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How to Find the Greatest Common Factor (GCF)

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The *greatest common factor* (GCF) of a set of numbers is the largest number that is a factor of all those numbers. For example, the GCF of the numbers 4 and 6 is 2 because 2 is the greatest number that's a factor of both 4 and 6. Here you will learn two ways to find the GCF.

Method 1: Use a list of factors to find the GCF

This method for finding the GCF is quicker when you're dealing with smaller numbers. To find the GCF of a set of numbers, list all the factors of each number. The greatest factor appearing on every list is the GCF. For example, to find the GCF of 6 and 15, first list all the factors of each number.

Factors of 6: 1, 2, 3, 6

Factors of 15: 1, 3, 5, 15

Because 3 is the greatest factor that appears on both lists, 3 is the GCF of 6 and 15.

As another example, suppose you want to find the GCF of 9, 20, and 25. Start by listing the factors of each:

Factors of 9: 1, 3, 9

Factors of 20: 1, 2, 4, 5, 10, 20

Factors of 25: 1, 5, 25

In this case, the only factor that appears on all three lists is 1, so 1 is the GCF of 9, 20, and 25.

Method 2: Use prime factorization to find the GCF

You can use prime factorization to find the GCF of a set of numbers. This often works better for large numbers, where generating lists of all factors can be time-consuming.

Here's how to find the GCF of a set of numbers using prime factorization:

1. List the prime factors of each number.
2. Circle every common prime factor — that is, every prime factor that's a factor of every number in the set.
3. Multiply all the circled numbers.

   The result is the GCF.

For example, suppose you want to find the GCF of 28, 42, and 70. Step 1 says to list the prime factors of each number. Step 2 says to circle every prime factor that's common to all three numbers (as shown in the following figure).

$$28 = \boxed{2} \cdot 2 \cdot \boxed{7}$$
$$42 = \boxed{2} \cdot 3 \cdot \boxed{7}$$
$$70 = \boxed{2} \cdot 5 \cdot \boxed{7}$$

As you can see, the numbers 2 and 7 are common factors of all three numbers. Multiply these circled numbers together:

$2 \cdot 7 = 14$

Thus, the GCF of 28, 42, and 70 is 14.

What is Greatest Common Factor (GCF)?

The GCF (Greatest Common Factor) of two or more numbers is the greatest number among all the common factors of the given numbers. The GCF of two natural numbers x and y is the largest possible number that divides both x and y. To calculate GCF, there are three common ways- division, multiplication, and prime factorization.

Example: Let us find the greatest common factor of 18 and 27.

Solution:

First, we list the factors of 18 and 27 and then we find out the common factors.

Factors of 18: 1, 2, 3, 6, 9, 18

Factors of 27: 1, 3, 9, 27

The common factors of 18 and 27 are 1,3, and 9. Among these numbers, 9 is the greatest (largest) number. Thus, the GCF of 18 and 27 is 9. This is written as: GCF(18 , 27) = 9.

A factor of a number is its divisor as well. Hence the greatest common factor is also called the Greatest Common Divisor (or) GCD**.** In the above example, the greatest common divisor (GCD) of 18 and 27 is 9 which can be written as:

GCD (18 , 27) = 9.

How to Find Greatest Common Factor(GCF)?

Following are 3 methods for finding the greatest common factor of two numbers.

- Listing Out Common Factors
- Prime Factorization
- Division Method

GCF by Listing Out the Common Factors

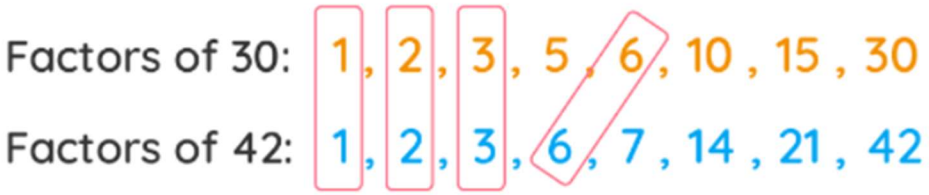
In this method, common factors of both the numbers can be listed, it then becomes easy to check for the common factors. By marking the common factors, we can choose the greatest one amongst all of them. Let's look at the example given below:

Example: What is the GCF of 30 and 42?

Solution:

- Step 1 - List out the factors of each number.
- Step 2 - Mark all the common factors.
- Step 3 - 6 is the common factor and the greatest one.

GCF by Listing Common Factors



Factors of 30 - 1, 2, 3, 5, 6, 10, 15, 30.

Factors of 42 - 1, 2, 3, 6, 7, 14, 21, 42.

Therefore, GCF of 30 and 42 = 6.

