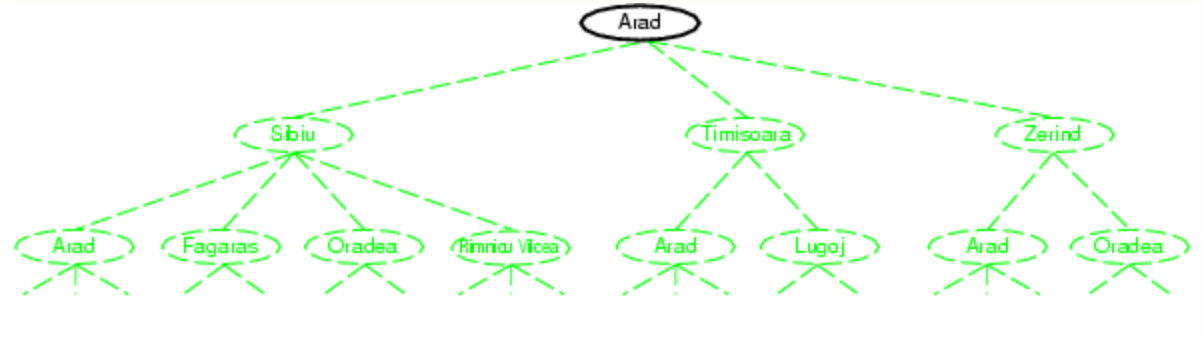
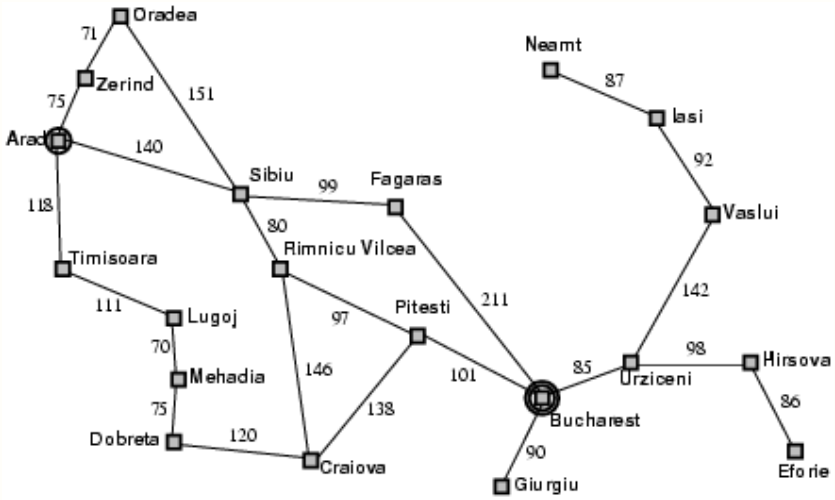
A* Search

Tree search algorithms

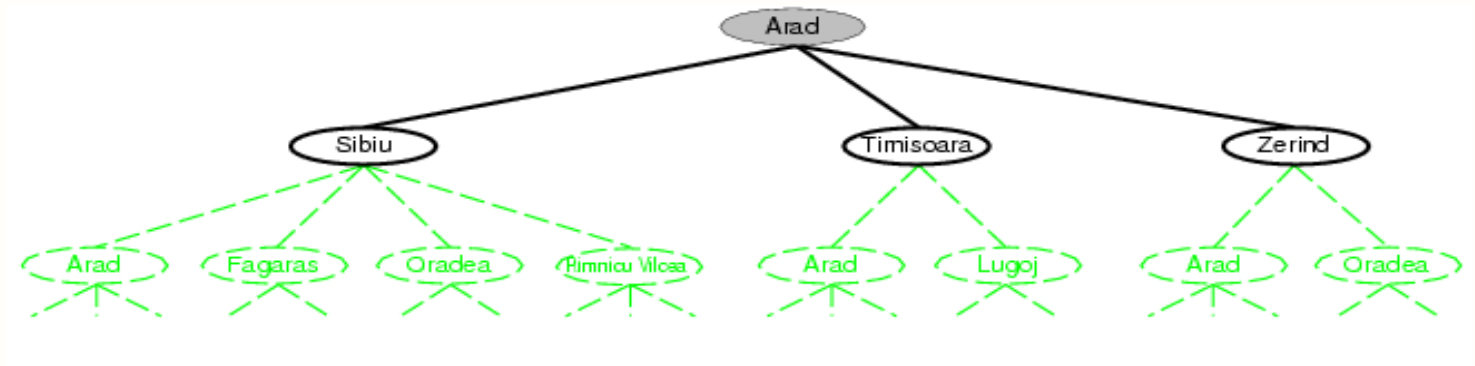
- Basic idea:
  - Exploration of state space by generating successors of already-explored states (a.k.a.~expanding states).
  - Every states is evaluated: *is it a goal state*?

