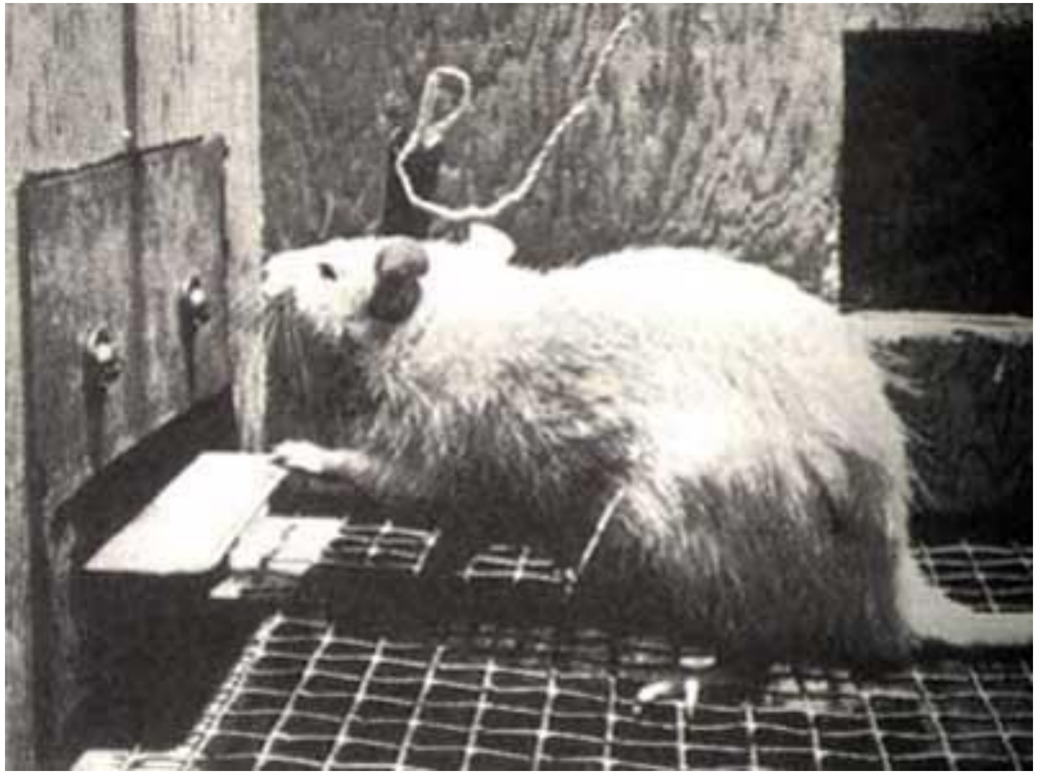
Introduction to deep reinforcement learning

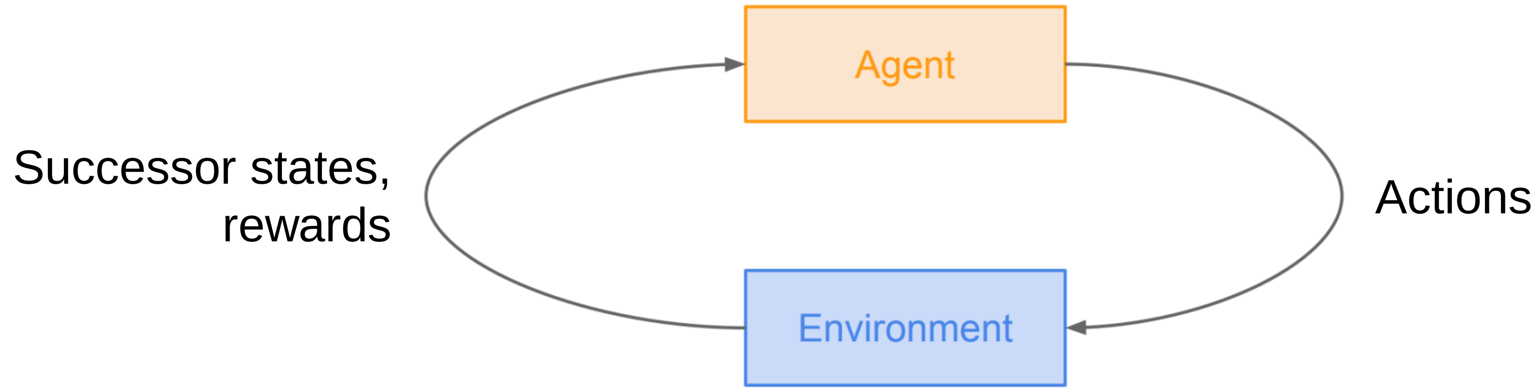


Outline

- Introduction to reinforcement learning
- Markov Decision Process (MDP) formalism
- The Bellman equation
- Q-learning
- Deep Q networks (DQN)
- Policy Gradient Methods

