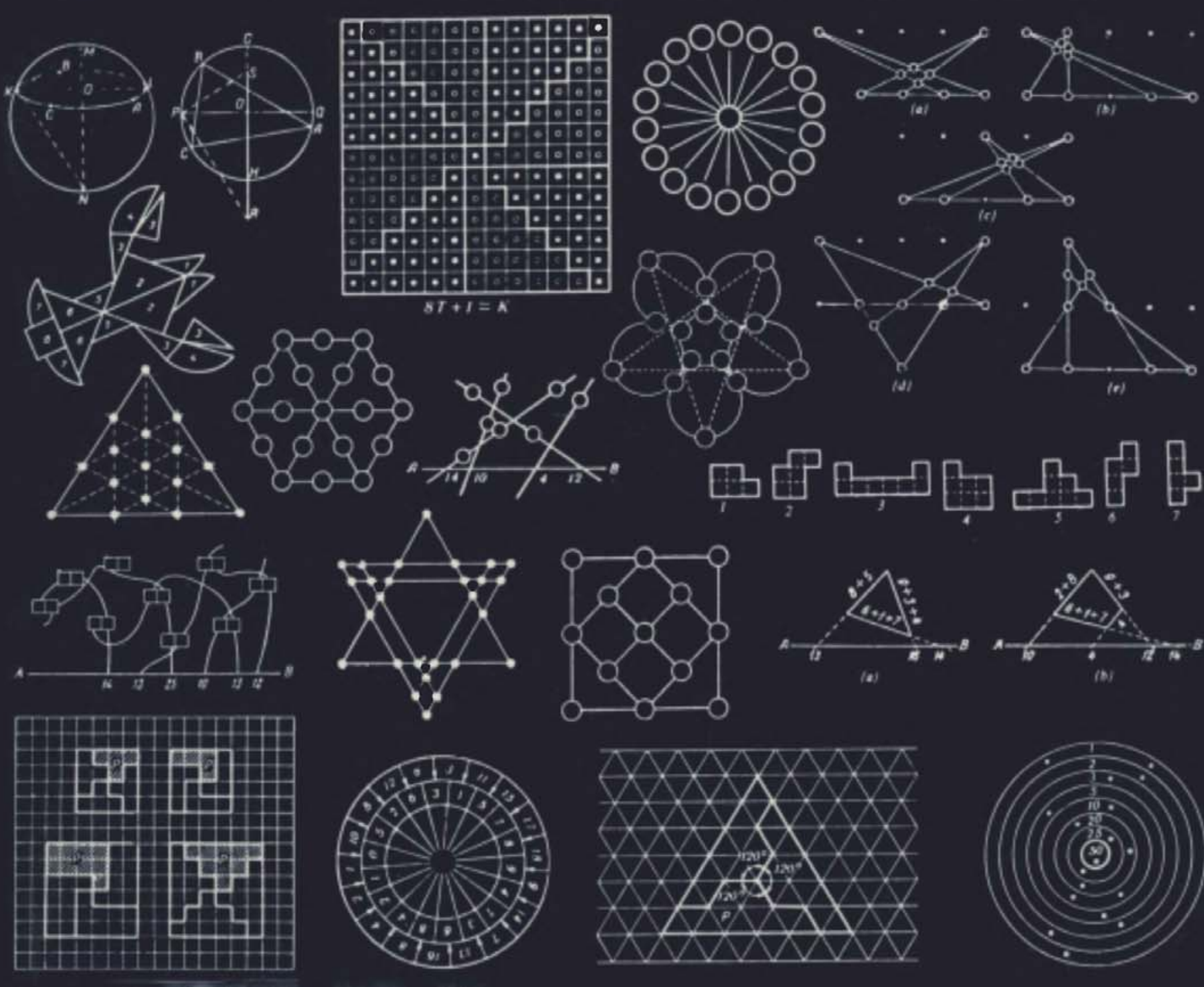
BORIS A. KORDEMSKY

The Moscow Puzzles

359 MATHEMATICAL RECREATIONS

Edited and with an introduction by MARTIN GARDNER

Editor of the Mathematical Games Department, SCIENTIFIC AMERICAN



Translated by ALBERT PARRY,
Professor Emeritus of Russian Civilization and Language
Colgate University

The Moscow Puzzles

359 Mathematical Recreations

BORIS A. KORDEMSKY

Edited and with an introduction by MARTIN GARDNER,
Editor of the Mathematical Games Department,
SCIENTIFIC AMERICAN

CHARLES SCRIBNER'S SONS / NEW YORK

Contents

Introduction

The book now in the reader's hands is the first English translation of *Mathematical Know-how,* the best and most popular puzzle book ever published in the Soviet Union. Since its first appearance in 1956 there have been eight editions, as well as translations from the original Russian into Ukranian, Estonian, Lettish, and Lithuanian. Almost a million copies of the Russian version alone have been sold. Outside the U.S.S.R. the book has been published in Bulgaria, Rumania, Hungary, Czechoslovakia, Poland, Germany, France, China, Japan, and Korea.

The author, Boris A. Kordemsky, who was born in 1907, is a talented high school mathematics teacher in Moscow. His first book on recreational mathematics, *The Wonderful Square,* a delightful discussion of curious properties of the ordinary geometric square, was published in Russian in 1952. In 1958 his *Essays on Challenging Mathematical Problems* appeared. In collaboration with an engineer he produced a picture book for children, *Geometry Aids Arithmetic* (1960), which by lavish use of color overlays, shows how simple diagrams and graphs can be used in solving arithmetic problems. His *Foundations of the Theory of Probabilities* appeared in 1964, and in 1967 he collaborated on a textbook about vector algebra and analytic geometry. But it is for his mammoth puzzle collection that Kordemsky is best known in the Soviet Union, and rightly so, for it is a marvelously varied assortment of brain teasers.

Admittedly many of the book's puzzles will be familiar in one form or another to puzzle buffs who know the Western literature, especially the books of England's Henry Ernest Dudeney and America's Sam Loyd. However, Kordemsky has given the old puzzles new angles and has presented them in such amusing and charming story forms that it is a pleasure to come upon them again, and the story backgrounds incidentally convey a valuable impression of contemporary Russian life and customs. Moreover, mixed with the known puzzles are many that will be new to Western readers, some of them no doubt invented by Kordemsky himself.

The only other Russian writer on recreational mathematics and science who can be compared with Kordemsky is Yakov I. Perelman (1882-1942), who in addition to books on recreational arithmetic, algebra, and geometry, wrote similar books on mechanics, physics, and astronomy. Paperback editions of Perelman's works are still widely sold throughout the U.S.S.R., but Kordemsky's book is now regarded as *the* outstanding puzzle collection in the history of Russian mathematics.

The translation of Kordemsky's book was made by Dr. Albert Parry, former chairman of Russian Studies at Colgate University, and more recently at Case Western Reserve University. Dr. Parry is a distinguished American scholar of Russian origin whose many books range from the early *Garrets and Pretenders* (a colorful history of American bohemianism) and a biography entitled *Whistler's Father* (the father of the painter was a pioneer railroad builder in prerevolutionary Russia) to *The New Class Divided,* a comprehensive, authoritative account of the growing conflict in the Soviet Union between its scientific-technical elite and its ruling bureaucracy.

As editor of this translation I have taken certain necessary liberties with the text. Problems involving Russian currency, for example, have been changed to problems about dollars and cents wherever this could be done without damaging the puzzle. Measurements in the metric system have been altered to miles, yards, feet, pounds, and other units more familiar to readers in a nation where, unfortunately, the metric system is still used only by scientists. Throughout, wherever Kordemsky's original text could be clarified and sometimes simplified, I have not hesitated to rephrase, cut, or add new sentences. Occasionally a passage or footnote referring to a Russian book or article not available in English has been omitted. Toward the end of his volume, Kordemsky included some problems in number theory that have been omitted because they seemed so difficult and technical, at least for American readers, as to be out of keeping with the rest of the collection. In a few instances where puzzles were inexplicable without a knowledge of Russian words, I substituted puzzles of a similar nature using English words.

The original illustrations by Yevgeni Konstantinovich Argutinsky have been retained, retouched where necessary and with Russian letters in the diagrams replaced by English letters.

In brief, the book has been edited to make it as easy as possible for an English-reading public to understand and enjoy. More than 90 percent of the original material has been retained, and every effort has been made to convey faithfully its warmth and humor. I hope that the result will provide many weeks or even months of entertainment for all who enjoy such problems.

Martin Gardner

The Moscow Puzzles

I
Amusing Problems

Using Elementary Operations

To see how good your brain is, let's first put it to work on problems that require only perseverance, patience, sharpness of mind, and the ability to add, subtract, multiply, and divide whole numbers.

1. OBSERVANT CHILDREN

A schoolboy and a schoolgirl have just completed some meteorological measurements. They are resting on a knoll. A freight train is passing, its locomotive fiercely fuming and huffing as it pulls the train up a slight incline. Along the railroad bed the wind is wafting evenly, without gusts.

"What wind speed did our measurements show?" the boy asked.

"Twenty miles per hour."

"That is enough to tell me the train's speed."

"Well now." The girl was dubious.

"All you have to do is watch the movement of the train a bit more closely."

The girl thought awhile and also figured it out.