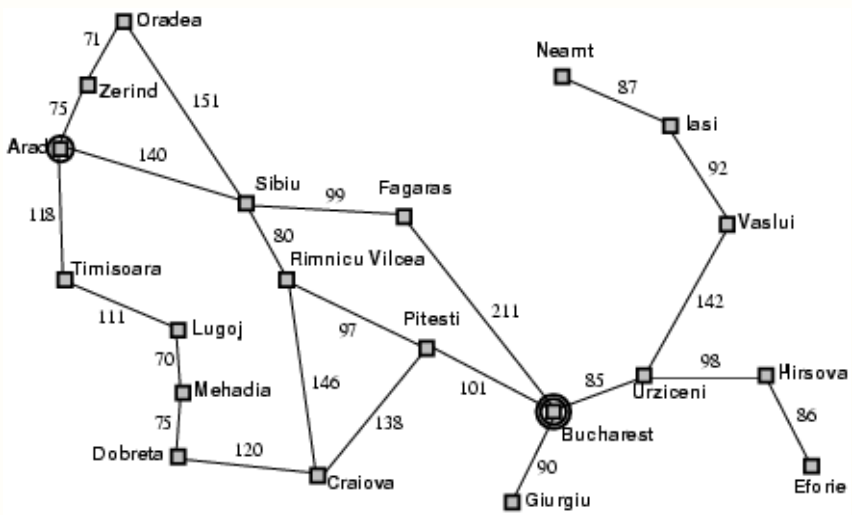
Tree search example



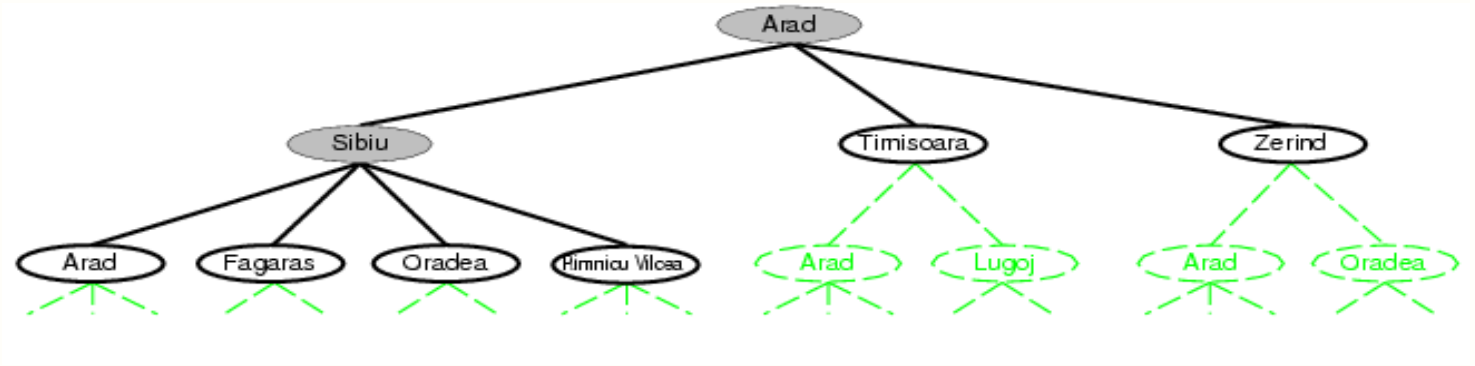


Tree search example





Tree search example



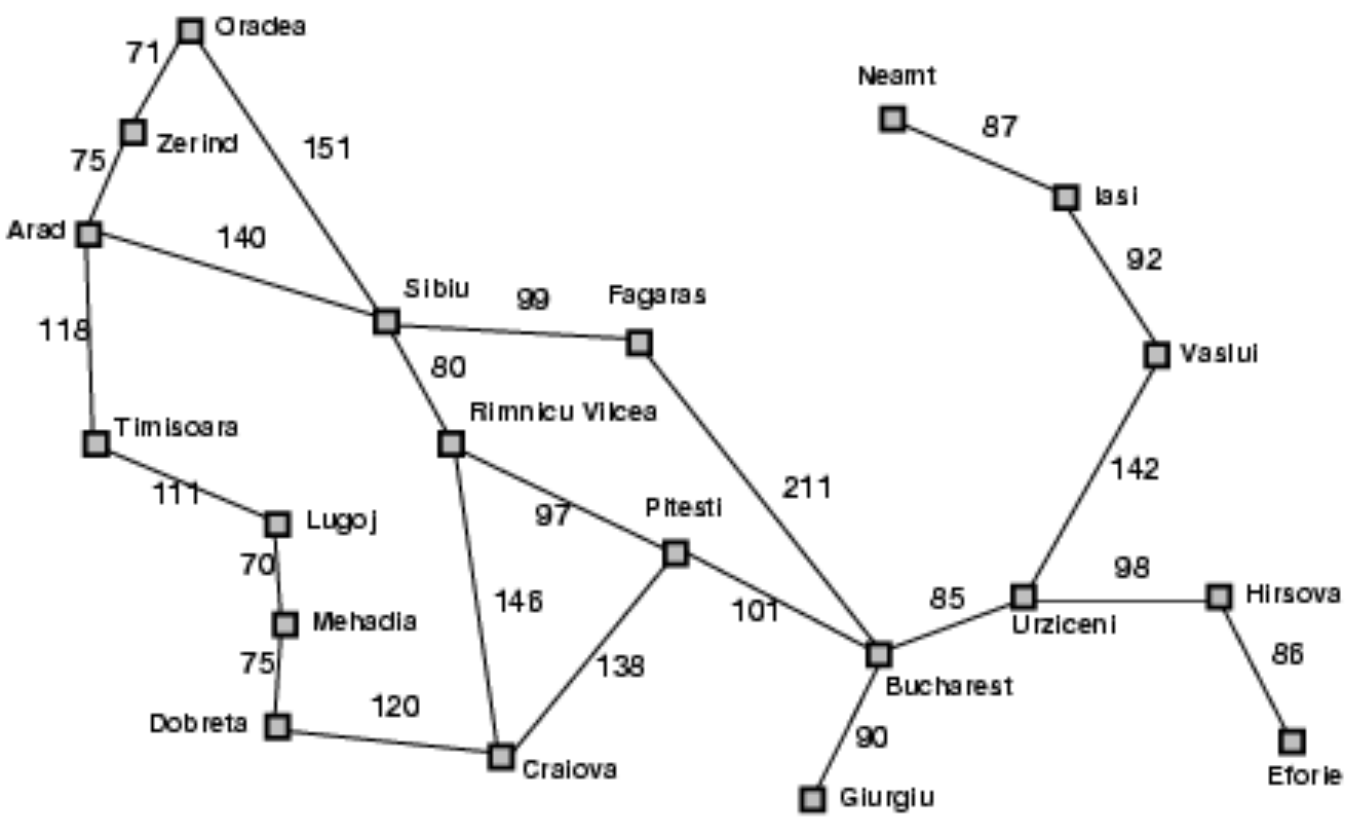

```
function TREE-SEARCH( problem, strategy) returns a solution, or failure
    initialize the search tree using the initial state of problem
    loop do
        if there are no candidates for expansion then return failure
        choose a leaf node for expansion according to strategy
        if the node contains a goal state then return the corresponding solution
        else expand the node and add the resulting nodes to the search tree
```

