# 1 High-level Problem Description

The objective of the solution is to prevent and solve congestion in the MV networks by providing some type of signal to the aggregator. This signal can be price, firm/non-firm capacity contracts, etc. The problem can be divided into two layers, upper-level layer (DSO operation) and a lower-level layer (set of aggregators). By approaching the problem with bi-level modelling approach, both, DSO and aggregators can be modelled independently and can maximize their own objectives while satisfying each others constraints.

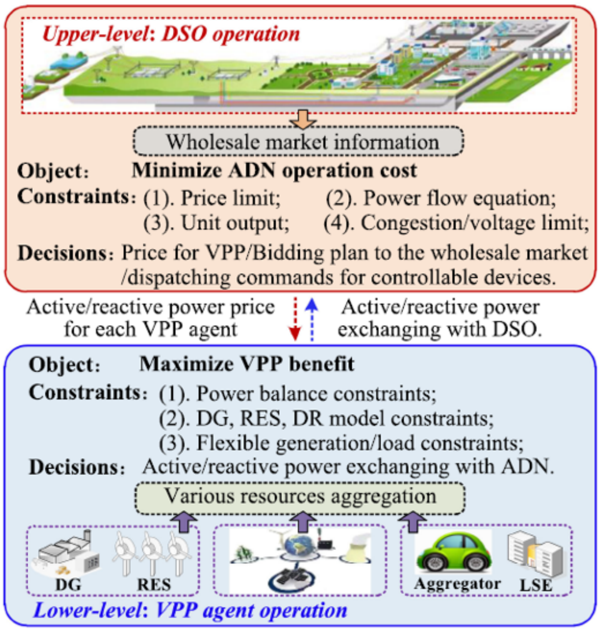


Figure 1: Graphical overview of the problem [1].

The objective of the DSO is to prevent congestion in the medium voltage network. This can be indirectly achieved by minimizing losses or operation costs in the network. DSO is subjected to a number of network constraints. The network is modeled as a steady-state radial distribution network with a pi-model of a line (including shunt components). Second-order cone approximation is implemented in order to obtain a convex representation of the non-linear system which can be solved with commercial solvers.

Same as DSO, aggregators all have an objective such as to maximize profits, minimize discomfort or minimize costs, etc. The advantage of the bi-level approach is that in practice, aggregator does not need to have information about the network. The aggregator decides on the optimal power dispatch (consumption/generation) based on the price signal obtained from DSO and communicates back the aggregated power to be exchanged with the medium voltage network. Aggregators base their decision based on price of energy, their energy needs and utility to consume energy.

A graphical overview of the problem description is depicted in Fig.1. Following sections provide derivation of the upper and lower-level problems.

## 2 Modeling - Upper Level Model

Decision vector $X_{DSO} = \{P_{ij,t}, P_{i,t}^{S}, P_{g,t}^{DG}, P_{s,t}^{EES,dc}, P_{s,t}^{EES,ch}, SOC_{s,t}, u_{s,t}^{EES}, F_{g,t}, u_{s,t}^{DG}, Q_{ij,t}, Q_{i,t}^{S}, Q_{g,t}^{DG}, I_{ij,t}^{sqr}, V_{ij,t}^{sqr}\}$

### Objective Function

$$\begin{aligned} \min_{X_{DSO}} \quad & \Delta T \sum_{t\in\Omega_T} \sum_{ij\in\Omega_l} c_t^{DR} R_{ij,a(d_{ij})} I_{ij,t}^{sqr} \\ \text{s.t.} \quad & (2) - (22), (25) \end{aligned} \tag{1}$$

### Nodal Power Balance

$$\begin{aligned} &\sum_{ij\in\Omega_l|_{j=k}} P_{ij,t} + \sum_{p\in\Omega_{PV}|_{p=k}} P_{p,t}^{PV} + \sum_{\omega\in\Omega_{WT}|_{\omega=k}} P_{\omega,t}^{WT} + \sum_{s\in\Omega_{EES}|_{s=k}} P_{s,t}^{EES,dc} + \sum_{g\in\Omega_{DG}|_{g=k}} P_{g,t}^{DG} \\ &+ P_{i,t}^{S} = P_{i,t}^{D} + \sum_{ij\in\Omega_l|_{i=k}} (P_{ij} + R_{ij,a(d_{ij})} I_{ij,t}^{sqr}) + \sum_{s\in\Omega_{EES}|_{s=k}} P_{s,t}^{EES,ch} \qquad \forall k\in\Omega_b, \forall t\in\Omega_T \end{aligned} \tag{2}$$

