Pre-Algebra Workbook Solutions

Mixed numbers

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MIXED NUMBERS AND IMPROPER FRACTIONS

■ 1. Complete the statement.

Improper fractions are fractions where the numerator is _____ than or equal to the denominator.

Solution:

greater

■ 2. Complete the statement.

All improper fractions represent a fraction with a value greater than _____.

Solution:

or equal to 1

■ 3. Complete the statement.

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If we're looking at the fraction $8/3$, imagine taking a circle and dividing it into thirds, there would be _____ of those size pieces to represent this fraction.

Solution:

8

■ 4. Mixed numbers are a representation of what operation (addition, subtraction, multiplication, division)?

Solution:

Addition, because each mixed number is the sum of a whole number and a fraction.

■ 5. Convert $15/4$ into a mixed number.

Solution:

4 goes into 15 three times. $4 \cdot 3 = 12$, and $15 - 12 = 3$, so 4 goes into 15 three times, with a remainder of 3.

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$$\frac{15}{4} = 3\frac{3}{4}$$

■ 6. Convert $20/3$ into a mixed number.

Solution:

3 goes into 20 six times. $3 \cdot 6 = 18$, and $20 - 18 = 2$, so 3 goes into 20 six times, with a remainder of 2.

$$\frac{20}{3} = 6\frac{2}{3}$$

■ 7. Convert $34/6$ into a mixed number.

Solution:

6 goes into 34 five times. $6 \cdot 5 = 30$, and $34 - 30 = 4$, so 6 goes into 34 five times, with a remainder of 4.

$$\frac{34}{6} = 5\frac{4}{6} = 5\frac{2}{3}$$

■ 8. Write $22/7$ as a mixed number.

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Solution:

7 goes into 22 three times. $7 \cdot 3 = 21$, and $22 - 21 = 1$, so 7 goes into 22 three times, with a remainder of 1.

$$\frac{22}{7} = 3\frac{1}{7}$$

■ 9. Write $135/11$ as a mixed number.

Solution:

11 goes into 135 twelve times. $11 \cdot 12 = 132$, and $135 - 132 = 3$, so 11 goes into 135 twelve times, with a remainder of 3.

$$\frac{135}{11} = 12\frac{3}{11}$$

■ 10. Write $114/25$ as a mixed number.

Solution:

25 goes into 114 four times. $25 \cdot 4 = 100$, and $114 - 100 = 14$, so 25 goes into 114 four times, with a remainder of 14.

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$$\frac{114}{25} = 4\frac{14}{25}$$

11. Write the mixed number as an improper fraction.

$$2\frac{2}{5}$$

Solution:

The equivalent improper fraction will have the same denominator, 5. The numerator is the product of the denominator and the whole number, added to the numerator, $5 \cdot 2 + 2 = 10 + 2 = 12$.

$$\frac{12}{5}$$

12. Convert the mixed number into an improper fraction.

$$2\frac{1}{6}$$

Solution:

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The equivalent improper fraction will have the same denominator, 6. The numerator is the product of the denominator and the whole number, added to the numerator, $6 \cdot 2 + 1 = 12 + 1 = 13$.

$$\frac{13}{6}$$

■ 13. Convert the mixed number into an improper fraction.

$$8\frac{4}{9}$$

Solution:

The equivalent improper fraction will have the same denominator, 9. The numerator is the product of the denominator and the whole number, added to the numerator, $9 \cdot 8 + 4 = 72 + 4 = 76$.

$$\frac{76}{9}$$

■ 14. Convert the mixed number into an improper fraction.

$$5\frac{2}{7}$$

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Solution:

The equivalent improper fraction will have the same denominator, 7. The numerator is the product of the denominator and the whole number, added to the numerator, $7 \cdot 5 + 2 = 35 + 2 = 37$.

$$\frac{37}{7}$$

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ADDING AND SUBTRACTING MIXED NUMBERS

■ 1. Simplify the expression.

$$2\frac{1}{2}+3\frac{1}{6}$$

Solution:

