1 Modeling - Upper Level Model

Decision vector $X_{DSO} = \{P_{ij,t}, P^S_{i,t}, P^{DG}_{g,t}, P^{EES,dc}_{s,t}, P^{EES,ch}_{s,t}, SOC_{s,t}, u^{EES}_{s,t}, F_{g,t}, u^{DG}_{s,t}, Q_{ij,t}, Q^S_{i,t}, Q^{DG}_{g,t}, I^{sqr}_{ij,t}, V^{sqr}_{ij,t}\}$

Objective Function

$$\begin{aligned} \min_{X_{DSO}} \quad & \Delta T \sum_{t\in\Omega_T} \sum_{ij\in\Omega_l} c^{DR}_t R_{ij,a(d_{ij})} I^{sqr}_{ij,t} \\ \text{s.t.} \quad & (2)-(18),(21) \end{aligned} \tag{1}$$

Nodal Power Balance

$$\begin{aligned} &\sum_{ij\in\Omega_l|_{j=k}} P_{ij,t} + \sum_{p\in\Omega_{PV}|_{p=k}} P^{PV}_{p,t} + \sum_{\omega\in\Omega_{WT}|_{\omega=k}} P^{WT}_{\omega,t} + \sum_{s\in\Omega_{EES}|_{s=k}} P^{EES,dc}_{s,t} + \sum_{g\in\Omega_{DG}|_{g=k}} P^{DG}_{g,t} \\ &+ P^S_{i,t} = P^D_{i,t} + \sum_{ij\in\Omega_l|_{i=k}} (P_{ij} + R_{ij,a(d_{ij})} I^{sqr}_{ij,t}) + \sum_{s\in\Omega_{EES}|_{s=k}} P^{EES,ch}_{s,t} \qquad \forall k\in\Omega_b, \forall t\in\Omega_T \end{aligned} \tag{2}$$

$$\begin{aligned} &\sum_{ij\in\Omega_l|_{j=k}} (Q_{ij,t} + \frac{B_{jj,a(d_{jj})}}{2} V^{sqr}_j) + \sum_{p\in\Omega_{PV}|_{p=k}} Q^{PV}_{p,t} + \sum_{\omega\in\Omega_{WT}|_{\omega=k}} Q^{WT}_{\omega,t} + \sum_{g\in\Omega_{DG}|_{g=k}} Q^{DG}_{g,t} \\ &+ Q^S_{i,t} = Q^D_{i,t} + \sum_{ij\in\Omega_l|_{i=k}} (Q_{ij} + X_{ij,a(d_{ij})} I^{sqr}_{ij,t} + \frac{B_{ii,a(d_{ii})}}{2} V^{sqr}_i) \qquad \forall k\in\Omega_b, \forall t\in\Omega_T \end{aligned} \tag{3}$$

Network Constraints

$$V^{sqr}_{i,t} - V^{sqr}_{j,t} = 2(R_{ij,a(d_{ij})} P_{ij,t} + X_{ij,a(d_{ij})} Q_{ij,t}) + Z^2_{ij,a(d_{ij})} I^{sqr}_{ij,t} \qquad \forall ij\in\Omega_l, \forall t\in\Omega_T \tag{4}$$

$$V^{sqr}_{j,t} I^{sqr}_{ij,t} \ge P^2_{ij,t} + Q^2_{ij,t} \qquad \forall ij\in\Omega_l, \forall t\in\Omega_T \tag{5}$$

$$\underline{V}^2 \le V^{sqr}_{i,t} \le \overline{V}^2 \qquad \forall i\in\Omega_b, \forall t\in\Omega_T \tag{6}$$

$$0 \le I^{sqr}_{ij,t} \qquad \forall ij\in\Omega_l, \forall t\in\Omega_T \tag{7}$$

Energy Storage System

$$SOC_{s,t} = SOC_{s,t-1} + \frac{\Delta T}{\overline{SOE}_s}(\eta_s P^{ch}_{s,t} - \frac{P^{dc}_{s,t}}{\eta_s}) \qquad \forall s \in \Omega_{EES}, \forall t \in \Omega_T \tag{8}$$

$$\underline{SOC}_s \leq SOC_{s,t} \leq \overline{SOC}_s \qquad \forall s \in \Omega_{EES}, \forall t \in \Omega_T \tag{9}$$

$$P^{EES,ch}_{s,t} \leq \overline{P}^{EES}_s u^{EES}_{s,t} \qquad \forall s \in \Omega_{EES}, \forall t \in \Omega_T \tag{10}$$

$$P^{EES,dc}_{s,t} \leq \overline{P}^{EES}_s (1 - u^{EES}_{s,t}) \qquad \forall s \in \Omega_{EES}, \forall t \in \Omega_T \tag{11}$$

$$u^{EES}_{s,t} \in \{0, 1\} \qquad \forall s \in \Omega_{EES}, \forall t \in \Omega_T \tag{12}$$

