Thoughts on Micrometeoroid Impact Model

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Introduction

This note is meant to convey some thoughts about how to improve the realism of the LPF micrometeorite impact model that a group of us (Ira, Tyson, John B) are studying.

Impact "waveforms"

One thing we need is a simple "waveform" model that maps the parameters of the micrometeorite impact that we want to measure with the signals measured by the test masses. The impact is modeled by 7 parameters: impact time, transferred momentum (scalar), impact direction (parameterized as 2 sky angles), and impact location (3 parameters). Note that the search strategy will use a geometrical model of the spacecraft to reduce the number of parameters describing impact location from 3 to 2 (e.g. the impact has to occur on the surface of the spacecraft). Here we write the waveform in terms of the 3 parameters.

We also assume that the search is performed in a data product known as "g", which is basically the equivalent free-body acceleration that the spacecraft would have had in the absence of forces applied by the control system. To give a specific example, if we're operating in a drag-free mode and experience an impact, the acceleration of the spacecraft caused by that impact will be compensated by applying a force to the spacecraft using the microthrusters. The "g" product will include that commanded force divided by the mass of the spacecraft as part of the "equivalent free-body acceleration".

With two test masses, there are two sources for g, named g_1 and g_2. It's simplest to write the dynamics equations down for the center of mass of the spacecraft. For the linear degrees of freedom (x,y,z) the impact will have waveform that depends only on impact time (tau), transferred momentum (P), direction (unit vector e-hat), and spacecraft mass (M):

$$\vec{h}_{x,B}(t) = P\, M^{-1} \delta(t-\tau)\hat{e}$$

For the angular DoFs, we have additional dependence on the impact location (r) and the know, fixed parameters corresponding to the location of the center of mass within the spacecraft (rB) and the moment of the inertia tensor for the spacecraft for rotations about the spacecraft center of mass (I):

$$\vec{h}_{\theta,B}(t) = P\, \mathbf{I}^{-1} \delta(t-\tau)(\vec{r}-\vec{r}_B) \times \hat{e}$$

Because the test masses aren't at the spacecraft center of mass, we need to transform these from the spacecraft body frame to the TM housing frames.

$$\vec{h}_{x,TM}(t) = \vec{h}_{x,B} + (\vec{r}_{TM} - \vec{r}_B) \times \vec{h}_{\theta,B}$$
$$\vec{h}_{\theta,TM}(t) = \vec{h}_{\theta,B}$$

This is an approximation for small angular accelerations. We could try and take things like centripetal acceleration, etc. into account but I don't think we need to.

Noise sources & correlations

In our simple model, we have two basic types of noise sources. Force noise, which mainly comes from the thruster system and sensor noise, which comes from either the capacitive inertial sensors or the interferometer for those DoFs that are measured by the interferometer.

Force Noise

There are two thruster systems on board LPF: the European cold-gas thrusters and the US colloidal micro-Newton thrusters (CMNTs). The plan is to run the cold gas thrusters during LTP ops (first 90 days) and the CMNTs during DRS ops (second 90 days). In each case you get N (N=6 for cold gas, N= 8 for CMNTs) individual thrusters with uncorrelated noise of roughly the same amplitude. To convert this to noise in our g1 and g2 signals, we basically use the same model we used to model the waveform for the impacts with the difference being that we use the known thruster directions and locations instead of the unknown impact directions and locations. As before, we need to move through the spacecraft center of mass (B-frame) to get the accelerations correct.

For one individual thruster denoted by *i*, the contribution in the B-frame would be:

$$\vec{n}_{Fx,i,B}(f) = S_{F,i}(f)\, M^{-1}\hat{e}_{T,i}$$
$$\vec{n}_{F\theta,i,B}(f) = S_{F,i}(f)\, \mathbf{I}^{-1}(\vec{r}_{T,i} - \vec{r}_B) \times \hat{e}_{T,i}$$

where $S_{F,i}$ is the force noise in thruster i, $e_{T,i}$ is the direction of thruster i, and $r_{T,i}$ is the location of thruster i. We expect the thruster noise to be white, with an amplitude spectral density of roughly 0.1 uN/rtHz. As with the impacts, this needs to get translated into the housing frame

$$\vec{n}_{Fx,i}(f) = \vec{n}_{Fx,i,B}(f) + (\vec{r}_{TM} - \vec{r}_B) \times \vec{n}_{F\theta,i,B}$$
$$\vec{n}_{F\theta,i}(f) = \vec{n}_{F\theta,i,B}(f)$$

If we want to include any noise parameter in our search, we could either (A) assume all thrusters have the same noise and fit one overall noise amplitude parameter or (B) fit an amplitude parameter for each thruster and try and identify any noisy thrusters.

Sensor Noise

There are two systems for sensing the TM positions relative to the spacecraft: a capacitive system that measures all 6 DoFs and a interferometric one that measures 3 DoFs (longitudinal position in x, and angles about y and z which are known as eta and phi respectively). For all of these measurements, the noise spectrum is approximately white and independent for each channel (there might be some correlations between some of the linear and angular readouts in the capacitive system that use common electrodes but I think we can safely ignore that for now). The equivalent acceleration noise is obtained by taking two time derivatives

$$\vec{n}_x(f) = \vec{S}_x(f)(2\pi f)^2$$

where, Sx are the noise spectral densities for the various DoFs. We will get better information on these noise levels as the mission progresses so we could choose to either make these variable or fixed. Again, we'd expect that TM1 and TM2 levels would be independent. I think we decided that there would be no reason to form the g1 and g2 signals using the capacitive data for DoFs where we have IFO data.