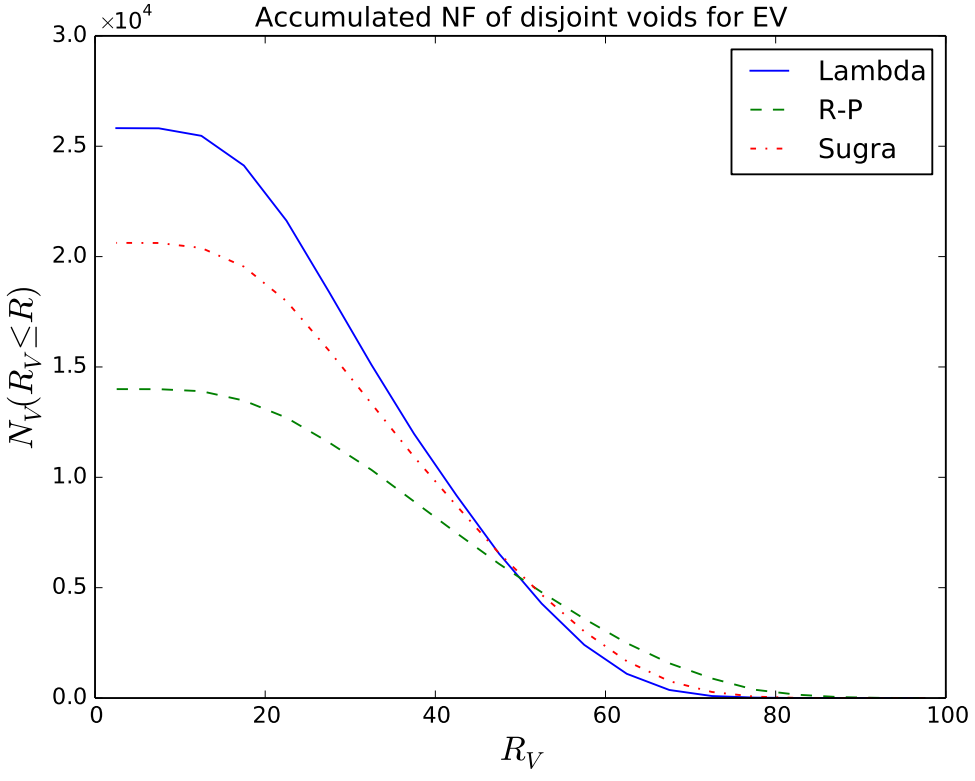

Figure: Similar to the case of *all voids*, there is an intersection region $R_V = 45 \sim 50 \mathrm{Mpc} \cdot \mathrm{h}^{-1}$. ($[R_V] = \mathrm{Mpc} \cdot h^{-1}$)

Trial results from Group 1 simulations

Guidelines for further investigations

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- For EV, differences among the three DE models are more significant, however, difficult to be related to the results from observation data.

Trial results from Group 1 simulations
Guidelines for further investigations

- It is more difficult to do void statistics for high-redshift snapshots, especially for *disjoint voids*.
  - Many structures had not enough time to grow in earlier epochs of the universe.
  - The number of *disjoint voids* is less than that of *all voids* by one magnitude or two.
- For ED, the differences between Lambda and R-P are much more significant than those between Lambda and Sugra.
- For EV, differences among the three DE models are more significant, however, difficult to be related to the results from observation data.

Narrow down the analysis

Void statistics of all voids for ED in Lambda and R-P