$$\begin{aligned} &\sum_{ij\in\Omega_l|_{j=k}} (Q_{ij,t} + \frac{B_{jj,a(d_{jj})}}{2} V_j^{sqr}) + \sum_{p\in\Omega_{PV}|_{p=k}} Q_{p,t}^{PV} + \sum_{\omega\in\Omega_{WT}|_{\omega=k}} Q_{\omega,t}^{WT} + \sum_{g\in\Omega_{DG}|_{g=k}} Q_{g,t}^{DG} \\ &+ Q_{i,t}^{S} = Q_{i,t}^{D} + \sum_{ij\in\Omega_l|_{i=k}} (Q_{ij} + X_{ij,a(d_{ij})} I_{ij,t}^{sqr} + \frac{B_{ii,a(d_{ii})}}{2} V_i^{sqr}) \qquad \forall k\in\Omega_b, \forall t\in\Omega_T \end{aligned} \tag{3}$$

### Network Constraints

$$V_{i,t}^{sqr} - V_{j,t}^{sqr} = 2(R_{ij,a(d_{ij})} P_{ij,t} + X_{ij,a(d_{ij})} Q_{ij,t}) + Z_{ij,a(d_{ij})}^2 I_{ij,t}^{sqr} \qquad \forall ij\in\Omega_l, \forall t\in\Omega_T \tag{4}$$

$$V_{j,t}^{sqr} I_{ij,t}^{sqr} \geq P_{ij,t}^2 + Q_{ij,t}^2 \qquad \forall ij\in\Omega_l, \forall t\in\Omega_T \tag{5}$$

$$\underline{V}^2 \leq V_{i,t}^{sqr} \leq \overline{V}^2 \qquad \forall i\in\Omega_b, \forall t\in\Omega_T \tag{6}$$

$$0 \leq I_{ij,t}^{sqr} \qquad \forall ij\in\Omega_l, \forall t\in\Omega_T \tag{7}$$

$$\begin{aligned} P_{i,t}^{Exch} &= P_{i,t}^{S} - P_{i,t}^{D} + \sum_{p\in\Omega_{PV}|_{p=i}} P_{p,t}^{PV} + \sum_{\omega\in\Omega_{WT}|_{\omega=i}} P_{\omega,t}^{WT} + \sum_{s\in\Omega_{EES}|_{s=i}} (P_{s,t}^{EES,dc} - P_{s,t}^{EES,ch}) \\ &+ \sum_{g\in\Omega_{DG}|_{g=i}} P_{g,t}^{DG} \quad \forall i\in\Omega_b, \forall t\in\Omega_T \end{aligned} \tag{8}$$

$$Q_{i,t}^{Exch} = Q_{i,t}^{S} - Q_{i,t}^{D} + \sum_{p\in\Omega_{PV}|_{p=i}} Q_{p,t}^{PV} + \sum_{\omega\in\Omega_{WT}|_{\omega=i}} Q_{\omega,t}^{WT} + \sum_{g\in\Omega_{DG}|_{g=i}} Q_{g,t}^{DG} \tag{9}$$

$$\forall i\in\Omega_b, \forall t\in\Omega_T \tag{10}$$

$$\left(P_{i,t}^{Exch}\right)^2 + \left(Q_{i,t}^{Exch}\right)^2 \leq \left(\overline{S}_i^{Exch}\right)^2 \quad \forall i \in \Omega_b, \forall t \in \Omega_T \tag{11}$$

