

Collaboration policy

Programs: Do not use someone else's code unless specifically authorized

Exceptions

- Code from course materials OK [cite source]
- Coding with partner OK **after** first assignment [stay tuned]

Where to get help

- Email (but no code in email)
- Office hours
- Lab TAs in Friend 008/009
- Bounce ideas (but not code) off classmates

Note: Programming in groups except as above is a serious violation.

Exercises: Write up your own solutions (no copying)

- working with classmates is encouraged
- checking solutions is OK

Stacks and Queues

- ▶ stacks
- ▶ dynamic resizing
- ▶ queues
- ▶ generics
- ▶ applications

Stacks and Queues

Fundamental data types.

- Values: sets of objects
- Operations: **insert**, **remove**, **test if empty**.
- Intent is clear when we insert.
- Which item do we remove?

Stack.

- Remove the item **most recently added**.
- Analogy: cafeteria trays, Web surfing.

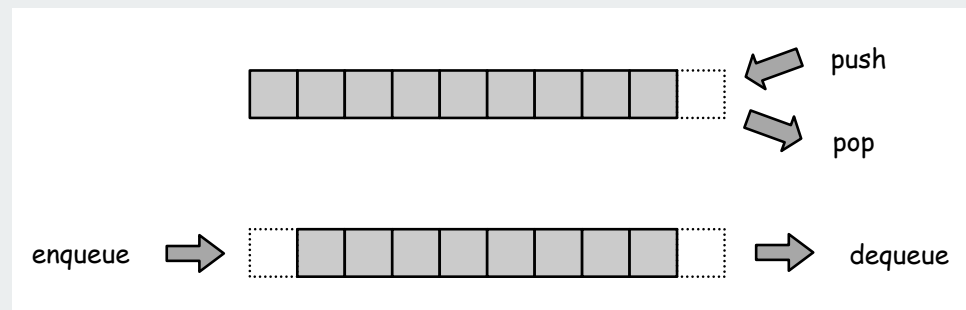
LIFO = "last in first out"



Queue.

- Remove the item **least recently added**.
- Analogy: Registrar's line.

FIFO = "first in first out"



Client, Implementation, Interface

Separate interface and implementation so as to:

- Build layers of abstraction.
- Reuse software.
- Ex: stack, queue, symbol table.

Interface: description of data type, basic operations.

Client: program using operations defined in interface.

Implementation: actual code implementing operations.

Client, Implementation, Interface

Benefits.

- Client can't know details of implementation \Rightarrow client has many implementation from which to choose.
- Implementation can't know details of client needs \Rightarrow many clients can re-use the same implementation.
- **Design:** creates modular, re-usable libraries.
- **Performance:** use optimized implementation where it matters.

Interface: description of data type, basic operations.

Client: program using operations defined in interface.

Implementation: actual code implementing operations.

▶ stacks

- ▶ dynamic resizing
- ▶ queues
- ▶ generics
- ▶ applications

Stacks

Stack operations.

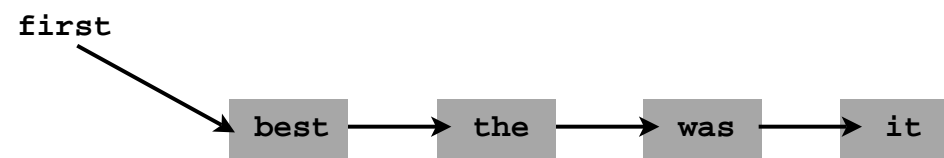
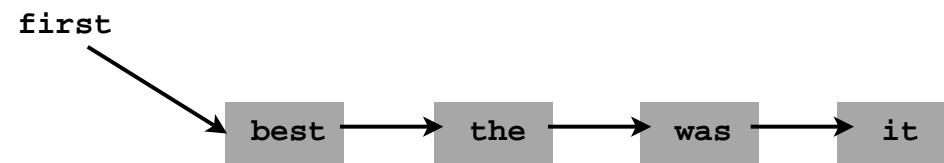
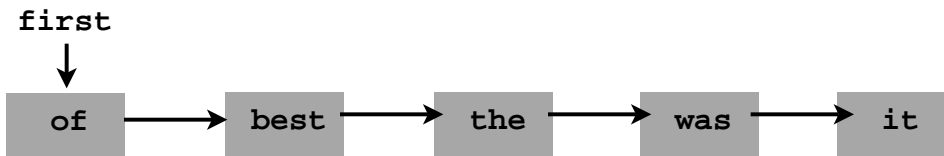
- `push()` **Insert** a new item onto stack.
- `pop()` **Remove** and return the item most recently added.
- `isEmpty()` Is the stack empty?



```
public static void main(String[] args)
{
    StackOfStrings stack = new StackOfStrings();
    while(!StdIn.isEmpty())
    {
        String s = StdIn.readString();
        stack.push(s);
    }
    while(!stack.isEmpty())
    {
        String s = stack.pop();
        StdOut.println(s);
    }
}
```