Reinforcement learning (RL)

- Setting: agent that can take *actions* affecting the *state* of the environment and observe occasional *rewards* that depend on the state
- Goal: learn a *policy* (mapping from states to actions) to maximize expected reward over time



RL vs. supervised learning

- **Reinforcement learning loop**
  - From state s, take action a determined by *policy* $\pi(s)$
  - Environment selects next state s' based on *transition model* $P(s'|s,a)$
  - Observe s' and reward $r(s')$, update policy

- **Supervised learning loop**
  - Get input x_i sampled from data distribution
  - Use model with parameters w to predict output y
  - Observe target output y_i and loss $l(w, x_i, y_i)$
  - Update w to reduce loss: $w \leftarrow w - \eta \nabla l(w, x_i, y_i)$

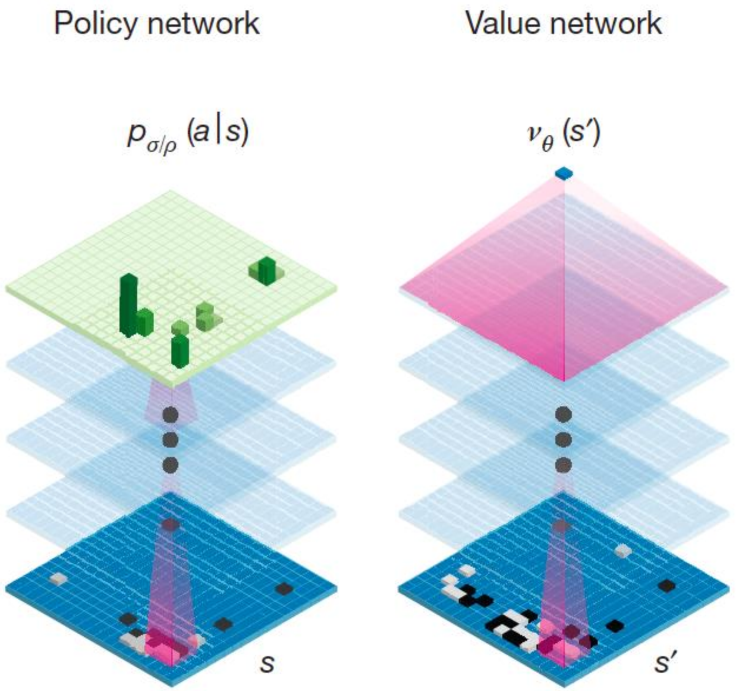
RL vs. supervised learning

- **Reinforcement learning**
  - Agent’s actions affect the environment and help to determine next observation
  - Rewards may be sparse
  - Rewards are not differentiable w.r.t. model parameters

- **Supervised learning**
  - Next input does not depend on previous inputs or agent’s predictions
  - There is a supervision signal at every step
  - Loss is differentiable w.r.t. model parameters