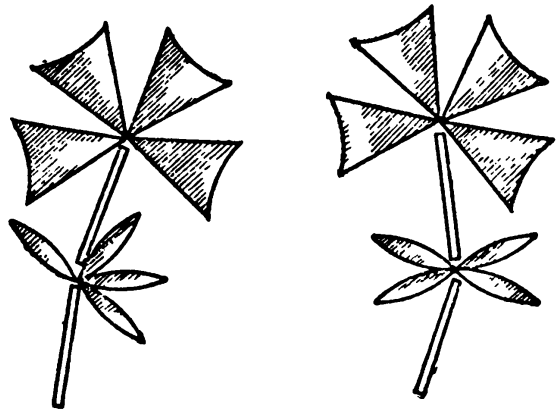
What they saw was precisely what the artist has drawn. What was the train's speed?

2. THE STONE FLOWER

Do you remember the smart craftsman Danila from P. Bazhov's fairy tale, "The Stone Flower"?

They tell in the Urals that Danila, while still an apprentice, took semiprecious Ural stones and chiseled two flowers whose leaves, stems, and petals could be separated. From the parts of these flowers it was possible to make a circular disk.

Take a piece of paper or cardboard, copy Danila's flowers from the diagram, then



cut out the petals, stems, and leaves and see if you can put them together to make a circle.

3. MOVING CHECKERS

Place 6 checkers on a table in a row, alternating them black, white, black, white, and so on, as shown.

Leave a vacant place large enough for 4 checkers on the left.

Move the checkers so that all the white ones will end on the left, followed by all the black ones. The checkers must be moved in pairs, taking 2 adjacent checkers at a time, without disturbing their order, and sliding them to a vacant place. To solve this problem, only three such moves are necessary.

The theme of this problem is further developed in Problems 94–97.

If no checkers are available, use coins, or cut pieces out of paper or cardboard.

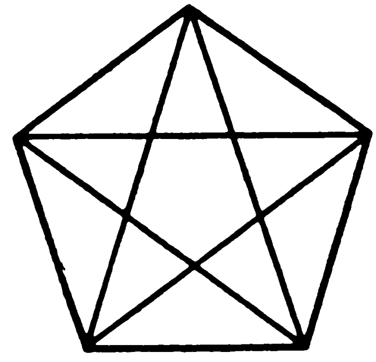
4. THREE MOVES

Place three piles of matches on a table, one with 11 matches, the second with 7, and the third with 6. You are to move matches so that each pile holds 8 matches. You may add to any pile only as many matches as it already contains, and all the matches must come from one other pile. For example, if a pile holds 6 matches, you may add 6 to it, no more or less.

You have three moves.

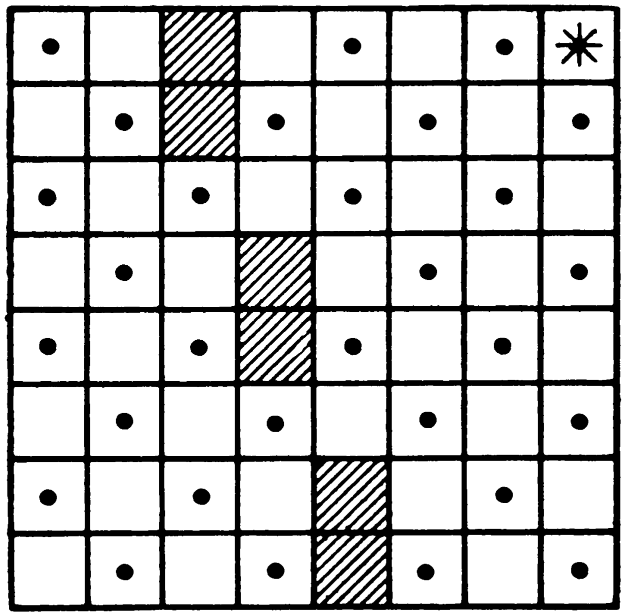
5. COUNT!

How many different triangles are there in the figure?



6. THE GARDENER'S ROUTE

The diagram shows the plan of an apple orchard (each dot is an apple tree). The gardener started with the square containing a star, and he worked his way through



all the squares, with or without apple trees, one after another. He never returned to a square previously occupied. He did not walk diagonally and he did not walk through the six shaded squares (which contain buildings). At the end of his route the gardener found himself on the starred square again.

Copy the diagram and see if you can trace the gardener's route.

7. FIVE APPLES

Five apples are in a basket. How do you divide them among five girls so that each girl gets an apple, but one apple remains in the basket?

8. DON'T THINK TOO LONG

How many cats are in a small room if in each of the four corners a cat is sitting, and opposite each cat there sit 3 cats, and at each cat's tail a cat is sitting?

9. DOWN AND UP

A boy presses a side of a blue pencil to a side of a yellow pencil, holding both pencils vertically. One inch of the pressed side of the blue pencil, measuring from its lower end, is smeared with paint. The yellow pencil is held steady while the boy slides the blue pencil down 1 inch, continuing to press it against the yellow one. He returns the blue pencil to its former position, then again slides it down 1 inch. He continues until he has lowered the blue pencil 5 times and raised it 5 times—10 moves in all.

Suppose that during this time the paint neither dries nor diminishes in quantity. How many inches of each pencil will be smeared with paint after the tenth move?

This problem was thought up by the mathematician Leonid Mikhailovich Rybakov while on his way home after a successful duck hunt. What led him to make up this puzzle is explained in the answer, but don't read it until you have solved the problem.

10. CROSSING A RIVER

A detachment of soldiers must cross a river. The bridge is broken, the river is deep. What to do? Suddenly the officer in charge spots 2 boys playing in a rowboat by the shore. The boat is so tiny, however, that it can only hold 2 boys or 1 soldier. Still, all the soldiers succeed in crossing the river in the boat. How?

Solve this problem either in your mind or practically—that is, by moving checkers, matches, or the like on a table across an imaginary river.

11. WOLF, GOAT, AND CABBAGE

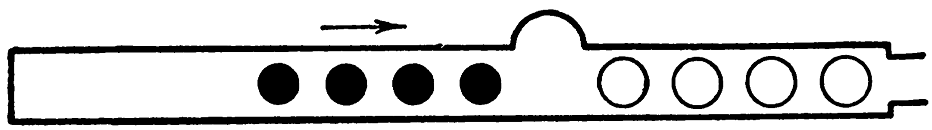
This problem can be found in eighth-century writings.

A man has to take a wolf, a goat, and some cabbage across a river. His rowboat has enough room for the man plus either the wolf or the goat or the cabbage. If he takes the cabbage with him, the wolf will eat the goat. If he takes the wolf, the goat will eat the cabbage. Only when the man is present are the goat and the cabbage safe from their enemies. All the same, the man carries wolf, goat, and cabbage across the river.

How?

12. ROLL THEM OUT

In a long, narrow chute there are 8 balls: 4 black ones on the left, and 4 white ones—slightly larger—on the right. In the middle of the chute there is a small niche

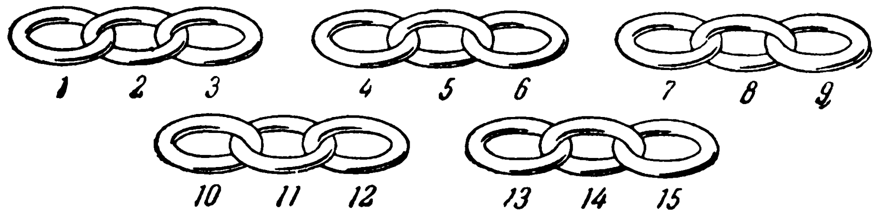


that can hold 1 ball of either color. The chute's right end has an opening large enough for a black but not a white ball.

Roll all the black balls out of the chute. (No, you can't pick them up.)

13. REPAIRING A CHAIN

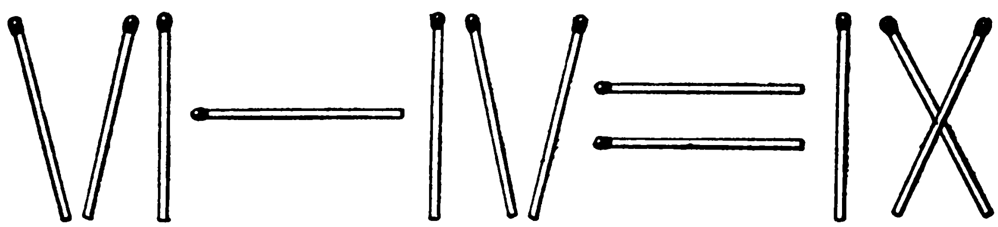
Do you know why the young craftsman in the picture is so deep in thought? He has 5 short pieces of chain that must be joined into a long chain. Should he open ring 3



(first operation), link it to ring 4 (second operation), then unfasten ring 6 and link it to ring 7, and so on? He could complete his task in 8 operations, but he wants to do it in 6. How does he do it?

14. CORRECT THE ERROR

With 12 matches form the "equation" shown.



The equation shows that $6 - 4 = 9$. Correct it by shifting just 1 match.

15. FOUR OUT OF THREE (A JOKE)

Three matches are on a table. Without adding another, make 4 out of 3. You are not allowed to break the matches.

16. THREE AND TWO IS EIGHT (ANOTHER JOKE)

Place 3 matches on a table. Ask a friend to add 2 more matches to make 8.

