5) $\left(A(x_i) \circ P(x_i, y_j) \right) \circ Q(y_j, z_k)$

$= \max_j \left(\max_i \left(A(x_i) \cdot P(x_i, y_j) \right) \cdot Q(y_j, z_k) \right)$

$= \max_i \left(\max_j \left(\max_i \left(A(x_i) \cdot P(x_i, y_j) \right) \circ Q(y_j, z_k) \right) \right)$

$\hookrightarrow \max_i (x_i) = \max_i \left(\max_i (x_i) \right)$

$= \max_i \left(\max_j \left(A(x_i) \cdot P(x_i, y_j) \cdot Q(y_j, z_k) \right) \right)$

$\hookrightarrow \max_i \left(\max_i (x_i) \right) = \max_i (x_i)$

Since A does not include j, take it outside.

$= \max_i \left(A(x_i) \circ \max_j \left(P(x_i, y_j) \cdot Q(y_j, z_k) \right) \right)$

$= A(x_i) \circ \left(P(x_i, y_j) \circ Q(y_j, z_k) \right)$

This implies that the max product composition is associative. It can be used to parallelize the sequential control that uses max product composition.