Distributed Generation

$$-P^{DG}_{g,t} \tan\left[\cos^{-1}(pf_g)\right] \leq Q^{DG}_{g,t} \leq P^{DG}_{g,t} \tan\left[\cos^{-1}(pf_g)\right] \qquad \forall g \in \Omega_{DG}, \forall t \in \Omega_T \tag{13}$$

$$\left(P^{DG}_{g,t}\right)^2 + \left(Q^{DG}_{g,t}\right)^2 \leq \left(\overline{S}^{DG}_g\right)^2 \qquad \forall g \in \Omega_{DG}, \forall t \in \Omega_T \tag{14}$$

$$R^d_g \leq P^{DG}_{g,t} - P^{DG}_{g,t-1} \leq R^u_g \qquad \forall g \in \Omega_{DG}, \forall t \in \Omega_T \tag{15}$$

$$F_{g,t} = F_{g,t-1} - \frac{\Delta T P^{DG}_{g,t}}{\eta^f_g \mathrm{FC}_g \mathrm{H}_g} \qquad \forall g \in \Omega_{DG}, \forall t \in \Omega_T \tag{16}$$

$$F_{g,t} \geq \underline{F}_g \qquad \forall g \in \Omega_{DG}, \forall t \in \Omega_T \tag{17}$$

$$u^{DG}_{g,t} \in \{0, 1\} \qquad \forall g \in \Omega_{DG}, \forall t \in \Omega_T \tag{18}$$

2 Modeling - Lower Level Model ($\forall i \in \Omega_b$)

Decision vector $X_{AGG} = \{P^D_{i,t}, Q^D_{i,t}, P^D_{i,m,t}\}$

Objective Function

$$\max_{X_{AGG}} \quad \sum_{t \in \Omega_T} \left[U_{i,t}(P^D_{i,t}) - \Delta T c^{DR}_t P^D_{i,t}\right] \tag{19}$$

$$U_{i,t}(P^D_{i,t}) = \Delta T \sum_{m \in \Omega_M} u^{DR}_{i,m,t} P^D_{i,m,t} \tag{20}$$

$$\begin{aligned} \min_{X_{AGG}} \quad & \Delta T \sum_{t \in \Omega_T} \left[c^{DR}_t P^D_{i,t} - \sum_{m \in \Omega_M} u^{DR}_{i,m,t} P^D_{i,m,t} \right] \\ \text{s.t.} \quad & (22) - (27) \end{aligned} \tag{21}$$

Demand Response

$$P_{i,t}^{D} = \sum_{m \in \Omega_M} P_{i,m,t}^{D} \qquad \forall t \in \Omega_T \quad (\lambda_{i,t}^{1}) \tag{22}$$

$$P_{i,m,t}^{D} \leq \overline{P}_{i,m,t}^{D} \qquad \forall m \in \Omega_M, \forall t \in \Omega_T \quad (\mu_{i,m,t}^{2}) \tag{23}$$

$$\Delta T \sum_{t \in \Omega_T} P_{i,t}^{D} \geq E_i^{D} \qquad (\mu_i^{3}) \tag{24}$$

$$P_{i,t}^{D} \geq \underline{P}_{i,t}^{D} \qquad \forall t \in \Omega_T \quad (\mu_{i,t}^{4}) \tag{25}$$

$$R_i^{d} \leq P_{i,t}^{D} - P_{i,t-1}^{D} \leq R_i^{u} \qquad \forall t \in \Omega_T \quad (\mu_{i,t}^{5}) \tag{26}$$

$$Q_{i,t}^{D} = P_{i,t}^{D} \tan\left(\arccos\left(pf_i\right)\right) \qquad \forall t \in \Omega_T \quad (\lambda_{i,t}^{2}) \tag{27}$$