a sample stack client

Stack pop: Linked-list implementation

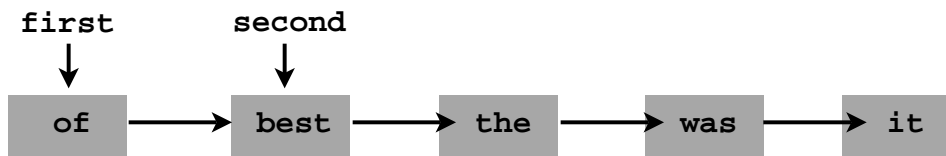
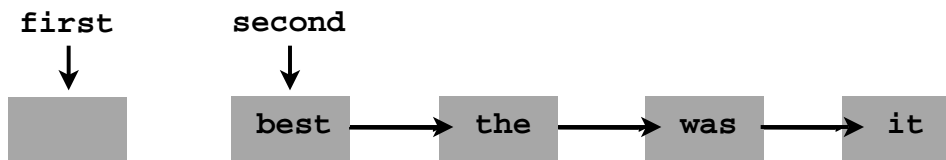
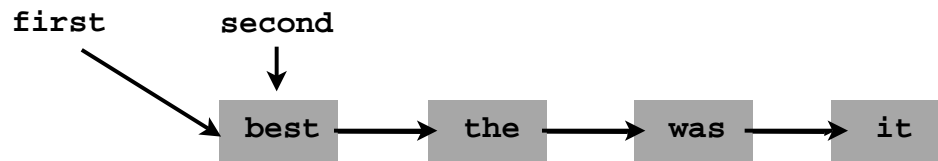
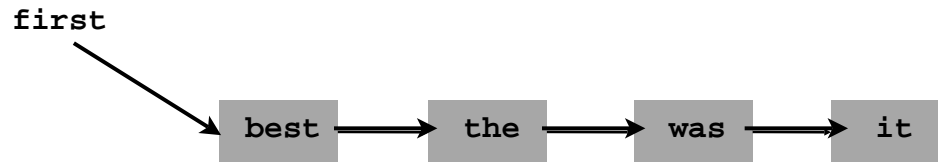


```
item = first.item;
```

```
first = first.next;
```

```
return item;
```


Stack push: Linked-list implementation



```
second = first;
```

```
first = new Node();
```

```
first.item = item;  
first.next = second;
```

Stack: Linked-list implementation

```
public class StackOfStrings
{
    private Node first = null;

    private class Node
    {
        String item;
        Node next;
    }

    public boolean isEmpty()
    { return first == null; }

    public void push(String item)
    {
        Node second = first;
        first = new Node();
        first.item = item;
        first.next = second;
    }

    public String pop()
    {
        String item = first.item;
        first = first.next;
        return item;
    }
}
```

← "inner class"

Error conditions?

Example: pop() an empty stack

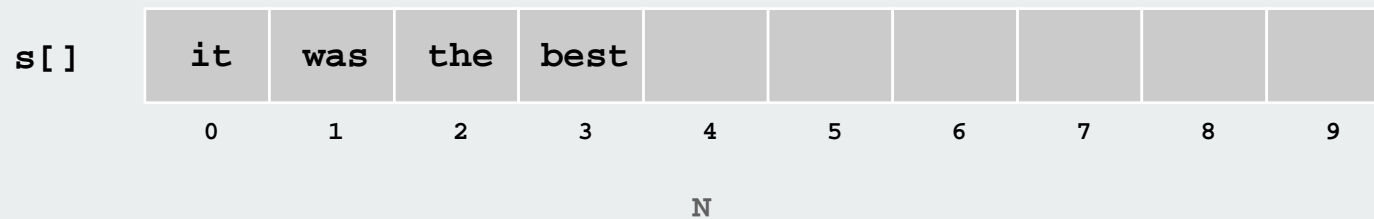
COS 217: bulletproof the code

COS 226: first find the code we want to use

Stack: Array implementation

Array implementation of a stack.

- Use array `s[]` to store `N` items on stack.
- `push()` add new item at `s[N]`.
- `pop()` remove item from `s[N-1]`.



Stack: Array implementation

```
public class StackOfStrings
{
    private String[] s;
    private int N = 0;

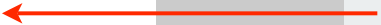
    public StringStack(int capacity)
    { s = new String[capacity]; }

    public boolean isEmpty()
    { return N == 0; }

    public void push(String item)
    { s[N++] = item; }

    public String pop()
    {
        String item = s[N-1];
        s[N-1] = null;
        N--;
        return item;
    }
}
```

avoid **loitering**
(garbage collector only reclaims memory
if no outstanding references)



- ▶ stacks
- ▶ **dynamic resizing**
- ▶ queues
- ▶ generics
- ▶ applications

Stack array implementation: Dynamic resizing

Q. How to grow array when capacity reached?

Q. How to shrink array (else it stays big even when stack is small)?

First try:

- **push()**: increase size of `s[]` by 1
- **pop()** : decrease size of `s[]` by 1

Too expensive

- Need to copy all of the elements to a new array.
- Inserting N elements: time proportional to $1 + 2 + \dots + N \approx N^2/2$.

↑
infeasible for large N

Need to **guarantee** that array resizing happens **infrequently**

Stack array implementation: Dynamic resizing

Q. How to grow array?

A. Use **repeated doubling**:

if array is full, create a new array of twice the size, and copy items

no-argument
constructor

```
public StackOfStrings()  
{ this(8); }  
  
public void push(String item)  
{  
    if (N >= s.length) resize();  
    s[N++] = item;  
}
```

create new array
copy items to it

```
private void resize(int max)  
{  
    String[] dup = new String[max];  
    for (int i = 0; i < N; i++)  
        dup[i] = s[i];  
    s = dup;  
}
```

Consequence. Inserting N items takes time proportional to N (not N^2).

$$\begin{array}{c} \uparrow \\ 8 + 16 + \dots + N/4 + N/2 + N \approx 2N \end{array}$$

Stack array implementation: Dynamic resizing

Q. How (and when) to shrink array?

How: create a new array of **half** the size, and copy items.

When (first try): array is half full?

