Title

April 12, 2017

1 Introduction

2 Modeling LISA Pathfinder data

Assuming the data $\tilde{d}$ is a signal $\tilde{h}$ plus additive noise $\tilde{n}$, and $\tilde{n}$ is zero-mean Gaussian distributed, the likelihood for data $\tilde{d}$ and parameters $\boldsymbol{\lambda}$

$$p(\tilde{d}|\boldsymbol{\lambda}) = \frac{1}{\det C_{ij}^{-1}} e^{-\frac{1}{2}\sum_{ij}\tilde{r}_i C_{ij}^{-1}\tilde{r}_j} \tag{1}$$

where the residual $\tilde{r} = \tilde{d} - \tilde{h}(\boldsymbol{\lambda})$ and $\tilde{h}(\boldsymbol{\lambda})$ is the modeled LISA Pathfinder response to an impact with parameters $\boldsymbol{\lambda}$ and the one-sided noise correlation matrix $C_{ij} \equiv \langle \tilde{n}_i \tilde{n}_j \rangle$. The indices i and j sum over different data channels, i.e. the 6 degrees of freedom $i := (x, y, z, \theta, \eta, \phi)$ where θ is a rotation about x, η is a rotation about y, and ϕ is a rotation about z. The x, y, z triad's origin is

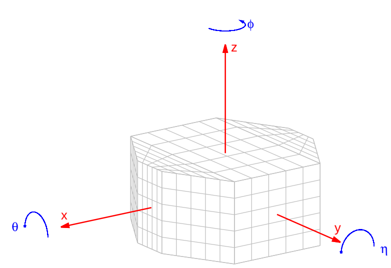

Figure 1: Schematic showing the naming conventions for the six kinematic degrees of freedom used in this work.

the center of the GRS housing with $\hat{z}$ in the sunward facing direction, $\hat{x}$ in the interferometric sensing direction (towards the other GRS) and $\hat{y}$ completing the triad.

2.1 Impact Response

To characterize micrometeorite strikes in the data we construct a model which maps parameters $\boldsymbol{\lambda}$ of the impact to the signal measured $\tilde{h}$ at the test mass. The impact is modeled by six parameters: the impact time t_0, the magnitude of the transferred momentum P, the incident impact direction $\hat{\boldsymbol{k}}$ parameterized by a polar and azimuthal angle δ and α, and the cartesian impact location $\boldsymbol{r}$ constrained to be on the surface of spacecraft (reducing the three dimensional vector to two degrees of freedom).

The search is performed on a data product known as "g", which is the equivalent free-body acceleration that the spacecraft would have had in the absence of forces applied by the control system. To give a specific example, if the spacecraft is operating in a drag-free mode and experience an impact, the acceleration of the spacecraft caused by that impact will be compensated by applying a force to the spacecraft using the thrusters. The g product will include that commanded force divided by the mass of the spacecraft as part of the equivalent free-body acceleration.

With two test masses on LISA Pathfinder, there are two sources for g, named g_1 and g_2. To derive the LISA Pathfinder response in the g data products to micrometeorite impacts we begin with the dynamics equations at the spacecraft center of mass C.

For the linear degrees of freedom ($\boldsymbol{x} = \{x, y, z\}$) the signal at the center of mass $\boldsymbol{h}_{\boldsymbol{x}}^C$ will depend only on t_0, P, $\hat{\boldsymbol{k}}$, and the spacecraft mass m:

$$\boldsymbol{h}_{\boldsymbol{x}}^C(t) = \delta(t - t_0) P m^{-1} \hat{k}. \tag{2}$$

The angular degrees of freedom ($\boldsymbol{\omega} = \{\theta, \eta, \phi\}$) introduce dependence on the impact location, the location of the center of mass $\boldsymbol{r}^C$, and the moment of inertia tensor $\boldsymbol{I}$ of the spacecraft for rotations about the center of mass

$$\boldsymbol{h}_{\boldsymbol{\omega}}^C(t) = \delta(t - t_0) P \boldsymbol{I}^{-1} (\boldsymbol{r} - \boldsymbol{r}^C) \times \hat{k}. \tag{3}$$

The coordinate system in which the various spacecraft locations are defined is known as the "body frame" with origin located at the bottom center of the spacecraft.