3 KKT - Lower Level Model ($\forall i \in \Omega_b$)

Lagrangian of Aggregators

$$\begin{aligned}
L_i =& \Delta T \sum_{t=1} \left[c_t^{DR} P_{i,t}^{D} - \sum_{m=1} u_{i,m,t}^{DR} P_{i,m,t}^{D} \right] + \sum_{t=1} \lambda_{i,t}^{1} \left(P_{i,t}^{D} - \sum_{m=1} P_{i,m,t}^{D} \right) + \sum_{t=1} \sum_{m=1} \mu_{i,m,t}^{1} (P_{i,m,t}^{D} - \overline{P}_{i,m,t}^{D}) \\
&+ \sum_{t=1} \sum_{m=1} \mu_{i,m,t}^{2} (-P_{i,m,t}^{D}) + \mu_i^{3} \left(E_i^{D} - \Delta T \sum_{t=1} P_{i,t}^{D} \right) + \sum_{t=1} \mu_{i,t}^{4} (\underline{P}_{i,t}^{D} - P_{i,t}^{D}) \\
&+ \sum_{t=1} \mu_{i,t}^{5} (P_{i,t}^{D} - P_{i,t-1}^{D} - R_i^{U}) + \sum_{t=1} \mu_{i,t}^{6} (R_i^{D} - P_{i,t}^{D} + P_{i,t-1}^{D}) \\
&+ \sum_{t=1} \lambda_{i,t}^{2} \left[Q_{i,t}^{D} - P_{i,t}^{D} \tan\left(\arccos\left(pf_i\right)\right) \right]
\end{aligned} \tag{28}$$

Stationarity

$$\frac{\partial L_i}{\partial P^D_{i,t}} = \Delta T c^{DR}_t + \lambda^1_{i,t} - \Delta T \mu^3_i - \mu^4_{i,t} + \mu^5_{i,t} - \mu^6_{i,t} - \mu^5_{i,t+1} + \mu^6_{i,t+1} - \lambda^2_{i,t}\zeta_i = 0 \qquad \forall t < NT \tag{29}$$

$$\frac{\partial L_i}{\partial P^D_{i,t}} = \Delta T c^{DR}_t + \lambda^1_{i,t} - \Delta T \mu^3_i - \mu^4_{i,t} + \mu^5_{i,t} - \mu^6_{i,t} - \lambda^2_{i,t}\zeta_i = 0 \qquad t = NT \tag{30}$$

$$\frac{\partial L_i}{\partial Q^D_{i,t}} = \lambda^2_{i,t} = 0 \qquad \forall t \in \Omega_T \tag{31}$$

$$\frac{\partial L_i}{\partial P^D_{i,m,t}} = -\Delta T u^{DR}_{i,m,t} - \lambda^1_{i,t} + \mu^1_{i,m,t} - \mu^2_{i,m,t} = 0 \qquad \forall t \in \Omega_T \tag{32}$$

Where $\tan(\arccos(pf_i)) = \zeta_i$

Primal Feasibility ($h(x) = 0$, $g(x) \leq 0$)

$$P^D_{i,t} - \sum_{m=1} P^D_{i,m,t} = 0 \qquad \forall t \in \Omega_T \quad (\lambda^1_{i,t}) \tag{33}$$

$$P^D_{i,m,t} - \overline{P}^D_{i,m,t} \leq 0 \qquad \forall m \in \Omega_M, \forall t \in \Omega_T \quad (\mu^1_{i,m,t}) \tag{34}$$

$$-P^D_{i,m,t} \leq 0 \qquad \forall m \in \Omega_M, \forall t \in \Omega_T \quad (\mu^2_{i,m,t}) \tag{35}$$

$$E^D_i - \Delta T \sum_{t=1} P^D_{i,t} \leq 0 \qquad (\mu^3_i) \tag{36}$$

$$\underline{P}^D_{i,t} - P^D_{i,t} \leq 0 \qquad \forall t \in \Omega_T \quad (\mu^4_{i,t}) \tag{37}$$

$$P^D_{i,t} - P^D_{i,t-1} - R^u_i \leq 0 \qquad \forall t \in \Omega_T \quad (\mu^5_{i,t}) \tag{38}$$

$$R^d_i - P^D_{i,t} + P^D_{i,t-1} \leq 0 \qquad \forall t \in \Omega_T \quad (\mu^6_{i,t}) \tag{39}$$

$$Q^D_{i,t} - P^D_{i,t} \tan(\arccos(pf_i)) = 0 \qquad \forall t \in \Omega_T \quad (\lambda^2_{i,t}) \tag{40}$$

