Trees 6B

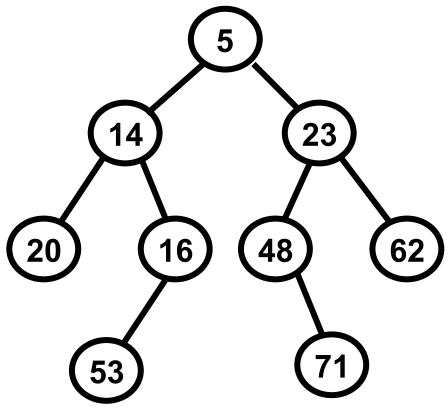
Heaps & Other Trees

Heap

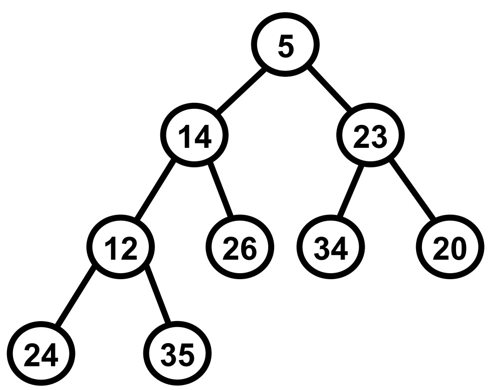
- A min-heap is a binary tree such that
  - the data contained in each node is less than (or equal to) the data in that node's children.
  - the binary tree is complete

  - A max-heap is a binary tree such that
    - the data contained in each node is greater than (or equal to) the data in that node's children.
    - the binary tree is complete

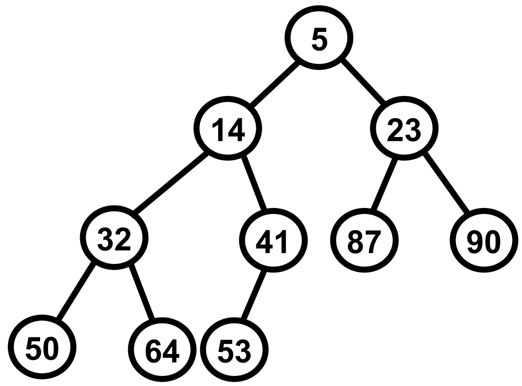
Is it a min-heap?



Is it a min-heap?



Is it a min-heap?



Using heaps

What are min-heaps good for?
(What operation is extremely fast when using a min-heap?)

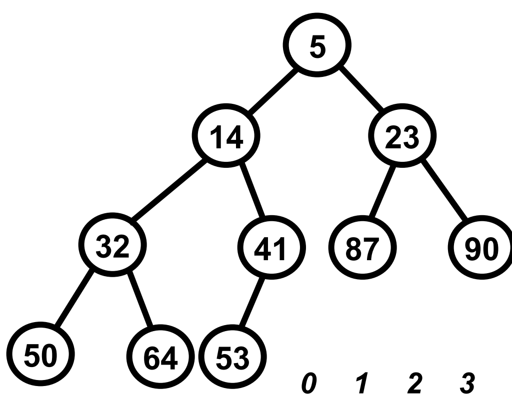
The difference in level between any two leaves in a heap is at most what?

Storage of a heap

- Use an array to hold the data.
- Store the root in position 1.
  - We won’t use index 0 for this implementation.
- For any node in position i,
  - its left child (if any) is in position 2i
  - its right child (if any) is in position 2i + 1
  - its parent (if any) is in position i/2

    (use integer division)

Storage of a heap



For node at i:
Left child is at 2i
Right child is at 2i12
Parent is at ⌊i/2⌋

0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
	5	**14**	**23**	**32**	**41**	**87**	**90**	**50**	**64**	**53**

Inserting into a min-heap

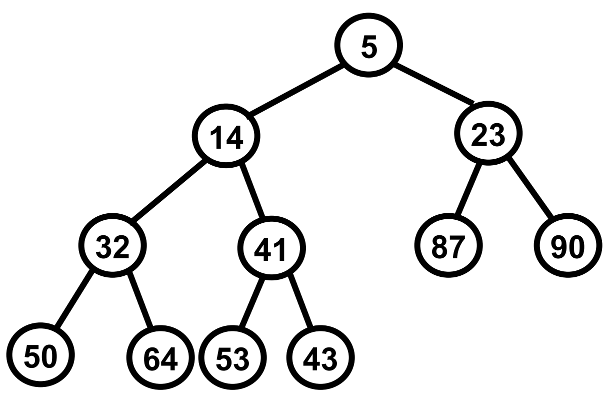
- Place the new element in the next available position in the array.
- Compare the new element with its parent. If the new element is smaller, than swap it with its parent.
- Continue this process until either