Because the test masses are not at the spacecraft center of mass, the spacecraft dynamics must be transformed into the test-mass housing frame with origin at $\boldsymbol{r}^G$. Assuming small angular accelerations, the signals from micrometeorite impacts in the linear and angular degrees of freedom for the g data product are

$$\begin{aligned} \boldsymbol{h}_{\boldsymbol{x}}(t) &= \boldsymbol{h}_{\boldsymbol{x}}^C + (\boldsymbol{r}^G - \boldsymbol{r}^C) \times \boldsymbol{h}_{\boldsymbol{\omega}}^C \\ \boldsymbol{h}_{\boldsymbol{\omega}}(t) &= \boldsymbol{h}_{\boldsymbol{\omega}}^C. \end{aligned} \tag{4}$$

2.2 Thruster Noise

The noise measured in one of the D degrees of freedom labeled by latin indices $\tilde{n}_i$ has contributions from the noise in each thruster labeled by greek indices $\tilde{n}^\mu$.

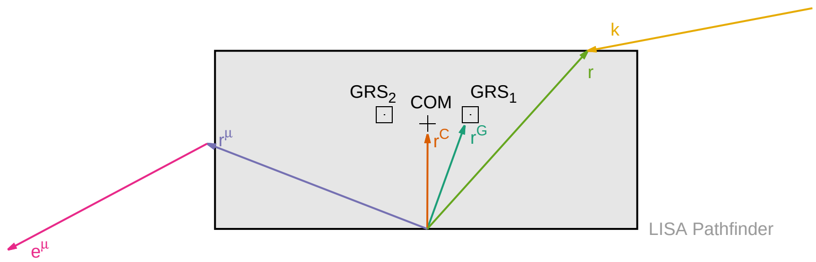

Figure 2: Schematic showing the different vector quantities that enter into the derivation of the detector response to impacts and thruster noise.

For N thrusters the mapping from the noise in the thrusters to the noise in a data channel i can be calculated by a $D \times N$ matrix M:

$$\tilde{n}_i = M_{i\mu} \tilde{n}^\mu \tag{5}$$

with an implied sum μ over the N thrusters. M depends on the thruster location and orientation relative to the spacecraft center of mass and GRS. The noise correlation matrix is

$$\begin{aligned} C_{ij} &= \langle M_{i\mu} \tilde{n}^\mu M_{j\nu} \tilde{n}^\nu \rangle \\ &= M_{i\mu} M_{j\nu} \langle \tilde{n}^\mu \tilde{n}^\nu \rangle . \end{aligned} \tag{6}$$

Assuming independent noise in each thruster head (possibly a bad assumption...) $\langle \tilde{n}^\mu \tilde{n}^\nu \rangle = \frac{T}{2} S_n^\mu \delta_{\mu\nu}$ where S_n^μ is the noise power spectral density of thruster μ and $\delta_{\mu\nu}$ is the Kronecker delta. Finally the noise correlation matrix is computed by

$$C_{ij} = \frac{T}{2} \sum_\mu^N M_{i\mu} M_{j\mu} S_n^\mu . \tag{7}$$

The derivation of the matrix elements $M_{i\mu}$ follows closely to Equations 2-4. For each thruster μ the contribution to the overall noise in each degree of freedom is found by replacing the impact momentum P with the noise in the thruster $\tilde{n}^\mu$; substituting the impact direction $\hat{\boldsymbol{k}}$ with the thrust unit vector $\hat{\boldsymbol{e}}^\mu$; and swapping the impact location $\boldsymbol{r}$ with the thruster location $\boldsymbol{r}^\mu$.

Considering these substitutions and Eq. 5, and defining $\boldsymbol{\tau}^\mu \equiv (\boldsymbol{r}^\mu - \boldsymbol{r}^C) \times \hat{\boldsymbol{e}}^\mu$, $\boldsymbol{\alpha}^\mu \equiv \boldsymbol{I}^{-1} \boldsymbol{\tau}^\mu$, then

$$\begin{aligned} M_{x\mu} &= \hat{e}_x^\mu / m + [(\boldsymbol{r}^G - \boldsymbol{r}^C) \times \boldsymbol{\alpha}^\mu]_x \\ M_{y\mu} &= \hat{e}_y^\mu / m + [(\boldsymbol{r}^G - \boldsymbol{r}^C) \times \boldsymbol{\alpha}^\mu]_y \\ M_{z\mu} &= \hat{e}_z^\mu / m + [(\boldsymbol{r}^G - \boldsymbol{r}^C) \times \boldsymbol{\alpha}^\mu]_z \\ M_{\theta\mu} &= \alpha_x^\mu \end{aligned}$$

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\begin{aligned}
M_{\eta\mu} &= \alpha_y^\mu \\
M_{\phi\mu} &= \alpha_z^\mu
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\tag{8}
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