Leader Election in Distributed Systems

Course: Distributed Computing

Faculty: Dr. Rajendra Prasath

About this topic

This course covers the essential aspects of **Leader Election in Distributed Systems and its related concepts**

RECAP

What did you learn so far?

- Challenges in Message Passing systems
- Distributed Sorting
- Space-Time Diagram
- Partial Ordering / Total Ordering
- Causal Ordering
- Causal Precedence Relation
  - Happens Before
- Concurrent Events
- Local Clocks and Vector Clocks
- Distributed Snapshots
- Termination Detection using Dist. Snapshots

Topics to focus on …

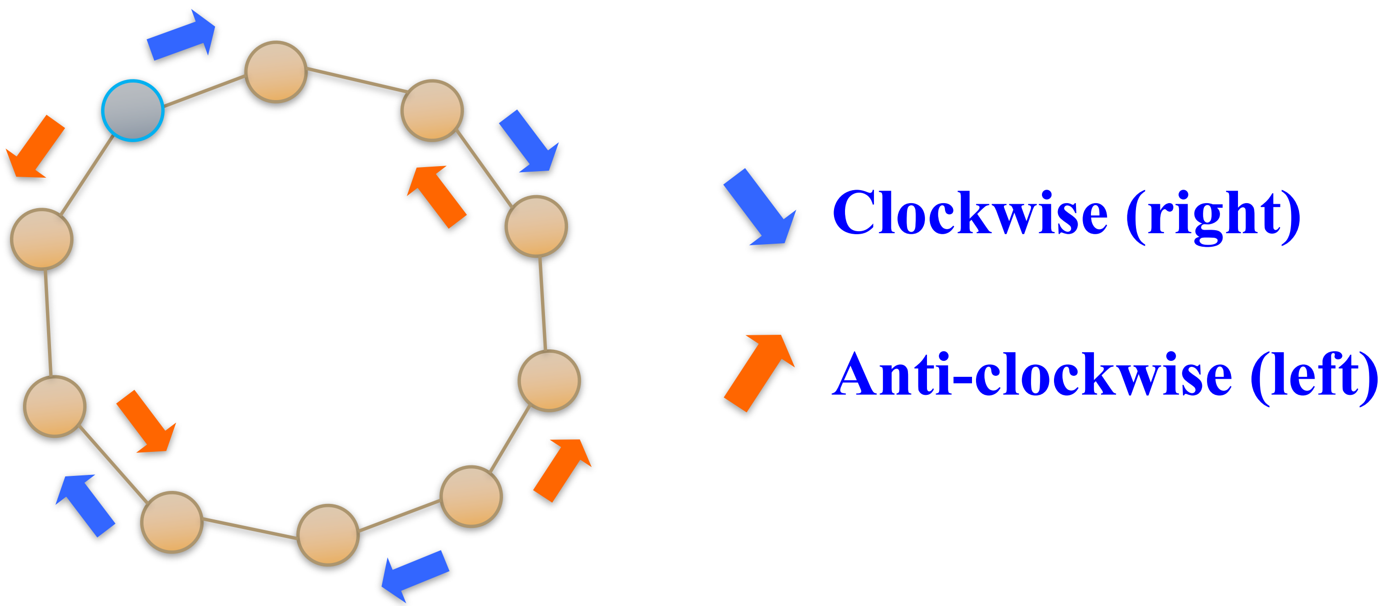
- ➔ Leader Election in Distributed Systems

- ➔ Topology Abstraction and Overlays
- ➔ Message Ordering
- ➔ Group Communication
- ➔ Distributed Mutual Exclusion
- ➔ Deadlock Detection
- ➔ Check pointing and rollback recovery

Leader Election in Distributed Systems

Ring Networks

- In an oriented ring, processes have a consistent notion of left and right



- For example, if messages are forwarded on right channel, they will cycle clockwise around the ring

Why Study Rings?

