Die hard puzzle

An interesting Math and Computer Science problem

A puzzle for you!

Suppose you have an unlimited amount of apple cider in a large tank. You want to measure 3 gallons of apple cider for a customer. You have only a 4-gallon and 5-gallon measuring device. How could you measure 3 gallons?

Another problem!

You are at the side of a river. You have a 3 liter jug and a 5 liter jug. The jugs do not have markings to allow measuring smaller quantities. How can you use the jugs to measure 4 liters of water?

Observations!

- Which jar is on the left?
- How many pours if the other way round?

Another!

Given two unmarked jugs, one which holds 7 liters, and another which holds 11 liters, an unlimited supply of water, and no need to conserve, how do you measure exactly 6 liters?

There are a 3-litre jug and a 7-litre jug. We want to use them to measure 5-litre of water.

Try this!

3 litre jar and 6 litre jar and measure 8 litres

Observations!

- Final measurement is always less than the total
- Divisible by greatest common divisor
- It is multiples (whole numbers) of poring into one jar and pouring out of another
- Pouring in is a positive and pouring out is negative and all fit into say, 4x+5y=3 , 4(2)+5(-1)=8-5=3
- 2 times of pouring into 4 litre jar and on time pouring out of 5 litre jar
- (4,0)->(0,4)->(4,4)->(3,5)->(3,0)

Math behind it!

- GCD/HCF of 3,6 is 3
- GCD/HCF of 24, 36 is 12

Euclid’s algorithm for finding GCD:

GCD(420,96). Not so obvious, right?

Repeatedly do (a,b) = (b, a mod b) till b is 0 and a gives the GCD

GCD(420,96)=GCD(96, (96*4+36)%96) = GCD(96,36)

GCD(36, 24)= GCD(24, 12)=GCD(12, 0)

Hence GCD is 12 (12*35, 12*8) = (420,96)

Generalize

First try without calculating the minimum value

- Fill the left jar
- Empty the right jar

Till we reach the required litres:

- Calculate the minimum of (left jar contents, what is remaining in the right jar)
- Subtract from left jar the value calculated and add the value to the right jar
- Check if solution reached
- If left jar is empty fill it
- If right jar is full, empty it

Diophantine equations

- All problems are of the form ax+by=c
- 4x+5y=3
- Solvable if 3 is divisible by the GCD(4,5) =1
- Not solvable 8 is not divisible by GCD(3,6)= 3
- All follow the extended euclidean algorithm -> ax+by= gcd(a,b)
- 4x+5y=1, Solve it and find answers for 4x+5y=3, say u,v
- Other solutions of the form u+bn*/gcd(a,b) and v-an/gcd(a,b)

Try this!

$51x+21y=18$

Enjoyed?