No, causes **thrashing**

(push-pop-push-pop... sequence: time proportional to N for each op)

When (solution): array is 1/4 full (then new array is half full).

```
public String pop(String item)
{
    String item = s[--N];
    sa[N] = null;
    if (N == s.length/4)
        resize(s.length/2);
    return item;
}
```

Not `a.length/2`
to avoid thrashing

Consequences.

- any sequence of N ops takes time proportional to N
- array is always between 25% and 100% full

Stack Implementations: Array vs. Linked List

Stack implementation tradeoffs. Can implement with either array or linked list, and client can use interchangeably. Which is better?

Array.

- Most operations take constant time.
- Expensive doubling operation every once in a while.
- Any sequence of N operations (starting from empty stack) takes time proportional to N .

← "amortized" bound

Linked list.

- Grows and shrinks gracefully.
- Every operation takes constant time.
- Every operation uses extra space and time to deal with references.

Bottom line: tossup for stacks

but differences are significant when other operations are added

Stack implementations: Array vs. Linked list

Which implementation is more convenient?

array?

linked list?

return count of elements in stack

remove the kth most recently added

sample a random element

- ▶ stacks
- ▶ dynamic resizing
- ▶ **queues**
- ▶ generics
- ▶ applications

Queues

Queue operations.

- `enqueue()` Insert a new item onto queue.
- `dequeue()` Delete and return the item least recently added.
- `isEmpty()` Is the queue empty?

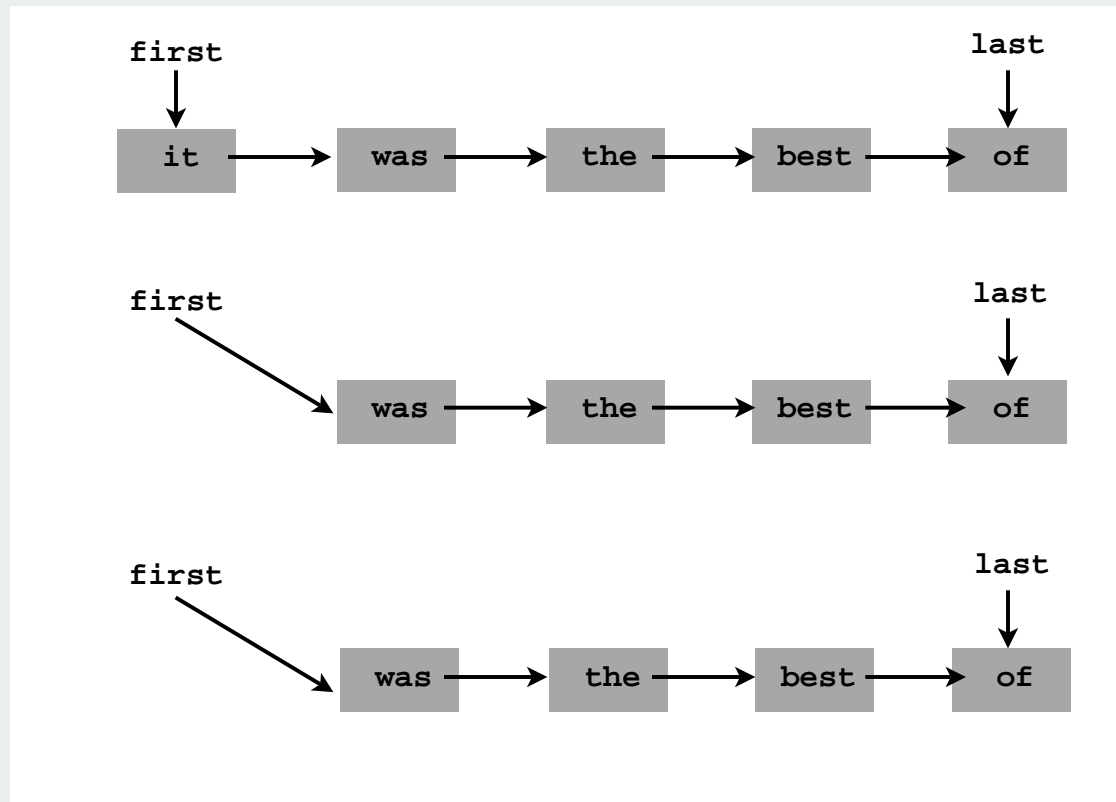
```
public static void main(String[] args)
{
    QueueOfStrings q = new QueueOfStrings();
    q.enqueue("Vertigo");
    q.enqueue("Just Lose It");
    q.enqueue("Pieces of Me");
    q.enqueue("Pieces of Me");
    System.out.println(q.dequeue());
    q.enqueue("Drop It Like It's Hot");

    while(!q.isEmpty())

        System.out.println(q.dequeue());
}
```



Deque: Linked List Implementation



```
item = first.item;
```

```
first = first.next;
```