- Simple starting point, easy to analyze
- Abstraction of a token ring
- Lower bounds and impossibility results for ring topology also apply to arbitrary topologies

Leader Election - Definition

- Each processor has a set of **elected** (won) and **not-elected** (lost) states.
- Once an elected state is entered, processor is always in an elected state (and similarly for not-elected): i.e., irreversible decision
- In every admissible execution:
  - every processor eventually enters either an elected or a not-elected state
  - exactly one processor (the **leader**) enters an elected state

Uses of Leader Election

- A leader can be used to coordinate activities of the system:
  - find a spanning tree using the leader as the root
  - reconstruct a lost token in a token-ring network

- We will study leader election in rings.

Anonymous Rings

- How to model situation when processes do not have unique identifiers?
- First attempt: Does each process require to be in the same state machine?
- Subtle point: Does the algorithm rely on knowing the ring size (number of processes)?

Uniform (Anonymous) Algorithms

- A **uniform** algorithm does not use the ring size (same algorithm for each size ring)
  - Formally, every processor in every size ring is modeled with the same state machine
- A **non-uniform** algorithm uses the ring size (different algorithm for each size ring)
  - Formally, for each value of n, every processor in a ring of size n is modeled with the same state machine A_n .
- Note the lack of unique ids.

Leader Election in Anonymous Rings

- **Theorem:** There is *no* leader election algorithm for anonymous rings, even if
  - algorithm knows the ring size (non-uniform)
  - synchronous model

- **Proof Sketch:**
  - Every processor begins in same state with same outgoing messages (since anonymous)
  - Every processor receives same messages, does same state transition, and sends same messages in round 1
  - Do the same for rounds 2, 3, …
  - Eventually some processor is supposed to enter an elected state.

Leader Election in Anonymous Rings

- Proof sketch shows that either safety (never elect more than one leader) or liveness (eventually elect at least one leader) is violated

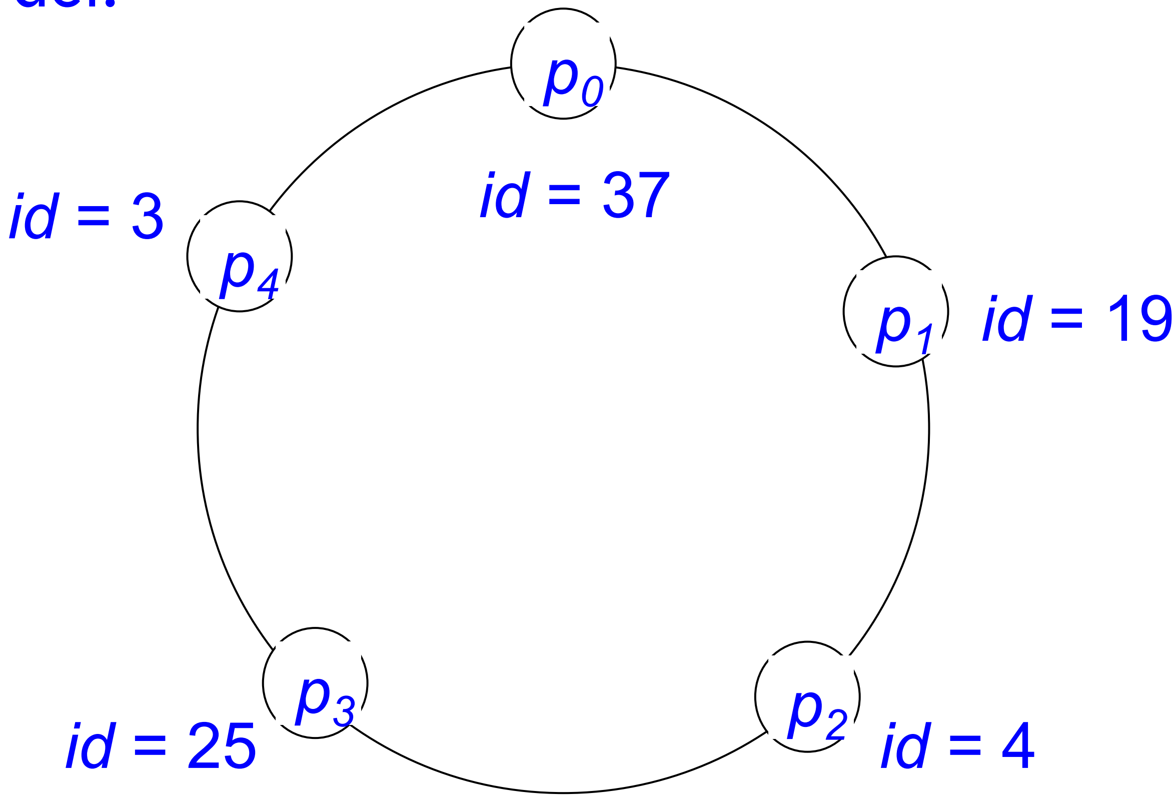
- Since the theorem was proved for non-uniform and synchronous rings, the same result holds for weaker (less well-behaved) models:
  - uniform
  - asynchronous

