

2P2

$$\|z\| = r^2$$

$$\min_{x=(s,r)} z$$

$$p^i T p^i - 2(p^i)^T s + s^T s \leq z, \quad i=1, \dots, m$$

$$L(s, z, \lambda) = z + \sum \lambda_i (-z + p^i T p^i - 2(p^i)^T s + s^T s)$$

WKT

$$\textcircled{1} \nabla_{s, z} L = \begin{bmatrix} \sum \lambda_i (-2p^i + 2s) \\ 1 - \sum \lambda_i \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\textcircled{2} \nabla_{\lambda} L = m \cdot (p - z + p^i T p^i - 2(p^i)^T s + s^T s) \leq 0$$

$$\textcircled{3} \lambda_i (p - z + p^i T p^i - 2(p^i)^T s + s^T s) = 0$$

$$\textcircled{4} \lambda_i \geq 0$$

2.1

$$\sum \lambda_i = 1$$

$$\sum \lambda_i (-2p^i + 2s) = 0$$

$$2s \sum \lambda_i - 2 \sum \lambda_i p^i = 0$$

$$2s = 2 \sum \lambda_i p^i$$

$$s = \sum \lambda_i p^i = Py$$

$$y = \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_m \end{bmatrix}$$

$$\max_{\lambda} \min_{s, z} L(s, z, \lambda) = \max_{\lambda} \left(z + \sum \lambda_i (-z + p^i T p^i - 2(p^i)^T s + s^T s) \right)$$

$$\max_{\lambda} \min_{s, z} \left(z(1 - \sum \lambda_i) + \sum \lambda_i (p^i T p^i - 2(p^i)^T s + s^T s) \right)$$

$$\max_{\lambda} \min_{s \in \mathbb{R}} \left(\sum \lambda_i (p_i^T p_i - 2p_i^T s + s^T s) \right)$$

$$\max_{\lambda} \min_{s \in \mathbb{R}} \left(\sum \lambda_i p_i^T p_i - 2 \sum \lambda_i p_i^T s + \underbrace{\left(\sum \lambda_i \right)}_{=1} s^T s \right) \quad \square$$

$$\max_{\lambda} \min_{s \in \mathbb{R}} \left(\sum \lambda_i p_i^T p_i - 2s \left(\sum \lambda_i p_i^T \right) + s^T s \right)$$

$$\max_{\lambda} \min_{s \in \mathbb{R}} \left(\sum \lambda_i p_i^T p_i - s^T s \right) = \max_{\lambda} \min_{s \in \mathbb{R}} \left(\sum \lambda_i p_i^T p_i - y^T P^T P y \right)$$

(nie ma $s \in \mathbb{R}$)

$$= \max_{\lambda} \left(-y^T P^T P y + \sum \lambda_i p_i^T p_i \right) \quad \square$$

$$\underline{s = \sum \lambda_i p_i} \quad (\text{było wae s nie})$$

final = 2 = r² (zadanie 2P2 i 2D ma to same wyniki)