To add the mixed numbers, add the whole numbers separately from the fractions.

$$2+3+\frac{1}{2}+\frac{1}{6}$$

To add the fractions, we'll need to start by making the denominators the same, which we can do by multiplying the first fraction by $3/3$.

$$5+\frac{1}{2}\left(\frac{3}{3}\right)+\frac{1}{6}$$

$$5+\frac{1\cdot 3}{2\cdot 3}+\frac{1}{6}$$

$$5+\frac{3}{6}+\frac{1}{6}$$

Then add the numerators, while keeping the denominator the same.

$$5+\frac{3+1}{6}$$

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$$5\frac{4}{6}$$

$$5\frac{2}{3}$$

■ 2. Simplify the expression.

$$5\frac{2}{3}+1\frac{1}{12}$$

Solution:

To add the mixed numbers, add the whole numbers separately from the fractions.

$$5+1+\frac{2}{3}+\frac{1}{12}$$

To add the fractions, we'll need to start by making the denominators the same, which we can do by multiplying the first fraction by $4/4$.

$$6+\frac{2}{3}\left(\frac{4}{4}\right)+\frac{1}{12}$$

$$6+\frac{2\cdot 4}{3\cdot 4}+\frac{1}{12}$$

$$6+\frac{8}{12}+\frac{1}{12}$$

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Then add the numerators, while keeping the denominator the same.

$$6+\frac{8+1}{12}$$

$$6\frac{9}{12}$$

$$6\frac{3}{4}$$

3. Simplify the expression.

$$10\frac{3}{7}+12\frac{1}{8}$$

Solution:

To add the mixed numbers, add the whole numbers separately from the fractions.

$$10+12+\frac{3}{7}+\frac{1}{8}$$

To add the fractions, we'll need to start by making the denominators the same, which we can do by multiplying the first fraction by $8/8$ and the second fraction by $7/7$.

$$22+\frac{3}{7}\left(\frac{8}{8}\right)+\frac{1}{8}\left(\frac{7}{7}\right)$$

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$$22+\frac{3\cdot 8}{7\cdot 8}+\frac{1\cdot 7}{8\cdot 7}$$

$$22+\frac{24}{56}+\frac{7}{56}$$

Then add the numerators, while keeping the denominator the same.

$$22+\frac{24+7}{56}$$

$$22\frac{31}{56}$$

■ 4. Simplify the expression.

$$8\frac{7}{8}-2\frac{1}{8}$$

Solution:

To find the difference of the mixed numbers, subtract the whole numbers separately from the fractions, but add those differences.

$$8-2+\frac{7}{8}-\frac{1}{8}$$

The fractions already have the same denominator, so we can find the difference directly, which we'll do by finding the difference of the numerators while keeping the denominator the same.

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$$6+\frac{7-1}{8}$$

$$6\frac{6}{8}$$

$$6\frac{3}{4}$$

■ 5. Simplify the expression.

$$7\frac{4}{5}-6\frac{1}{15}$$

Solution:

To find the difference of the mixed numbers, subtract the whole numbers separately from the fractions, but add those differences.

$$7-6+\frac{4}{5}-\frac{1}{15}$$

To find the difference of the fractions, we'll need to start by making the denominators the same, which we can do by multiplying the first fraction by $3/3$.

$$1+\frac{4}{5}\left(\frac{3}{3}\right)-\frac{1}{15}$$

$$1+\frac{4\cdot 3}{5\cdot 3}-\frac{1}{15}$$

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$$1+\frac{12}{15}-\frac{1}{15}$$

To find the difference of the fractions, we'll find the difference of the numerators while keeping the denominator the same.

$$1+\frac{12-1}{15}$$

$$1\frac{11}{15}$$

■ 6. Simplify the expression.

$$15\frac{1}{2}-11\frac{1}{4}$$

Solution:

To find the difference of the mixed numbers, subtract the whole numbers separately from the fractions, but add those differences.