Rings with Identifiers

- Assume each processor has a unique ID.
- Don't confuse indices and IDs:
  - **indices** are 0 to $n - 1$; used only for analysis, not available to the processors
  - **IDs** are arbitrary nonnegative integers; are available to the processors through local variable *ID*

Specifying a Ring

- Start with the smallest ID and list IDs in clockwise order.



- Example: 3, 37, 19, 4, 25

Uniform (Non-anonymous) Algorithms

- **Uniform** algorithm: there is one state machine for every id, no matter what size ring
- **Non-uniform** algorithm: there is one state machine for every id and every different ring size
- These definitions are tailored for leader election in a ring.

Overview of LE in Rings with IDs

- There exist algorithms when nodes have unique IDs.
- We will evaluate them according to their *message complexity*.
- asynchronous ring:
  - $\Theta(n \log n)$ messages
- synchronous ring:
  - $\Theta(n)$ messages under certain conditions
  - otherwise $\Theta(n \log n)$ messages
- All bounds are asymptotically tight.

The LCR algorithm

- Lelann-Chang-Robert (LCR) Algorithm, 1979
- The network graph is a directed ring (uni-directed or bi-directed) consisting of n nodes (n may be unknown to the processes … Does this condition really require for rings?)
- Processes run the same deterministic algorithm
- The only piece of information supplied to the processes is a unique identifier (ID).
- IDs may be used
- In comparisons only (comparison-based algorithms)
- In comparisons and other calculations (non-comparison-based)

LCR algorithm - Description

- Each process sends its ID around the ring.
- When a process receives a ID, it compares this one to its own
- If the incoming ID is greater, then it passes this ID to the next process.
- If the incoming ID is smaller, then it discards it.
- If it is equal, then the process declares itself the leader.

LCR Algorithm for Leader Election

Alternative Algorithm:

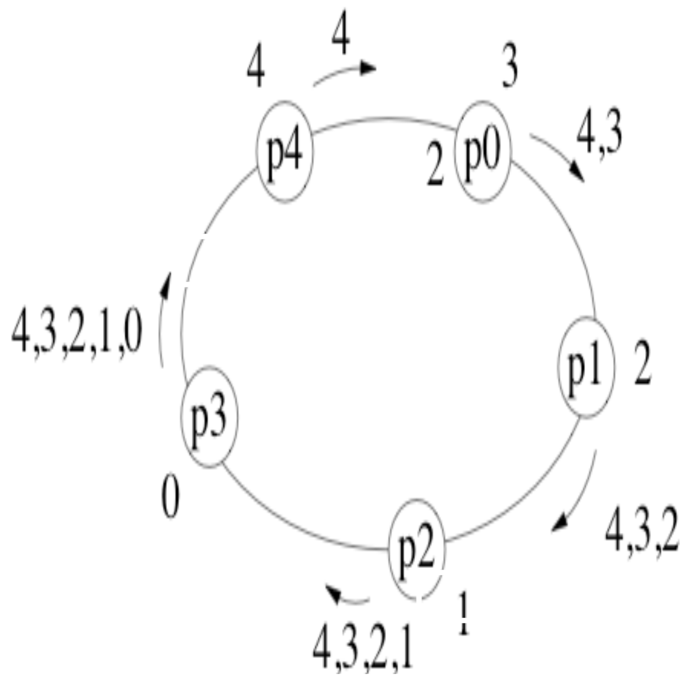
- send value of own id to the left
- when receive an ID j (from the right):
- if j > id then
    - forward j to the left (this processor has lost)
  - if j < id then
    - do nothing
  - if j = id then
    - elect self as leader
      (this processor has won)