Example applications of deep RL

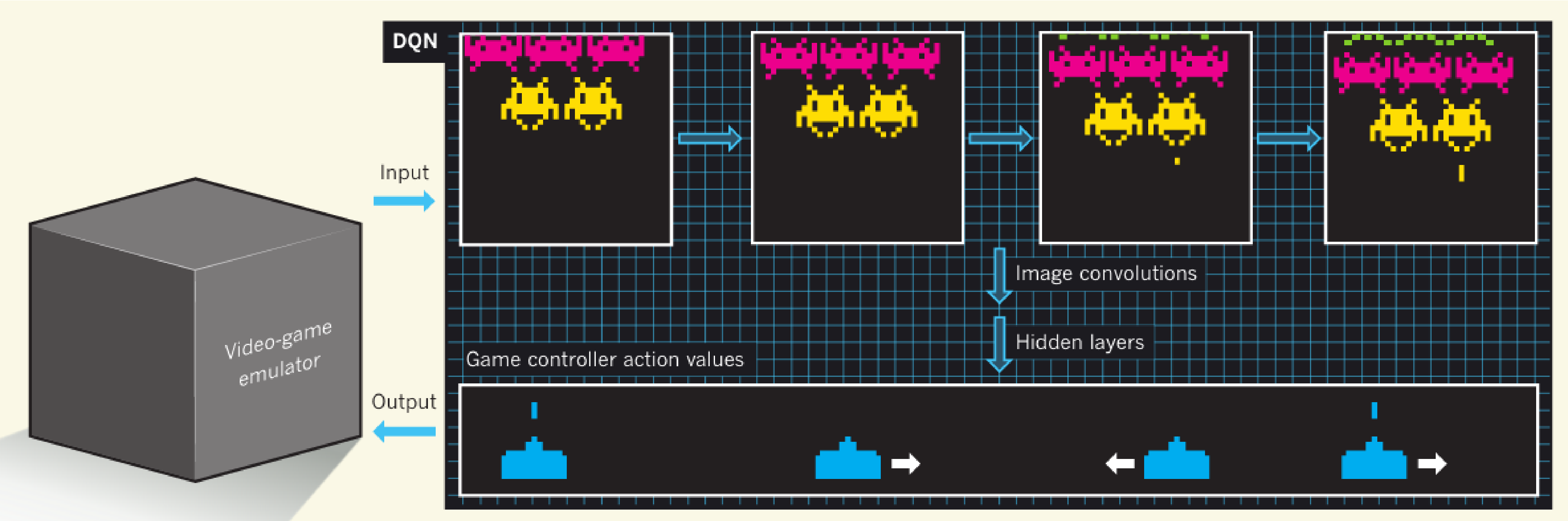
- AlphaGo and AlphaZero



https://deepmind.com/research/alphago/

Example applications of deep RL

- Playing video games



[Video]

V. Mnih, K. Kavukcuoglu, D. Silver, A. Graves, I. Antonoglou, D. Wierstra, M. Riedmiller, [Human-level control through deep reinforcement learning], *Nature* 2015