17. THREE SQUARES

Take 8 small sticks (or matches), 4 of which are half the length of the other 4. Make three equal squares out of the 8 sticks (or matches).

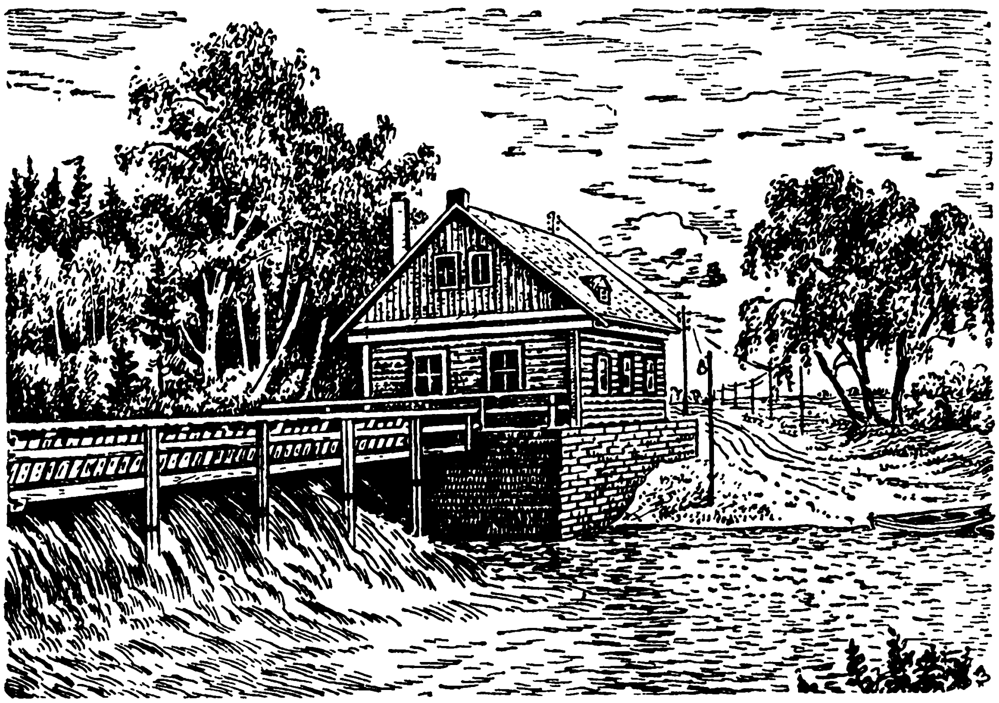
18. HOW MANY ITEMS?

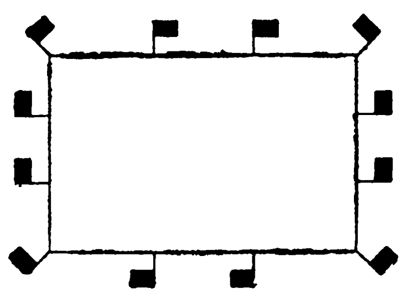
An item is made from lead blanks in a lathe shop. Each blank suffices for 1 item.

Lead shavings accumulated from making 6 items can be melted and made into a blank. How many items can be made from 36 blanks?

19. ARRANGING FLAGS

Komsomol youths have built a small hydroelectric powerhouse. Preparing for its opening, young Communist boys and girls are decorating the powerhouse on all four sides with garlands, electric bulbs, and small flags. There are 12 flags.





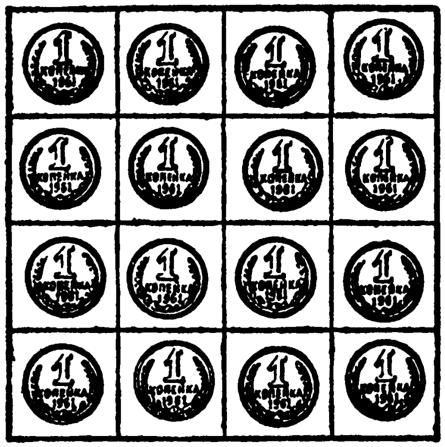
At first they arrange the flags 4 to a side, as shown, but then they see that the flags can be arranged 5 or even 6 to a side. How?

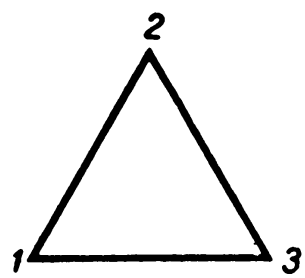
20. TEN CHAIRS

In a rectangular dance hall, how do you place 10 chairs along the walls so that there are an equal number of chairs along each wall?

21. KEEP IT EVEN

Take 16 objects (pieces of paper, coins, plums, checkers) and put them in four rows of 4 each. Remove 6, leaving an even number of objects in each row and each column. (There are many solutions.)





22. A MAGIC TRIANGLE

I have placed the numbers 1, 2, and 3 at the vertices of a triangle. Arrange 4, 5, 6, 7, 8, and 9 along the sides of the triangle so that the numbers along each side add to 17.

This is harder: without being told which numbers to place at the vertices, make a similar arrangement of the numbers from 1 through 9, adding to 20 along each side. (Several solutions are possible.)

23. GIRLS PLAYING BALL

Twelve girls in a circle began to toss a ball, each girl to her neighbor on the left. When the ball completed the circle, it was tossed in the opposite direction.

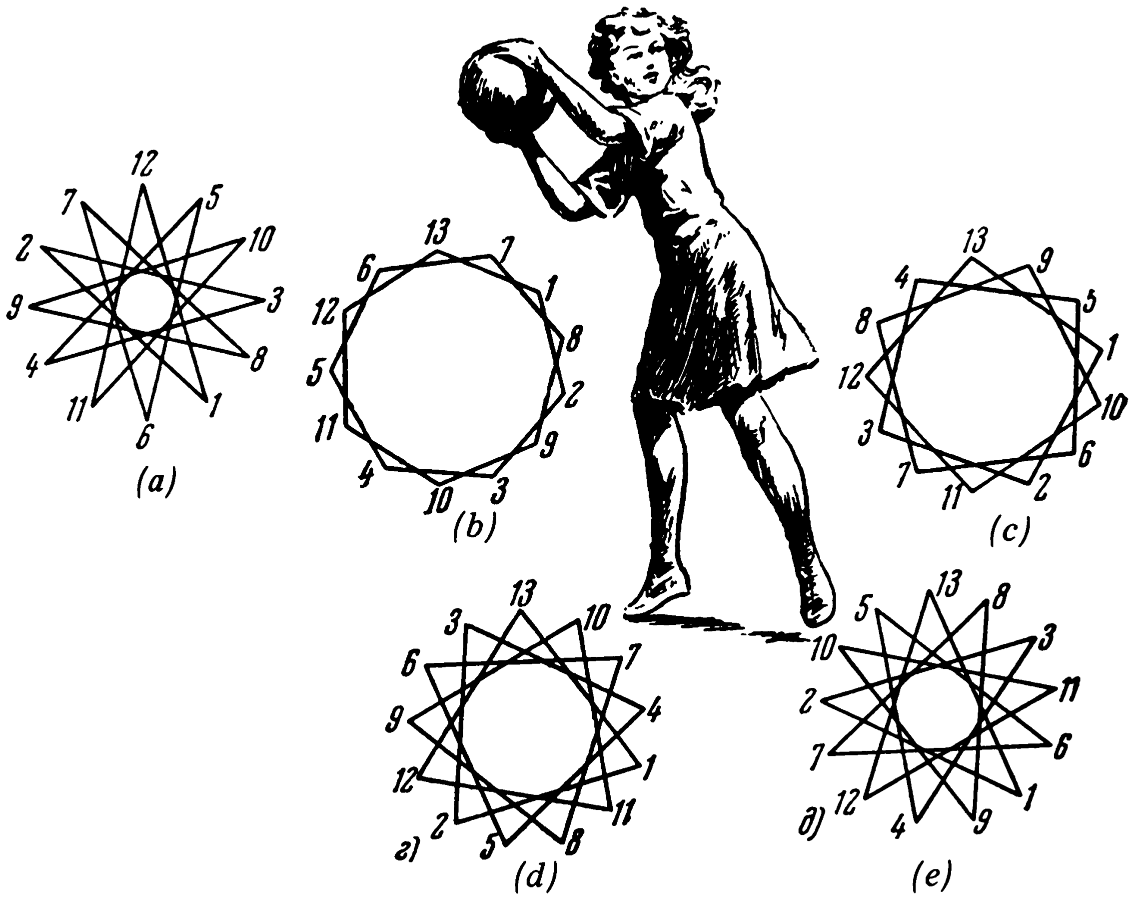
After a while one of the girls said: "Let's skip 1 girl as we toss the ball."

"But since there are 12 of us, half the girls will not be playing," Natasha objected.

"Well, let's skip 2 girls!"

"This would be even worse—only 4 would be playing. We should skip 4 girls—the fifth would catch it. There is no other combination."

"And if we skip 6?"



"It is the same as skipping 4, only the ball goes in the opposite direction," Natasha answered.

"And if we skip 10 girls each time, so that the eleventh girl catches it?"

