1 Gauss - Markov Therom Proof

$$Var(d'y) > Var(c'\hat{\beta})$$

$$Var(d'y) = Var(d'y - c'\hat{\beta} + c'\hat{\beta})$$
$$= Var(d'y - c'\hat{\beta}) + Var(c'\hat{\beta}) + 2Cov(d'y - c'\hat{\beta}, c'\hat{\beta})$$

$$Var(d'y - c'\hat{\beta}) = Var(d'y - l'y)$$
$$= Var((d' - l')y)$$
$$= Var((d - l)'y)$$
$$= (d - l)' Var(y)(d - l)$$
$$= (d - l)'(\sigma^2 I)(d - l)$$
$$= \sigma^2 (d - l)' I (d - l)$$
$$= \sigma^2 (d - l)'(d - l) > 0 \text{ by } (1)$$

$$Cov(d'y - c'\hat{\beta}, c'\hat{\beta}) = Cov(d'y - l_y, l'y)$$
$$= Cov((d - l)'y, l'y)$$
$$= (d - l)' Var(y) l$$
$$= \sigma^2 (d - l)' l$$
$$= \sigma^2 (d - l)' X [(X'X)^-]' c = 0 \text{ by } (2)$$

$$Var(d'y) = Var(d'y - c'\hat{\beta}) + Var(c'\hat{\beta})$$

$$Var(d'y) > Var(c'\hat{\beta}) \quad \text{- QED}$$

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