Dual Feasibility

$$\mu^1_{i,m,t} \geq 0, \quad \mu^2_{i,m,t} \geq 0 \qquad \forall m \in \Omega_M, \forall t \in \Omega_T \tag{41}$$

$$\mu^3_i \geq 0 \tag{42}$$

$$\mu^4_{i,t} \geq 0, \quad \mu^5_{i,t} \geq 0, \quad \mu^6_{i,t} \geq 0 \qquad \forall t \in \Omega_T \tag{43}$$

$$\lambda^1_{i,t}, \quad \lambda^2_{i,t} \quad \text{free} \qquad \forall t \in \Omega_T \tag{44}$$

Complementary Slackness ($(\mu \geq 0) + (g(x) \leq 0) + (\mu g(x) = 0) \rightarrow \mu(-g(x) \geq 0)$)

$$0 \leq \mu \perp -g(x) \geq 0$$

$$0 \leq \mu^1_{i,m,t} \perp (-P^D_{i,m,t} + \overline{P}^D_{i,m,t}) \geq 0 \qquad \forall m \in \Omega_M, \forall t \in \Omega_T \tag{45}$$

$$0 \leq \mu^2_{i,m,t} \perp (P^D_{i,m,t}) \geq 0 \qquad \forall m \in \Omega_M, \forall t \in \Omega_T \tag{46}$$

$$0 \leq \mu^3_i \perp (-E^D_i + \Delta T \sum_{t=1} P^D_{i,t}) \geq 0 \tag{47}$$

$$0 \leq \mu^4_{i,t} \perp (-\underline{P}^D_{i,t} + P^D_{i,t}) \geq 0 \qquad \forall t \in \Omega_T \tag{48}$$

$$0 \leq \mu^5_{i,t} \perp (-P^D_{i,t} + P^D_{i,t-1} + R^u_i) \geq 0 \qquad \forall t \in \Omega_T \tag{49}$$

$$0 \leq \mu^6_{i,t} \perp (-R^d_i + P^D_{i,t} - P^D_{i,t-1}) \geq 0 \qquad \forall t \in \Omega_T \tag{50}$$

Fortuny-Amat Transformation

$$0 \leq \mu \perp -g(x) \geq 0$$

$$0 \leq -g(x) \leq Mz$$

$$0 \leq \mu \leq Mz$$

$$z \in \{0, 1\}$$

Equations in primal and dual feasibility capture lower limit of the Fortuny-Amat transformation. This leads to set of equations $\mathbf{g}(\mathbf{x}) \geq -\mathbf{M}\mathbf{z}$ and $\mu \leq \mathbf{M}(\mathbf{1}-\mathbf{z})$.

$$P^D_{i,m,t} - \overline{P}^D_{i,m,t} \geq -M^1 z^1_{i,m,t} \qquad \mu^1_{i,m,t} \leq M^1(1 - z^1_{i,m,t}) \quad \forall m \in \Omega_M, \forall t \in \Omega_T \tag{51}$$

$$-P^D_{i,m,t} \geq -M^2 z^2_{i,m,t} \qquad \mu^2_{i,m,t} \leq M^2(1 - z^2_{i,m,t}) \quad \forall m \in \Omega_M, \forall t \in \Omega_T \tag{52}$$

$$E^D_i - \Delta T \sum_{t=1} P^D_{i,t} \geq -M^3 z^3_i \qquad \mu^3_i \leq M^3(1 - z^3_i) \tag{53}$$

$$\underline{P}^D_{i,t} - P^D_{i,t} \geq -M^4 z^4_{i,t} \qquad \mu^4_{i,t} \leq M^4(1 - z^4_{i,t}) \quad \forall t \in \Omega_T \tag{54}$$

$$P^D_{i,t} - P^D_{i,t-1} - R^u_i \geq -M^5 z^5_{i,t} \qquad \mu^5_{i,t} \leq M^5(1 - z^5_{i,t}) \quad \forall t \in \Omega_T \tag{55}$$

$$R^d_i - P^D_{i,t} + P^D_{i,t-1} \geq -M^6 z^6_{i,t} \qquad \mu^6_{i,t} \leq M^6(1 - z^6_{i,t}) \quad \forall t \in \Omega_T \tag{56}$$

$$z^1_{i,m,t}, \quad z^2_{i,m,t} \qquad \in \{0,1\} \tag{57}$$

$$z^3_i \qquad \in \{0,1\} \tag{58}$$

$$z^4_{i,t}, \quad z^5_{i,t}, \quad z^6_{i,t} \qquad \in \{0,1\} \tag{59}$$

4 Mixed-integer Linear Programming (MILP) Model

Decision vector $X = \{X_{DSO}, X_{AGG}, \lambda^{(1-2)}_{i,t}, \mu^{(1-2)}_{i,m,t}, \mu^3_i, \mu^{(4-6)}_{i,t}, z^{(1-2)}_{i,m,t}, z^3_i, z^{(4-6)}_{i,t}\}$