Best-first search

- Idea: use an evaluation function *f(n)* for each node
  - f(n) provides an estimate for the total cost.
  - → Expand the node n with smallest f(n).

- <u>Implementation</u>:
  Order the nodes in fringe increasing order of cost.

Romania with straight-line dist.



Straight–line distance to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	176
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	10
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

A* search

- Idea: avoid expanding paths that are already expensive
- Evaluation function $f(n) = g(n) + h(n)$
- $g(n)$ = cost so far to reach n
- $h(n)$ = estimated cost from n to goal
- $f(n)$ = estimated total cost of path through n to goal
- Best First search has $f(n)=h(n)$
- Uniform Cost search has $f(n)=g(n)$

Admissible heuristics

- A heuristic $h(n)$ is admissible if for every node n, $h(n) \leq h^*(n)$, where $h^*(n)$ is the true cost to reach the goal state from n.
- An admissible heuristic never overestimates the cost to reach the goal, i.e., it is optimistic
- Example: $h_{SLD}(n)$ (never overestimates the actual road distance)
- Theorem: If $h(n)$ is admissible, A* using `TREE-SEARCH` is optimal

Dominance

- If $h_2(n) \geq h_1(n)$ for all n (both admissible)
- then h_2 dominates h_1
- h_2 is better for search: it is guaranteed to expand less or equal nr of nodes.

- Typical search costs (average number of nodes expanded):

- *d=12* IDS = 3,644,035 nodes
  $A^*(h_1)$ = 227 nodes
  $A^*(h_2)$ = 73 nodes
- *d=24* IDS = too many nodes
  $A^*(h_1)$ = 39,135 nodes
  $A^*(h_2)$ = 1,641 nodes

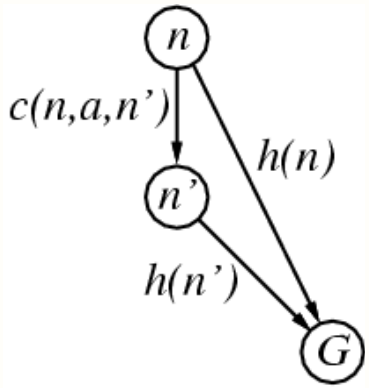
Relaxed problems

- A problem with fewer restrictions on the actions is called a relaxed problem
- The cost of an optimal solution to a relaxed problem is an admissible heuristic for the original problem
- If the rules of the 8-puzzle are relaxed so that a tile can move anywhere, then $h_1(n)$ gives the shortest solution
- If the rules are relaxed so that a tile can move to any adjacent square, then $h_2(n)$ gives the shortest solution

Consistent heuristics

- A heuristic is consistent if for every node *n*, every successor *n'* of *n* generated by any action *a*,

$$h(n) \leq c(n,a,n') + h(n')$$



- If *h* is consistent, we have

$$\begin{aligned} f(n') &= g(n') + h(n') && \text{(by def.)} \\ &= g(n) + c(n,a,n') + h(n') && (g(n')=g(n)+c(n.a.n')) \\ &\geq g(n) + h(n) = f(n) && \text{(consistency)} \\ f(n') &\geq f(n) \end{aligned}$$

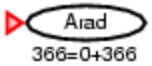
It's the triangle inequality !

