Splay trees

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Binary search trees

- A *binary search tree* is a binary tree storing entries (i.e., key-value pairs) at its internal nodes satisfying:

  If u, v, and w are nodes such that u is in the left subtree of v and w is in the right subtree of v, then

  $$key(u) < key(v) < key(w).$$

- External nodes do not store entries.

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Performance

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- *This can be improved by keeping the binary tree "balanced".*

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- A *splay* tree is a self-balancing binary search tree in which *splaying* is performed after every search, insertion and removal.
- While splay trees are often “balanced”, there is no explicit bound on the height of the tree (unlike AVL trees).
- With a splay tree of n entries, search, insertion and removal all take $O(\log n)$-time *on average*.

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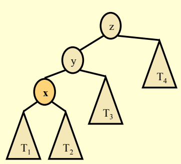
- *Splaying an internal node x* means raising x up to the root through a sequence of rotations.
- There are three types of rotation: *zig-zig*, *zig-zag* and *zig*.

Zig-zig rotation

x is a left (resp. right) child of y, and y is a left (resp. right) child of z.



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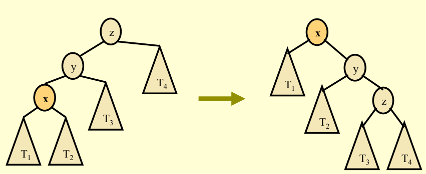
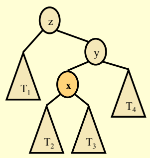


Zig-zag rotation

x is a left (resp. right) child of *y*, and *y* is a right (resp. left) child of *z*.



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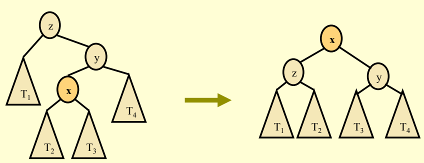

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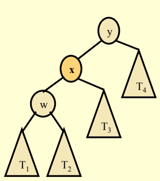

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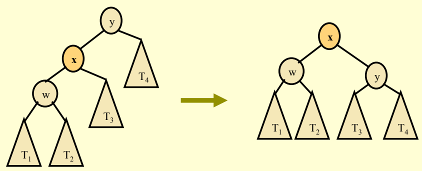

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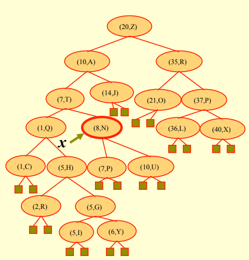
- remove(k): Splay the parent of the node removed (not necessarily the node that had the removed entry).

Splaying

Example. Splaying $x = (8, \mathrm{N})$.



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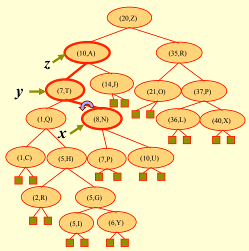

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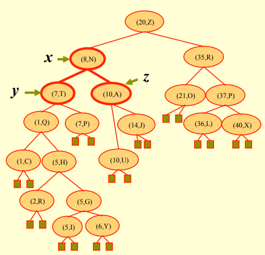

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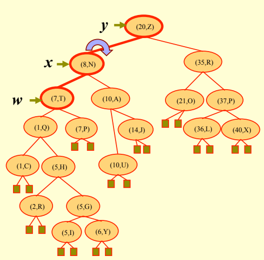

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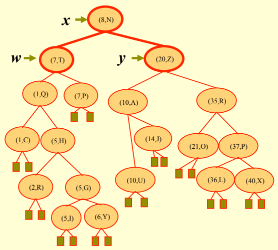

Splaying Visualization

https://www.cs.usfca.edu/~galles/visualization/SplayTree.html

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- Starting from the empty map, consider performing a sequence of m search/ insertion/removal operations on n entries. The total running time of these m operations is $O(m \log n)$.
- Thus, *on average*, each operation takes $O(\log n)$ time.