Simple sorting algorithms and their complexity

- Bubble sort
- Selection sort
- Insertion sort

Bubble sort

```
void  bubbleSort(int arr[]){
   int i;
   int j;
   int temp;
   for(i = arr.length-1; i > 0; i--){
      for(j = 0; j < i; j++){
         if(arr[j] > arr[j+1]){
            temp = arr[j];
            arr[j] = arr[j+1];
            arr[j+1] = temp;
         }//
      }// end inner loop
   }//end outer loop}// end bubble sort
```

swap adjacent elements, if in the wrong order

Complexity of bubble sort

- For an array of size n, in the worst case:
  1st passage through the inner loop: n-1 comparisons and n-1 swaps
- ...
- (n-1)st passage through the inner loop: one comparison and one swap.
- All together: c ((n-1) + (n-2) + ... + 1), where c is the time required to do one comparison, one swap, check the inner loop condition and increment j.
- We also spend constant time k declaring i,j,temp and initialising i. Outer loop is executed n-1 times, suppose the cost of checking the loop condition and decrementing i is c_1.

Complexity of bubble sort

c ((n-1) + (n-2) + ... + 1) + k + c_1(n-1)

(n-1) + (n-2) + ... + 1 = n(n-1)/2

so our function equals
c n*(n-1)/2 + k + c_1(n-1) = 1/2c (n^2-n) + c(n-1) + k
complexity O(n^2).

Complexity of bubble sort

Need to find n_0 and K, such that for all $n \geq n_0$,
1/2c (n^2-n) + c_1(n-1) + k ≤ K* n^2

1/2c n^2 – 1/2c n + c_1n – c_1+ k ≤
1/2c n^2 + c_1n + k ≤
c n^2 + c_1 n^2 + k n^2 (if $n \geq 1$)
Take K = c + c_1+ k and n_0 = 1.

Bubble sort of lists

- Bubble sort is just as efficient (or rather inefficient) on linked lists.
- We can easily bubble sort even a singly linked list.
- Assume we have a class Node with fields: element of type E and next of type Node.
- (Strictly speaking getter/setter methods would be better, but this is just for the sake of brevity…)
- The List class just has head field.

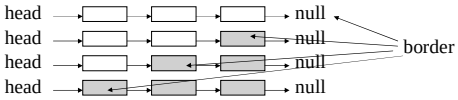
Bubble sort of a linked list

```
Node border = null; // first node in the sorted part
while (border != head) {
  Node current = head; // start from the first node
  while (current.next != border) {
    if (current.element > current.next.element) {
      E element v = current.element;
      current.element = current.next.element;
      current.next.element = v;
    }
    current = current.next;
  }
  border = current; // the sorted part increases by one
}
```

swap with the next node if elements out of order

Complexity of bubble sort on lists

- Same complexity as for arrays $O(n^2)$:
  - First time we iterate until we see a null (swapping elements)
  - Second time we iterate until we see the last node;
  - … each time the border is one link closer to the head of the list
  - until border == head.



Selection sort

```
void  selectionSort(int arr[]){
  int i, j, temp, pos_greatest;
  for( i = arr.length-1; i > 0; i--){
     pos_greatest = 0;
     for(j = 0; j <= i; j++){
       if( arr[j] > arr[pos_greatest])
          pos_greatest =  j;
     }//end inner for loop
     temp = arr[i];
     arr[i] = arr[pos_greatest];
     arr[pos_greatest] = temp;
}//end outer for loop}//end selection sort
```

compare the current element to the largest seen so far; if it is larger, remember its index

swap the largest element to the end of range

Complexity of selection sort

- Same number of iterations
- Same number of comparisons in the worst case
- fewer swaps (one for each outer loop = n-1)
- also $O(n^2)$

Selection sort on linked lists

- Implementation similar to bubble sort; also $O(n^2)$
- Instead of pos_greatest, have a variable Node largest which keeps the reference of the node with the largest element we have seen so far
- Swap elements once in every iteration through the unsorted part of the list:

  E element v = current.element;
  current.element =largest.element;
  largest.element= v;