- **NOTE**: $V_{j,t}^{sqr} I_{ij,t}^{sqr} \geq P_{ij,t}^2 + Q_{ij,t}^2$ is the second-order cone relaxation.
- **NOTE**: Considering voltage drops in HV/LV, MV/LV transformers and at the low voltage side gives Stedin ±4% room for voltage variations in the medium voltage network.
- **NOTE**: $I_{ij,t}^{sqr}$ does not have upper limit yet.
- **NOTE**: $P_{i,t}^S$ and $Q_{i,t}^S$ are set to 0 for all buses except the slack bus. Since there is no local consumption or generation at the slack bus, the equation (11) breaks down to $\left(P_{i,t}^S\right)^2 + \left(Q_{i,t}^S\right)^2 \leq \left(\overline{S}_i^{Exch}\right)^2 \quad$ if $i$=1, $\forall t \in \Omega_T$
- **NOTE**: Bus $i$ at time $t$ is consuming active and reactive power when $P_{i,t}^{Exch} > 0$ and $Q_{i,t}^{Exch} > 0$ and is injecting active and reactive power into MV network when $P_{i,t}^{Exch} < 0$ and $Q_{i,t}^{Exch} < 0$. Local demand and supply are in balance when $P_{i,t}^{Exch} = 0$ and $Q_{i,t}^{Exch} = 0$.

### Energy Storage System

$$SOC_{s,t} = SOC_{s,t-1} + \frac{\Delta T}{\overline{SOE}_s}(\eta_s P_{s,t}^{ch} - \frac{P_{s,t}^{dc}}{\eta_s}) \qquad \forall s \in \Omega_{EES}, \forall t \in \Omega_T \tag{12}$$

$$\underline{SOC}_s \leq SOC_{s,t} \leq \overline{SOC}_s \qquad \forall s \in \Omega_{EES}, \forall t \in \Omega_T \tag{13}$$

$$P_{s,t}^{EES,ch} \leq \overline{P}_s^{EES} u_{s,t}^{EES} \qquad \forall s \in \Omega_{EES}, \forall t \in \Omega_T \tag{14}$$

$$P_{s,t}^{EES,dc} \leq \overline{P}_s^{EES}(1 - u_{s,t}^{EES}) \qquad \forall s \in \Omega_{EES}, \forall t \in \Omega_T \tag{15}$$

$$u_{s,t}^{EES} \in \{0, 1\} \qquad \forall s \in \Omega_{EES}, \forall t \in \Omega_T \tag{16}$$

### Distributed Generation

$$-P^{DG}_{g,t} \tan\left[\cos^{-1}(pf_g)\right] \leq Q^{DG}_{g,t} \leq P^{DG}_{g,t} \tan\left[\cos^{-1}(pf_g)\right] \qquad \forall g \in \Omega_{DG}, \forall t \in \Omega_T \tag{17}$$

$$\left(P^{DG}_{g,t}\right)^2 + \left(Q^{DG}_{g,t}\right)^2 \leq \left(\overline{S}^{DG}_g\right)^2 \qquad \forall g \in \Omega_{DG}, \forall t \in \Omega_T \tag{18}$$

$$R^d_g \leq P^{DG}_{g,t} - P^{DG}_{g,t-1} \leq R^u_g \qquad \forall g \in \Omega_{DG}, \forall t \in \Omega_T \tag{19}$$

$$F_{g,t} = F_{g,t-1} - \frac{\Delta T P^{DG}_{g,t}}{\eta^f_g \mathrm{FC}_g \mathrm{H}_g} \qquad \forall g \in \Omega_{DG}, \forall t \in \Omega_T \tag{20}$$

$$F_{g,t} \geq \underline{F}_g \qquad \forall g \in \Omega_{DG}, \forall t \in \Omega_T \tag{21}$$

$$u^{DG}_{g,t} \in \{0,1\} \qquad \forall g \in \Omega_{DG}, \forall t \in \Omega_T \tag{22}$$

- **NOTE**: All distributed generation is modelled in the upper level. This should eventually be moved to the lower level since DSO is not allowed to operate RES. Capacity constraint for MV/LV transformers should be moved to lower-level as well.

## 3 Modeling - Lower Level Model ($\forall i \in \Omega_b$)

Decision vector $X_{AGG} = \{P^D_{i,t}, Q^D_{i,t}, P^D_{i,m,t}\}$

### Objective Function

$$\max_{X_{AGG}} \quad \sum_{t \in \Omega_T} \left[U_{i,t}(P^D_{i,t}) - \Delta T c^{DR}_t P^D_{i,t}\right] \tag{23}$$

$$U_{i,t}(P^D_{i,t}) = \Delta T \sum_{m \in \Omega_M} u^{DR}_{i,m,t} P^D_{i,m,t} \tag{24}$$

$$\begin{aligned} \min_{X_{AGG}} \quad & \Delta T \sum_{t \in \Omega_T} \left[c^{DR}_t P^D_{i,t} - \sum_{m \in \Omega_M} u^{DR}_{i,m,t} P^D_{i,m,t}\right] \\ \text{s.t.} \quad & (26) - (32) \end{aligned} \tag{25}$$