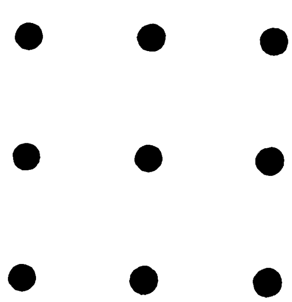
"But we have already played that way," said Natasha.

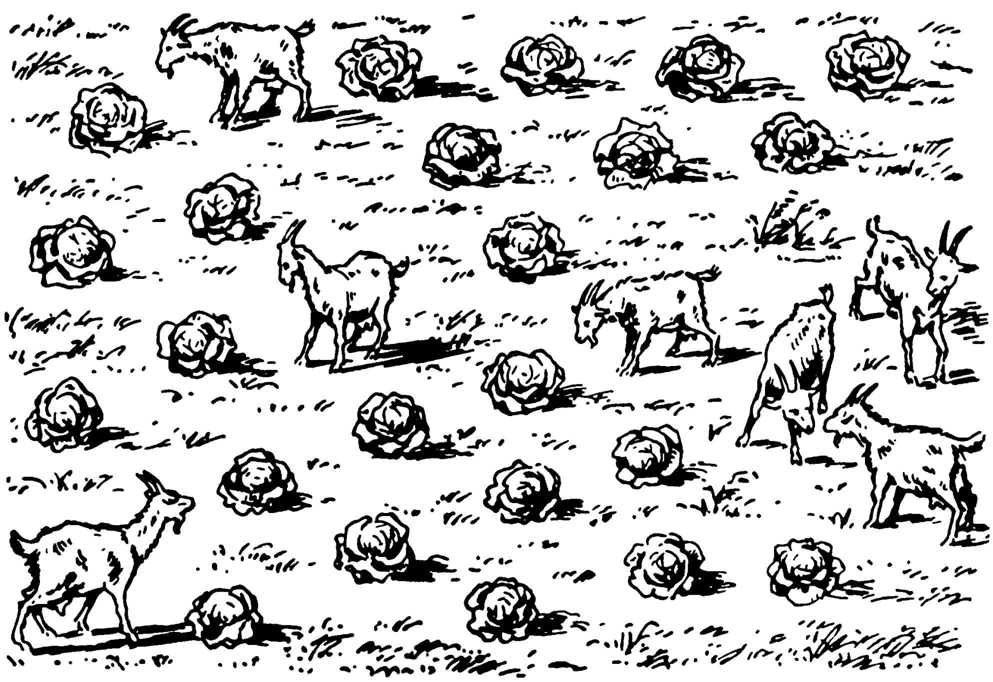
They began to draw diagrams of every such way to toss the ball, and were soon convinced that Natasha was right. Besides skipping none, only skipping 4 (or its mirror image 6) let all the girls participate (see *a* in the picture).

If there had been 13 girls, the ball could have been tossed skipping 1 girl *(b)*, or 2 *(c)*, or 3 *(d)*, or 4 *(e)*, without leaving any girls out. How about 5 and 6? Draw diagrams.

24. FOUR STRAIGHT LINES

Make a square with 9 dots as shown. Cross all the dots with 4 straight lines without taking your pencil off the paper.





25. GOATS FROM CABBAGE

Now, instead of joining points, separate all the goats from the cabbage in the picture by drawing 3 straight lines.

26. TWO TRAINS

A nonstop train leaves Moscow for Leningrad at 60 miles per hour. Another nonstop train leaves Leningrad for Moscow at 40 miles an hour.

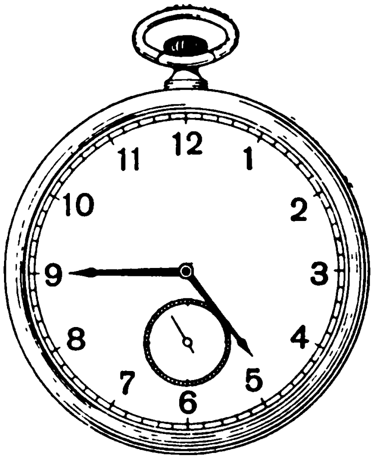
How far apart are the trains 1 hour before they pass each other?

27. THE TIDE COMES IN (A JOKE)

Not far off shore a ship stands with a rope ladder hanging over her side. The rope has 10 rungs. The distance between each rung is 12 inches. The lowest rung touches the water. The ocean is calm. Because of the incoming tide, the surface of the water rises 4 inches per hour. How soon will the water cover the third rung from the top rung of the rope ladder?

28. A WATCH FACE

Can you divide the watch face with 2 straight lines so that the sums of the numbers in each part are equal?



Can you divide it into 6 parts so that each part contains 2 numbers and the six sums of 2 numbers are equal?

29. A BROKEN CLOCK FACE

In a museum I saw an old clock with Roman numerals. Instead of the familiar IV there was an old-fashioned IIII. Cracks had formed on the face and divided it into 4 parts. The picture shows unequal sums of the numbers in each part, ranging from 17 to 21.

Can you change one crack, leaving the others untouched, so that the sum of the numbers in each of 4 parts is 20?

(Hint: The crack, as changed, does not have to run through the center of the clock.)

30. THE WONDROUS CLOCK

A watchmaker was telephoned urgently to make a house call to replace the broken hands of a clock. He was sick, so he sent his apprentice.

The apprentice was thorough. When he finished inspecting the clock it was dark. Assuming his work was done, he hurriedly attached the new hands and set the clock by his pocket watch. It was six o'clock, so he set the big hand at 12 and the little hand at 6.

The apprentice returned, but soon the telephone rang. He picked up the receiver only to hear the client's angry voice:

"You didn't do the job right. The clock shows the wrong time."

Surprised, he hurried back to the client's house. He found the clock showing not much past eight. He handed his watch to the client, saying: "Check the time, please. Your clock is not off even by 1 second."

The client had to agree.

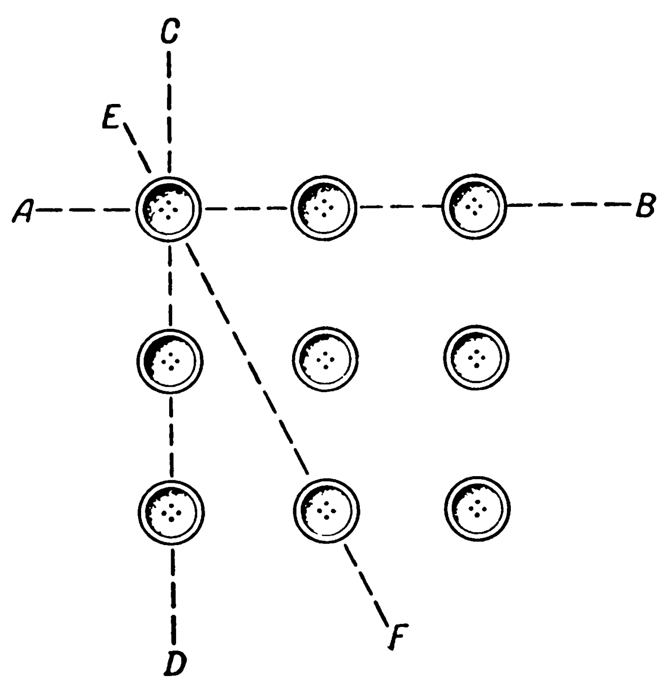
Early the next morning the client telephoned to say that the clock hands, apparently gone berserk, were moving around the clock at will. When the apprentice rushed over, the clock showed a little past seven. After checking with his watch, the apprentice got angry:

"You are making fun of me! Your clock shows the right time!"

Have you figured out what was going on?

31. THREE IN A ROW

On a table, arrange 9 buttons in a 3-by-3 square. When 2 or more buttons are in a



straight line we will call it a row. Thus rows *AB* and *CD* have 3 buttons, and row *EF* has 2.

How many 3- and 2-button rows are there?

Now remove 3 buttons. Arrange the remaining 6 buttons in 3 rows so that each row contains 3 buttons. (Ignore the subsidiary 2-button rows this time.)

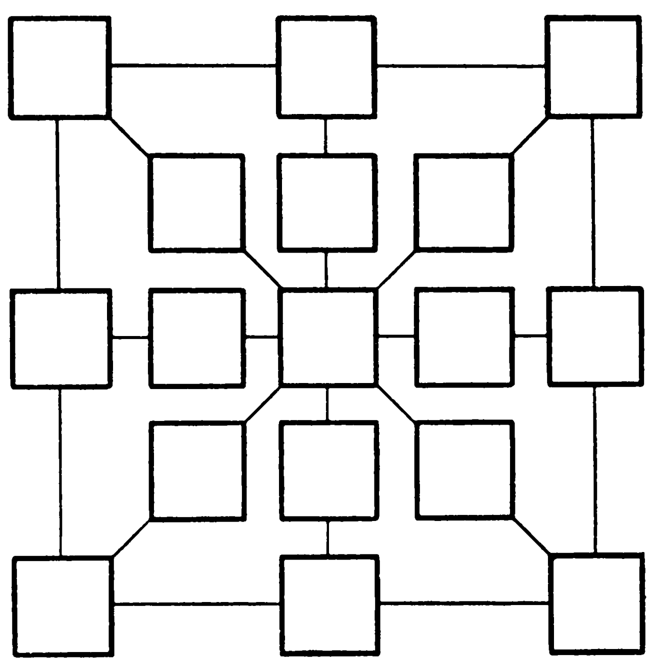
32. TEN ROWS

It is easy to arrange 16 checkers in 10 rows of 4 checkers each, but harder to arrange 9 checkers in 6 rows of 3 checkers each. Do both.