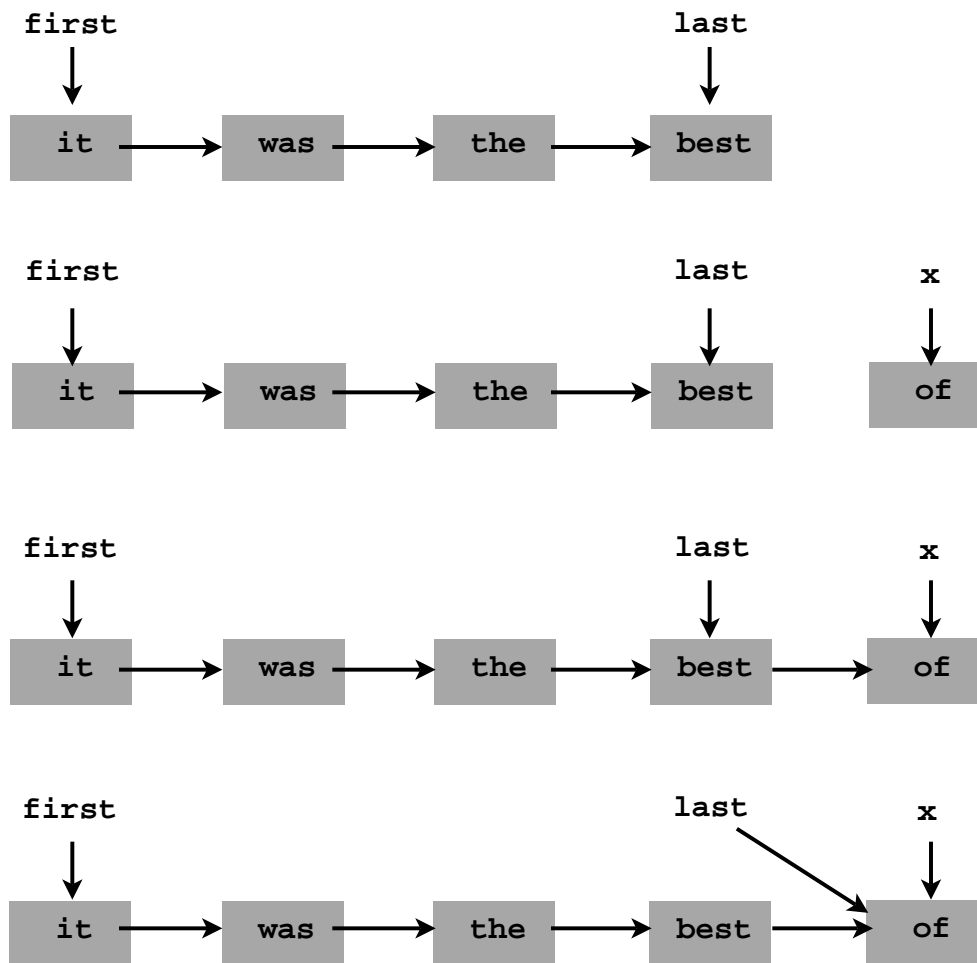
```
return item;
```

Aside:

deque (pronounced "DQ") means "remove from a queue"

deque (pronounced "deck") is a **data structure** (see PA 1)

Enqueue: Linked List Implementation



```
x = new Node();  
x.item = item;  
x.next = null;
```

```
last.next = x;
```

```
last = x;
```

Queue: Linked List Implementation

```
public class QueueOfStrings
{
    private Node first;
    private Node last;

    private class Node
    { String item; Node next; }

    public boolean isEmpty()
    { return first == null; }

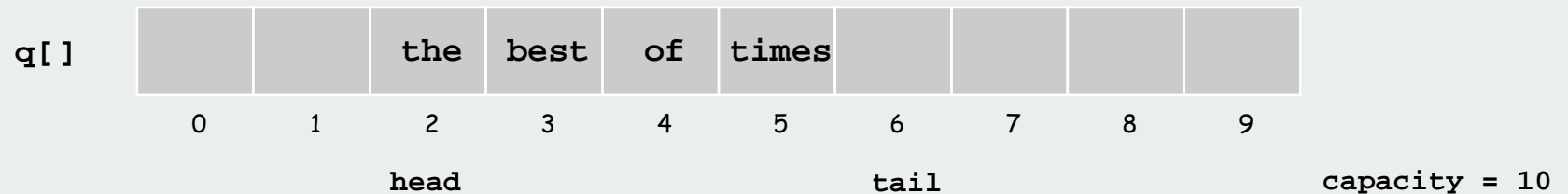
    public void enqueue(String item)
    {
        Node x = new Node();
        x.item = item;
        x.next = null;
        if (isEmpty()) { first = x; last = x; }
        else            { last.next = x; last = x; }
    }

    public String dequeue()
    {
        String item = first.item;
        first = first.next;
        return item;
    }
}
```

Queue: Array implementation

Array implementation of a queue.

- Use array `q[]` to store items on queue.
- `enqueue()`: add new object at `q[tail]`.
- `dequeue()`: remove object from `q[head]`.
- Update `head` and `tail` modulo the capacity.



[details: good exercise or exam question]

- ▶ stacks
- ▶ dynamic resizing
- ▶ queues
- ▶ **generics**
- ▶ applications

Generics (parameterized data types)

We implemented: `StackOfStrings`, `QueueOfStrings`.

We also want: `StackOfURLs`, `QueueOfCustomers`, etc?

Attempt 1. Implement a separate stack class for each type.

- Rewriting code is tedious and **error-prone**.
- Maintaining cut-and-pasted code is tedious and **error-prone**.

@#*\$! most reasonable approach until Java 1.5 [hence, used in `AlgsJava`]

Stack of Objects

We implemented: `StackOfStrings`, `QueueOfStrings`.

We also want: `StackOfURLs`, `QueueOfCustomers`, etc?

Attempt 2. Implement a stack with items of type object.

- Casting is required in client.
- Casting is error-prone: **run-time error** if types mismatch.

```
Stack s = new Stack();  
Apple a = new Apple();  
Orange b = new Orange();  
s.push(a);  
s.push(b);  
a = (Apple) (s.pop()); ← run-time error
```

Generics

Generics. Parameterize stack by a single type.

- Avoid casting in both client and implementation.
- Discover type mismatch errors at **compile-time** instead of run-time.

```
Stack<Apple> s = new Stack<Apple>();  
Apple a = new Apple();  
Orange b = new Orange();  
s.push(a);  
s.push(b);           compile-time error  
a = s.pop();
```

parameter



no cast needed in client

Guiding principles.