  - the new element's parent is smaller than or equal to the new element, or

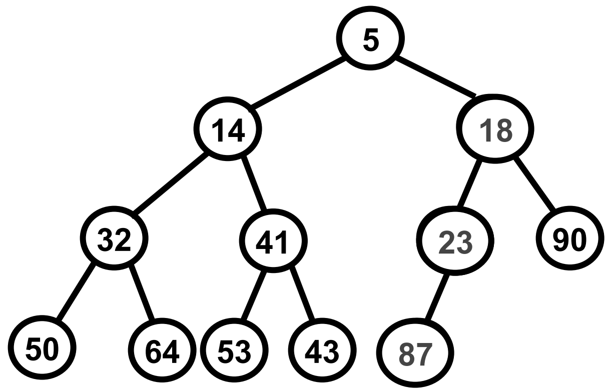
  - the new element reaches the root (index 0 of the array)

Inserting into a min-heap

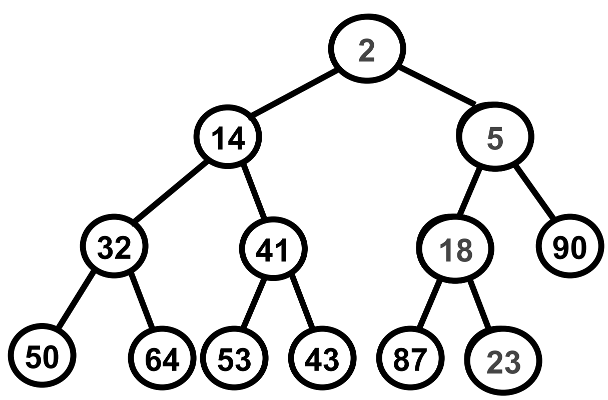
Insert 43



Inserting into a min-heap
Insert 18



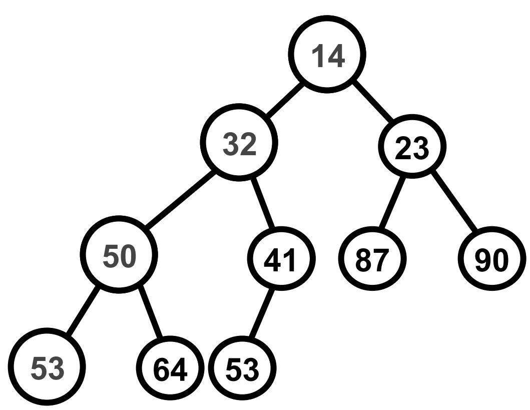
Inserting into a min-heap
Insert 2



Removing from a heap

- Place the root element in a variable to return later.
- Remove the last element in the deepest level and move it to the root.
- While the moved element has a value greater than at least one of its children, swap this value with the smaller-valued child.
- Return the original root that was saved.

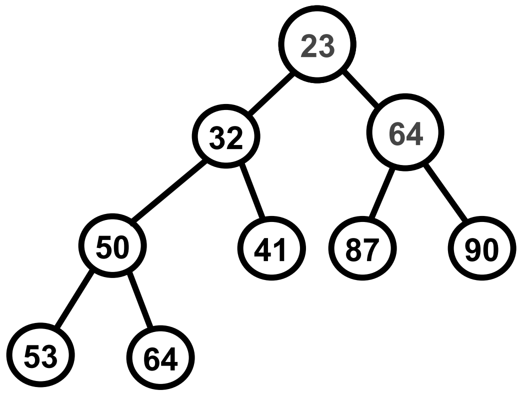
Removing from a min-heap
Remove min



returnValue 5

Removing from a min-heap

Remove min



returnValue 14

Efficiency of heaps

Assume the heap has N nodes.

Then the heap has $\lceil \log_2(N+1) \rceil$ levels.

- Insert

  Since the insert swaps at most once per level, the order of complexity of insert is O(log N)

- Remove

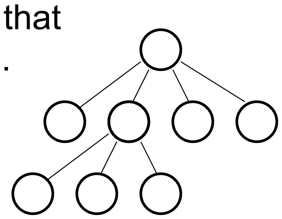
  Since the remove swaps at most once per level, the order of complexity of remove is also O(log N)

Priority Queues

- A priority queue PQ is like an ordinary queue except that we can only remove the “maximum” element at any given time (not the “front” element necessarily).
- If we use an array to implement a PQ,
  enqueue is O(______)   dequeue is O(______)
- If we use a sorted array to implement a PQ
  enqueue is O(______)   dequeue is O(______)
- If we use a max-heap to implement a PQ
  enqueue is O(______)   dequeue is O(______)

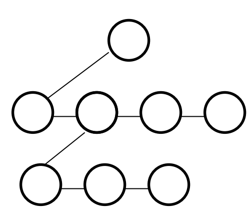
General Trees

- A general tree consists of nodes that can have any number of children.