33. PATTERN OF COINS

Take a sheet of paper, copy the diagram on it, enlarging it two or three times, and have ready 17 coins:

20 kopeks	5
15 kopeks	3
10 kopeks	3
5 kopeks	6

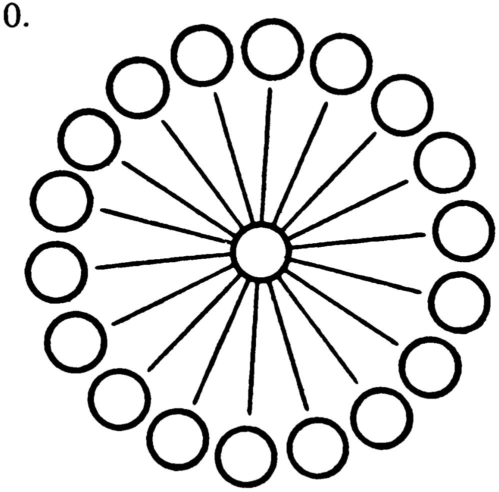


Place a coin in each square so that the number of kopeks along each straight line is 55.

[This problem cannot be translated into United States coinage, but you can work on it by writing the kopek values on pieces of paper—*M.G.*]

34. FROM 1 THROUGH 19

Write the numbers from 1 through 19 in the circles so that the numbers in every 3 circles on a straight line total 30.



35. SPEEDILY YET CAUTIOUSLY

The title of the problem tells you how to approach these four questions.

(A) A bus leaves Moscow for Tula at noon. An hour later a cyclist leaves Tula for Moscow, moving, of course, slower than the bus. When bus and bicycle meet, which of the two will be farther from Moscow?

(B) Which is worth more: a pound of $10 gold pieces or half a pound of $20 gold pieces?

(C) At six o'clock the wall clock struck 6 times. Checking with my watch, I noticed that the time between the first and last strokes was 30 seconds. How long will the clock take to strike 12 at midnight?

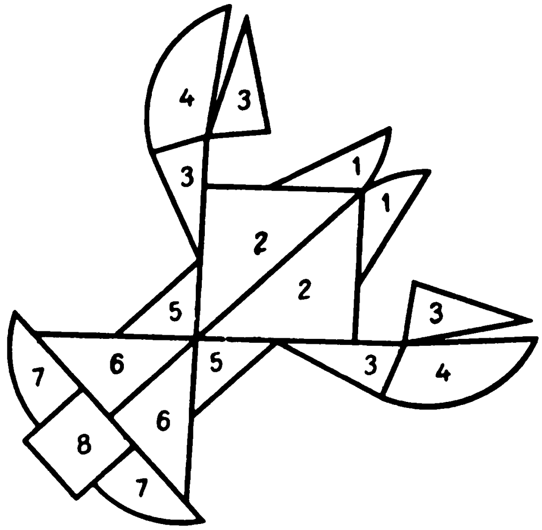
(D) Three swallows fly outward from a point. When will they all be on the same plane in space?

Now check the Answers. Did you fall into any of the traps which lurk in these simple problems?

The attraction of such problems is that they keep you on your toes and teach you to think cautiously.

36. A CRAYFISH FULL OF FIGURES

The crayfish is made of 17 numbered pieces. Copy them on a sheet of paper and cut them out.



Using all the pieces, make a circle and, by its side, a square.

37. THE PRICE OF A BOOK

A book costs \$1 plus half its price. How much does it cost?

38. THE RESTLESS FLY

Two cyclists began a training run simultaneously, one starting from Moscow, the other from Simferopol.

When the riders were 180 miles apart, a fly took an interest. Starting on one cyclist's shoulder, the fly flew ahead to meet the other cyclist. On reaching the latter, the fly at once turned back.

The restless fly continued to shuttle back and forth until the pair met; then it settled on the nose of one of the cyclists.

The fly's speed was 30 miles per hour. Each cyclist's speed was 15 miles per hour.

How many miles did the fly travel?

39. UPSIDE-DOWN YEAR

When was the latest year that is the same upside down?

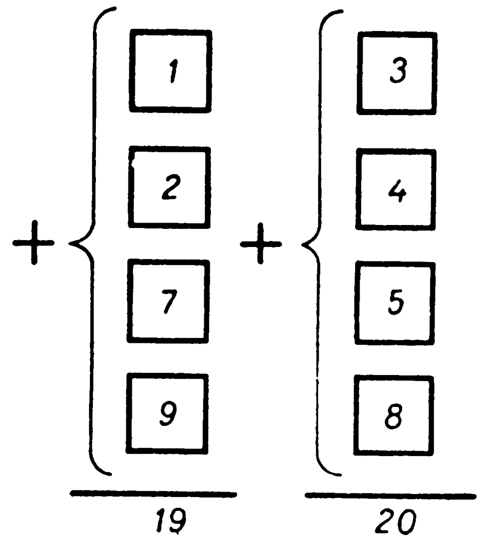
40. TWO JOKES

(A) A man phoned his daughter to ask her to buy a few things he needed for a trip. He told her she would find enough dollar bills for the purchases in an envelope on his desk. She found the envelope with 98 written on it.

In a store she bought \$90 worth of things, but when it was time to pay she not only didn't have \$8 left over but she was short.

By how much, and why?

(B) Mark 1, 2, 3, 4, 5, 7, 8, and 9 on 8 pieces of paper and place them in 2 rows as shown.



Move 2 pieces so as to make the sums of the two columns equal.

41. HOW OLD AM I?

When my father was 31 I was 8. Now he is twice as old as I am. How old am I?

42. TELL "AT A GLANCE"

Here are two columns of numbers:

123456789	1
12345678	21
1234567	321
123456	4321
12345	54321
1234	654321
123	7654321
12	87654321
1	987654321

Look closely: the numbers on the right are the same as on the left, but reversed and in reverse order. Which column has the larger total? (First answer "at a glance"; then check by adding.)

43. A QUICK ADDITION

(A) These six-digit numbers:

328,645
491,221
816,304
117,586
671,355
508,779
183,696
882,414

can be grouped mentally and added in 8 seconds. How?

(B) Say to a friend: "Write down as many four-digit numbers as you like. Then I will jot down just as many numbers and add them all up, yours and mine, in an instant."

Suppose he writes:

7,621
3,057
2,794
4,518

For your first number, match his fourth number: his 4 with a 5, his 5 with a 4, his 1 with an 8, and his 8 with a 1. His 4,518 plus your 5,481 equals 9,999. Match his other numbers the same way. The complete list is:

7,621
3,057
2,794
4,518
5,481
7,205
6,942
2,378

How can you know, in just a few seconds, that the correct sum is 39,996?

(C) Say: "Write down any two numbers. I will write a third and at once write (from left to right) the sum of the three numbers."

If he writes:

72,603,294
51,273,081

what number should you write, and how do you find the total so quickly?

44. WHICH HAND?

Give a friend an "even" coin (say, a dime—ten is an even number) and an "odd" coin (say, a nickel). Ask him to hold one coin in his right hand and the other in his left.

Tell him to triple the value of the coin in his right hand and double the value of the coin in his left, then add the two.

If the sum is even, the dime is in his right hand; if odd, in his left.

Explain, and think up some variations.

45. HOW MANY?

A boy has as many sisters as brothers, but each sister has only half as many sisters as brothers.

How many brothers and sisters are there in the family?

46. WITH THE SAME FIGURES

Combine plus signs and five 2s to get 28. Combine plus signs and eight 8s to get 1,000.

47. ONE HUNDRED

Express 100 with five 1s. Express 100 three ways with five 5s. You can use brackets, parentheses, and these signs: +, −, ×, ÷ .

48. A DUEL IN ARITHMETIC

The Mathematics Circle in our school had this custom: Each applicant was given a simple problem to solve—a little mathematical nut to crack, so to speak. You became a full member only if you solved the problem.

An applicant named Vitia was given this array:

$$\begin{matrix} 1 & 1 & 1 \\ 3 & 3 & 3 \\ 5 & 5 & 5 \\ 7 & 7 & 7 \\ 9 & 9 & 9 \end{matrix}$$

He was asked to replace 12 digits with zeros so that the sum would be 20.

Vitia thought a little, then wrote rapidly:

$$\begin{array}{r} 0\,1\,1 \\ 0\,0\,0 \\ 0\,0\,0 \\ 0\,0\,0 \\ \underline{0\,0\,9} \\ 2\,0 \end{array} \qquad \begin{array}{r} 0\,1\,0 \\ 0\,0\,3 \\ 0\,0\,0 \\ 0\,0\,7 \\ \underline{0\,0\,0} \\ 2\,0 \end{array}$$

He smiled and said: "If you substitute just ten zeros for digits, the sum will be 1,111. Try it!"

The Circle's president was taken aback briefly. But he not only solved Vitia's problem, he improved on it:

"Why not replace only 9 digits with zeros–and still get 1,111?"

As the debate continued, ways of getting 1,111 by replacing 8, 7, 6, and 5 digits with zeros were found.

Solve the six forms of this problem.