### Demand Response

$$P_{i,t}^{D} = \sum_{m\in\Omega_M} P_{i,m,t}^{D} \qquad \forall t \in \Omega_T \quad (\lambda_{i,t}^{1}) \tag{26}$$

$$P_{i,m,t}^{D} \leq \overline{P}_{i,m,t}^{D} \qquad \forall m \in \Omega_M, \forall t \in \Omega_T \quad (\mu_{i,m,t}^{1}) \tag{27}$$

$$P_{i,m,t}^{D} \geq 0 \qquad \forall m \in \Omega_M, \forall t \in \Omega_T \quad (\mu_{i,m,t}^{2}) \tag{28}$$

$$\Delta T \sum_{t\in\Omega_T} P_{i,t}^{D} \geq E_i^{D} \qquad (\mu_i^{3}) \tag{29}$$

$$P_{i,t}^{D} \geq \underline{P}_{i,t}^{D} \qquad \forall t \in \Omega_T \quad (\mu_{i,t}^{4}) \tag{30}$$

$$R_i^{d} \leq P_{i,t}^{D} - P_{i,t-1}^{D} \leq R_i^{u} \qquad \forall t \in \Omega_T \quad (\mu_{i,t}^{5}) \tag{31}$$

$$Q_{i,t}^{D} = P_{i,t}^{D} \tan\left(\arccos\left(pf_i\right)\right) \qquad \forall t \in \Omega_T \quad (\lambda_{i,t}^{2}) \tag{32}$$

- **NOTE**: $\overline{P}_{i,m,t}^{D}$ is initialized as $\frac{1}{4}$ of the estimated load at time $t$. This ultimately does not allow consumer to consume more than is estimated which should probably be changed. The maximum could be set to $\frac{1}{4}$ of the maximum allowable consumption which is constrained by the physical connection capacity. **DONE**
- **NOTE**: $\underline{P}_{i,m,t}^{D}$ is initialized with 0 which has to be changed to some minimum value based on aggregated amount of inflexible load. **DONE**
- **NOTE**: Aggregated load operates at constant power factor ($\tan\left(\arccos\left(pf_i\right)\right) = \zeta_i$).

# 4 KKT - Lower Level Model ($\forall i \in \Omega_b$)

### Lagrangian of Aggregators

$$\begin{aligned}
L_i =& \Delta T \sum_{t=1} \left[ c_t^{DR} P_{i,t}^{D} - \sum_{m=1} u_{i,m,t}^{DR} P_{i,m,t}^{D} \right] + \sum_{t=1} \lambda_{i,t}^{1} \left( P_{i,t}^{D} - \sum_{m=1} P_{i,m,t}^{D} \right) + \sum_{t=1}\sum_{m=1} \mu_{i,m,t}^{1} (P_{i,m,t}^{D} - \overline{P}_{i,m,t}^{D}) \\
&+ \sum_{t=1}\sum_{m=1} \mu_{i,m,t}^{2} (-P_{i,m,t}^{D}) + \mu_i^{3} \left( E_i^{D} - \Delta T \sum_{t=1} P_{i,t}^{D} \right) + \sum_{t=1} \mu_{i,t}^{4} (\underline{P}_{i,t}^{D} - P_{i,t}^{D}) \\
&+ \sum_{t=1} \mu_{i,t}^{5} (P_{i,t}^{D} - P_{i,t-1}^{D} - R_i^{U}) + \sum_{t=1} \mu_{i,t}^{6} (R_i^{D} - P_{i,t}^{D} + P_{i,t-1}^{D}) \\
&+ \sum_{t=1} \lambda_{i,t}^{2} \left[ Q_{i,t}^{D} - P_{i,t}^{D} \tan\left(\arccos\left(pf_i\right)\right) \right]
\end{aligned} \tag{33}$$

