Coordinate transformations

From the LPF spacecraft reference frame, the colatitude and azimuth sky angles, defined as the angle of the impact vector from the +z direction and the angle in the x-y plane from the +x axis respectively, were converted to Cartesian Science Spacecraft Control Frame (O_ss_CF) coordinates with the origin at the SCM center of mass, the z-axis pointing towards the solar array, the x-axis parallel to the LTP center-line with positive towards Test Mass 1, and the y-axis forming an orthogonal set[1].

The attitude quaternions (q) for the mission were in the form q = [q_1 q_2 q_3 q_4] where the vector or imaginary part is represented by [q_1 q_2 q_3] and the scalar or real part is represented by q_4. Any ASU delivered quaternion represents the attitude of O_ss_CF with respect to the Earth Centered Inertial J2000 (true of date) reference frame, such that the positive x-axis in the direction of the first point of Aries on 01/01/2000 at 12 noon, the z-axis in the direction of the Earth's North Pole on 01/01/2000 at 12 noon, and the y-axis forming an orthogonal set[1].

To adjust for the attitude of LPF throughout the mission and to rotate into the inertial J2000 reference frame, the following transformation was used:

$$v_{J2000} = R_i^{\ J2000}(q_i^{\ J2000})v_{\mathrm{O\_ss\_CF}}$$

such that $v_{\mathrm{O\_ss\_CF}}$ represents the O_ss_CF coordinates and v_{J2000} represents the transformed Cartesian O_ss_CF coordinates in the J2000 frame. $R_i^{\ J2000}(q_i^{\ J2000})$ represents the attitude quaternion rotation matrix from from O_ss_CF to the inertial J2000 frame, defined as:

$$\begin{bmatrix} {q_4}^2 + {q_1}^2 - {q_2}^2 - {q_3}^2 & 2q_1q_2 - 2q_4q_3 & 2q_1q_3 + 2q_4q_2 \\ 2q_1q_2 + 2q_4q_3 & {q_4}^2 - {q_1}^2 + {q_2}^2 - {q_3}^2 & 2q_2q_3 - 2q_4q_1 \\ 2q_1q_3 - 2q_4q_2 & 2q_2q_3 + 2q_4q_1 & {q_4}^2 - {q_1}^2 - {q_2}^2 + {q_3}^2 \end{bmatrix}$$

where $q_i^{\ J2000}$ represents the attitude quaternion at time i from O_ss_CF to the inertial J2000 frame and $q_i^{\ J2000}$ = [q_1 q_2 q_3 q_4]. To reverse the rotation, the conjugate of the attitude quaternion ($q_i^{\ SC}$) is used to create the matrix, where $q_i^{\ SC}$= [$-q_1$ $-q_2$ $-q_3$ q_4] if $q_i^{\ J2000}$= [q_1 q_2 q_3 q_4]. Since attitude quaternions by definition are unit quaternions, the conjugate is constructed by negating the vector component of the quaternion, as seen above[2].

As defined, the inertial J2000 frame is oriented with respect to the Earth's North Pole, not with the ecliptic plane. Thus, the equatorial J2000 coordinates were rotated into ecliptic coordinates through the following matrix multiplication:

$$v_{ecliptic} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\epsilon) & \sin(\epsilon) \\ 0 & -\sin(\epsilon) & \cos(\epsilon) \end{bmatrix} v_{equatorial}$$

such that ϵ is the obliquity of the ecliptic, $v_{equatorial}$ is v_{J2000} from the prior rotation, and $v_{ecliptic}$ is the resultant coordinates for the inertial J2000 frame but now oriented about the ecliptic plane. The Cartesian coordinates in the ecliptic inertial Earth-centered coordinates were then transformed back into spherical coordinates, where 0° latitude is the ecliptic plane, and longitude represents the rotation of LPF about the sun on this plane. To make the ecliptic plane 0° latitude, 90° was subtracted from the ecliptic inertial J2000 latitude values. To center the sun at the origin by transforming the frame from inertial to rotating, the true anomaly of LPF about the sun (v) was then offset. The following equation was used to find the true anomaly offset:

$$v = 2 \arctan\left(\sqrt{\frac{1+e}{1-e}} \tan\left(\frac{E}{2}\right)\right)$$

where e is the ellipticity of LPF's orbit and E is the eccentric anomaly, which can be determined through Kepler's Equation:

$$M = E - e \sin(E)$$

where M is the mean anomaly. M is found through the following:

$$rad_{offset} = 2\pi \frac{t - t_0}{T}$$

where t is the time of the event, t_0 is 1 Jan, 2000 at 12 noon, and T is the orbit period. t_0 was chosen to correct for any spatial changes over time in the inertial J2000 frame, while also fulfilling the assumption that t_0 is at periapsis of LPF's orbit about the sun. M is the remainder of $\frac{rad_{offset}}{2\pi}$. Kepler's relation between M and E is a transcendental equation and cannot be solved analytically. For small e, E can be approximated for through expanding it into a converging series by Lagrange's Theorem[3]. The following relations were used to solve for E:

$$E_0 = \{M + e\sin(M) + \frac{e^2}{2}\sin(2M) + \frac{e^3}{6}\sin(3M) + \cdots\}$$

$$M_0 = E_0 - e\sin(E_0)$$

$$E = E_0 + \frac{M - M_0}{1 - e\cos(E_0)}$$

Once E—and consequently v—was found, the longitude values in the ecliptic inertial J2000 frame were offset by v. To center the sun at 0° longitude, the longitude values were then offset using the same scaling in the previous rad_{offset} calculation, where t is 21 March, 2010 at 12 noon, t_0 is 1 Jan, 2000 at 12 noon, and T is the orbit period. The implemented offset once again was the remainder of $\frac{rad_{offset}}{2\pi}$.

Thus, the final reference frame centered the sun at 0° latitude and longitude. The final ranges in latitude and longitude were $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $[-\pi, \pi]$, respectively. Impacts in orbit with LPF in the longitude-latitude frame were at $(-\frac{\pi}{2}, 0)$, while impacts in retrograde were $(\frac{\pi}{2}, 0)$.

**ok, so this has bothered me for a while: in the README file from Petr, he said the sun was at +90º lon and prograde is 0º, but when I spoke to Diego he said the sun should be at 0º and prograde at -90º. I'm not sure about this disconnect, but hopefully it was a simple typo (either way it's simple to correct; just changes interpretation ☹).

*if you have any questions, feel free to email me at npagane1@jhu.edu and sorry in advance for any confusing notation or poorly developed reasoning for rotations.

References

1. *I'm not sure, but Jake sent me an email with "6 ATTITUDE CONTROL INTERFACE 6.1 Definitions" that defined the constructed LPF spacecraft frame*
2. *http://www.chrobotics.com/library/understanding-quaternions* (quaternion matrix)
3. *http://articles.adsabs.harvard.edu//full/1895PA......3..136M/0000138.000.html* (true anomaly offset)

*sorry for not using proper citations, but I'm sure you won't need the references for these things; but I certainly did.