49. TWENTY

There are three ways to add four odd numbers and get 10:

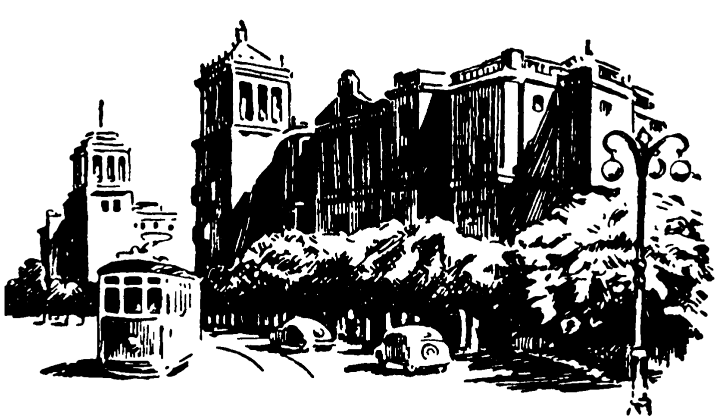
$$1 + 1 + 3 + 5 = 10;$$
$$1 + 1 + 1 + 7 = 10;$$
$$1 + 3 + 3 + 3 = 10.$$

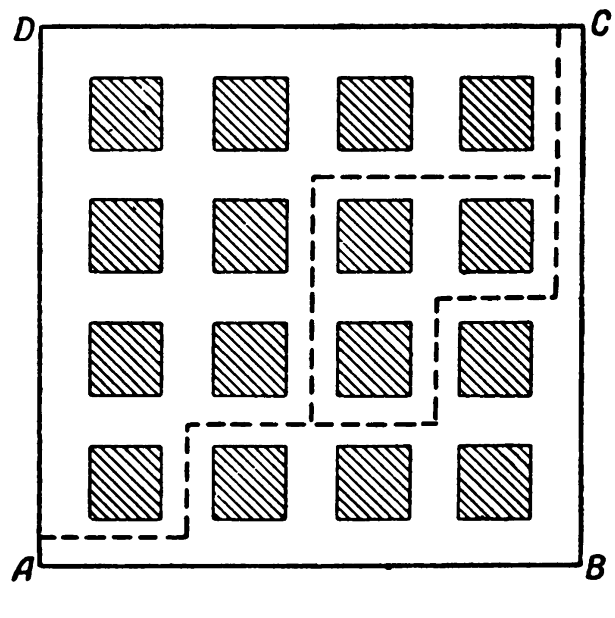
Changes in the order of numbers do not count as new solutions.

Now add eight odd numbers to get 20. To find all eleven solutions you will need to be systematic.

50. HOW MANY ROUTES?

"In our Mathematics Circle we diagramed 16 blocks of our city. How many different routes can we draw from *A* to *C* moving only upward and to the right?





Different routes may, of course, have portions that coincide (as in the diagram).
"This problem is not easy. Have we solved it by counting 70 different routes?"
What answer should we give these students?

51. ORDER THE NUMBERS

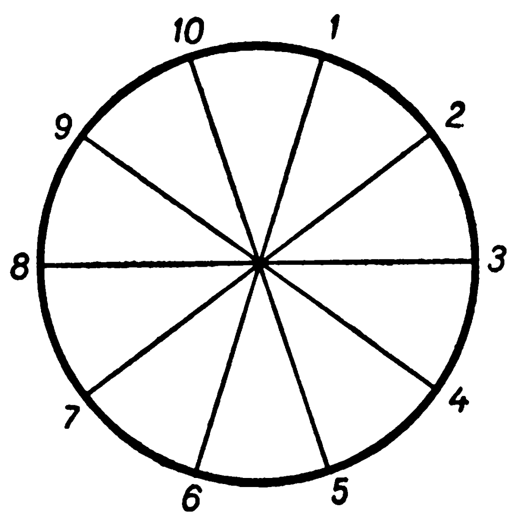
The diagram shows 1 through 10 (in order) at the tips of five diameters. Only once does the sum of two adjacent numbers equal the sum of the opposite two numbers:

$$10 + 1 = 5 + 6.$$

Elsewhere, for example:

$$1 + 2 \neq 6 + 7;$$
$$2 + 3 \neq 7 + 8.$$

Rearrange the numbers so that all such sums are equal. You can expect more than one solution to this problem. How many basic solutions are there? How many variants (not including simple rotations of variants)?



52. DIFFERENT ACTIONS, SAME RESULT

Given two 2s, "plus" can be changed to "times" without changing the result: $2 + 2 = 2 \times 2$. The solution with 3 numbers is easy too: $1 + 2 + 3 = 1 \times 2 \times 3$.
Now find the answer for 4 numbers and the answer(s) for 5 numbers.

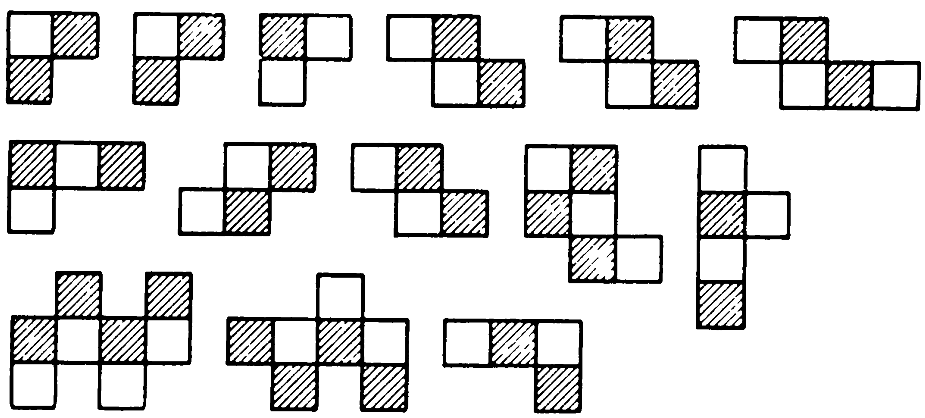
53. NINETY-NINE AND ONE HUNDRED

How many pluses should we put between the digits of 987,654,321 to get a total of 99, and where?

There are two solutions. To find even one is not easy. But the experience will help you put pluses between 1, 2, 3, 4, 5, 6, and 7, in order to get a total of 100. (A schoolgirl from Kemerovo, central Siberia, has found two solutions.)

54. A CUT-UP CHESSBOARD

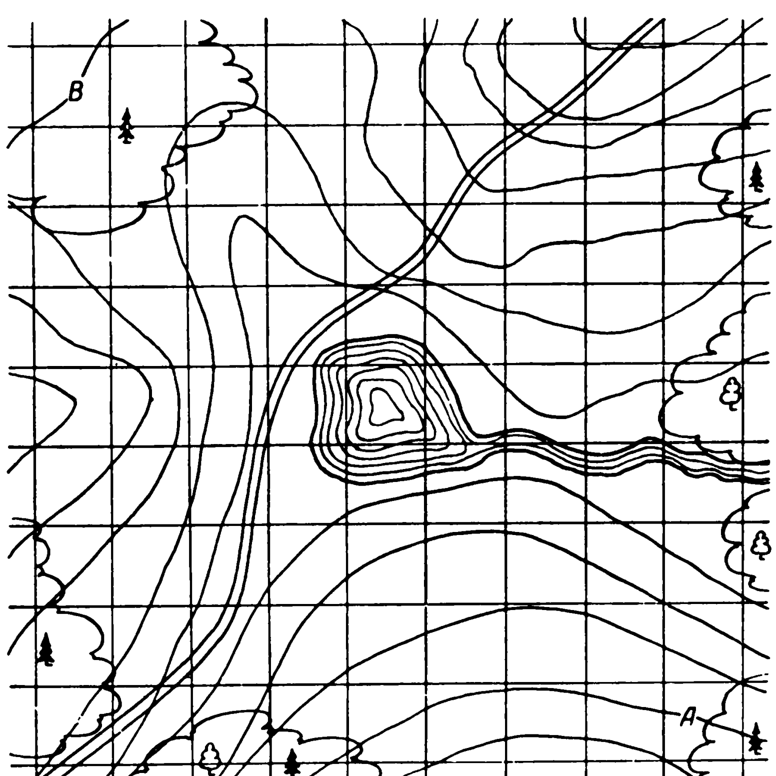
A merry chess player cut his cardboard chessboard into 14 parts, as shown. Friends who wanted to play chess with him had to put the parts back together again first.