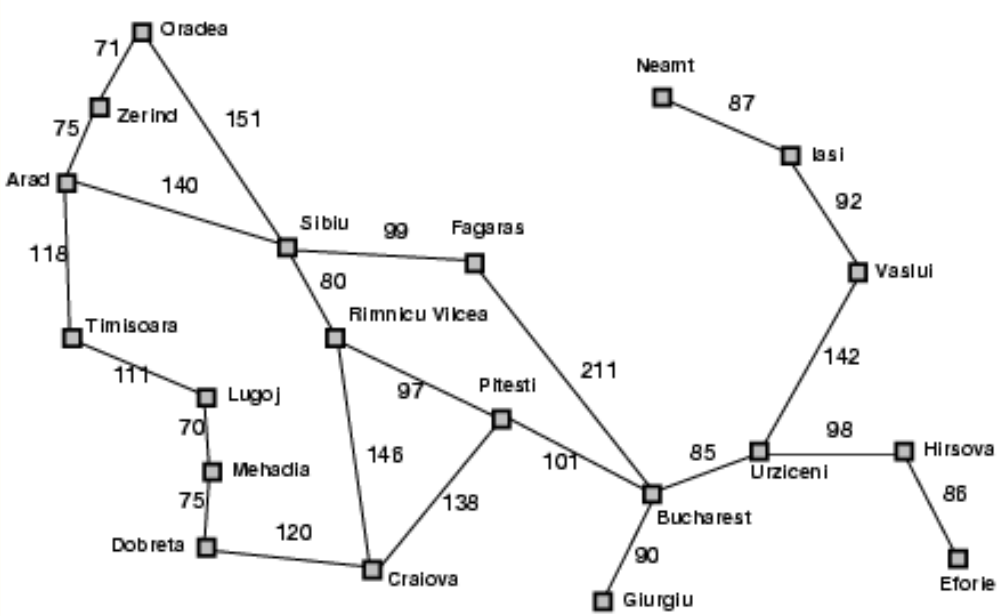
- i.e., *f(n)* is non-decreasing along any path.

- Theorem:
  If *h(n)* is consistent, A* using `GRAPH-SEARCH` is optimal

keeps all checked nodes in memory to avoid repeated states

A* search example

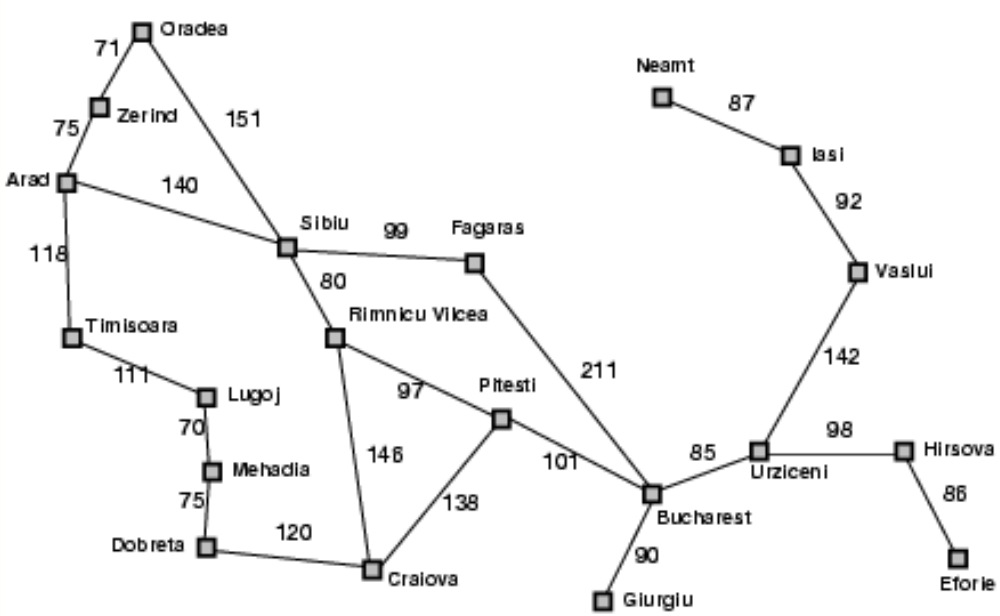




Straight–line distance to Bucharest

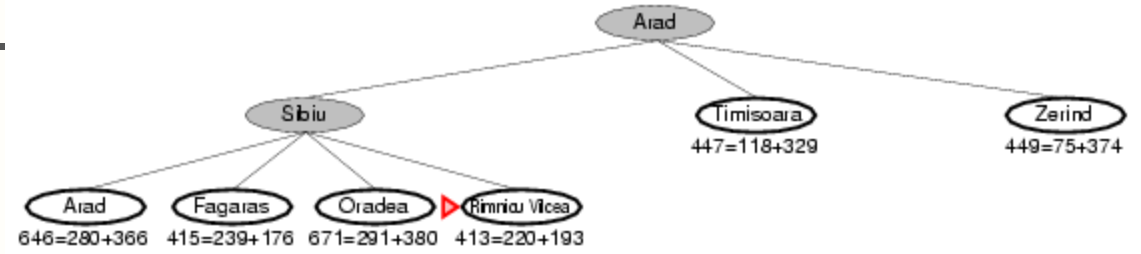
Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	176
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	10
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