Example applications of deep RL

- Sensorimotor learning

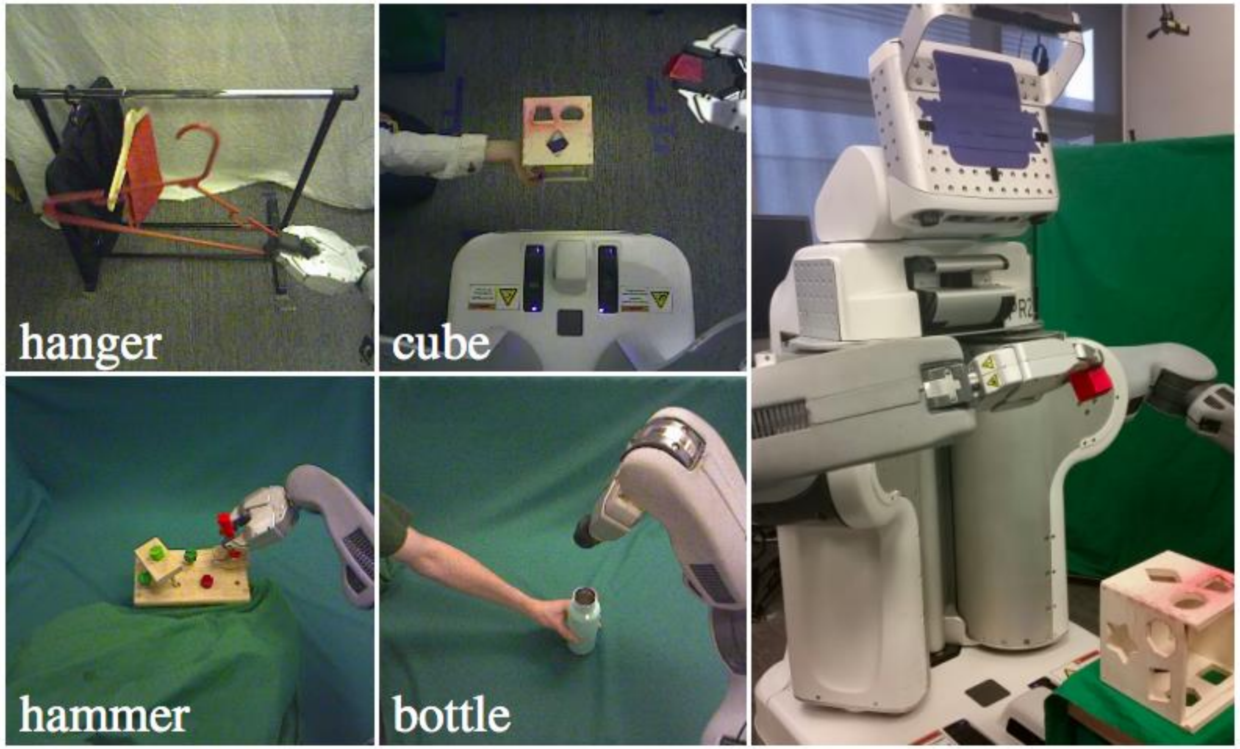


Fig. 1: Our method learns visuomotor policies that directly use camera image observations (left) to set motor torques on a PR2 robot (right).

Video

S. Levine, C. Finn, T. Darrell and P. Abbeel, End-to-End Training of Deep Visuomotor Policies, JMLR 2016

Example applications of deep RL

- Sensorimotor learning

(a) Urban driving

(b) Off-road traversal

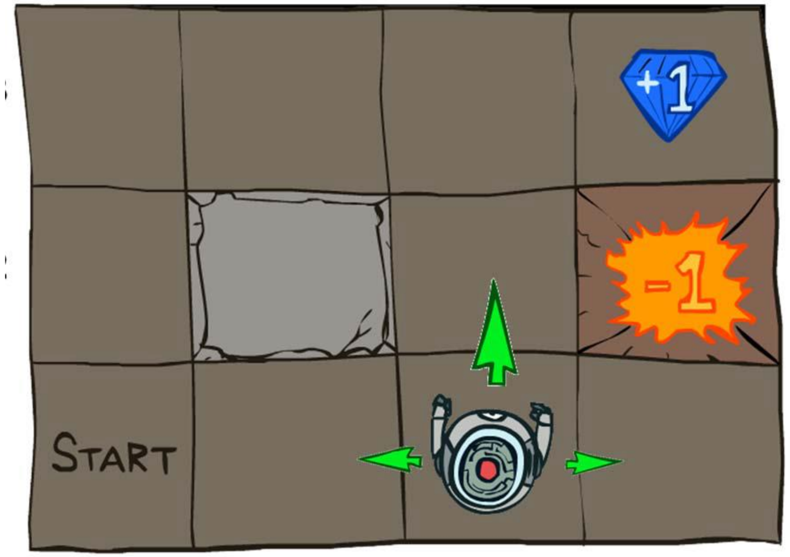
(c) Battle

B. Zhou, P. Krähenbühl, and V. Koltun, [Does Computer Vision Matter for Action?](#) Science Robotics, 4(30), 2019

Formalism: Markov Decision Processes

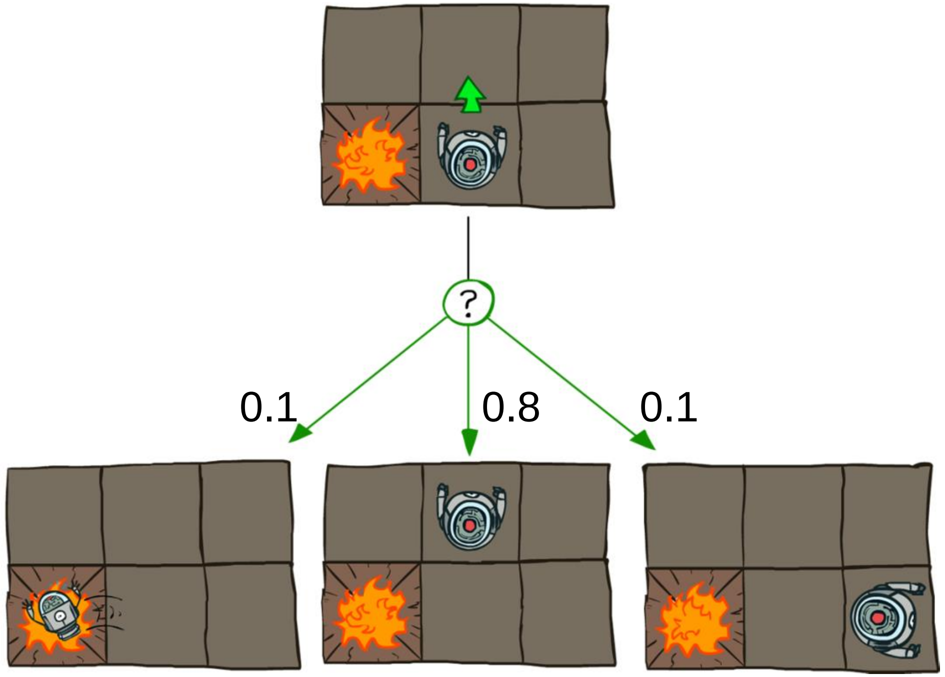
- Components:
  - **States** s, beginning with initial state s_0
  - **Actions** a
  - **Transition model** $P(s' \mid s, a)$
    - *Markov assumption*: the probability of going to s' from s depends only on s and a and not on any other past actions or states
  - **Reward function** $r(s)$
- **Policy** $\pi(s)$: the action that an agent takes in any given state
  - The "solution" to an MDP