Analysis of $O(n^2)$ Algorithm

- **Correctness:** Elects processor with largest id.
  - message containing largest id passes through every processor
- **Time:** $O(n)$
- **Message complexity:** Depends how the ids are arranged.
  - largest id travels all around the ring (n messages)
  - 2nd largest id travels until reaching largest
  - 3rd largest id travels until reaching largest or second largest
  - And so on

Analysis of $O(n^2)$ Algorithm

- Worst way to arrange the ids is in decreasing order:
  - 2nd largest causes $n - 1$ messages
  - 3rd largest causes $n - 2$ messages and so on
- Total number of messages is $n + (n-1) + (n-2) + \ldots + 1 = \Theta(n^2)$.



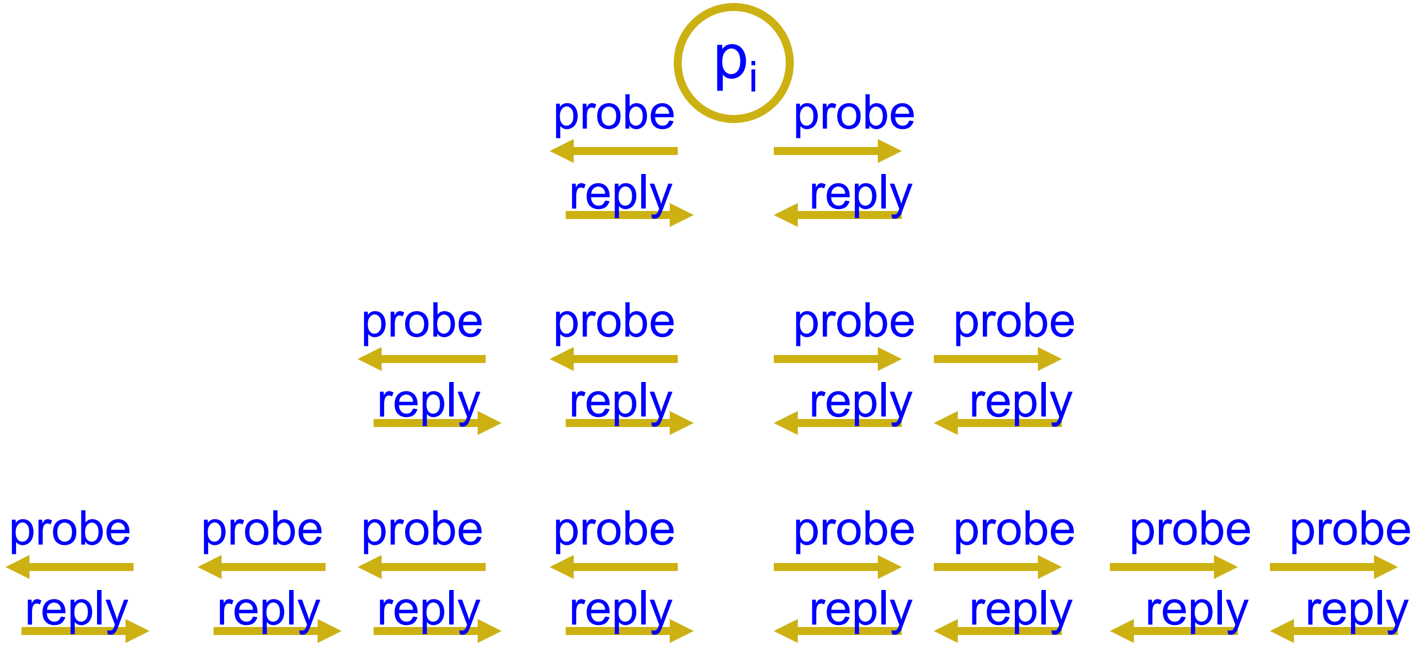
Can We Use Fewer Messages?