- Welcome compile-time errors
- Avoid run-time errors

Why?

Generic Stack: Linked List Implementation

```
public class StackOfStrings
{
    private Node first = null;

    private class Node
    {
        String item;
        Node next;
    }

    public boolean isEmpty()
    { return first == null; }

    public void push(String item)
    {
        Node second = first;
        first = new Node();
        first.item = item;
        first.next = second;
    }

    public String pop()
    {
        String item = first.item;
        first = first.next;
        return item;
    }
}
```

```
public class Stack<Item>
{
    private Node first = null;

    private class Node
    {
        Item item;
        Node next;
    }

    public boolean isEmpty()
    { return first == null; }

    public void push(Item item)
    {
        Node second = first;
        first = new Node();
        first.item = item;
        first.next = second;
    }

    public Item pop()
    {
        Item item = first.item;
        first = first.next;
        return item;
    }
}
```

Generic type name



Generic stack: array implementation

The way it should be.

```
public class Stack<Item>
{
    private Item[] s;
    private int N = 0;

    public Stack(int cap)
    { s = new Item[cap]; }

    public boolean isEmpty()
    { return N == 0; }

    public void push(Item item)
    { s[N++] = item; }

    public String pop()
    {
        Item item = s[N-1];
        s[N-1] = null;
        N--;
        return item;
    }
}
```

```
public class StackOfStrings
{
    private String[] s;
    private int N = 0;

    public StackOfStrings(int cap)
    { s = new String[cap]; }

    public boolean isEmpty()
    { return N == 0; }

    public void push(String item)
    { s[N++] = item; }

    public String pop()
    {
        String item = s[N-1];
        s[N-1] = null;
        N--;
        return item;
    }
}
```

@#*\$! generic array creation not allowed in Java

Generic stack: array implementation

The way it is: an **ugly cast** in the implementation.

```
public class Stack<Item>
{
```

```
    private Item[] s;
    private int N = 0;
```

```
    public Stack(int cap)
    { s = (Item[]) new Object[cap]; }
```

← the ugly cast

```
    public boolean isEmpty()
    { return N == 0; }
```

```
    public void push(Item item)
    { s[N++] = item; }
```

```
    public String pop()
    {
        Item item = s[N-1];
        s[N-1] = null;
        N--;
        return item;
    }
```

```
}
```

Number of casts in good code: 0

Generic data types: autoboxing

Generic stack implementation is object-based.

What to do about primitive types?

Wrapper type.

- Each primitive type has a **wrapper** object type.
- Ex: Integer is wrapper type for int.

Autoboxing. Automatic cast between a primitive type and its wrapper.

Syntactic sugar. Behind-the-scenes casting.

```
Stack<Integer> s = new Stack<Integer>();  
s.push(17);           // s.push(new Integer(17));  
int a = s.pop();      // int a = ((int) s.pop()).intValue();
```

Bottom line: Client code can use generic stack for **any** type of data

- ▶ stacks
- ▶ dynamic resizing
- ▶ queues
- ▶ generics
- ▶ applications

Stack Applications

Real world applications.

- Parsing in a compiler.
- Java virtual machine.
- Undo in a word processor.
- Back button in a Web browser.
- PostScript language for printers.
- Implementing function calls in a compiler.

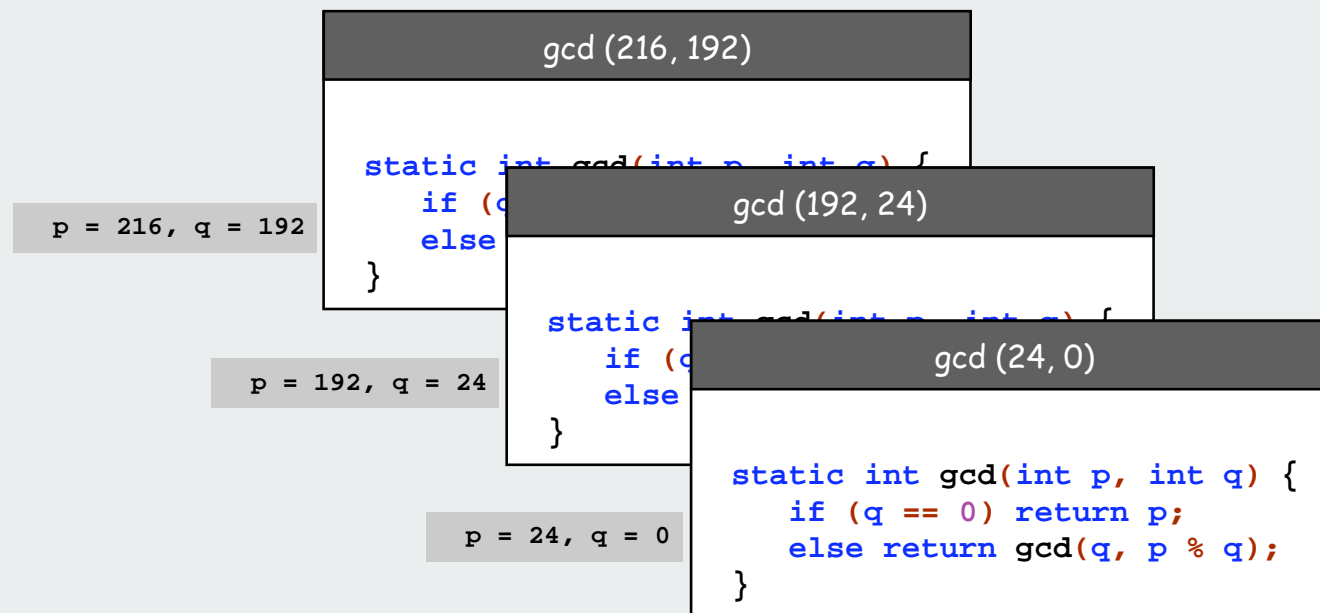
Function Calls

How a compiler implements functions.

