Power rule for exponents

In this section we're going to dive into the power rule for exponents. Think about this one as the "power to a power" rule. In other words, what happens when we raise an exponential expression (a base raised to some power) to another power (when one exponential expression becomes the base of another exponential expression)?

The trick to these problems is to get back to the basics of exponents and remember that the exponent simply tells us how many times to multiply the base by itself. So if we're given

$$(3^2)^3$$

it means that we're supposed to multiply 3^2 by itself 3 times, since 3^2 is the base and 3 is the exponent. So we could rewrite the expression as

$$(3^2)(3^2)(3^2)$$

From here, we remember that when we multiply exponential expressions with the same base, we add the exponents. Since the base of each factor of 3^2 is 3, all our bases are the same, so we just add the exponents and we get

$$3^{2+2+2}$$

$$3^6$$

But this is the long way of expanding the expression $(3^2)^3$. What we actually want to do is use the power rule for exponents. The power rule tells us that when we raise an exponential expression to a power, we can

just multiply the exponents. In $(3^2)^3$, the first exponent is 2 and the second exponent is 3. The power rule tells us that we can just multiply those exponents and get $2 \cdot 3 = 6$, which means that

$$(3^2)^3 = 3^6$$

Let's do some examples.

Example

Use the power rule for exponents to simplify the expression.

$$(2^2)^4$$

To use the power rule, we just multiply the exponents.

$$2^{2 \cdot 4}$$

$$2^8$$

$$256$$

We'll try an example with a negative exponent.

Example

Use the power rule for exponents to simplify the expression.

$$(3^2)^{-2}$$

Here we have a base 3 that's positive, so it doesn't matter that one of the exponents is negative. We can apply the power rule, and multiply the exponents.

$$(3^2)^{-2}$$

$$3^{2(-2)}$$

$$3^{-4}$$

To simplify the rest of the way, remember that we can make the exponent positive instead of negative by moving the resulting expression to the denominator of a fraction and setting the numerator to 1.

$$\frac{1}{3^4}$$

$$\frac{1}{81}$$