- Implementation using a binary tree:

  Each node has 2 fields:
  firstChild, nextSibling



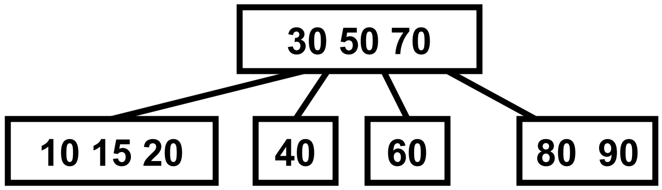
Balanced Trees

- Binary search trees can become quite unbalanced, with some branches being much longer than others.
  - Searches can become O(n) operations
- These variants allow for searching while keeping the tree (nearly) balanced:
  - 2-3-4 trees
  - Red-black trees

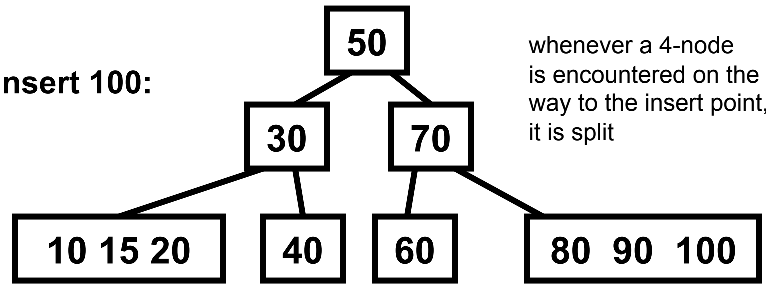
2-3-4-trees

- A 2-3-4 Tree is a tree in which each internal node (nonleaf) has two, three, or four children, and all leaves are at the same depth.
  - A node with 2 children is called a "2-node".
  - A node with 3 children is called a "3-node".
  - A node with 4 children is called a "4-node".

Sample 2-3-4-tree



Insert 100:

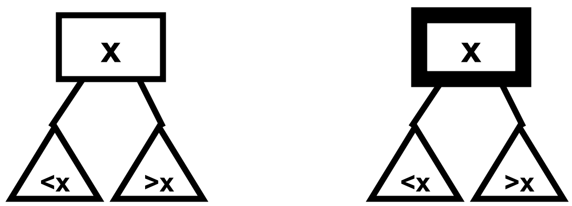


whenever a 4-node is encountered on the way to the insert point, it is split

Red-Black Trees

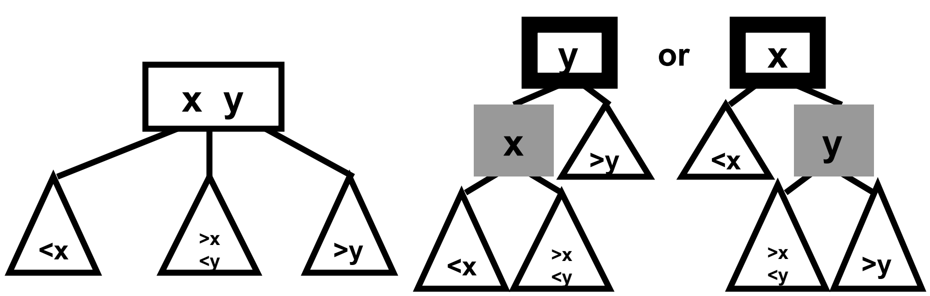
- A red-black tree has the advantages of a 2-3-4 tree but requires less storage.
- Red-black tree rules:
  - Every node is colored either red or black.
  - The root is black.
  - If a node is red, its children must be black.
  - Every path from a node to a null link must contain the same number of black nodes.

2-3-4 Trees vs. Red-Black Trees



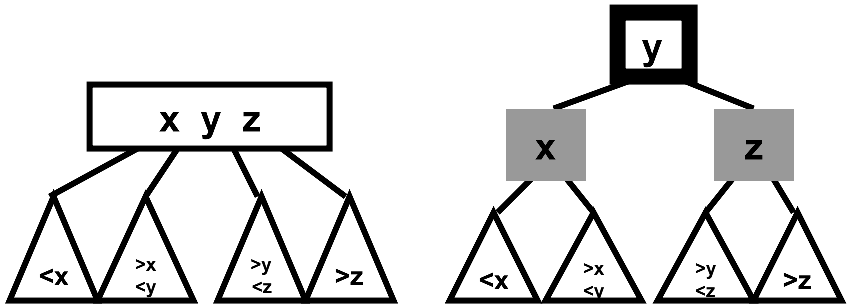
"2-node" in a 2-3-4 tree   equivalent red-black tree configuration

