

Instructions: The exam is 3 hours long and contains 6 questions. Write your answers clearly in the notebook provided. You may quote any result/theorem seen in the lectures without proving it. **Justify all your answers.**

- Q1** Let G be the graph depicted in Figure 1.
- a) Is G planar?
 - b) Find $\tau(G)$.
 - c) Show that $\chi(G) = 4$.
(*Hint:* To prove that $\chi(G) > 3$, show that in any 3-coloring of the graph $G \setminus v$, all three colors will appear on the neighborhood of v .)
- Q2** Let G be the graph with weights $w : E(G) \rightarrow \mathbb{Z}_+$ depicted in Figure 2.
- a) Find the min-cost spanning tree in G .
 - b) Find a shortest path spanning tree for the vertex A .
- Q3** Let $k \geq 1$ be an integer, and let G be a connected k -regular bipartite graph. Show that G is 2-connected.
- Q4** Let G be a planar graph. Show that if G does not contain any cycles of length five or less then $\chi(G) \leq 3$.
- Q5** Let G be a 3-connected graph with $|V(G)| \geq 5$. Show that C_5 is a minor of G .
- Q6** Let G be a simple graph. Suppose that there exists a clique $X \subseteq V(G)$ such that $V(G) - X$ is an independent set. Show that G is perfect without using the strong perfect graph theorem.

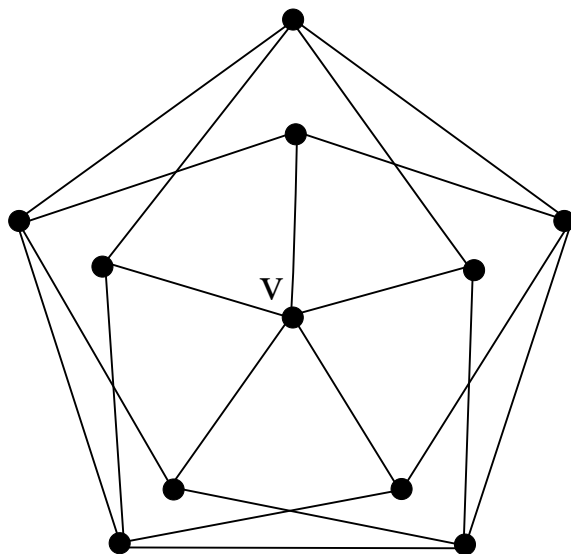


Figure 1: The graph in the question Q1.

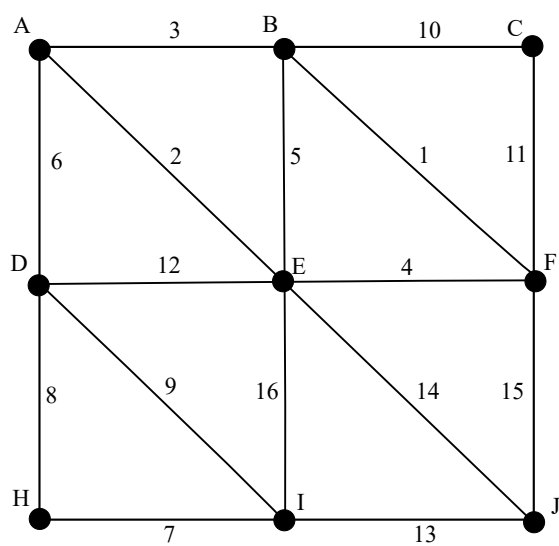


Figure 2: The graph in the question Q2.