Finding the greatest common factor by listing factors may be difficult if the numbers are bigger. In such cases, we use the prime factorization and division methods for finding GCF.

Using Prime Factorization to Find GCF

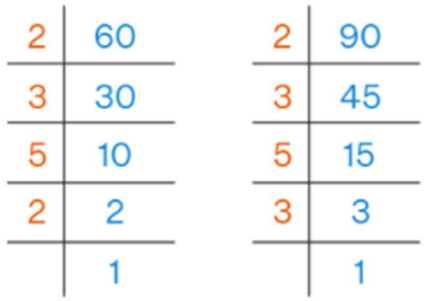
Prime factorization is a way of expressing a number as a product of its prime factors, starting from the smallest prime factor of that number. Let's look at the example given below:

Example: What is the GCF of 60 and 90?

Solution:

- Step 1 - Represent the numbers in the prime factored form.
- Step 2 - GCF is the product of the factors that are common to each of the given numbers.

GCF by Prime Factorization



2	60
3	30
5	10
2	2
	1

2	90
3	45
5	15
3	3
	1

$60 = 2 \times 3 \times 5 \times 2$

$90 = 2 \times 3 \times 5 \times 3$

GCF of 60 and 90 $= 2 \times 3 \times 5 = 30$

Thus, GCF (60,90) = $2^1 \times 3^1 \times 5^1 = 30$.

Therefore, GCF of 60 and 90 = 30

Using Division Method to Find GCF

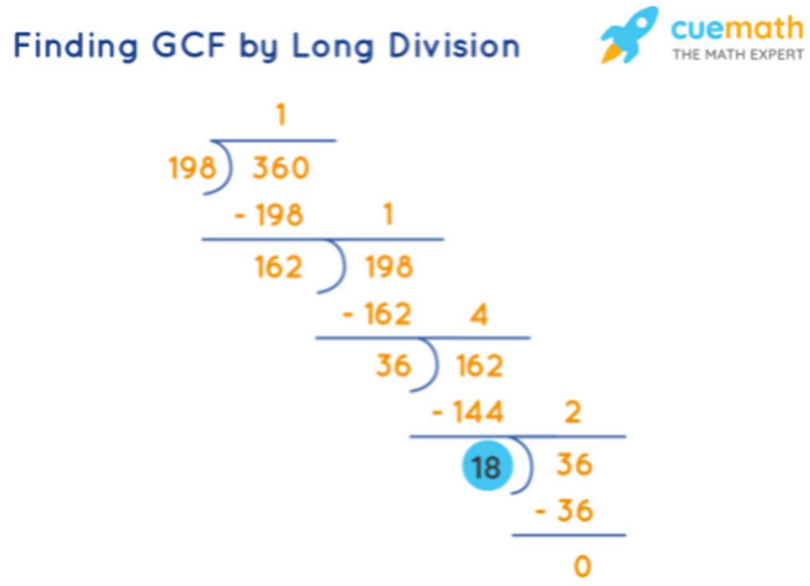
The division is a method of grouping objects in equal groups, whereas for large numbers we follow long division, which breaks down a division problem into a series of easier steps. The greatest common factor (GCF) of a set of whole numbers is the largest positive integer that divides evenly into all the given numbers, without leaving any remainder. Let's look at the example given below:

Example: Find the GCF of 198 and 360 using the division method.

Solution:

Among the given two numbers, 360 is the larger number and 198 is the smaller number.

- Step 1 - Divide the larger number by the smaller number using long division.
- Step 2 - If the remainder is 0, then the divisor is the GCF. If the remainder is not 0, then make the remainder of the above step as the divisor and the divisor of the above step as the dividend and perform long division again.
- Step 3 - If the remainder is 0, then the divisor of the last division is the GCF. If the remainder is not 0, then we have to repeat step 2 until we get the remainder 0.



Therefore, the GCF of the given two numbers is the divisor of the last division. In this case, the divisor of the last division is 18. Therefore, the GCF of 198 and 360 is 18.

Greatest Common Factor of Multiple Numbers

We have discussed finding the GCF of two numbers. Now, what if there are more than two numbers. For multiple numbers, listing out common factors becomes difficult. Thus, we can use either of the methods: Prime Factorization or Long Division.