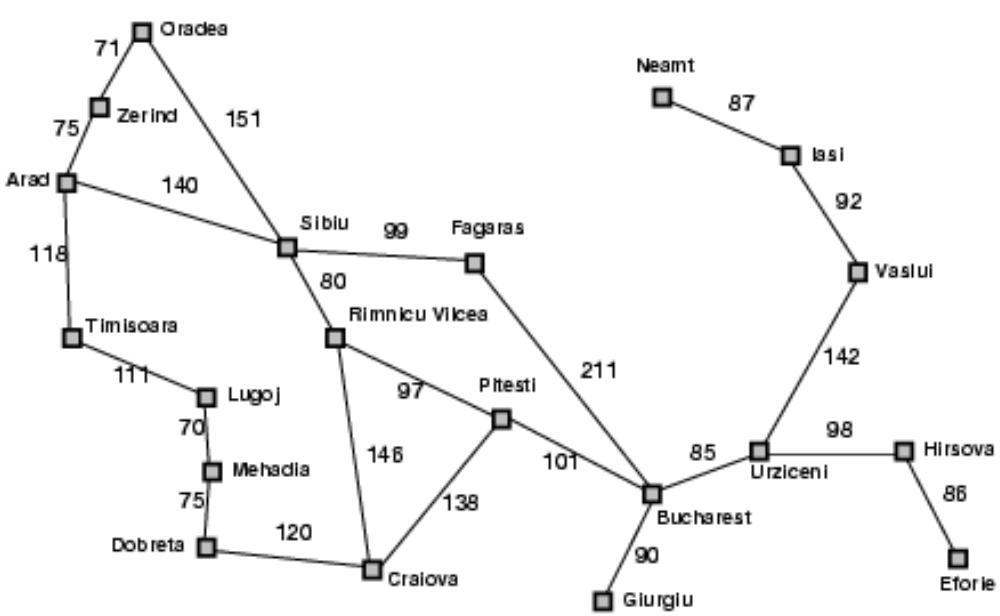
A* search example



Straight–line distance to Bucharest	
Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	176
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	10
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

