Recursion

- A recursive function, as you saw in CS100, is one that calls itself.
- Classic textbook example: factorial

  Mathematical definition:

  $$0! = 1$$
  $$N! = N \times (N-1)! \text{ if } N > 0$$

How would you implement this?

Recursion vs. Iteration

- Roughly speaking, recursion and iteration perform the same kinds of tasks:

  → Solve a complicated task one piece at a time, and combine the results.

- Emphasis of iteration:

  → keep repeating until a task is "done"

  e.g., loop counter reaches limit,
  linked list reaches null pointer,
  `instream.eof()` becomes true

- Emphasis of recursion:

  → Solve a large problem by breaking it up into smaller and smaller pieces until you can solve it; combine the results.

  e.g., recursive factorial funtion

Which is Better?[a]

- No clear answer, but there are known trade-offs.
- "Mathematicians" often prefer recursive approach.

  → Solutions often shorter, closer in spirit to abstract mathematical entity.

  → Good recursive solutions *may* be more difficult to design and test.

- "Programmers", esp. w/o college CS training, often prefer iterative solutions.

  → Somehow, it seems more appealing to many.

  → Control stays local to loop, less "magical".

Compare iterative to recursive versions of `factorial`!

[a] Some of these statements are personal opinion.

Which Approach Should You Choose?

- Depends on the problem.

  → The factorial example is pretty artificial; it's so simple that it really doesn't matter which version you choose.

- Many ADTs (*e.g.,* trees) are simpler & more natural if methods are implemented recursively.
- Recursive isn't always better, 'tho:

```
// Recursively compute nth Fibonacci number.
// assumes n>=0
public static int fib (int n) {









}
```

→ This takes $O(2^n)$ steps! Unusable for large n.

```
// Iteratively compute nth Fibonacci number.
// assumes n>=0
public static int ifib (int n) {









}
```

→ This iterative approach is "linear"; it takes $O(n)$ steps.

Moral: No substitute for careful thought.

Moral: "Obvious" and "natural" solutions aren't always practical.

Basic Idea of Recursion

1. Know how to solve a problem immediately for "small" of trivial cases.

   → These are called the *basis* cases.

   → Often there are only one or two of these.

2. If input is non-trivial, can break it up smaller and smaller until chunks until you reach something you know how to deal with.

   Eventually, you make enough recursive calls that the input reaches a "basis case".

```
    public static int factorial (int n) {
        if (n==0) {
            return 1;
        } else {
            return n * factorial (n-1);
        }
    }
```


Problem: Infinite Recursive Calls

- If you are not careful with the program logic, you may miss a basis case and go off into an infinite recursion.
- This is similar to an infinite loop!
- Example: call to factorial with $N < 0$
  - → Either you must ensure that `factorial` is never, ever called with a negative N, or you must build in a check somehow.
- Moral: When you are designing your recursive calls, make sure that at least one of the basis cases MUST be reached eventually.
  - → This is often pretty hard!

More Recursive Examples

```
class RecursionTest {
    // Find max value in an unsorted array of ints.
    public static int findMax (int [] A, int startIndex) {
        if (startIndex == A.length - 1) {
            return A[startIndex];
        } else {
            return Math.max (A[startIndex],
                findMax(A, startIndex+1)) ;
        }
    }

    public static void main (String[] args) {
        int [] A = new int[5];
        A[0] = 3; A[1] = 37; A[2] = -5;
        A[3] = 12; A[4] = 7;
        System.out.println ("Max = " + findMax (A, 0));
    }
}
```

Recursive Implementations of BST Routines

- Can implement many BST routines recursively.
- Recursive method implementations are more elegant than iterative, but no more or less efficient:
  - → Recursion is a big win for printing full BSTs.
  - → Search is a little nicer.
  - → Insert would be nicer recursively ... if only Java allowed changes to parameters to percolate back to the caller.
  - → Delete still complicated; left to student.
- Assume `BSTNode` class is the same as before. We will implement a new class called `recBST` for recursive BSTs.
- Again, will assume only integers values are stored. Can make BSTs of more interesting entities if they support ideas of `lessThan`/`greaterThan`.
  - → BST routines will have to be altered slightly, but the basic ideas are the same.


```
public class recBST {
    BSTNode root;

    public recBST () {
        root = null;
    }

    // Public sees this function, which starts
    // a recursive search at the root of the tree.
    public boolean search (int value) {
        return rSearch (value, root);
    }

    private boolean rSearch (int value, BSTNode curNode) {
        if (curNode == null) {
            return false;
        } else if (curNode.value == value) {
            return true;
        } else if (value < curNode.value) {
            return rSearch (value, curNode.left);
        } else {
            return rSearch (value, curNode.right);
        }
    }

    ...
```


An Incorrect Approach to `insert`

```
    public void insert (int value) {
        rInsert (value, root);
    }

    private void rInsert (int value, BSTNode node) {
        if (node == null) {
            node = new BSTNode (value);
        } else if (value < node.value) {
            rInsert (value, node.left);
        } else if (value > node.value) {
            rInsert (value, node.right);
        }
    }
```

- This approach will work in some programming languages ... but not Java.
-
-

Correct Recursive BST `insert`

```
    public void insert (int value) {
        root = rInsert (value, root);
    }

    private BSTNode rInsert (int value, BSTNode node) {
        if (node == null) {
            node = new BSTNode (value);
        } else if (value < node.value) {
            node.left = rInsert (value, node.left);
        } else if (value > node.value) {
            node.right = rInsert (value, node.right);
        }
        return node;
    }
```

- Solution is to send back a reference to the new node as the value of the function!
- Somewhat unintuitive.