- Function call: **push** local environment and return address.
- Return: **pop** return address and local environment.

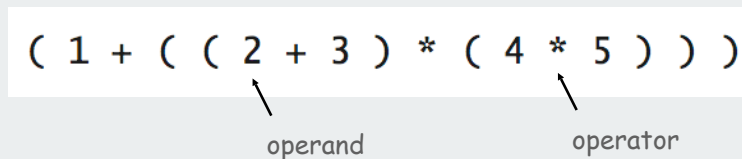
Recursive function. Function that calls itself.

Note. Can always use an explicit stack to remove recursion.



Arithmetic Expression Evaluation

Goal. Evaluate infix expressions.

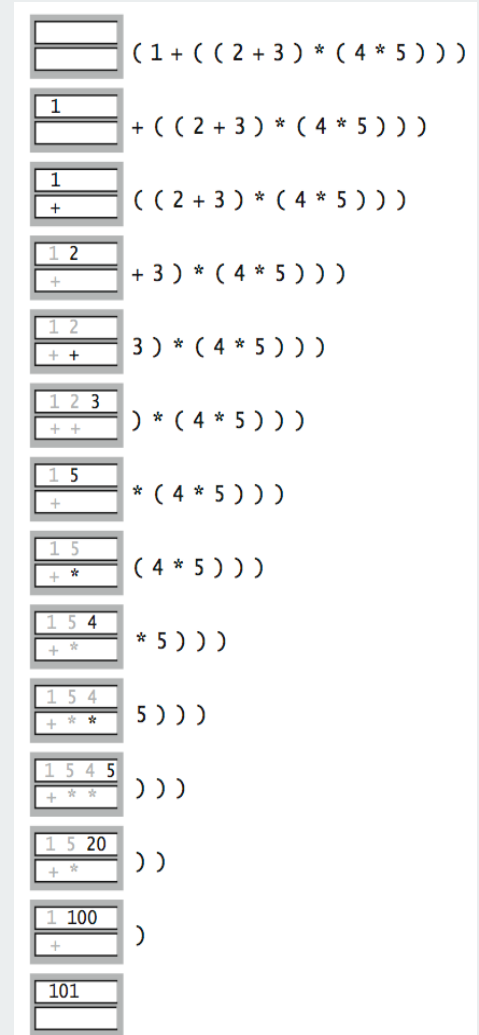


Two-stack algorithm. [E. W. Dijkstra]

- Value: push onto the value stack.
- Operator: push onto the operator stack.
- Left parens: ignore.
- Right parens: pop operator and two values; push the result of applying that operator to those values onto the operand stack.

Context. An interpreter!

value stack
operator stack



Arithmetic Expression Evaluation

```
public class Evaluate {
    public static void main(String[] args) {
        Stack<String> ops = new Stack<String>();
        Stack<Double> vals = new Stack<Double>();
        while (!StdIn.isEmpty()) {
            String s = StdIn.readString();
            if (s.equals("(")) ;
            else if (s.equals("+")) ops.push(s);
            else if (s.equals("*")) ops.push(s);
            else if (s.equals(")") {
                String op = ops.pop();
                if (op.equals("+")) vals.push(vals.pop() + vals.pop());
                else if (op.equals("*")) vals.push(vals.pop() * vals.pop());
            }
            else vals.push(Double.parseDouble(s));
        }
        StdOut.println(vals.pop());
    }
}
```

```
% java Evaluate
( 1 + ( ( 2 + 3 ) * ( 4 * 5 ) ) )
101.0
```

Note: Old books have two-pass algorithm because generics were not available!

Correctness

Why correct?

When algorithm encounters an operator surrounded by two values within parentheses, it leaves the result on the value stack.

$$(1 + ((2 + 3) * (4 * 5)))$$

as if the original input were:

$$(1 + (5 * (4 * 5)))$$

Repeating the argument:

$$(1 + (5 * 20))$$
$$(1 + 100)$$
$$101$$

Extensions. More ops, precedence order, associativity.

$$1 + (2 - 3 - 4) * 5 * \text{sqrt}(6 + 7)$$

Stack-based programming languages

Observation 1.

Remarkably, the 2-stack algorithm computes the same value if the operator occurs **after** the two values.

(1 ((2 3 +) (4 5 *) *) +)

Observation 2.

All of the parentheses are redundant!

1 2 3 + 4 5 * * +



Jan Lukasiewicz

Bottom line. Postfix or "reverse Polish" notation.

Applications. Postscript, Forth, calculators, Java virtual machine, ...