$$15-11+\frac{1}{2}-\frac{1}{4}$$

To find the difference of the fractions, we'll need to start by making the denominators the same, which we can do by multiplying the first fraction by $2/2$.

$$4+\frac{1}{2}\left(\frac{2}{2}\right)-\frac{1}{4}$$

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$$4+\frac{1\cdot 2}{2\cdot 2}-\frac{1}{4}$$

$$4+\frac{2}{4}-\frac{1}{4}$$

To find the difference of the fractions, we'll find the difference of the numerators while keeping the denominator the same.

$$4+\frac{2-1}{4}$$

$$4\frac{1}{4}$$

■ 7. Joey and Alex are both solving the following problem.

$$2\frac{1}{3}+1\frac{3}{5}$$

Joey takes $2+1=3$ and then takes

$$\frac{1}{3}+\frac{3}{5}=\frac{14}{15}$$

Then he adds them together to get

$$3\frac{14}{15}$$

Alex decides to change both into improper fractions before adding. He gets

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$$2\frac{1}{3} = \frac{7}{3} \text{ and } 1\frac{3}{5} = \frac{8}{5}$$

Then she finds common denominators and adds them together to get

$$\frac{59}{15}$$

Who solved this problem correctly?

Solution:

They both solved it correctly. We get Joey's answer when we convert Alex's answer to a mixed number, and we get Alex's answer when we convert Joey's answer to an improper fraction.

■ 8. Simplify the expression.

$$3\frac{2}{5} + \frac{3}{10} - 2\frac{3}{5}$$

Solution:

We'll start by adding the first two mixed numbers. To add the mixed numbers, add the whole numbers separately from the fractions.

$$3 + \frac{2}{5} + \frac{3}{10} - 2\frac{3}{5}$$

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To add the fractions, we'll need to start by making the denominators the same, which we can do by multiplying the first fraction by $2/2$.

$$3+\frac{2}{5}\left(\frac{2}{2}\right)+\frac{3}{10}-2\frac{3}{5}$$

$$3+\frac{2\cdot 2}{5\cdot 2}+\frac{3}{10}-2\frac{3}{5}$$

$$3+\frac{4}{10}+\frac{3}{10}-2\frac{3}{5}$$

Then add the numerators, while keeping the denominator the same.

$$3+\frac{4+3}{10}-2\frac{3}{5}$$

$$3\frac{7}{10}-2\frac{3}{5}$$

To find the difference of the mixed numbers, subtract the whole numbers separately from the fractions, but add those differences.

$$3-2+\frac{7}{10}-\frac{3}{5}$$

To find the difference of the fractions, we'll need to start by making the denominators the same, which we can do by multiplying the second fraction by $2/2$.

$$1+\frac{7}{10}-\frac{3}{5}\left(\frac{2}{2}\right)$$

$$1+\frac{7}{10}-\frac{3\cdot 2}{5\cdot 2}$$

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$$1+\frac{7}{10}-\frac{6}{10}$$

To find the difference of the fractions, we'll find the difference of the numerators while keeping the denominator the same.

$$1+\frac{7-6}{10}$$

$$1\frac{1}{10}$$

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MULTIPLYING AND DIVIDING MIXED NUMBERS

■ 1. Complete the statement.

When you multiply and divide mixed numbers, you need to change the mixed numbers into _____ fractions before you do the multiplication or division.

Solution:

improper

■ 2. Simplify the expression.

$$2\frac{4}{5} \cdot 4\frac{1}{2}$$

Solution:

Convert both mixed numbers to improper fractions.

$$\frac{5 \cdot 2 + 4}{5} \cdot \frac{2 \cdot 4 + 1}{2}$$

$$\frac{14}{5} \cdot \frac{9}{2}$$

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To multiply the fractions, multiply the numerators to find the new numerator, and multiply the denominators to find the new denominator.

$$\frac{14 \cdot 9}{5 \cdot 2}$$

$$\frac{126}{10}$$

$$12\frac{6}{10}$$

$$12\frac{3}{5}$$

■ 3. Simplify the expression.