**Stationarity**

$$\frac{\partial L_i}{\partial P_{i,t}^D} = \Delta T c_t^{DR} + \lambda_{i,t}^1 - \Delta T \mu_i^3 - \mu_{i,t}^4 + \mu_{i,t}^5 - \mu_{i,t}^6 - \mu_{i,t+1}^5 + \mu_{i,t+1}^6 - \lambda_{i,t}^2 \zeta_i = 0 \qquad \forall t < NT \tag{34}$$

$$\frac{\partial L_i}{\partial P_{i,t}^D} = \Delta T c_t^{DR} + \lambda_{i,t}^1 - \Delta T \mu_i^3 - \mu_{i,t}^4 + \mu_{i,t}^5 - \mu_{i,t}^6 - \lambda_{i,t}^2 \zeta_i = 0 \qquad t = NT \tag{35}$$

$$\frac{\partial L_i}{\partial Q_{i,t}^D} = \lambda_{i,t}^2 = 0 \qquad \forall t \in \Omega_T \tag{36}$$

$$\frac{\partial L_i}{\partial P_{i,m,t}^D} = -\Delta T u_{i,m,t}^{DR} - \lambda_{i,t}^1 + \mu_{i,m,t}^1 - \mu_{i,m,t}^2 = 0 \qquad \forall t \in \Omega_T \tag{37}$$

Where $\tan\left(\arccos\left(pf_i\right)\right) = \zeta_i$

**Primal Feasibility ($h(x) = 0$, $g(x) \leq 0$)**

$$P_{i,t}^D - \sum_{m=1} P_{i,m,t}^D = 0 \qquad \forall t \in \Omega_T \quad (\lambda_{i,t}^1) \tag{38}$$

$$P_{i,m,t}^D - \overline{P}_{i,m,t}^D \leq 0 \qquad \forall m \in \Omega_M, \forall t \in \Omega_T \quad (\mu_{i,m,t}^1) \tag{39}$$

$$-P_{i,m,t}^D \leq 0 \qquad \forall m \in \Omega_M, \forall t \in \Omega_T \quad (\mu_{i,m,t}^2) \tag{40}$$

$$E_i^D - \Delta T \sum_{t=1} P_{i,t}^D \leq 0 \qquad (\mu_i^3) \tag{41}$$

$$\underline{P}_{i,t}^D - P_{i,t}^D \leq 0 \qquad \forall t \in \Omega_T \quad (\mu_{i,t}^4) \tag{42}$$

$$P_{i,t}^D - P_{i,t-1}^D - R_i^u \leq 0 \qquad \forall t \in \Omega_T \quad (\mu_{i,t}^5) \tag{43}$$

$$R_i^d - P_{i,t}^D + P_{i,t-1}^D \leq 0 \qquad \forall t \in \Omega_T \quad (\mu_{i,t}^6) \tag{44}$$

$$Q_{i,t}^D - P_{i,t}^D \tan\left(\arccos\left(pf_i\right)\right) = 0 \qquad \forall t \in \Omega_T \quad (\lambda_{i,t}^2) \tag{45}$$

**Dual Feasibility**

$$\mu_{i,m,t}^1 \geq 0, \quad \mu_{i,m,t}^2 \geq 0 \qquad \forall m \in \Omega_M, \forall t \in \Omega_T \tag{46}$$

$$\mu_i^3 \geq 0 \tag{47}$$

$$\mu_{i,t}^4 \geq 0, \quad \mu_{i,t}^5 \geq 0, \quad \mu_{i,t}^6 \geq 0 \qquad \forall t \in \Omega_T \tag{48}$$

$$\lambda_{i,t}^1, \quad \lambda_{i,t}^2 \quad \text{free} \qquad \forall t \in \Omega_T \tag{49}$$

**Complementary Slackness** $((\mu \geq 0) + (g(x) \leq 0) + (\mu g(x) = 0) \rightarrow \mu(-g(x) \geq 0))$

$$0 \leq \mu \perp -g(x) \geq 0$$

$$0 \leq \mu_{i,m,t}^1 \perp (-P_{i,m,t}^D + \overline{P}_{i,m,t}^D) \geq 0 \qquad \forall m \in \Omega_M, \forall t \in \Omega_T \tag{50}$$

$$0 \leq \mu_{i,m,t}^2 \perp (P_{i,m,t}^D) \geq 0 \qquad \forall m \in \Omega_M, \forall t \in \Omega_T \tag{51}$$

$$0 \leq \mu_i^3 \perp (-E_i^D + \Delta T \sum_{t=1} P_{i,t}^D) \geq 0 \tag{52}$$

$$0 \leq \mu_{i,t}^4 \perp (-\underline{P}_{i,t}^D + P_{i,t}^D) \geq 0 \qquad \forall t \in \Omega_T \tag{53}$$

$$0 \leq \mu_{i,t}^5 \perp (-P_{i,t}^D + P_{i,t-1}^D + R_i^u) \geq 0 \qquad \forall t \in \Omega_T \tag{54}$$

$$0 \leq \mu_{i,t}^6 \perp (-R_i^d + P_{i,t}^D - P_{i,t-1}^D) \geq 0 \qquad \forall t \in \Omega_T \tag{55}$$

