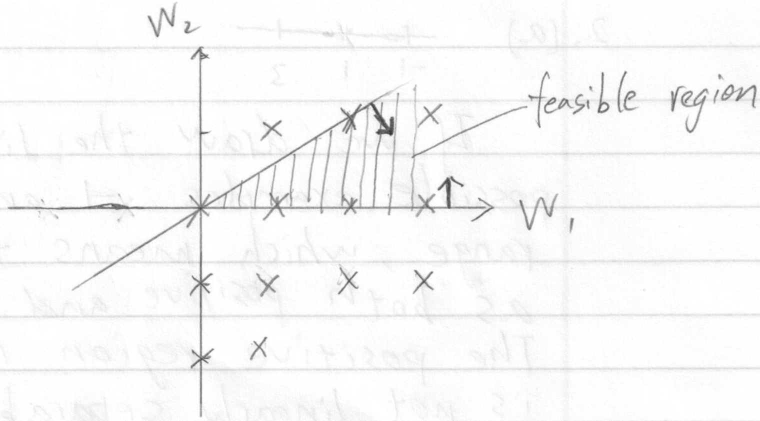
HW 2

1. a) $w_1 - 2w_2 > 0$

$-w_2 < 0$



(b) $w_1 = 0, \quad w_2 = -2$

$z^{(1)}t^{(1)} = 4 > 0$

$z^{(2)}t^{(2)} = -2 \le 0$

$w = \begin{pmatrix} 0 \\ -2 \end{pmatrix} - \begin{pmatrix} 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$

$z^{(1)}t^{(1)} = 2 > 0$

$z^{2}t^{(2)} = -1 < 0$

$w = \begin{pmatrix} 0 \\ -1 \end{pmatrix} - \begin{pmatrix} 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$z^{(1)}t^{(1)} = 0 \le 0$

$w = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ -2 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

$z^{(2)}t^{(2)} = -2 \le 0$

$w = \begin{pmatrix} 1 \\ -2 \end{pmatrix} - \begin{pmatrix} 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$z^{(1)}t^{(1)} = 2 > 0$

$z^{(2)}t^{(2)} = -1 \le 0$

$w = \begin{pmatrix} 1 \\ -1 \end{pmatrix} - \begin{pmatrix} 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$z^{(1)}t^{(1)} = 1 > 0$

$z^{(2)}t^{(2)} = 0 \le 0$

$w = \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$z^{(1)}t^{(1)} = -1 \le 0$

$w = \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

$w = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$

$w = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$w = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$

$w = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$

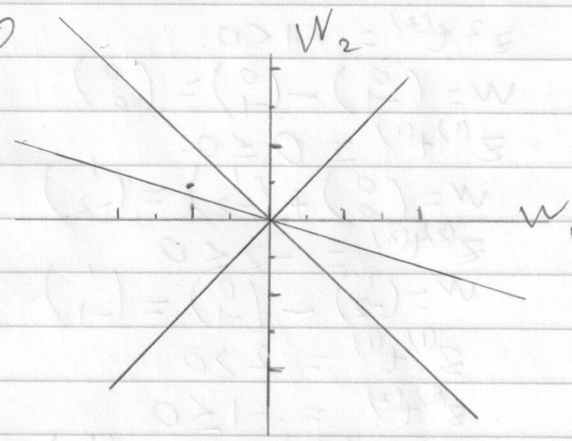
$\boxed{w = \begin{pmatrix} 3 \\ 1 \end{pmatrix}}$

2. (a) 

If we draw the line segment connecting the two possible examples -1 and 3, but 1 fall within this range, which means this point must be classified as both positive and negative, which is impossible. The positive region is covex. Therefore, this dataset is not linearly separable.

(b)

$\phi_1(x)$	$\phi_2(x)$	t
-1	1	1
1	1	0
3	9	1



$-w_1 + w_2 > 0$

$w_1 + w_2 < 0$

$3w_1 + 9w_2 > 0$

$w_1 = -1 \quad w_2 = \frac{1}{2}$

3. $\mathbf{y} = \mathbf{X}\mathbf{w} + \mathbf{b}$

$$\frac{\partial \mathcal{E}}{\partial \mathbf{y}} = \frac{\partial}{\partial \mathbf{y}}\left(\frac{1}{N}\sum_{i=1}^{N} 1 - \cos(y^{(i)} - t^{(i)})\right) = \frac{1}{N}\sin(\mathbf{y} - \mathbf{t})$$

$$\frac{\partial \mathcal{E}}{\partial \mathbf{w}} = \frac{\partial}{\partial \mathbf{y}}\frac{\partial \mathbf{y}}{\partial \mathbf{w}} = \frac{1}{N}\mathbf{X}^T\sin(\mathbf{y} - \mathbf{t})$$

$$\frac{\partial \mathcal{E}}{\partial b} = \frac{\partial}{\partial \mathbf{y}}\frac{\partial \mathbf{y}}{\partial b} = \frac{1}{N}\mathbf{1}^T\sin(\mathbf{y} - \mathbf{t}) \quad \text{where } \mathbf{1} = \begin{pmatrix}1\\ \vdots \\ 1\end{pmatrix} \; N \times 1 \text{ vector}$$