55. LOOKING FOR A LAND MINE

A colonel gave a group of military school cadets a puzzle to solve. He pointed to a field map and said:

"Two sappers with mine detectors must search this area to find enemy mines and defuse them. They have to examine every square on the diagram except the central

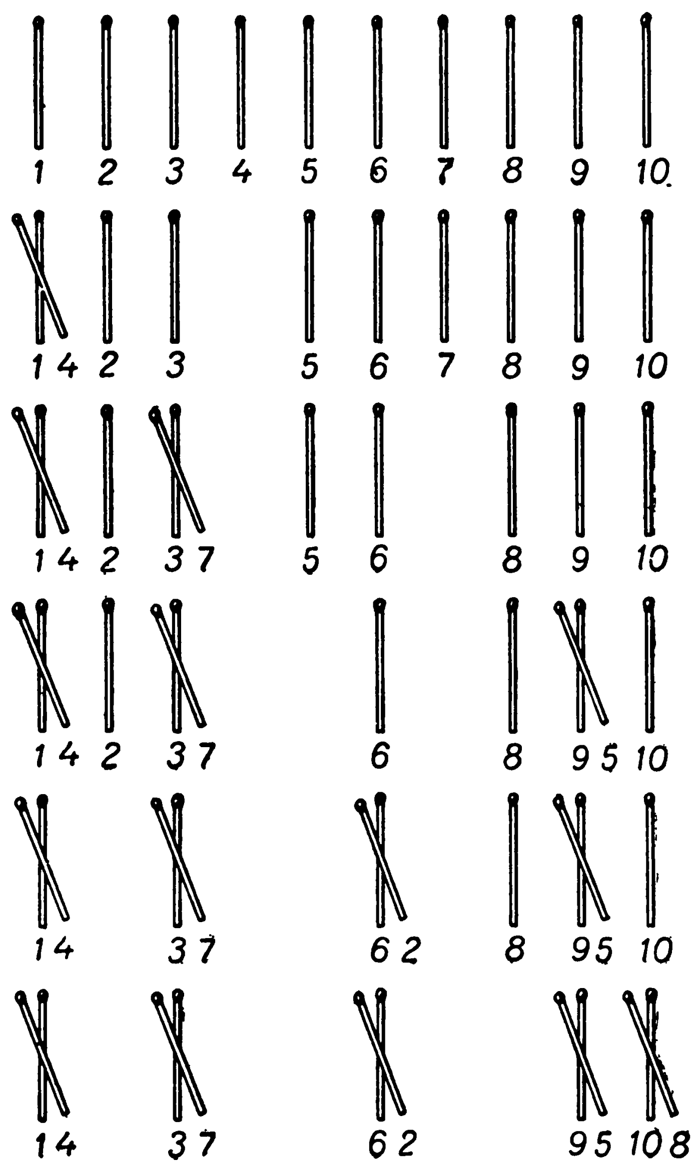


square, which is a small pond. They can proceed horizontally and vertically, but not diagonally, and only one sapper can visit each square, once. The first soldier goes from *A* to *B,* the other from *B* to *A.* Draw their paths so that each one passes through the same number of squares."

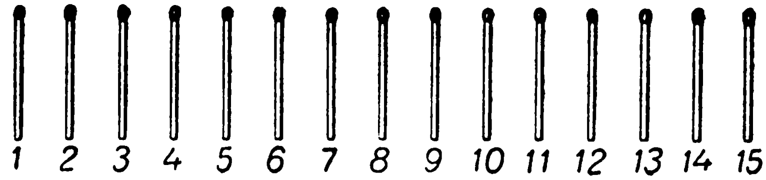
Can you, too, solve the colonel's puzzle?

56. GROUPS OF TWO

Ten matches are in a row. I can group them in five pairs, each time moving a match across 2 matches, and placing it on top of a third one, as shown.



Do it in 10 moves, writing them down.
Find another solution.



57. GROUPS OF THREE

There are 15 matches in a row. Put them in five groups of 3. In each move a match jumps over 3 matches.

58. THE STOPPED CLOCK

My only timepiece is a wall clock. One day I forgot to wind it and it stopped. I went to visit a friend whose watch is always correct, stayed awhile, and returned home. There I made a simple calculation and set the clock right.

How did I do this when I had no watch on me to tell how long it took me to return from my friend's house?

59. PLUS AND MINUS SIGNS

1 2 3 4 5 6 7 8 9 = 100

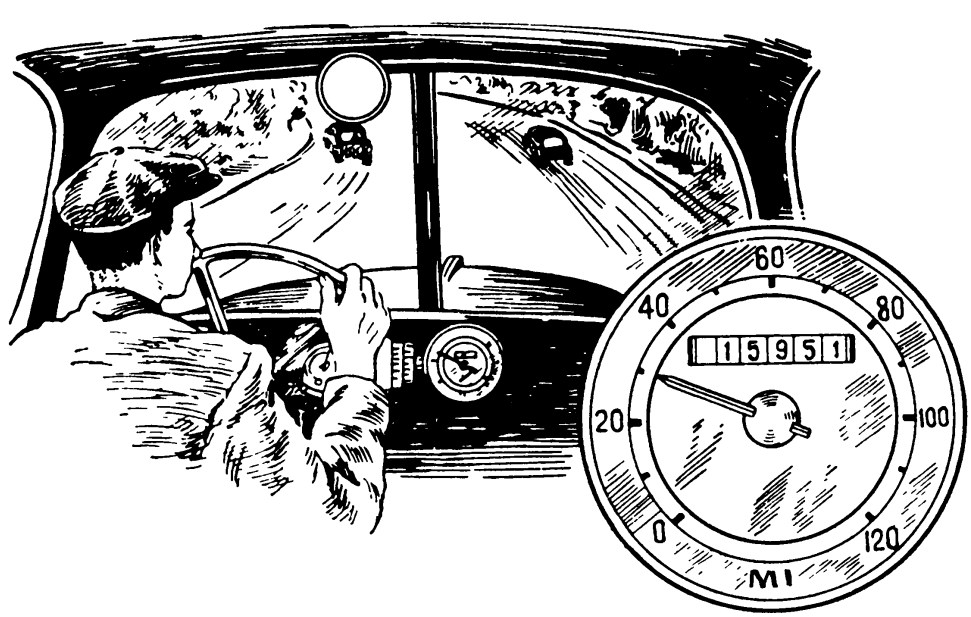
Here is the only way to insert 7 plus and minus signs between the digits on the left side to make the equation correct:

$$1 + 2 + 3 - 4 + 5 + 6 + 78 + 9 = 100.$$

Can you do it with only three plus or minus signs?

60. THE PUZZLED DRIVER

The odometer of the family car shows 15,951 miles. The driver noticed that this number is palindromic: it reads the same backward as forward.



"Curious," the driver said to himself. "It will be a long time before that happens again."

But 2 hours later, the odometer showed a new palindromic number.

How fast was the car traveling in those 2 hours?

61. FOR THE TSIMLYANSK POWER INSTALLATION

A factory making measuring equipment urgently needed by the famous Tsimlyansk power installation has a brigade of ten excellent workers: the chief (an older, experienced man) and 9 recent graduates of a manual training school.

Each of the nine young workers produces 15 sets of equipment per day, and their chief turns out 9 more sets than the average of all ten workers.

How many sets does the brigade produce in a day?

62. DELIVERING GRAIN ON TIME

A collective farm was due to deliver its quota of grain to the state authorities. The management of the kolkhoz decided the trucks should arrive in the city at exactly 11:00 A.M. If the trucks traveled at 30 miles per hour they would reach the city at ten, an hour early; at 20 miles an hour they would arrive at noon, an hour late.

How far is the kolkhoz from the city, and how fast should the trucks travel to arrive at 11:00 A.M.?

63. RIDING THE TRAIN TO A DACHA

Two schoolgirls were traveling from the city to a dacha (summer cottage) on an electric train.

"I notice," one of the girls said, "that the dacha trains coming in the opposite direction pass us every 5 minutes. What do you think–how many dacha trains arrive in the city in an hour, given equal speeds in both directions?"

"Twelve, of course," the other girl answered, "because 60 divided by 5 equals 12."

The first girl did not agree. What do you think?

64. FROM 1 TO 1,000,000,000

When the celebrated German mathematician Karl Friedrich Gauss (1777-1855) was nine he was asked to add all the integers from 1 through 100. He quickly added 1 to 100, 2 to 99, and so on for 50 pairs of numbers each adding to 101. Answer: $50 \times 101 = 5{,}050$.

Now find the sum of all the digits in the integers from 1 through 1,000,000,000. That's all the *digits* in all the numbers, not all the numbers themselves.

65. A SOCCER FAN'S NIGHTMARE

A soccer fan, upset by the defeat of his favorite team, slept restlessly. In his dream a goalkeeper was practicing in a large unfurnished room, tossing a soccer ball against a wall, then catching it.

But the goalkeeper grew smaller and smaller and then changed into a ping-pong ball while the soccer ball swelled up into a huge cast-iron ball. The iron ball circled around madly, trying to crush the ping-pong ball which darted desperately about.

Could the ping-pong ball find safety without leaving the floor?

Using Fractions and Decimals

To solve the following problems you must know how to use fractions and decimals.

If you have not studied fractions and decimals, skip this section and go on to Chapter II.

66. MY WATCH

As I traveled up and down our great and glorious country, I found myself in a place where the temperature goes up sharply in the day and down at night. This had an effect on my watch. I noticed it was 1/2 minute fast at nightfall, but at dawn it had lost 1/3 minute, making it only 1/6 minute fast.

One morning—May 1—my watch showed the right time. By what date was it 5 minutes fast?

67. STAIRS

A house has 6 stories, each the same height. How many times as long is the ascent to the sixth floor as the ascent to the third?

68. A DIGITAL PUZZLE

What arithmetic symbol can we place between 2 and 3 to make a number greater than 2 but less than 3?

69. INTERESTING FRACTIONS

If to the numerator and denominator of the fraction 1/3 you add its denominator, 3, the fraction will double.

Find a fraction which will triple when its denominator is added to its numerator and to its denominator; find one that will quadruple.