$$3\frac{3}{7} \cdot 1\frac{1}{7}$$

Solution:

Convert both mixed numbers to improper fractions.

$$\frac{7 \cdot 3 + 3}{7} \cdot \frac{7 \cdot 1 + 1}{7}$$

$$\frac{24}{7} \cdot \frac{8}{7}$$

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To multiply the fractions, multiply the numerators to find the new numerator, and multiply the denominators to find the new denominator.

$$\frac{24 \cdot 8}{7 \cdot 7}$$

$$\frac{192}{49}$$

$$3\frac{45}{49}$$

■ 4. Simplify the expression.

$$5\frac{1}{5} \cdot 2\frac{2}{3}$$

Solution:

Convert both mixed numbers to improper fractions.

$$\frac{5 \cdot 5 + 1}{5} \cdot \frac{3 \cdot 2 + 2}{3}$$

$$\frac{26}{5} \cdot \frac{8}{3}$$

To multiply the fractions, multiply the numerators to find the new numerator, and multiply the denominators to find the new denominator.

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$$\frac{26 \cdot 8}{5 \cdot 3}$$

$$\frac{208}{15}$$

$$13\frac{13}{15}$$

5. Simplify the expression.

$$2\frac{3}{4} \div 5\frac{1}{8}$$

Solution:

Convert both mixed numbers to improper fractions.

$$\frac{4 \cdot 2 + 3}{4} \div \frac{8 \cdot 5 + 1}{8}$$

$$\frac{11}{4} \div \frac{41}{8}$$

Convert the division to multiplication by flipping the second fraction.

$$\frac{11}{4} \cdot \frac{8}{41}$$

To multiply the fractions, multiply the numerators to find the new numerator, and multiply the denominators to find the new denominator.

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$$\frac{11 \cdot 8}{4 \cdot 41}$$

$$\frac{88}{164}$$

$$\frac{22}{41}$$

6. Simplify the expression.

$$4\frac{5}{9} \div 2\frac{1}{4}$$

Solution:

Convert both mixed numbers to improper fractions.

$$\frac{9 \cdot 4 + 5}{9} \div \frac{4 \cdot 2 + 1}{4}$$

$$\frac{41}{9} \div \frac{9}{4}$$

Convert the division to multiplication by flipping the second fraction.

$$\frac{41}{9} \cdot \frac{4}{9}$$

To multiply the fractions, multiply the numerators to find the new numerator, and multiply the denominators to find the new denominator.

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$$\frac{41 \cdot 4}{9 \cdot 9}$$

$$\frac{164}{81}$$

$$2\frac{2}{81}$$

7. Simplify the expression.

$$1\frac{4}{5} \div 3\frac{3}{8}$$

Solution:

Convert both mixed numbers to improper fractions.

$$\frac{5 \cdot 1 + 4}{5} \div \frac{8 \cdot 3 + 3}{8}$$

$$\frac{9}{5} \div \frac{27}{8}$$

Convert the division to multiplication by flipping the second fraction.

$$\frac{9}{5} \cdot \frac{8}{27}$$

To multiply the fractions, multiply the numerators to find the new numerator, and multiply the denominators to find the new denominator.

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$$\frac{9 \cdot 8}{5 \cdot 27}$$

$$\frac{72}{135}$$

$$\frac{8}{15}$$

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RELATIONSHIPS OF NUMBERS

■ 1. Which fraction is larger?

$$\frac{1}{8} \text{ or } \frac{1}{6}$$

Solution:

When the numerators are equivalent, the larger fraction is the one with the smaller denominator, so $1/6$ is the larger fraction.

■ 2. Which fraction is smaller?

$$\frac{3}{7} \text{ or } \frac{3}{8}$$

Solution:

When the numerators are equivalent, the smaller fraction is the one with the larger denominator, so $3/8$ is the smaller fraction.

■ 3. Find the number that's halfway between -3 and 5.