**Fortuny-Amat Transformation**

$$0 \leq \mu \perp -g(x) \geq 0$$

$$0 \leq -g(x) \leq Mz$$

$$0 \leq \mu \leq Mz$$

$$z \in \{0, 1\}$$

Equations in primal and dual feasibility capture lower limit of the Fortuny-Amat transformation. This leads to set of equations $\mathbf{g}(\mathbf{x}) \geq -\mathbf{M}\mathbf{z}$ and $\mu \leq \mathbf{M}(\mathbf{1}-\mathbf{z})$.

$$P^D_{i,m,t} - \overline{P}^D_{i,m,t} \geq -M^1 z^1_{i,m,t} \qquad \mu^1_{i,m,t} \leq M^1(1 - z^1_{i,m,t}) \quad \forall m \in \Omega_M, \forall t \in \Omega_T \tag{56}$$

$$-P^D_{i,m,t} \geq -M^2 z^2_{i,m,t} \qquad \mu^2_{i,m,t} \leq M^2(1 - z^2_{i,m,t}) \quad \forall m \in \Omega_M, \forall t \in \Omega_T \tag{57}$$

$$E^D_i - \Delta T \sum_{t=1} P^D_{i,t} \geq -M^3 z^3_i \qquad \mu^3_i \leq M^3(1 - z^3_i) \tag{58}$$

$$\underline{P}^D_{i,t} - P^D_{i,t} \geq -M^4 z^4_{i,t} \qquad \mu^4_{i,t} \leq M^4(1 - z^4_{i,t}) \quad \forall t \in \Omega_T \tag{59}$$

$$P^D_{i,t} - P^D_{i,t-1} - R^u_i \geq -M^5 z^5_{i,t} \qquad \mu^5_{i,t} \leq M^5(1 - z^5_{i,t}) \quad \forall t \in \Omega_T \tag{60}$$

$$R^d_i - P^D_{i,t} + P^D_{i,t-1} \geq -M^6 z^6_{i,t} \qquad \mu^6_{i,t} \leq M^6(1 - z^6_{i,t}) \quad \forall t \in \Omega_T \tag{61}$$

$$z^1_{i,m,t}, \quad z^2_{i,m,t} \qquad \in \{0,1\} \tag{62}$$

$$z^3_i \qquad \in \{0,1\} \tag{63}$$

$$z^4_{i,t}, \quad z^5_{i,t}, \quad z^6_{i,t} \qquad \in \{0,1\} \tag{64}$$

- **NOTE**: All M values are set to $k\frac{S_{base}}{S_{base}}$ where $k = 100$

# 5 Mixed-integer Linear Programming (MILP) Model

Decision vector $X = \{X_{DSO}, X_{AGG}, \lambda^{(1-2)}_{i,t}, \mu^{(1-2)}_{i,m,t}, \mu^3_i, \mu^{(4-6)}_{i,t}, z^{(1-2)}_{i,m,t}, z^3_i, z^{(4-6)}_{i,t}\}$

$$\begin{aligned} \min_X \quad & \Delta T \sum_{t \in \Omega_T} \sum_{ij \in \Omega_l} c^{DR}_t R_{ij,a(d_{ij})} I^{sqr}_{ij,t} \\ \text{s.t.} \quad & (2)-(22), (34)-(49), (56)-(64) \end{aligned} \tag{65}$$

# 6 Nomenclature

## Sets

| | |
|---|---|
| $\Omega_a$ | Set of conductor types |
| $\Omega_b$ | Set of buses (nodes). |
| $\Omega_{DG}$ | Set of nodes with distributed generation. |
| $\Omega_{EES}$ | Set of nodes with energy storage systems. |
| $\Omega_l$ | Set of lines. |
| $\Omega_{PV}$ | Set of nodes with photo-voltaic (PV) systems. |
| $\Omega_T$ | Set of time periods. |
| $\Omega_{WT}$ | Set of nodes with wind parks. |