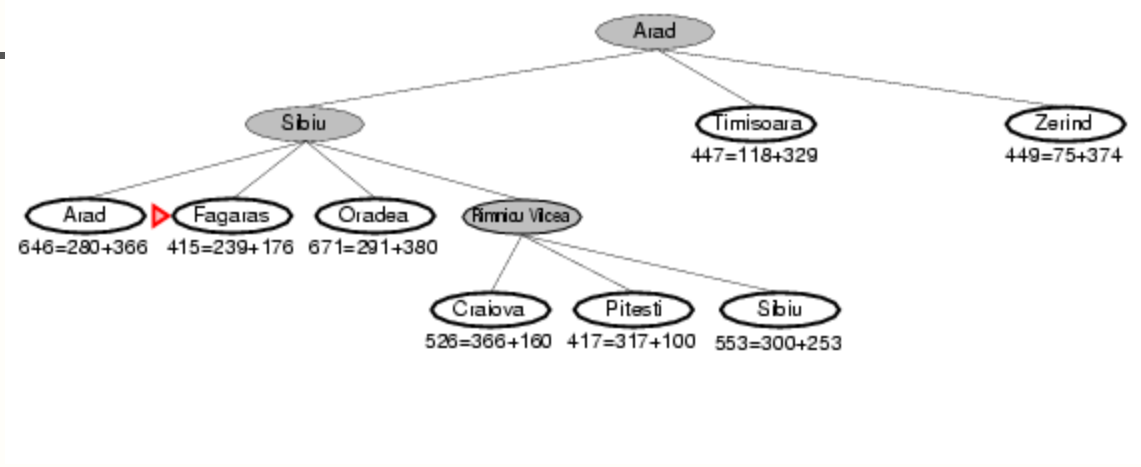
A* search example

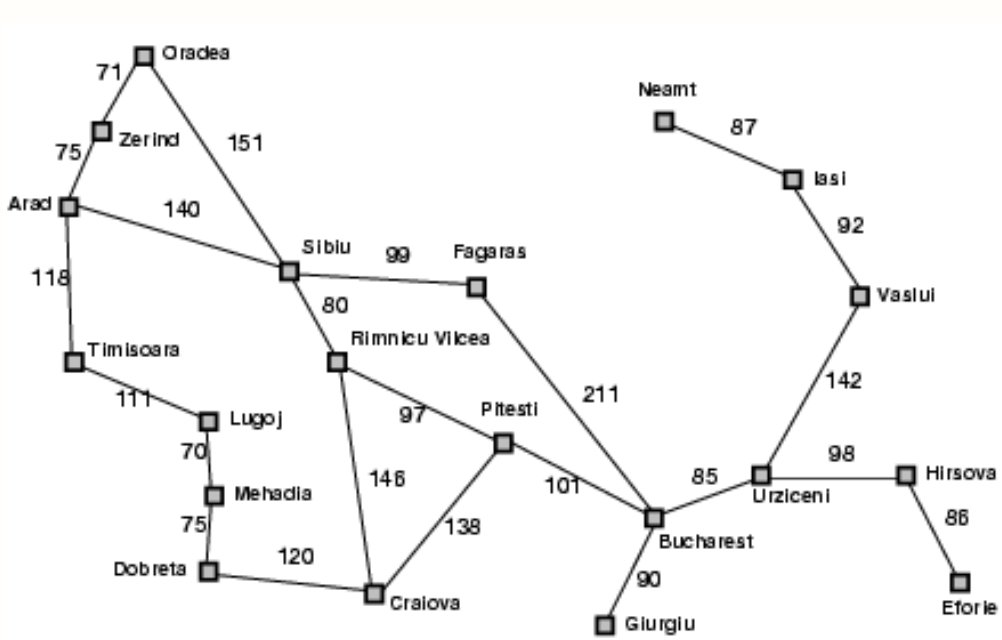




Straight–line distance to Bucharest	
Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	176
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	10
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

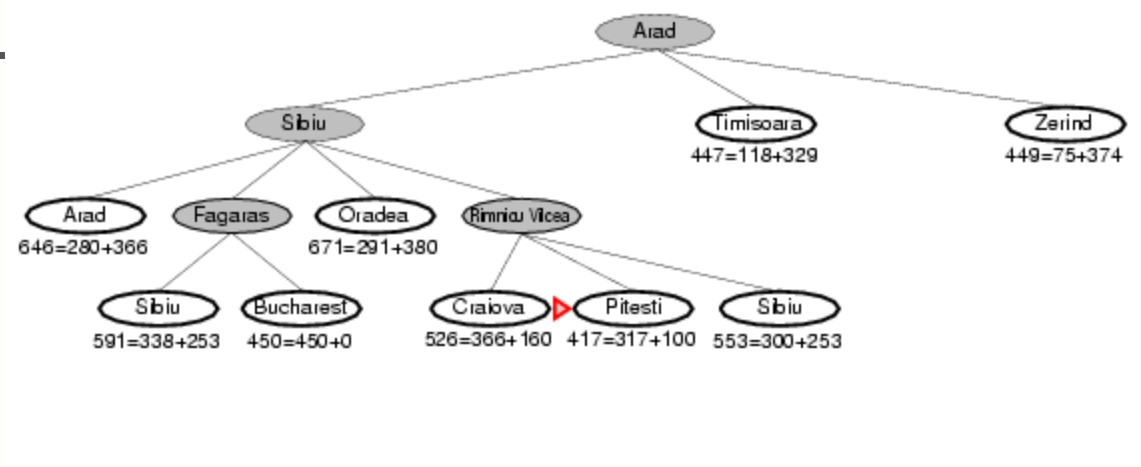
A* search example

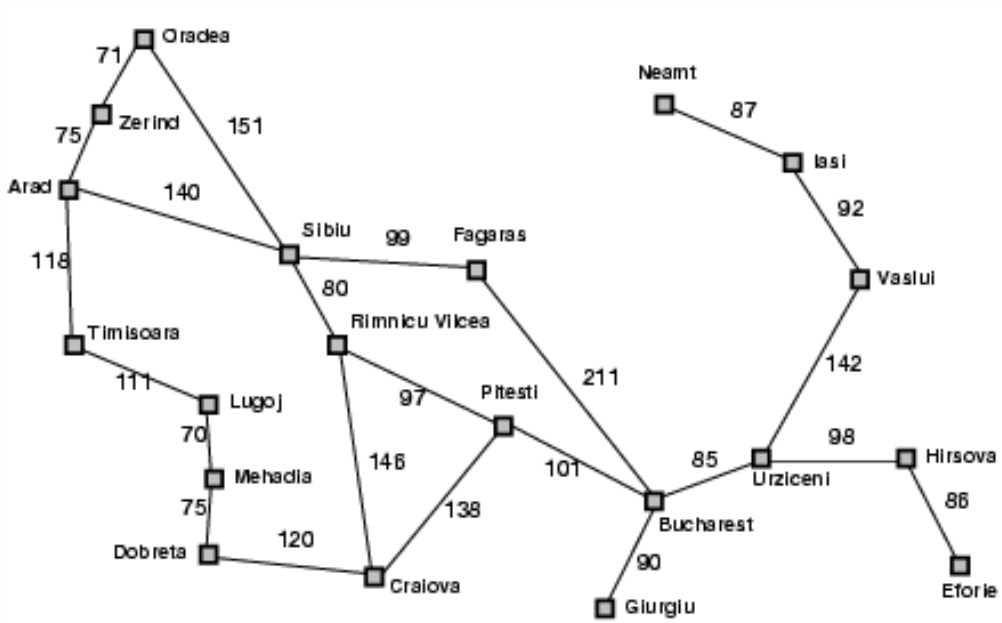




Straight–line distance to Bucharest	
Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	176
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	10
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