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Solution:

We can find the distance between these numbers by subtracting the smaller number from the larger number.

$$5-(-3)$$

$$5+3$$

$$8$$

We need half of this total distance, so we'll divide 8 by 2 to find that half of the distance is $8/2=4$. Now to find the value that's halfway between -3 and 5, we'll add this half distance to -3.

$$-3+4$$

$$1$$

■ 4. Find the number that's halfway between -5 and 2.

Solution:

We can find the distance between these numbers by subtracting the smaller number from the larger number.

$$2-(-5)$$

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$$2+5$$

$$7$$

We need half of this total distance, so we'll divide 7 by 2 to find that half of the distance is $7/2$. Now to find the value that's halfway between -5 and 2, we'll add this half distance to -5.

$$-5+\frac{7}{2}$$

$$-5\left(\frac{2}{2}\right)+\frac{7}{2}$$

$$-\frac{10}{2}+\frac{7}{2}$$

$$\frac{7-10}{2}$$

$$-\frac{3}{2}$$

■ 5. Find the fraction halfway between $1/2$ and $2/5$.

Solution:

If we find a common denominator among the fractions, $1/2$ converts to $5/10$, and $2/5$ converts to $4/10$. Now that the denominators are equivalent,

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we know the smaller fraction is the one with the smaller numerator, so $4/10$ is smaller than $5/10$.

We can now find the distance between these fractions by subtracting the smaller fraction from the larger fraction.

$$\frac{5}{10} - \frac{4}{10}$$

$$\frac{5-4}{10}$$

$$\frac{1}{10}$$

We need half of this total distance, so we'll divide $1/10$ by 2 to find that half of the distance is $1/20$. Now to find the fraction that's halfway between $4/10$ and $5/10$, we'll add this half distance to $4/10$.

$$\frac{4}{10} + \frac{1}{20}$$

$$\frac{4}{10}\left(\frac{2}{2}\right) + \frac{1}{20}$$

$$\frac{4 \cdot 2}{10 \cdot 2} + \frac{1}{20}$$

$$\frac{8}{20} + \frac{1}{20}$$

$$\frac{8+1}{20}$$

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$$\frac{9}{20}$$

■ 6. Find the fraction halfway between $1/10$ and $8/13$.

Solution:

If we find a common denominator among the fractions, $1/10$ converts to $13/130$, and $8/13$ converts to $80/130$. Now that the denominators are equivalent, we know the smaller fraction is the one with the smaller numerator, so $13/130$ is smaller than $80/130$.

We can now find the distance between these fractions by subtracting the smaller fraction from the larger fraction.

$$\frac{80}{130} - \frac{13}{130}$$

$$\frac{80 - 13}{130}$$

$$\frac{67}{130}$$

We need half of this total distance, so we'll divide $67/130$ by 2 to find that half of the distance is $67/260$. Now to find the fraction that's halfway between $13/130$ and $80/130$, we'll add this half distance to $13/130$.

$$\frac{13}{130} + \frac{67}{260}$$

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$$\frac{13}{130}\left(\frac{2}{2}\right)+\frac{67}{260}$$

$$\frac{13\cdot 2}{130\cdot 2}+\frac{67}{260}$$

$$\frac{26}{260}+\frac{67}{260}$$

$$\frac{93}{260}$$

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ADDING MIXED MEASURES

■ 1. Add the mixed measures.

4 seconds, 11 minutes, 3 hours, 35 minutes, 56 minutes, 35 seconds

Solution:

Organize and combine the measures we've been given.

3 hours, $11 + 35 + 56$ minutes, $4 + 35$ seconds

3 hours, 102 minutes, 39 seconds

Since we have more than 60 minutes, we want to convert some of those minutes into hours.

3 hours, $60 + 42$ minutes, 39 seconds

3 hours, 1 hour, 42 minutes, 39 seconds

$3 + 1$ hours, 42 minutes, 39 seconds

4 hours, 42 minutes, 39 seconds

■ 2. Add the mixed measures.

34 inches, 2 yards, 5 feet, 8 inches, 13 feet, 1 yard

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Solution:

Organize and combine the measures we've been given.

