Signs of fractions

Until now, every fraction we've dealt with has had a denominator that's positive, and a numerator that's either positive or 0. Now we're going to deal with fractions involving negative whole numbers as well, so we have to take positive and negative signs into account.

There are three signs associated with every fraction. But this can be hard to remember, because not all of the signs are always visible. Before we talk about signs of fractions, let's talk about signs of integers.

We know that -3 is a negative integer, because the negative sign is present. And we know that 4 is a positive integer, even though we haven't written a positive sign in front of the 4. The fact that we haven't written $+4$ doesn't mean that 4 isn't a positive integer.

Translating this to fractions, if we look at the fraction

$$\frac{3}{4}$$

we know that, even though there aren't any positive signs, this fraction has a positive 3 in the numerator and a positive 4 in the denominator. This gives us our first hint about the three signs that are associated with every fraction. Two of these signs are the sign of the numerator and the sign of the denominator. In the fraction above, the sign of the numerator is positive, and the sign of the denominator is positive, since we have both positive 3 and positive 4.

If we have the fraction

$$\frac{-3}{4}$$

then the sign of the numerator is negative since -3 is a negative integer, and the sign of the denominator is positive since 4 is a positive integer.

So we've talked about the sign of the numerator and the sign of the denominator, but what is the third sign associated with a fraction? That's the fraction's own sign. The numerator always has a sign, the denominator always has a sign, and the fraction always has a sign of its own as well. For example, you'll often see a fraction like

$$-\frac{3}{4}$$

In a fraction like this one, the sign of the numerator is positive since 3 is a positive integer, the sign of the denominator is positive since 4 is a positive integer, and the fraction's own sign is negative since we have a negative sign in front of the fraction.

Remember how we talked before about the fact that, even though there are no positive signs in the fraction

$$\frac{3}{4}$$

we still know that the sign of the numerator and the sign of the denominator are both positive? Well in the same way, even though there's no positive sign in front of this fraction, we still know that the fraction's own sign is positive. In other words, if there's a negative sign in front of a fraction, then the fraction's own sign is negative. If there's no negative sign

in front of a fraction (if there's a positive sign in front of the fraction or if there's no sign in front of it), then the fraction's own sign is positive.

Now that we have this foundation, the most important thing to know about signs of fractions is that you can always change exactly two signs of the fraction, and still keep the value of the fraction the same. This works because of the way that every two negative signs cancel each other out.

Example

Write the fraction in at least two other ways.

$$-\frac{4}{7}$$

Before we change anything, we can say that the sign of the numerator is positive since 4 is a positive integer, that the sign of the denominator is positive since 7 is a positive integer, and that the fraction's own sign is negative, since we have a negative sign in front of the fraction.

In order to keep the value of the fraction the same, we have to change two signs at the same time.

Let's change the fraction's own sign and the sign of the numerator.

$$-\frac{4}{7} \quad \text{becomes} \quad \frac{-4}{7}$$

Let's change the fraction's own sign and the sign of the denominator.

$$-\frac{4}{7} \quad \text{becomes} \quad \frac{4}{-7}$$

Or we could change just the sign of the numerator and the sign of the denominator, and keep the fraction's own sign the same.

$$-\frac{4}{7} \quad \text{becomes} \quad -\frac{-4}{-7}$$

So we rewrote the fraction three different ways, which means we can say that the values of all four fractions (the original fraction and the three fractions we found by changing the signs) are equal:

$$-\frac{4}{7} = \frac{-4}{7} = \frac{4}{-7} = -\frac{-4}{-7}$$

Therefore, these four fractions are all equivalent.