1. To find the GCF of three numbers

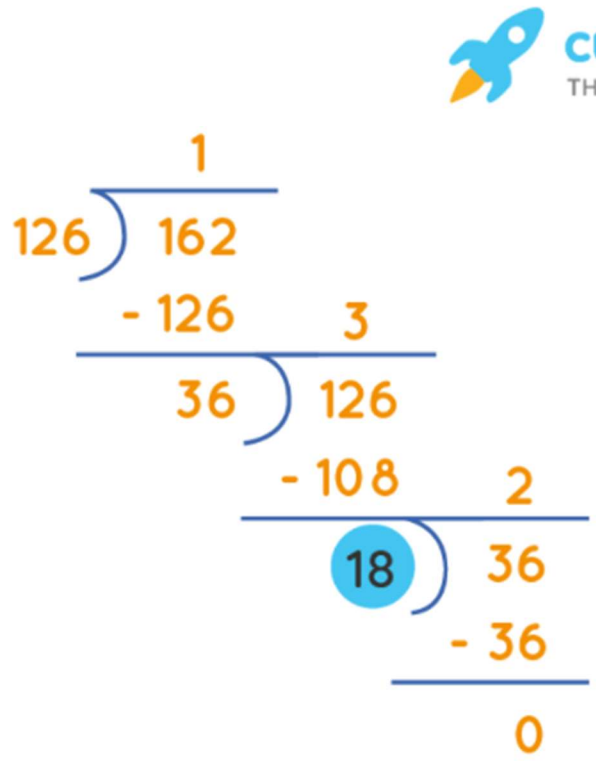
For finding the GCF of three numbers, we can use long division as well as prime factorization. In order to find the GCF by long division, the following steps are to be followed:

- First, we will find the GCF of two of the numbers.
- Next, we will find the GCF of the third number and the GCF of the first two numbers.

Example: Find the GCF of 126, 162, and 180.

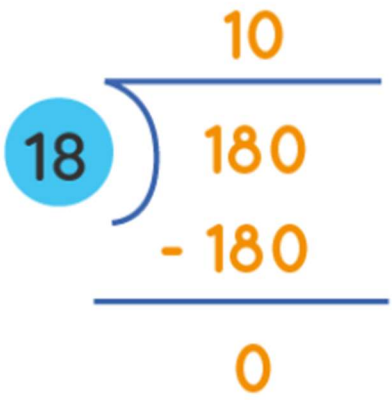
Solution:

First, we will find the GCF of the two numbers 126 and 162.



Thus, GCF of 126 and 162 = 18 .....(1)

Next, we will find the GCF of the third number, which is 180, and the above GCF18



Thus, GCF of 180 and 18 =18 .....(2)

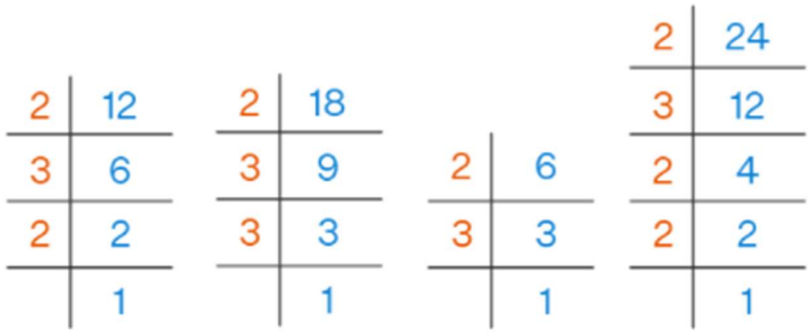
From (1) and (2), GCF(198,360) = 18. Therefore, GCF of 198 and 360 = 18

2. To find the GCF of four numbers

For finding the GCF of four numbers, we can use prime factorization as well as the division method. In the prime factorization method, we just take prime factors of the given numbers, specifically prime factors that divide all the given numbers. Let's have a look at the example given below, where prime factorization has been used to find the GCF of four numbers.

Example: Find the GCF of 12, 18, 6, and 24.

GCF of 12, 18, 6 and 24



$12 = 2 \times 3 \times 2$

$18 = 2 \times 3 \times 3$

$6 = 2 \times 3$

$24 = 2 \times 3 \times 2 \times 2$

GCF of 12, 18, 6 and 24 = $3 \times 2 = 6$

The prime factor which divides 12, 18, 6 and 24 are 3 and 2. Thus, GCF (12, 18, 6 and 24) = $2 \times 3 = 6$.

Therefore, GCF of 12, 18, 6 and 24 = 6.

Difference between GCF and LCM

The GCF or the greatest common factor of two or more numbers is the greatest factor among all the common factors of the given numbers, whereas the LCM or the least common multiple of two or more numbers is the smallest number among all common multiples of the given numbers. The following table shows the difference between GCF and LCM:

Greatest Common Factor(GCF)	Least Common Multiple(LCM)
The GCF of two natural numbers a and b is the greatest natural number x, which is a factor of both a and b.	The LCM of two natural numbers a and b is the smallest number y, which is a multiple of both a and b.
In the intersection of the sets of common factors, it is the greatest value.	In the intersection of the sets of multiple, it is the minimum value.
GCF(a, b) = x	LCM(a, b) = y