$2 + 1$ yards, $5 + 13$ feet, $34 + 8$ inches

3 yards, 18 feet, 42 inches

Since we have more than 12 inches, we want to convert some of those inches into feet.

3 yards, 18 feet, $36 + 6$ inches

3 yards, 18 feet, 3 feet, 6 inches

3 yards, $18 + 3$ feet, 6 inches

3 yards, 21 feet, 6 inches

Since we have more than 3 feet, we want to convert some of those feet into yards.

3 yards, 7 yards, 6 inches

10 yards, 6 inches

■ 3. Add the mixed measures.

25 seconds, 1 hour, 15 minutes, 45 seconds, 22 minutes

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Solution:

Organize and combine the measures we've been given.

1 hour, $15 + 22$ minutes, $25 + 45$ seconds

1 hour, 37 minutes, 70 seconds

Since we have more than 60 seconds, we want to convert some of those seconds into minutes.

1 hour, 37 minutes, $60 + 10$ seconds

1 hour, 37 minutes, 1 minute, 10 seconds

1 hour, $37 + 1$ minutes, 10 seconds

1 hour, 38 minutes, 10 seconds

■ 4. Add the mixed measures.

13 inches, 45 feet, 35 inches, 27 feet, 9 yards

Solution:

Organize and combine the measures we've been given.

9 yards, $45 + 27$ feet, $13 + 35$ inches

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9 yards, 72 feet, 48 inches

Since we have more than 12 inches, we want to convert some of those inches into feet.

9 yards, 72 feet, 4 feet

9 yards, $72 + 4$ feet

9 yards, 76 feet

Since we have more than 3 feet, we want to convert some of those feet into yards.

9 yards, $75 + 1$ feet

9 yards, 25 yards, 1 foot

34 yards, 1 foot

■ 5. How many inches are in 5 feet?

Solution:

There are 12 inches in 1 foot, so

$$\frac{12 \text{ inches}}{1 \text{ foot}} = \frac{x}{5 \text{ feet}}$$

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$$x = \frac{12 \text{ inches}}{1 \text{ foot}}(5 \text{ feet})$$

$$x = 60 \text{ inches}$$

■ 6. How many inches are in 2 yards?

Solution:

There are 3 feet in 1 yard, so

$$\frac{3 \text{ feet}}{1 \text{ yard}} = \frac{x}{2 \text{ yards}}$$

$$x = \frac{3 \text{ feet}}{1 \text{ yard}}(2 \text{ yards})$$

$$x = 6 \text{ feet}$$

There are 12 inches in 1 foot, so

$$\frac{12 \text{ inches}}{1 \text{ foot}} = \frac{x}{6 \text{ feet}}$$

$$x = \frac{12 \text{ inches}}{1 \text{ foot}}(6 \text{ feet})$$

$$x = 72 \text{ inches}$$

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■ 7. How much of a yard is 1 foot?

Solution:

There are 3 feet in 1 yard, so

$$\frac{3 \text{ feet}}{1 \text{ yard}} = \frac{1 \text{ foot}}{x}$$

$$x = (1 \text{ foot})\frac{1 \text{ yard}}{3 \text{ feet}}$$

$$x = \frac{1}{3} \text{ yards}$$

■ 8. How many seconds are in 1 hour?

Solution:

There are 60 seconds in 1 minute, and 60 minutes in 1 hour, so there are $(60)(60) = 3{,}600$ seconds in 1 hour.

■ 9. How much of an hour is 1 minute?

Solution:

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There 60 minutes in 1 hour, so

$$\frac{60 \text{ minutes}}{1 \text{ hour}} = \frac{1 \text{ minute}}{x}$$

$$x = (1 \text{ minute})\frac{1 \text{ hour}}{60 \text{ minutes}}$$

$$x = \frac{1}{60} \text{ hour}$$

■ 10. How many seconds are in 5 minutes?

Solution:

There 60 seconds in 1 minute, so there are $(60)(5) = 300$ seconds in 5 minutes.

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