- The O(n^2) algorithm is simple and works in both synchronous and asynchronous model.
- But can we solve the problem with fewer messages?
- Idea:
  - Try to have messages containing smaller ids travel smaller distance in the ring

O(n log n) Leader Election

- Each process tries to probe successively larger neighborhoods in both directions
  - size of neighborhood *doubles* in each phase
- If probe reaches a node with a larger id, the probe stops
- If probe reaches end of its neighborhood, then a reply is sent back to initiator
- If initiator gets back replies from both directions, then go to next phase
- If process receives a probe with its own id, it elects itself

O(n log n) Leader Election



Analysis of O(nlogn) Algorithm

- **Correctness:** Similar to $O(n^2)$ algorithm.
- **Message Complexity:**
  - Each message belongs to a particular phase and is initiated by a particular proc.
  - Probe distance in phase k is 2^k
  - Number of messages initiated by a proc. in phase k is at most $4*2^k$ (probes and replies in both directions)

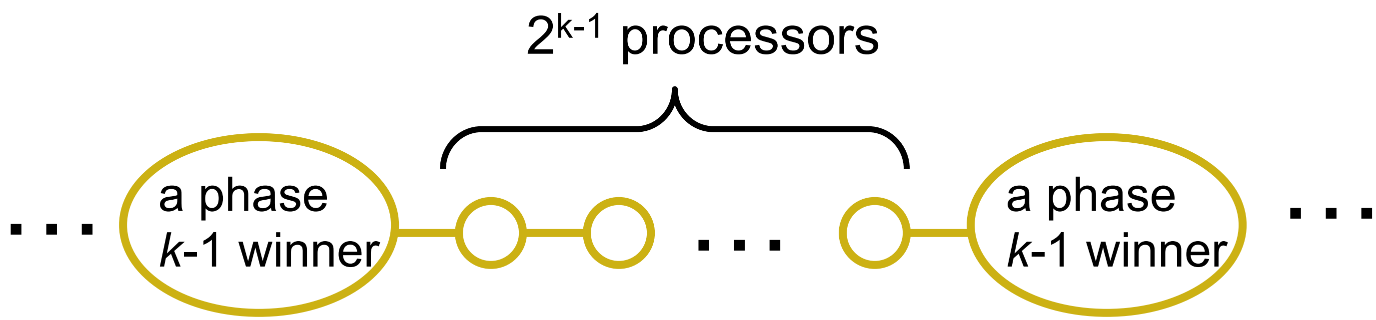
Analysis of O(n log n) Algo

- How many processes initiate probes in phase k ?
  - For $k = 0$, every process does
  - For $k > 0$, every process that is a "winner" in phase $k - 1$ does

  - "winner" means has largest id in its 2^{k-1} neighborhood

Analysis of O(n log n) Algo

- Maximum number of phase $k - 1$ winners occurs when they are packed as densely as possible:



- The total number of phase $k - 1$ winners is at most

$$n/(2^{k-1} + 1)$$

Analysis of O(n log n) Algo

- How many phases are there?
- At each phase the number of (phase) winners is cut approx. in half
  - from $n/(2^{k-1} + 1)$ to $n/(2^k + 1)$
- So after approx. $\log_2 n$ phases, only one winner is left.
  - more precisely, max phase is $\lceil \log(n-1) \rceil + 1$

Analysis of O(n log n) Algo

- Total number of messages is sum, over all phases, of number of winners at that phase times number of messages originated by that winner:

$$\leq 4n + n + \boxed{\sum_{k=1}^{\lceil \log(n-1) \rceil + 1} 4 \cdot 2^k \cdot n/(2^{k-1}+1)}$$

phase 0 msgs

termination msgs

msgs for phases 1 to $\lceil \log(n-1) \rceil + 1$

$$< 8n(\log n + 2) + 5n$$

$$= O(n \log n)$$

Can We Do Better?

- The O$(n \log n)$ algorithm is more complicated than the O(n^2) algorithm but uses fewer messages in the worst case.
- Works in both synchronous and asynchronous case.
- Can we reduce the number of messages even more?
- Not in the asynchronous model … !!

Summary

- Leader Election
  - Formulation of the problem
  - LCR algorithm
  - O(nlogn) algorithm using probes
  - Complexity Analysis
  - Many more to come up … stay tuned in !!