2-3-4 Trees vs. Red-Black Trees



"3-node" in a 2-3-4 tree   equivalent red-black tree configurations

2-3-4 Trees vs. Red-Black Trees

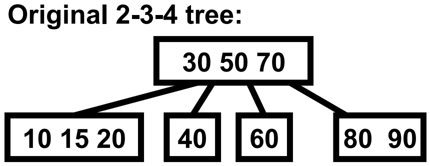


"4-node" in a 2-3-4 tree

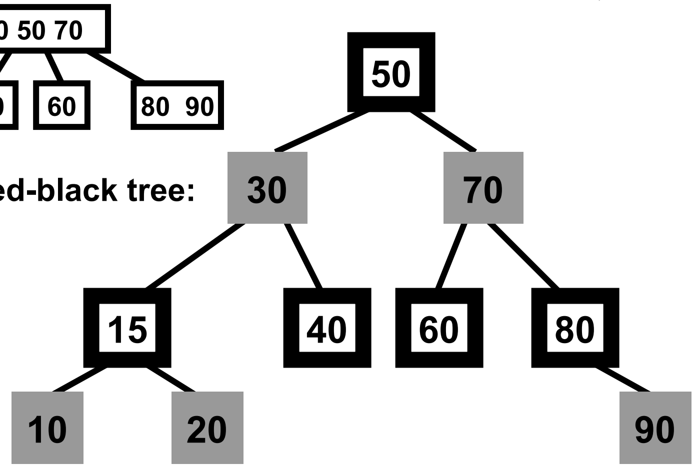
equivalent red-black tree configuration

Sample Red-Black Tree

Original 2-3-4 tree:

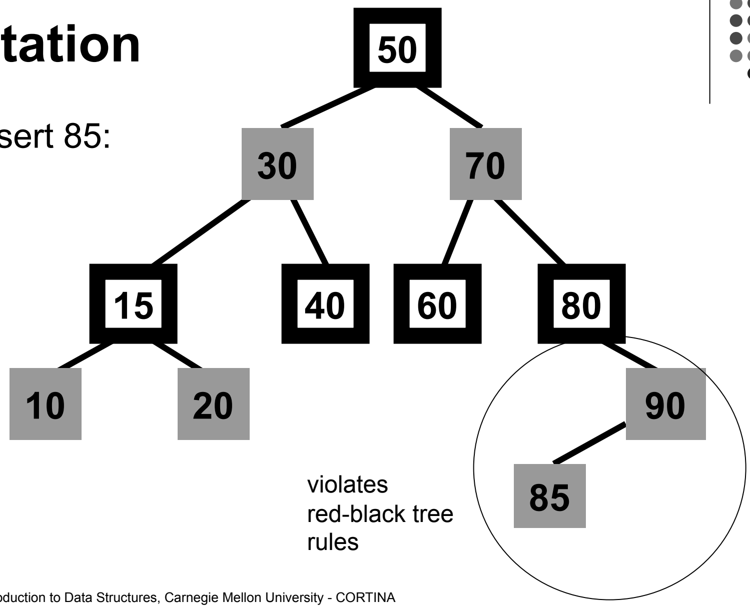


Equivalent red-black tree:

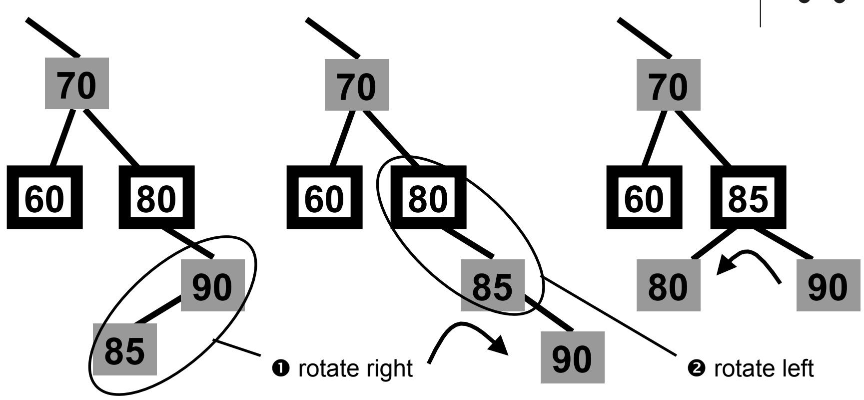


Rotation

- Insert 85:



Rotation (cont'd)



See textbook for additional cases where rotation is required.

Additional Self-Balancing Trees

- AVL Trees
- 2-3 Trees
- B-Trees
- Splay Trees
  - (co-invented by Prof. Danny Sleator at CMU)