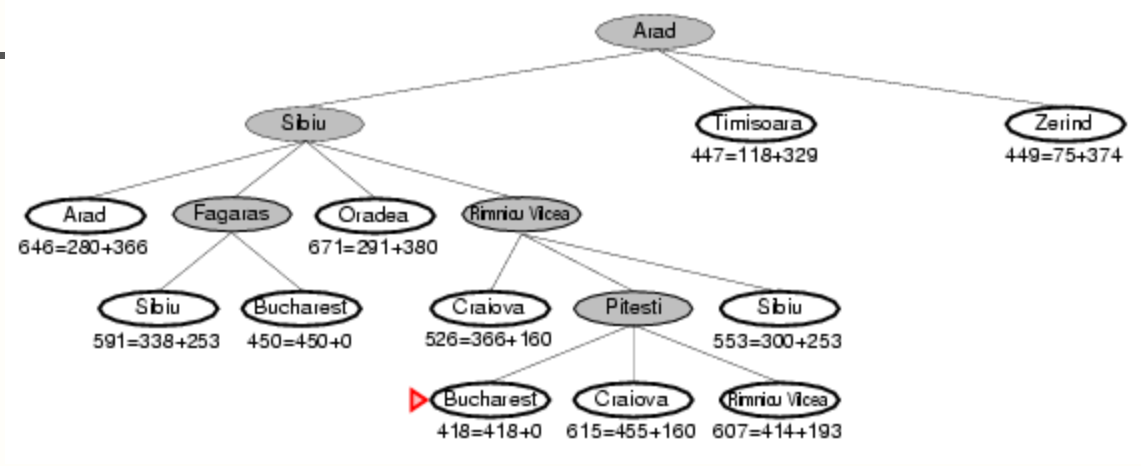
A* search example

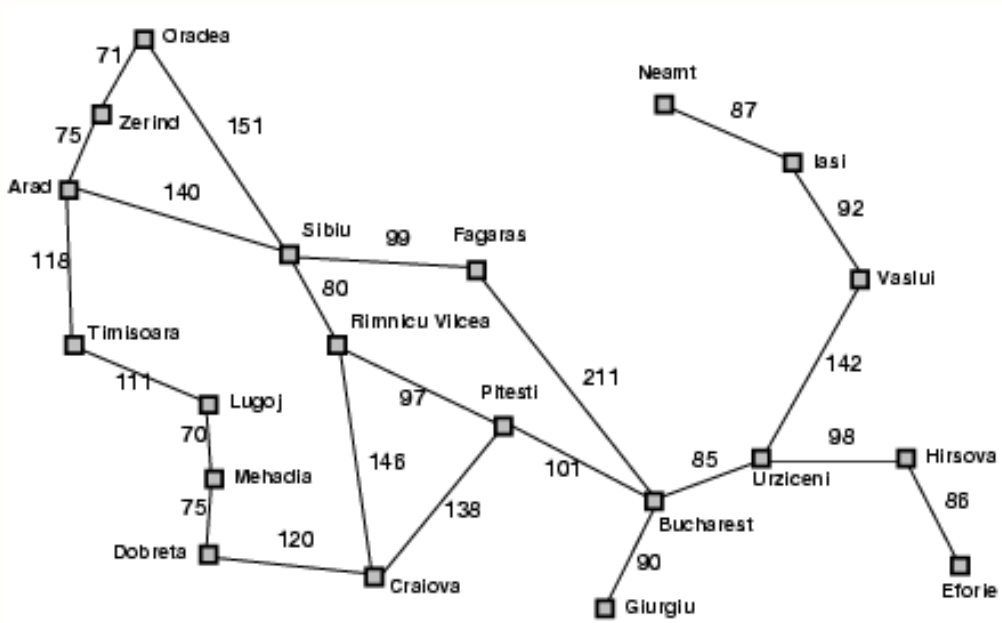




Straight–line distance to Bucharest	
Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	176
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	10
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

A* search example





Straight–line distance to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	176
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	10
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

Properties of A*

- Complete? Yes (unless there are infinitely many nodes with f $\leq f(G)$, i.e. step-cost > ε)
- Time/Space? Exponential: b^d

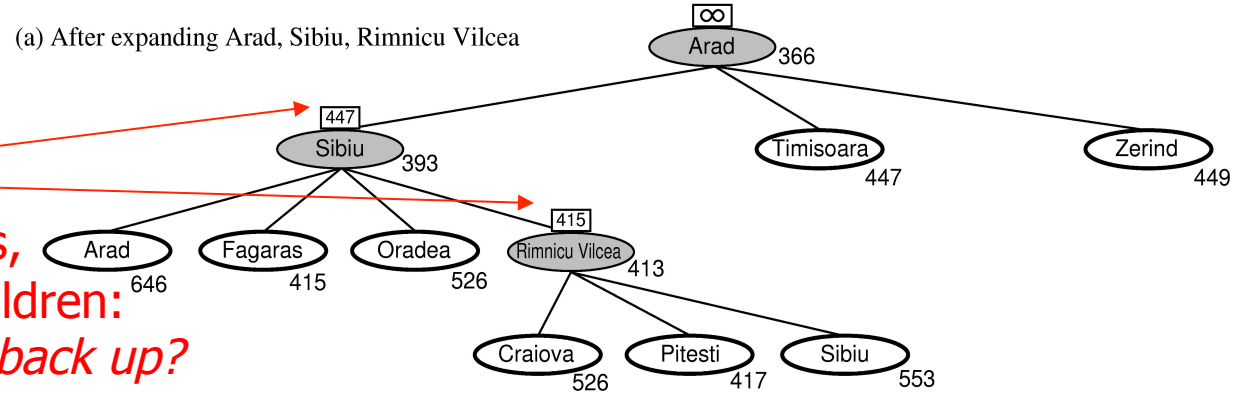
  except if: $|h(n) - h^*(n)| \leq O(\log h^*(n))$
- Optimal? Yes
- *Optimally Efficient*: Yes (no algorithm with the same heuristic is guaranteed to expand fewer nodes)