Stack-based programming languages: PostScript

Page description language

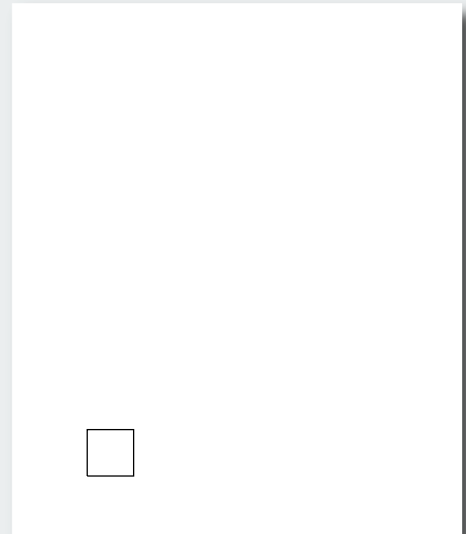
- explicit stack
- full computational model
- graphics engine

Basics

- `%!`: "I am a PostScript program"
- literal: "push me on the stack"
- function calls take args from stack
- turtle graphics built in

a PostScript program

```
%!  
72 72 moveto  
0 72 rlineto  
72 0 rlineto  
0 -72 rlineto  
-72 0 rlineto  
2 setlinewidth  
stroke
```



Stack-based programming languages: PostScript

Data types

- basic: integer, floating point, boolean, ...
- graphics: font, path,
- full set of built-in operators

Text and strings

- full font support
- **show** (display a string, using current font)
- **cvs** (convert anything to a string)

like System.out.print()

like toString()

```
%!  
/Helvetica-Bold findfont 16 scalefont setfont  
72 168 moveto  
(Square root of 2:) show  
72 144 moveto  
2 sqrt 10 string cvs show
```

Square root of 2:
1.4142

Stack-based programming languages: PostScript

Variables (and functions)

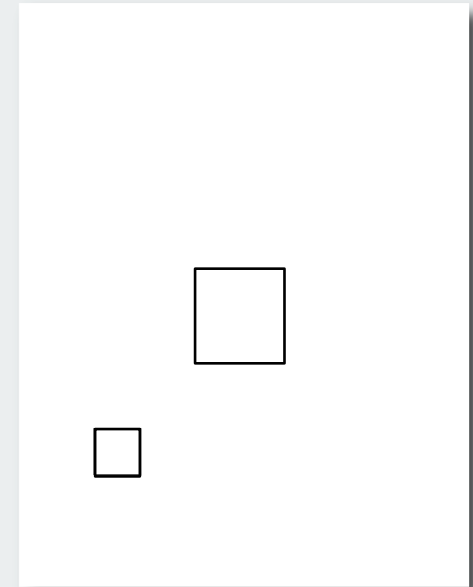
- identifiers start with /
- def operator associates id with value
- braces
- args on stack

function definition →

```
%!  
/box  
{  
  /sz exch def  
  0 sz rlineto  
  sz 0 rlineto  
  0 sz neg rlineto  
  sz neg 0 rlineto  
} def
```

function calls →

```
72 144 moveto  
72 box  
288 288 moveto  
144 box  
2 setlinewidth  
stroke
```



Stack-based programming languages: PostScript

for loop

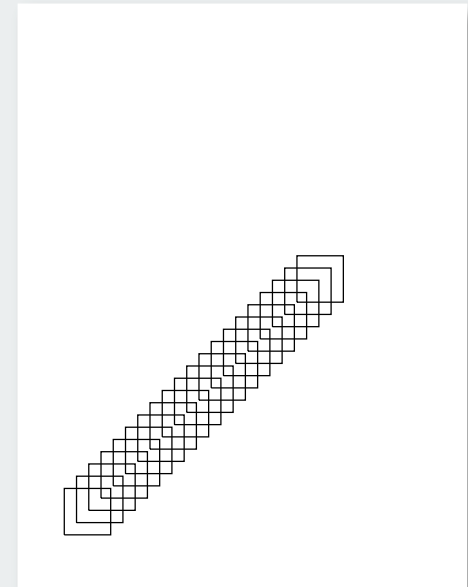
- "from, increment, to" on stack
- loop body in braces
- for operator

```
1 1 20  
{ 19 mul dup 2 add moveto 72 box }  
for
```

if-else

- boolean on stack
- alternatives in braces
- if operator

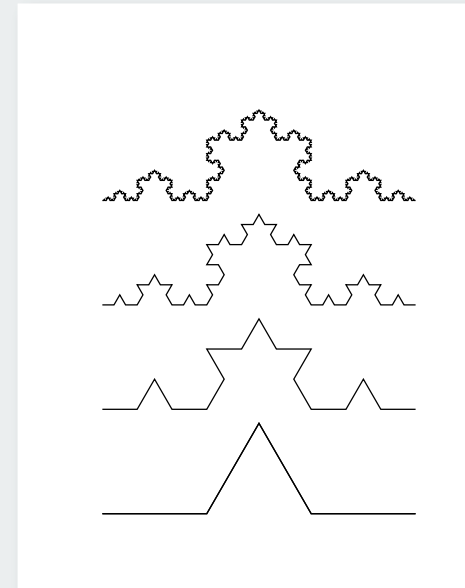
... (hundreds of operators)



Stack-based programming languages: PostScript

An application: **all figures in Algorithms in Java**

```
%!  
72 72 translate  
  
/kochR  
{  
  2 copy ge { dup 0 rlineto }  
  {  
    3 div  
    2 copy kochR 60 rotate  
    2 copy kochR -120 rotate  
    2 copy kochR 60 rotate  
    2 copy kochR  
  } ifelse  
  pop pop  
} def  
  
0 0 moveto 81 243 kochR  
0 81 moveto 27 243 kochR  
0 162 moveto 9 243 kochR  
0 243 moveto 1 243 kochR  
stroke
```



See page 218



Queue applications

Familiar applications.

- iTunes playlist.
- Data buffers (iPod, TiVo).
- Asynchronous data transfer (file IO, pipes, sockets).
- Dispensing requests on a shared resource (printer, processor).

Simulations of the real world.

- Traffic analysis.
- Waiting times of customers at call center.
- Determining number of cashiers to have at a supermarket.