$$\begin{aligned} \min_X \quad & \Delta T \sum_{t \in \Omega_T} \sum_{ij \in \Omega_l} c^{DR}_t R_{ij,a(d_{ij})} I^{sqr}_{ij,t} \\ \text{s.t.} \quad & (2)-(18), (29)-(44), (51)-(59) \end{aligned} \tag{60}$$

5 Nomenclature

Sets

Ω_a	Set of conductor types
Ω_b	Set of buses (nodes).
Ω_{DG}	Set of nodes with distributed generation.
Ω_{EES}	Set of nodes with energy storage systems.
Ω_l	Set of lines.
Ω_{PV}	Set of nodes with photo-voltaic (PV) systems.
Ω_T	Set of time periods.
Ω_{WT}	Set of nodes with wind parks.

Parameters

$B_{ij,a(d_{ij})}$	Shunt susceptance of node i of cable type a with length d [mS].
$R_{ij,a(d_{ij})}$	Resistance of line ij of cable type a with length d [mΩ].
$X_{ij,a(d_{ij})}$	Reactance of line ij of cable type a with length d [mΩ].
$Z_{ij,a(d_{ij})}$	Impedance of line ij of cable type a with length d [mΩ].
d_{ij}	Length of line ij [km].
R_g^d	Ramp-down rate of DG unit at node g [kW/h].
R_g^u	Ramp-up rate of DG unit at node g [kW/h].
FC_g	Fuel capacity of DG unit at node g [m^3].
H_g	Calorific value of DG unit at node g [kWh/m^3].
$\underline{F}_g$	Minimum fuel of DG unit at node g [m^3].
pf_g	Minimum power factor of DG unit at node g.
$P_{p,t}^{PV}$	Active power supplied by PV at node p in period t [kW].
$P_{i,t}^{D}$	Active power demand at node i in period t [kW].
$P_{\omega,t}^{WT}$	Active power supplied by wind park at node ω in period t [kW].
$\overline{P}_s^{EES}$	Maximum charging/discharging power of the EES at node s [kW].
$Q_{p,t}^{PV}$	Reactive power supplied by PV at node p in period t [kvar].
$Q_{\omega,t}^{WT}$	Reactive power supplied by wind park at node ω in period t[kvar].
$Q_{i,t}^{D}$	Reactive power demand a node i in period t [kvar].
$\overline{S}_g^{DG}$	Maximum capacity of DG at node g [kVA]
$\overline{S}_i^{S}$	Maximum substation capacity at node i [kVA]
$\overline{V}$	Maximum voltage magnitude [V].
$\underline{V}$	Minimum voltage magnitude [V].
$\overline{SOE}_s$	Maximum state-of-energy of the EES at node s [kWh].
$\overline{SOC}_s$	Maximum state-of-charge of the EES at node s [%].
$\underline{SOC}_s$	Minimum state-of-charge of the EES at node s [%].
η_g^f	
η_s	Charging/discharging of EES at node s.
ΔT	Time step [h].

Variables

$I_{ij,t}^{sqr}$	Squared current magnitude at line ij in period t $[A^2]$.
V_i^{sqr}	Squared voltage magnitude at node i in period t $[V^2]$.
$P_{ij,t}$	Active power flow at line ij in period t [kW].
$P_{i,t}^{S}$	Active power injection at node i in period t [kW].
$P_{g,t}^{DG}$	Active power supplied by DG unit at node g in period t [kW].
$Q_{i,t}^{S}$	Reactive power injection at node i in period t [kW].
$Q_{ij,t}$	Reactive power flow at line ij in period t [kvar].
$Q_{g,t}^{DG}$	Reactive power supplied by DG unit at node g in period t [kvar].
$P_{s,t}^{EES,ch}$	Charging power of EES at node s in period t [kW].
$P_{s,t}^{EES,dc}$	Discharging power of EES at node s in period t [kW].
$SOC_{s,t}$	Sate-of-charge of EES at node s in period t [kWh].
$u_{s,t}^{EES}$	Binary variable associated to the charging (1) or discharging (0) operation of EES at node s in period t.
$u_{g,t}^{DG}$	Commitment status of DG unit at node g in period t.