Rounding

When we're dealing with decimal numbers, rounding is an important tool that we need to know how to use. Let's take an extreme example to see why.

Given the decimal number

$$102.3740000000000000000000000000000000001$$

what we notice is that the 1 hanging way out there on the end makes only a tiny, tiny, tiny, tiny difference in the value of the number as a whole. So writing all of those extra 0's just for the sake of including that 1, doesn't make that much sense. It's much more practical to write the number as 102.374, because it saves so much time and effort not having to write all those extra digits, and 102.374 is so close to the actual number that it doesn't even really make a difference.

We round decimals (meaning we approximate them with fewer than the number of digits in their actual values) because it saves us a lot of time and lets us express really long numbers as shorter, simpler numbers.

When we round, we always round to a certain decimal place. This goes back to what we learned previously about place value. For example, we know that the second digit to the right of the decimal point is the hundredths place. This means that if we're asked to round to the hundredths place, we want the last digit in the number to be the digit in the hundredths place; we don't want to include any digits after that.

We do have to follow certain rules when we round. The rule you need to remember is:

> "If the next digit is less than 5, round the **previous digit down**; if it's 5 or greater, round the **previous digit up**."

To round a digit **down** means to leave it unchanged; to round a digit **up** means to increase it by one unit. For example, if we round down a 3, we leave it unchanged (at 3); if we round up a 3, we increase it to 4.

Example

Round to the nearest hundredth.

3.14159

Since we've been asked to round to the nearest hundredth, that means we're taking everything that comes after the hundredths place and rounding it into the hundredths place.

In 3.14159, the 4 is in the hundredths place, so we need to round everything that comes after that. The question we need to ask is, "What digit comes right after the 4?" Well, we have a 1 right after the 4 in the thousandths place. Since 1 is less than 5, that means we're rounding the 4 down, so the digit that was originally in the hundredths place (the 4) will be unchanged.

The digits 5 and 9 that come after the 1 that we used in deciding whether to round up or down don't matter. Since we're rounding to the nearest

hundredth, we need to consider only the digit that comes right after the hundredths place, which is the thousandths place.

So 3.14159, rounded to the nearest hundredth, is

3.14

Let's do another example. In this one we'll round up.

Example

Round to the nearest hundred.

$130,874.62$

Since we've been asked to round to the nearest hundred, that means we're taking everything that comes after the hundreds place and rounding it into the hundreds place.

In $130,874.62$, the 8 is in the hundreds place, so we need to round everything that comes after that. The question we need to ask is, "What digit comes right after the 8?" Well, we have a 7 right after the 8, and since 7 is greater than or equal to 5, that means we're rounding the 8 up, so the original digit in the hundreds place (the 8) needs to get bumped up to the next higher digit, 9.

The numbers 4, 6, and 2 that come after the 7 that we used in deciding whether to round up or down don't matter. Since we're rounding to the

nearest hundred, we need to consider only the digit that comes right after the hundreds place.

There's another aspect of rounding that we sometimes have to deal with. Whenever we're rounding to a digit that's left of the decimal point, we drop the decimal point and all the digits to the right of it. But we have to have some digit in each place between the decimal point and the place we're rounding to, so we put a 0 in each of those places. In this example, we have to put 0's in the ones and tens places.

So $130{,}874.62$, rounded to the nearest hundred, is

$$130{,}900$$

Now what if, in the last example, we'd had $130{,}974.62$ instead of $130{,}874.62$, and we wanted to round it to the nearest hundred? Well, we'd first look at the 7, and because 7 is greater than or equal to 5, we'd know that we need to round up the 9. But we can't round up the 9 to a 10 - that would be replacing one digit with two, which doesn't work.

What we do in the special case where we're rounding up a 9 is change the 9 to a 0 and increase the digit to the left of that 0 by one unit. So in rounding $130{,}974.62$ to the nearest hundred, we change the 9 to a 0 and increase the digit in the thousands place (the 0 that comes right after the 3) by one unit (we increase it from 0 to 1). Of course, we also have to drop the decimal point (and the digits to the right of it), and put 0's in the ones and tens places. Therefore, $130{,}974.62$ rounded to the nearest hundred, is

$$131{,}000$$