Reciprocals

Think about it this way: The reciprocal of a fraction is what you get when you turn the fraction upside down. We first saw the reciprocal when we learned about dividing by fractions, because that fraction division process required us to multiply by the reciprocal. In other words, what you get when you switch its numerator with its denominator. So the reciprocal of

$$\frac{3}{4}$$

is

$$\frac{4}{3}$$

Now there are a couple of situations with reciprocals that we should clarify. The first one is that integers have a reciprocal. We have to remember that a number like 2 can be written in fraction form as

$$\frac{2}{1}$$

Because when we divide by 1, it doesn't change the value at all. So $2/1$ is the same as just 2. But then we can take the reciprocal of 2, or $2/1$, and we see that it's

$$\frac{1}{2}$$

The second thing to know is that 0 doesn't have a reciprocal. Because 0 is the same as $0/1$, and the reciprocal of $0/1$ is $1/0$. But in $1/0$, we have a 0 in the denominator, which is undefined. So we can't take the reciprocal of 0.

The third thing we want to say about reciprocals is that whenever you multiply a fraction by its reciprocal, you'll always get 1 as the result. In other words, since $3/4$ and $4/3$ are reciprocals of one another,

$$\frac{3}{4} \times \frac{4}{3} = \frac{3 \times 4}{4 \times 3} = \frac{12}{12} = 1$$

Or, since $2/1$ and $1/2$ are reciprocals of one another,

$$\frac{2}{1} \times \frac{1}{2} = \frac{2 \times 1}{1 \times 2} = \frac{2}{2} = 1$$

Last, we need to know that if we're taking the reciprocal of a fraction that has at least one negative sign associated with it, then we need to include the negative sign(s) in the reciprocal as well. That way, when we multiply the original fraction by its reciprocal, all the negative signs will cancel and the result will be positive 1.

Example

Find the reciprocal, and then double-check your result by making sure that the product of the fraction and the reciprocal you found for it is 1.

$$-\frac{2}{9}$$

The reciprocal of $2/9$ is $9/2$, because that's what we get when we flip the fraction upside down. Because the original fraction's own sign is negative, we include that negative sign in the reciprocal as well. So the reciprocal of $-(2/9)$ is $-(9/2)$.

Let's check to make sure we get positive 1 when we multiply the original fraction by its reciprocal.

$$-\frac{2}{9} \times \left(-\frac{9}{2} \right)$$

The negative signs cancel and go away. Then, as always, we multiply the numerators and the denominators separately.

$$\frac{2 \times 9}{9 \times 2}$$

$$\frac{18}{18}$$

$$1$$

Since we get positive 1, we know that we found the correct reciprocal.