How to reach me?

- Please leave me an email:
  rajendra [DOT] prasath [AT] iiits [DOT] in

- Visit my homepage @

  - http://www.iiits.ac.in/FacPages/index-rajendra.html

  OR

  - http://rajendra.2power3.com

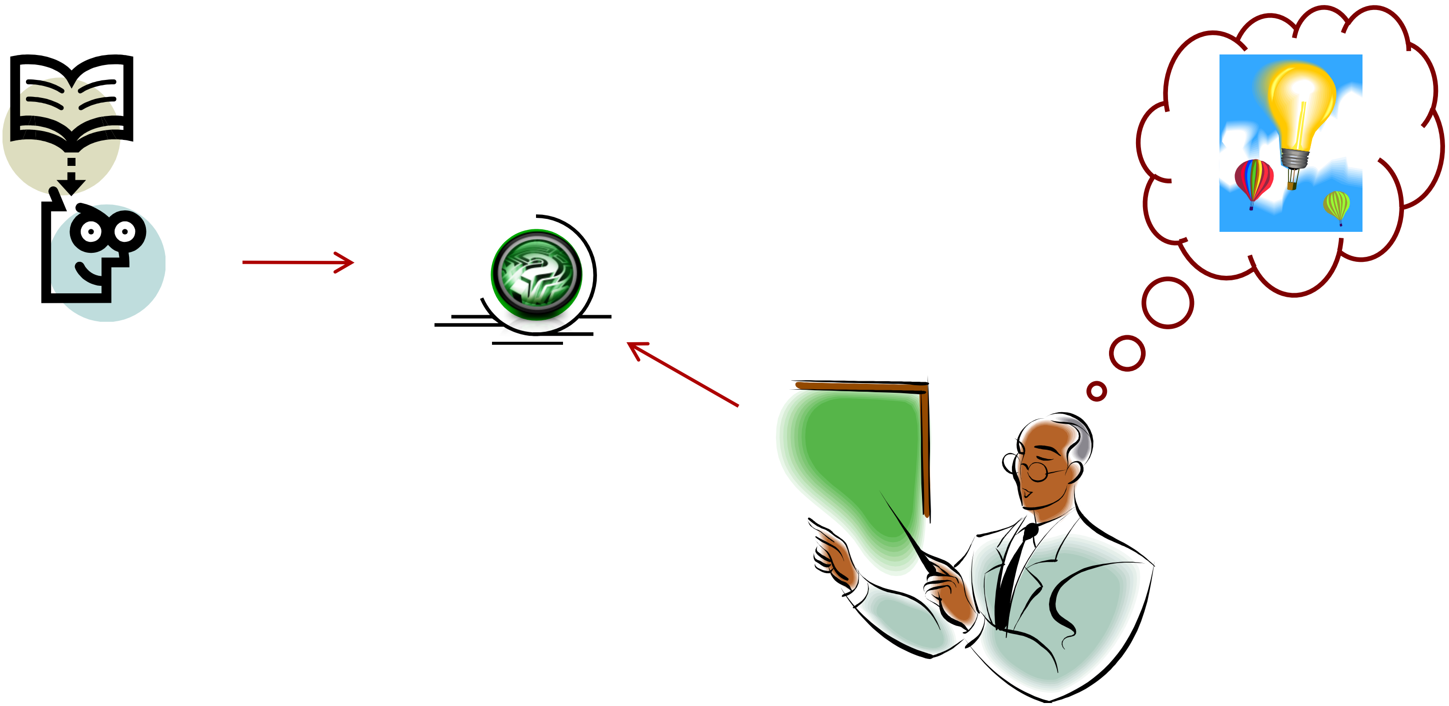
Help among Yourselves?

- **Perspective Students** (having CGPA above 8.5 and above)
- **Promising Students** (having CGPA above 6.5 and less than 8.5)

- **Needy Students** (having CGPA less than 6.5)
  - Can the above group help these students? (Your work will also be rewarded)

- You may grow a culture of **collaborative learning** by helping the needy students

Thanks …



… Questions ???