70. WHAT IS IT?

A half is a third of it. What is it?

71. THE SCHOOLBOY'S ROUTE

Each morning Boris walks to school. At one-fourth of the way he passes the machine and tractor station; at one-third of the way, the railroad station. At the machine and tractor station its clock shows 7:30, and at the railroad station its clock shows 7:35.

When does Boris leave his house, when does he reach school?

72. AT THE STADIUM

Twelve flags stand equidistant along the track at the stadium. The runners start at the first flag.

A runner reaches the eighth flag 8 seconds after he starts. If he runs at an even speed, how many seconds does he need altogether to reach the twelfth flag?

73. WOULD HE HAVE SAVED TIME?

Our man Ostap was going home from Kiev. He rode halfway—fifteen times as fast as he goes on foot. The second half he went by ox team. He can walk twice as fast as that.

Would he have saved time if he had gone all the way on foot? How much?

74. THE ALARM CLOCK

An alarm clock runs 4 minutes slow every hour. It was set right 3½ hours ago. Now another clock, which is correct, shows noon.

In how many minutes, to the nearest minute, will the alarm clock show noon?

75. LARGE SEGMENTS INSTEAD OF SMALL

In the Soviet machine industry a marker is a man who draws lines on a metal blank. The blank is cut along the lines to produce the desired shape.

A marker was asked to distribute 7 equal-sized sheets of metal among 12 workers, each worker to get the same amount of metal. He could not use the simple solution of dividing each sheet into 12 equal parts, for this would result in too many tiny pieces. What was he to do?

He thought awhile and found a more convenient method.

Later, he easily divided 5 sheets for 6 workers, 13 for 12, 13 for 36, 26 for 21, and so on.

What was his method?

76. A CAKE OF SOAP

If you place 1 cake of soap on a pan of a scale and 3/4 cake of soap and a 3/4-pound weight on the other, the pans balance.

How much does a cake of soap weigh?

77. ARITHMETICAL NUTS TO CRACK

(A) Use two digits to make the smallest possible positive integer.

(B) Five 3s can express 37:

$$37 = 33 + 3 + 3/3.$$

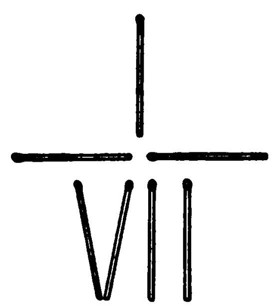
Find another way to do it.

(C) Use six identical digits to make 100. (Several solutions are possible.)

(D) Use five 4s to make 55.

(E) Use four 9s to make 20.

(F) Seven matches are shown that represent 1/7. Can you get a fraction that equals 1/3 without removing or adding any matches?



(G) Express 20 with plus signs and 1, 3, 5, and 7, using each digit three times.

(H) The sum of two numbers formed with plus signs and the digits 1, 3, 5, 7, and 9 equals the sum of two numbers formed with plus signs and the digits 2, 4, 6, and 8. Find these numbers, using each digit only once, and not using improper fractions.

(I) Name two numbers that have the same product and difference.

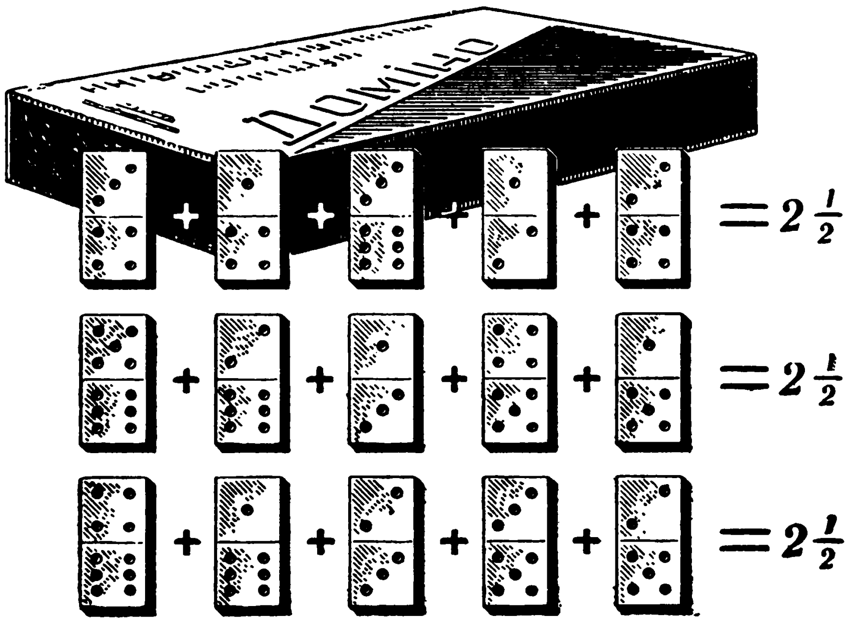
Such pairs are uncountably many. How are they formed?

(J) From the digits 0 through 9, each used once, form two equal fractions whose sum equals 1. (Several solutions are possible.)

(K) Using 0 through 9 once each, form two numbers—each an integer with a proper fraction—that add to 100. (Several solutions are possible.)

78. DOMINO FRACTIONS

From a box of dominoes remove the doubles (tiles with the same number at both ends), and the tiles that contain a blank. The remaining 15 tiles, regarded as fractions, are shown in three rows such that the sum of each row is $2\frac{1}{2}$.



Arrange the 15 tiles in three rows of 5 tiles each so that the sum of the fractions in each row is 10. (You can use improper fractions, such as 4/3, 6/1, 3/2.)

79. MISHA'S KITTENS

Every time young Misha sees a stray kitten he picks up the animal and brings it home. He is always raising several kittens, but he won't tell you how many because he is afraid you may laugh at him.

Someone will ask: "How many kittens do you have now?"

"Not many," he answers. "Three-quarters of their number plus three-quarters of a kitten."

His pals think he is joking. But he is really posing a problem—an easy one.

80. AVERAGE SPEED

A horse travels half his route, with no load, at 12 miles per hour. The rest of the way a load slows him to 4 miles per hour.

What is his average speed?

81. THE SLEEPING PASSENGER

A passenger fell asleep on a train halfway to his destination. He slept till he had half as far to go as he went while he slept. How much of the whole trip was he sleeping?

82. HOW LONG IS THE TRAIN?

A train moving 45 miles per hour meets and is passed by a train moving 36 miles per hour. A passenger in the first train sees the second train take 6 seconds to pass him. How long is the second train?

83. A CYCLIST

After a cyclist has gone two-thirds of his route, he gets a puncture. Finishing on foot, he spends twice as long walking as he did riding.

How many times as fast does he ride as walk?

84. A CONTEST

Volody A. and Kostya B., students in a metal-trade school, are doing lathework. Their foreman-teacher assigns them to make batches of metal parts. They want to finish simultaneously and ahead of the deadline, but after a while Kostya has only done half of what Volodya has left to do, and this is half what Volodya has already done.

How much faster than Volodya does Kostya have to work so they finish at the same time?

85. WHO IS RIGHT?

Masha had to find the product of three numbers in order to calculate the volume of some soil.

She multiplied the first number by the second correctly and was about to multiply the result by the third number when she noticed that the second number had been written incorrectly. It was one-third larger than it should be.

To avoid recalculating, Masha decided it would be safe to merely lower the third number by one-third of itself—particularly since it equaled the second number.

"But you shouldn't do that," a girl friend said to Masha. "If you do, you will be wrong by 20 cubic yards."

"Why?" said Masha.

Why indeed? And what is the correct soil volume?

86. THREE SLICES OF TOAST

Mother makes tasty toast in a small pan. After toasting one side of a slice, she turns it over. Each side takes 30 seconds.

The pan can only hold two slices. How can she toast both sides of three slices in 1½ instead of 2 minutes?

II
Difficult Problems

87. BLACKSMITH KHECHO'S INGENUITY

Last summer, as we traveled through the Georgian Republic, we would make up all sorts of unusual stories. Seeing a relic of old times often inspired us.

One day we came across an old and isolated tower. One of us, a student mathematician, invented an amusing puzzle story:

"Some three hundred years ago a prince lived here, a man of ill heart and much pride. His daughter, who was ripe for marriage, was named Daridjan. He had promised her to a rich neighbor, but she had a different plan: she fell in love with a plain lad, the blacksmith Khecho. The lovers tried to run off to the mountains, but were caught.

"Angered, the prince decided to execute them both the next day. He had them locked up in this tower—a somber structure, unfinished and abandoned. A young girl, a servant who had helped the lovers in their unsuccessful flight, was locked up with them.

"Khecho, calmly looking around, climbed the steps to the tower's upper part and glanced out the window. He realized it would be impossible to jump out and survive. But he saw a rope, forgotten by the masons, hanging near the window. The rope was thrown over a rusty tackle fastened to the tower wall above the window. Empty baskets were tied to each end of the rope. These baskets had been used by the masons to lift bricks and lower rubble. Khecho knew that if one load was 10 pounds more than the other, the heavier basket would descend smoothly to the ground while the other rose to the window.

"Looking at the two girls, Khecho guessed Daridjan's weight at 100 pounds and the servant's at 80. He himself weighed nearly 180. In the tower he found 13 separated pieces of chain, each weighing 10 pounds. Now all three prisoners