## Parameters

| | |
|---|---|
| $B_{ij,a(d_{ij})}$ | Shunt susceptance of node i of cable type a with length d [mS]. |
| $R_{ij,a(d_{ij})}$ | Resistance of line ij of cable type a with length d [mΩ]. |
| $X_{ij,a(d_{ij})}$ | Reactance of line ij of cable type a with length d [mΩ]. |
| $Z_{ij,a(d_{ij})}$ | Impedance of line ij of cable type a with length d [mΩ]. |
| $d_{ij}$ | Length of line ij [km]. |
| $R_g^d$ | Ramp-down rate of DG unit at node g [kW/h]. |
| $R_g^u$ | Ramp-up rate of DG unit at node g [kW/h]. |
| $FC_g$ | Fuel capacity of DG unit at node g [$m^3$]. |
| $H_g$ | Calorific value of DG unit at node g [kWh/$m^3$]. |
| $\underline{F}_g$ | Minimum fuel of DG unit at node g [$m^3$]. |
| $pf_g$ | Minimum power factor of DG unit at node g. |
| $P_{p,t}^{PV}$ | Active power supplied by PV at node p in period t [kW]. |
| $P_{i,t}^{D}$ | Active power demand at node i in period t [kW]. |
| $P_{\omega,t}^{WT}$ | Active power supplied by wind park at node $\omega$ in period t [kW]. |
| $\overline{P}_s^{EES}$ | Maximum charging/discharging power of the EES at node s [kW]. |
| $Q_{p,t}^{PV}$ | Reactive power supplied by PV at node p in period t [kvar]. |
| $Q_{\omega,t}^{WT}$ | Reactive power supplied by wind park at node $\omega$ in period t[kvar]. |
| $Q_{i,t}^{D}$ | Reactive power demand a node i in period t [kvar]. |
| $\overline{S}_g^{DG}$ | Maximum capacity of DG at node g [kVA] |
| $\overline{S}_i^{S}$ | Maximum substation capacity at node i [kVA] |
| $\overline{V}$ | Maximum voltage magnitude [V]. |
| $\underline{V}$ | Minimum voltage magnitude [V]. |
| $\overline{SOE}_s$ | Maximum state-of-energy of the EES at node s [kWh]. |
| $\overline{SOC}_s$ | Maximum state-of-charge of the EES at node s [%]. |
| $\underline{SOC}_s$ | Minimum state-of-charge of the EES at node s [%]. |
| $\eta_g^f$ | |
| $\eta_s$ | Charging/discharging of EES at node s. |
| $\Delta T$ | Time step [h]. |

**Variables**

| | |
|---|---|
| $I_{ij,t}^{sqr}$ | Squared current magnitude at line ij in period t $[A^2]$. |
| $V_i^{sqr}$ | Squared voltage magnitude at node i in period t $[V^2]$. |
| $P_{ij,t}$ | Active power flow at line ij in period t [kW]. |
| $P_{i,t}^S$ | Active power injection at node i in period t [kW]. |
| $P_{g,t}^{DG}$ | Active power supplied by DG unit at node g in period t [kW]. |
| $Q_{i,t}^S$ | Reactive power injection at node i in period t [kW]. |
| $Q_{ij,t}$ | Reactive power flow at line ij in period t [kvar]. |
| $Q_{g,t}^{DG}$ | Reactive power supplied by DG unit at node g in period t [kvar]. |
| $P_{s,t}^{EES,ch}$ | Charging power of EES at node s in period t [kW]. |
| $P_{s,t}^{EES,dc}$ | Discharging power of EES at node s in period t [kW]. |
| $SOC_{s,t}$ | Sate-of-charge of EES at node s in period t [kWh]. |
| $u_{s,t}^{EES}$ | Binary variable associated to the charging (1) or discharging (0) operation of EES at node s in period t. |
| $u_{g,t}^{DG}$ | Commitment status of DG unit at node g in period t. |

# References


[1] J. F. Franco, M. J. Rider, and R. Romero, "A mixed-integer quadratically-constrained programming model for the distribution system expansion planning," *International Journal of Electrical Power and Energy Systems*, vol. 62, pp. 265–272, 2014.