Memory Bounded Heuristic Search: Recursive BFS

- How can we solve the memory problem for A* search?
- Idea: Try something like depth first search, but let's not forget everything about the branches we have partially explored.
- *We remember the best f-value we have found so far in the branch we are deleting*.

RBFS:

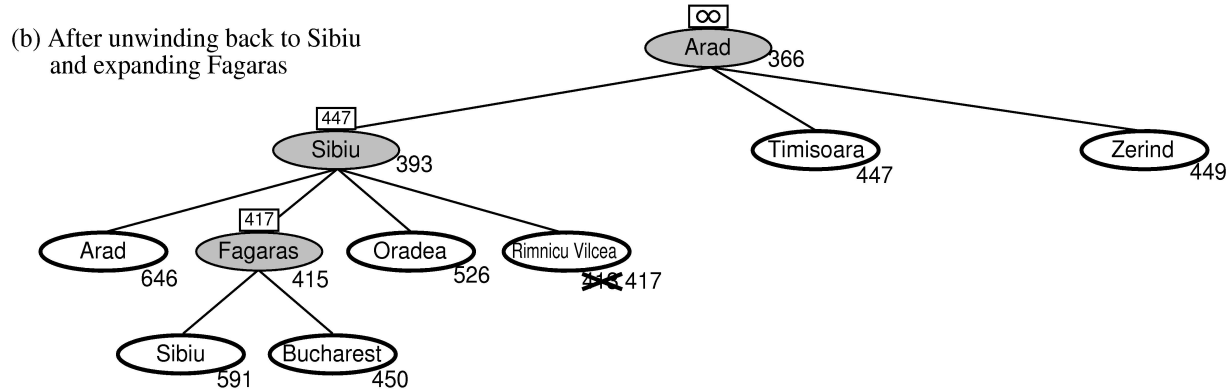
best alternative
over fringe nodes,
which are not children:
i.e. do I want to back up?

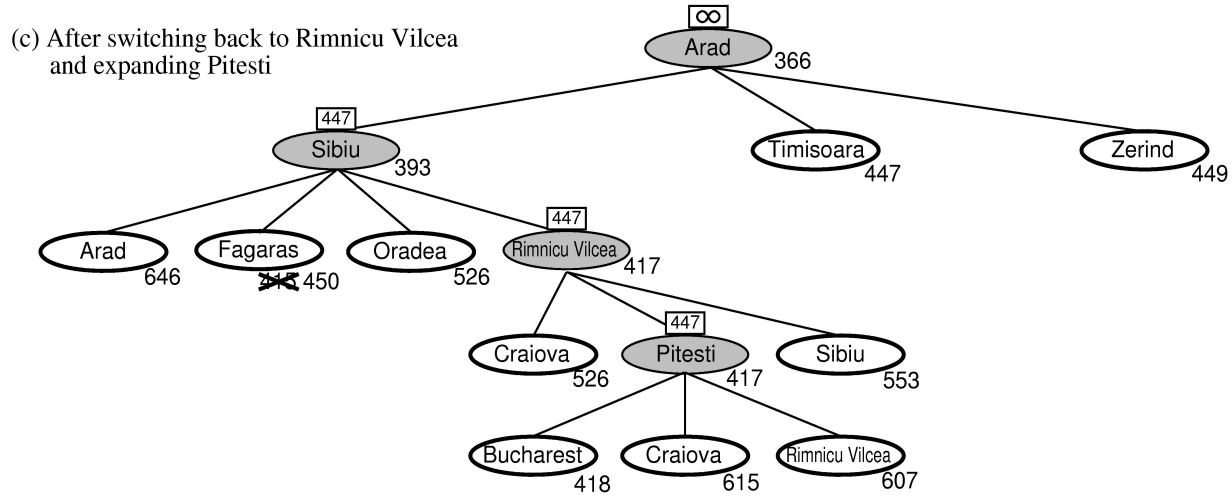


RBFS changes its mind very often in practice.

This is because the f=g+h become more accurate (less optimistic) as we approach the goal. Hence, higher level nodes have smaller f-values and will be explored first.

Problem: We should keep in memory whatever we can.





Simple Memory Bounded A*

- This is like A*, but when memory is full we delete the worst node (largest f-value).
- Like RBFS, we remember the best descendent in the branch we delete.
- If there is a tie (equal f-values) we delete the oldest nodes first.
- simple-MBA* finds the optimal *reachable* solution given the memory constraint.
- Time can still be exponential.

A Solution is not reachable
if a single path from root to goal
does not fit into memory