Insertion sort

```
void insertionSort(int arr[]){
  int i,j,temp;
  for(j=1; j < arr.length-1; j++){
    temp = arr[j];
    i = j; // range 0 to j-1 is sorted
    while(i > 0 && arr[i-1] >= temp){
      arr[i] = arr[i-1];
      i--;
    }
    arr[i] = temp;
  } // end outer for loop
}  // end insertion sort
```

Find a place to insert temp in the sorted range; as you are looking, shift elements in the sorted range to the right

Complexity of insertion sort

- In the worst case, has to make n(n-1)/2 comparisons and shifts to the right
- also $O(n^2)$ worst case complexity
- best case: array already sorted, no shifts.

Insertion sort on linked lists

- This is a suitable sorting method for doubly linked lists
- We can just insert a node in a sorted portion of linked list in constant time, don’t need to shift other nodes to make space for it (but need to find the place…):

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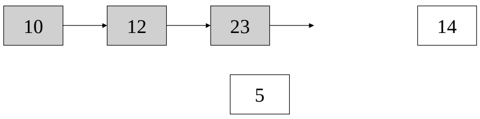

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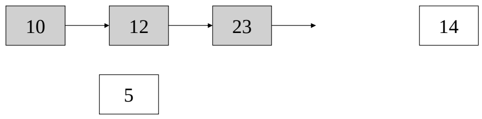

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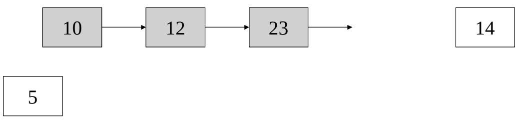

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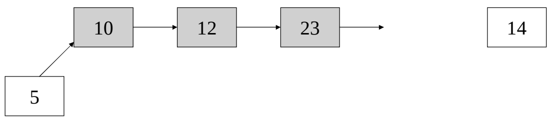

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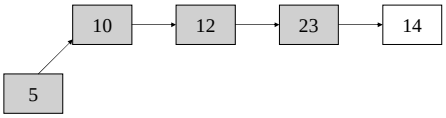


Implementation of insertion sort on linked lists

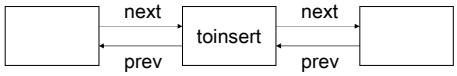
- So this time assume we have a doubly-linked list
- We will move nodes rather than swapping elements
- Assume we have DNode class which has fields element, next and prev
- DList class has head field

Implementation of insertion sort on linked lists

```
border = head; // last node in the sorted part
while (border.next ! = null) {
   DNode toinsert = border.next;
  // do we need to move it?
  if (toinsert.element >= border.element) {
     border = border.next;
  }
```

Implementation of insertion sort on linked lists

```
else { // toinsert.element < border.element
   // remove toinsert from the list
   toinsert.prev.next = toinsert.next;
   toinsert.next.prev = toinsert.prev;
```



Implementation of insertion sort on linked lists

```
// find a place between head and border to
insert toinsert, in order
if (head.element > toinsert.element) {
   toinsert.next = head;
   toinsert.prev = null;
   head.prev = toinsert;
   head = toinsert;
}
```

Implementation of insertion sort on linked lists

```
// find a place between head and border to insert
toinsert, in order
DNode current = head.next;
while (current.element < toinsert.element) {
   current = current.next; // this is O(n) in worst case
} // now current.element >= toinsert.element
toinsert.next = current;
toinsert.prev = current.prev;
current.prev.next = toinsert;
current.prev = toinsert;   // and close all brackets
```

Implementation of insertion sort on linked lists

- This is $O(n^2)$: in each iteration through the outer loop, we move border one step to the right
- In the inner loop, we iterate through the sorted portion, looking for a place to insert
- In the worst case (list sorted in reverse order) will have to do 1+2+…+n-1 comparisons in the sorted part.
- It would have been better (if we planned to use insertion sort for sorting almost sorted lists) to search for the insertion point not from the head of the list going towards the border, but from the border going towards the head; if the list is almost sorted, this would result in a shorter iteration.