# Rules of exponents

When it comes to dealing with exponents, we have to follow certain rules.

## Addition and subtraction

When we want to find the sum or difference of two exponential expressions, they must be "like terms," meaning that they must have the same base and the same exponent; otherwise, we can't add or subtract them.

For example, we can add or subtract $3x^2$ and $x^2$, because the bases are both $x$ and the exponents are both $2$. The $3$ is what we call a "coefficient"; that just tells us we have three $x^2$'s added together ($3x^2 = x^2 + x^2 + x^2$), so the sum of $3x^2$ and $x^2$ is found by adding one $x^2$ to three $x^2$'s.

$$3x^2 + x^2$$

$$\left(x^2 + x^2 + x^2\right) + x^2$$

$$x^2 + x^2 + x^2 + x^2$$

$$4x^2$$

Now let's subtract $x^2$ from $3x^2$.

$$3x^2 - x^2$$

$$\left(x^2 + x^2 + x^2\right) - x^2$$

$$x^2 + x^2 + x^2 - x^2$$

Now take a look at the last two terms in the expression we just found: $x^2 - x^2$. As you might guess, when we have $x^2 - x^2$ (when we want to subtract $x^2$ from $x^2$), we get $0$. That's because no matter what number the $x$ stands for, the number $-x^2$ is the opposite of $x^2$.

$$x^2 + x^2 + (x^2 - x^2)$$

$$x^2 + x^2 + 0$$

$$x^2 + x^2$$

$$2x^2$$

## Multiplication and division

Multiplication and division of exponential expressions is a little different. When we multiply and divide, we need only the bases to be the same. We do not need the exponents to be the same.

For example, if we want to multiply $x^4$ by $x^5$, we can do it because the bases are the same, even though the exponents are different.

$$x^4 \cdot x^5$$

$$(xxxx) \cdot (xxxxx)$$

$$(xxxxxxxxx)$$

$$x^9$$



From this example, we realize that we're really just adding the exponents when we multiply two exponential expressions with the same base. In other words, the rule for **multiplication** is

$$x^a \cdot x^b = x^{a+b}$$

Similarly, if we want to divide $x^5$ by $x^2$, we can do it because the bases are the same, even though the exponents are different.

$$\frac{x^5}{x^2}$$

$$\frac{x \cdot x \cdot x \cdot x \cdot x}{x \cdot x}$$

The factor that's common to the numerator and the denominator is $x \cdot x$, so we'll divide top and bottom by $x \cdot x$.

$$\frac{(x \cdot x \cdot x \cdot x \cdot x) \div (x \cdot x)}{(x \cdot x) \div (x \cdot x)}$$

$$\frac{x \cdot x \cdot x}{1}$$

$$x^3$$

From this example, we realize that we're really just subtracting the exponents when we divide two exponential expressions with the same base. In other words, the rule for **division** is

$$\frac{x^a}{x^b} = x^{a-b}$$