Printing a BST

- Want to print all values stored in a tree in increasing order.
- Recall BST property:

  →

- Thinking recursively ... what should the print routine look like?
- Again, if node has interesting value, may want to invoke `node.toString` or another customized routine instead of `System.out.println(node.value)`

```
public void print () {
    printSubtree(root);
}

private void printSubtree (BSTNode node) {





}
```


Example: Searching Through a List

- Want to search through an array for a sought element.
- Again, we assume integers for simplicity of example. In "real world", we would be searching through an array of generic `Sortable`s.

  i.e., use `equals` instead of `==`,
  use `lessThan` instead of `<`

- I hope you have already seen iterative versions of linear and binary search in CS100!

Linear Search

Simplest approach: linear search

→ Start at beginning, keep going until you find the element.

→ Works with sorted and unsorted lists
[If list is sorted, can exit once elements become larger that sought value.]

```
// Returns an index in array of soughtVal, if it occurs.
// Returns -1 is soughtVal is not present in array.
// Note: Array A need not be sorted.
public static int iterLinearSearch (int [] A,
        int soughtVal) {
    for (int i=0; i<A.length; i++) {
        if (A[i] == soughtVal) {
            return i;
        }
    }
    return -1;
}
```


Recursive Linear Search

Binary Search

Idea:

- Have a window or range of values we are currently considering.
  [Initially, the window is the whole array.]
- Look at midpoint in range and compare to soughtValue.
  - → If `A[mid] == soughtValue`, we're found it.
  - → If `A[mid] < soughtValue`, discard first half.
  - → If `A[mid] > soughtValue`, discard second half.
- Keep halving the list until either you find it or your sublist has no elements.
- Computational complexity of binary search is number of times you can halve the list.

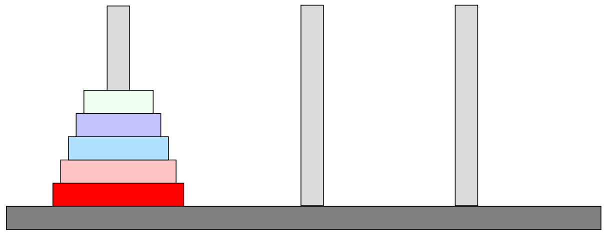
Iterative Binary Search

```
// Returns an index in array of soughtVal, if it occurs.
// Returns -1 is soughtVal is not present in array.
// Note: Array A MUST be sorted.
public static int iterBinarySearch (int [] A,
        int soughtVal) {
    int lo=0, hi=A.length-1;

    // Exit loop when lo>hi.  This will happen just
    // after the sublist has been reduced to one
    // element (lo==hi) and then we reset lo or hi
    // because we didn't find soughtVal there.
    while (lo <= hi) {
        // note integer division.
        final int mid = (lo + hi)/2;
        if (A[mid] == soughtVal) {
            return mid;
        } else if (A[mid] < soughtVal) {
            // discard first half
            lo = mid +1;
        } else {
            // discard second half
            hi = mid -1;
        }
    }
    return -1;
}
```


Recursive Binary Search

```
// Returns an index in array of soughtVal, if it occurs.
// Returns -1 is soughtVal is not present in array.
// Note: array A must be sorted.
// Also note extra parameters lo and hi.  Clients will
// make initial call with lo==0 and hi==A.length-1
public static int recBinarySearch (int [] A,
        int soughtVal, int lo, int hi) {









}
```


Example: Towers of Hanoi



Classic ancient problem:

- N rings in increasing size.
- 3 poles.
- Rings start stacked on pole 1. Goal is to move rings so that they are stacked on pole 3 ... BUT
  - → Can only move one ring at a time.
  - → Can't put larger ring on top of smaller.
- Iterative solution is "powerful ugly";
  recursive solution is "elegant".

Towers of Hanoi Solution

```
public class hanoi {
    public static void move (int N, int src, int dest) {
        if (N>0) {
            // neat trick to get index of other pole.
            final int temp = 6 - src - dest;
            move (N-1, src, temp);
            System.out.println ("Move ring from pole "
                    + src + " to pole " + dest);
            move (N-1, temp, dest);
        }
    }

    public static void main (String[] args) {
        // Move two rings from pole 1 to pole 3.
        System.out.println ("\nSoln for two rings:\n");
        move (2, 1, 3);
        // Move three rings from pole 1 to pole 3.
        System.out.println ("\nSoln for three rings:\n");
        move (3, 1, 3);
    }
}
```


How Recursion is Implemented by the Compiler

- Run-time *stack* is used to keep track of pending function calls (parameters and local variables).
- Storage for *objects* comes from another part of memory called the *heap*.
  - → However, params and local vars that refer to these objects are stored within the run-time stack somewhere.
- `static` variables are stored somewhere else.
- The set of params and local vars for a function call is stored in an *activation record* (AR).

- If one function calls another, a new AR is created and pushed onto the run-time stack. AR has storage for params and local vars PLUS remembers where to return to when done.
- When a function call finishes, the AR is popped off the stack and (eventually) destroyed. Return to appropriate spot and return the value of the function (if not `void`).

Tracing Through Hanoi