Unit multipliers

Unit multipliers are what we use to convert one set of units to another. A really easy example is using a unit multiplier to convert feet to inches. If we want to write 4 feet in terms of inches, we use a unit multiplier and write

$$4 \text{ feet} \cdot \frac{12 \text{ inches}}{1 \text{ foot}}$$

We cancel the units of "feet," and then simplify in order to get a value that's only in terms of inches.

$$4 \cdot \frac{12 \text{ inches}}{1}$$

$$\frac{48 \text{ inches}}{1}$$

$$48 \text{ inches}$$

Then we can say that a distance of 4 feet is the same as a distance of 48 inches. So what did we just do? Well, the fraction $(12 \text{ inches})/(1 \text{ feet})$ is the unit multiplier. Since we want to change our units from feet to inches, essentially what we're doing is using a fraction to relate feet to inches (we know that there are 12 inches in 1 foot), and then multiplying by that fraction in order to cancel the units we want to get rid of, and keep only the units we want.

When we set up our unit multiplier fraction, we put in the denominator the units we want to cancel and we put in the numerator the units we want to

keep. That way, the units we want to get rid of will cancel out, and we'll be left with only the units we want to keep.

Let's do another example so we can start to get the hang of this.

Example

Convert the value from meters to centimeters. Hint: there are 100 centimeters in 1 meter.

3.5 meters

Let's realize first that we want to get rid of the meters (the units we've been given), and we want to end up with only units of centimeters. We've been told that there are 100 centimeters in 1 meter, which means we need to use either the unit multiplier

$$\frac{1 \text{ meter}}{100 \text{ centimeters}}$$

or the unit multiplier

$$\frac{100 \text{ centimeters}}{1 \text{ meter}}$$

Remember that we put on the top (the numerator) the units we want to keep, and we put on the bottom (the denominator) the units we want to get rid of. Since we want to keep centimeters and get rid of meters, we'll put centimeters on the top and meters on the bottom, and use

$$\frac{100 \text{ centimeters}}{1 \text{ meter}}$$

Let's multiply this unit multiplier by the 3.5 meters we were given originally.

$$3.5 \text{ meters} \cdot \frac{100 \text{ centimeters}}{1 \text{ meter}}$$

Cancel the units of "meters," and then simplify in order to get a value that's only in terms of centimeters.

$$3.5 \cdot \frac{100 \text{ centimeters}}{1}$$

$$\frac{350 \text{ centimeters}}{1}$$

$$350 \text{ centimeters}$$

Then we can say that a distance of 3.5 meters is the same as a distance of 350 centimeters.

Let's do another example, but this time we'll have to do multiple conversions.

Example

Convert the value from inches to yards. Hint: there are 12 inches in 1 foot and 3 feet in 1 yard.

$$288 \text{ inches}$$

If we think ahead about our plan a little bit, we realize that what we want to do is convert first from inches to feet, and then from feet to yards. We can do this in two separate steps or all at once.

First, we know we want to get rid of the units of inches and keep the units of feet. So we can use a unit multiplier and write

$$288 \text{ inches} \cdot \frac{1 \text{ foot}}{12 \text{ inches}}$$

This will cancel out inches and leave us with feet. But we also want to convert feet to yards. To do that, we'll multiply by another unit multiplier - one that has feet in the denominator (since that's what we want to cancel) and yards in the numerator (since that's what we want to keep).

$$288 \text{ inches} \cdot \frac{1 \text{ foot}}{12 \text{ inches}} \cdot \frac{1 \text{ yard}}{3 \text{ feet}}$$

Now we'll start canceling units. Let's first cancel out the inches.

$$288 \cdot \frac{1 \text{ foot}}{12} \cdot \frac{1 \text{ yard}}{3 \text{ foot}}$$

Now we'll cancel out the feet.

$$288 \cdot \frac{1}{12} \cdot \frac{1 \text{ yard}}{3}$$

All we have left is yards, which is exactly what we want. So we'll just do the multiplication to simplify.

$$\frac{288 \cdot 1 \cdot 1}{12 \cdot 3} \text{ yards}$$

Combining the 1's with the 288, we have

$$\frac{288}{12 \cdot 3} \text{ yards}$$

To simplify this, we'll first divide 288 and 3 by 3.

$$\frac{288 \div 3}{12 \cdot (3 \div 3)} \text{ yards}$$

$$\frac{96}{12 \cdot 1} \text{ yards}$$

$$\frac{96}{12} \text{ yards}$$

$$8 \text{ yards}$$