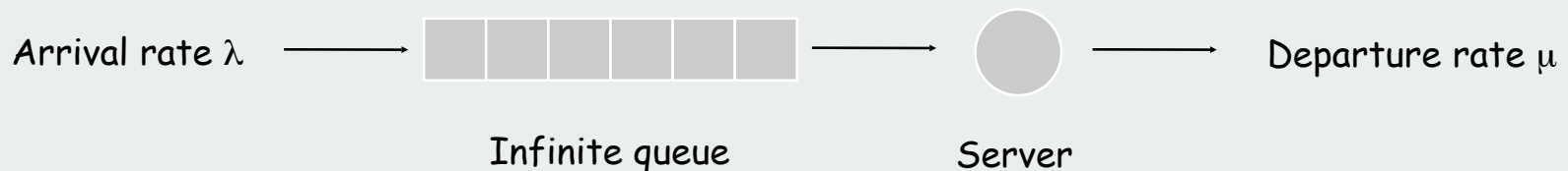
M/D/1 queuing model

M/D/1 queue.

- Customers are serviced at fixed rate of μ per minute.
- Customers arrive according to Poisson process at rate of λ per minute.

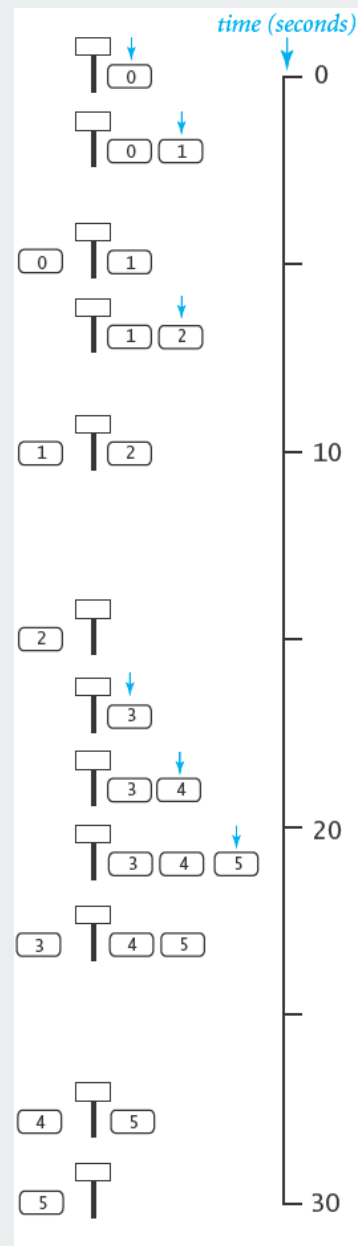
↙ inter-arrival time has exponential distribution

$$\Pr[X \leq x] = 1 - e^{-\lambda x}$$



- Q. What is average wait time W of a customer?
- Q. What is average number of customers L in system?

M/D/1 queuing model: example

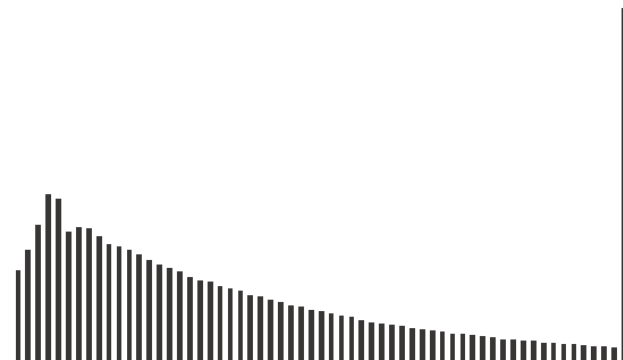
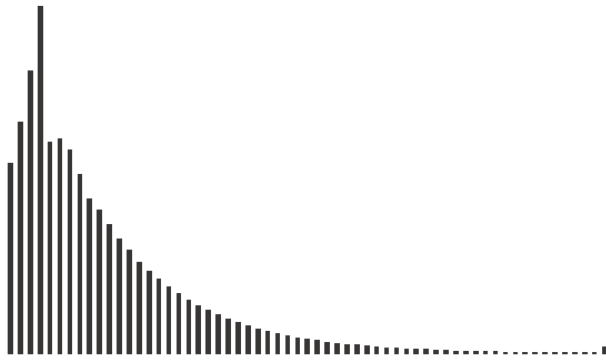


	arrival	departure	wait
0	0	5	5
1	2	10	8
2	7	15	8
3	17	23	6
4	19	28	9
5	21	30	9

M/D/1 queuing model: experiments and analysis

Observation.

As service rate μ approaches arrival rate λ , service goes to h^{**} .



Queueing theory (see ORFE 309).

$$W = \frac{\lambda}{2\mu(\mu - \lambda)} + \frac{1}{\mu}, \quad L = \lambda W$$

wait time W and queue length L approach infinity as service rate approaches arrival rate

M/D/1 queuing model: event-based simulation

```
public class MD1Queue
{
    public static void main(String[] args)
    {
        double lambda = Double.parseDouble(args[0]);    // arrival rate
        double mu      = Double.parseDouble(args[1]);    // service rate
        Histogram hist = new Histogram(60);
        Queue<Double> q = new Queue<Double>();
        double nextArrival = StdRandom.exp(lambda);
        double nextService = 1/mu;
        while (true)
        {
            while (nextArrival < nextService)
            {
                q.enqueue(nextArrival);
                nextArrival += StdRandom.exp(lambda);
            }
            double wait = nextService - q.dequeue();
            hist.addDataPoint(Math.min(60, (int) (wait)));
            if (!q.isEmpty())
                nextService = nextArrival + 1/mu;
            else
                nextService = nextService + 1/mu;
        }
    }
}
```