Example MDP: Grid world



$r(s) = -0.04$ for every non-terminal state

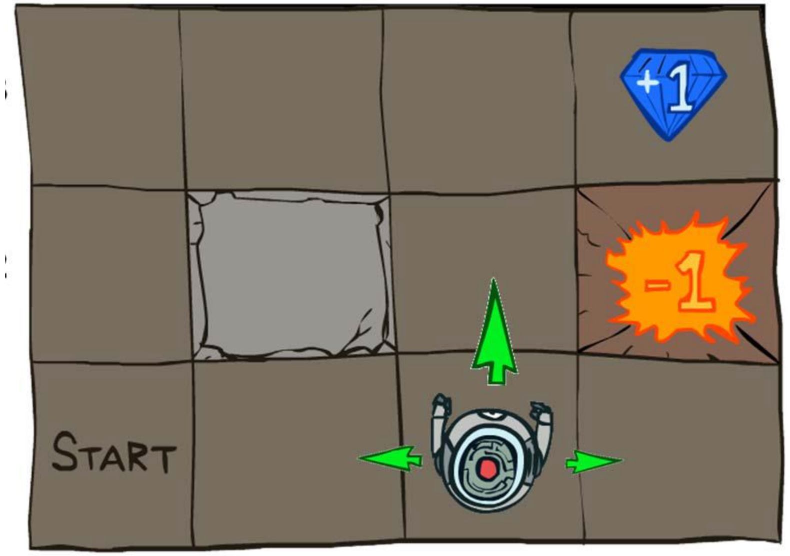
Transition model:

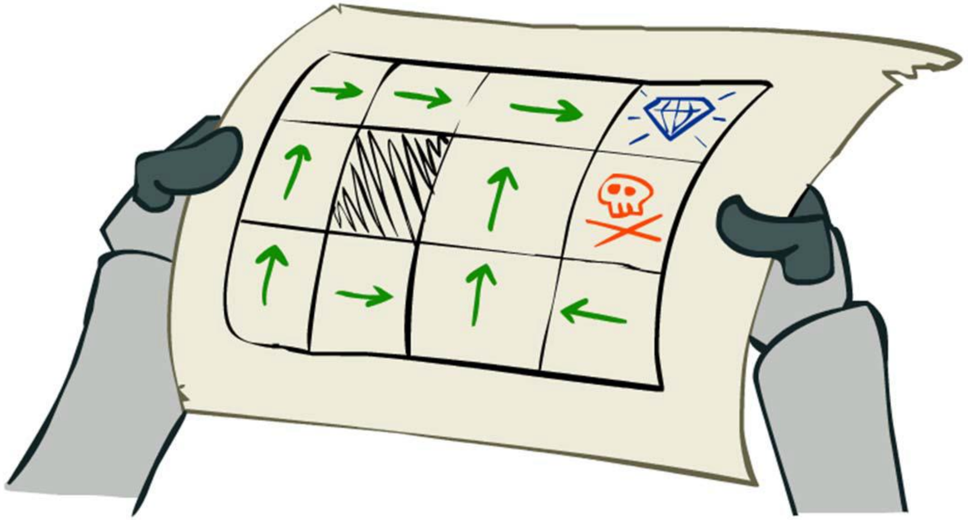


Source: P. Abbeel and D. Klein

Example MDP: Grid world

- Goal: find the best policy

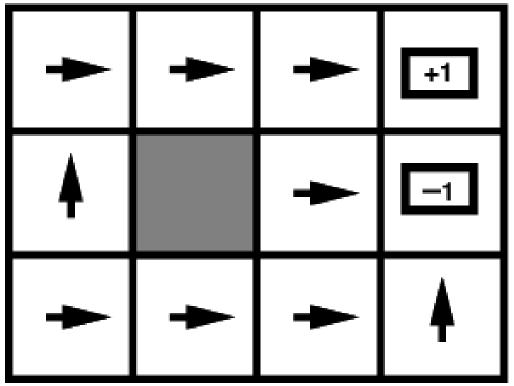




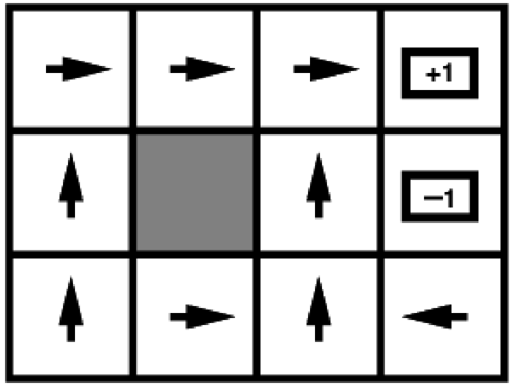
Source: P. Abbeel and D. Klein

Example MDP: Grid world

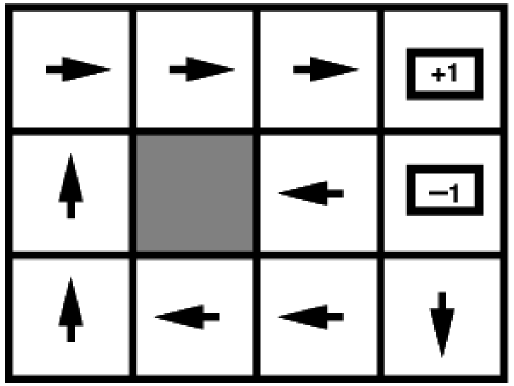
- Optimal policies for various values of $r(s)$:



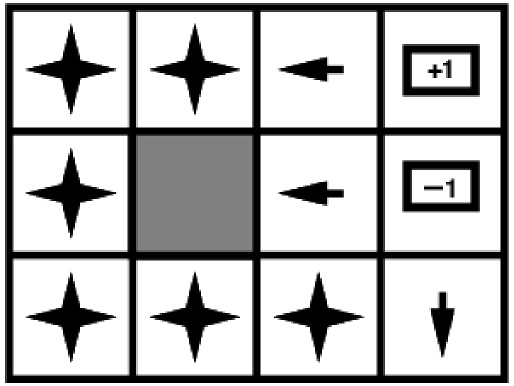
$R(s) < -1.6284$



$-0.4278 < R(s) < -0.0850$



$-0.0221 < R(s) < 0$



$R(s) > 0$

Rewards of state sequences

- Suppose that following policy π starting in state s_0 leads to a sequence $s_0, s_1, s_2, \ldots$
- The *cumulative reward* of the sequence is $\sum_{t \geq 0} r(s_t)$
- **Problem:** state sequences can vary in length or even be infinite
- **Solution:** redefine cumulative reward as sum of rewards *discounted* by a factor γ

Image source: P. Abbeel and D. Klein

Discounting

- Discounted cumulative reward:

$$r(s_0) + \gamma\, r(s_1) + \gamma^2 r(s_2) + \gamma^3 r(s_3) + \cdots$$
$$= \sum_{t \geq 0} \gamma^t\, r(s_t), \qquad 0 < \gamma \leq 1$$

- Sum is bounded by $\frac{r_{\max}}{1-\gamma}$
- Helps algorithms converge

Value function

- The *value function* $V^{\pi}(s)$ of a state s w.r.t. policy π is the expected cumulative reward of following that policy starting in s:

$$V^{\pi}(s) = \mathbb{E}\left[\sum_{t\geq 0} \gamma^t\, r(s_t) \mid s_0 = s, \pi\right]$$

with $a_t = \pi(s_t), s_{t+1} \sim P(\cdot\,|s_t, a_t)$

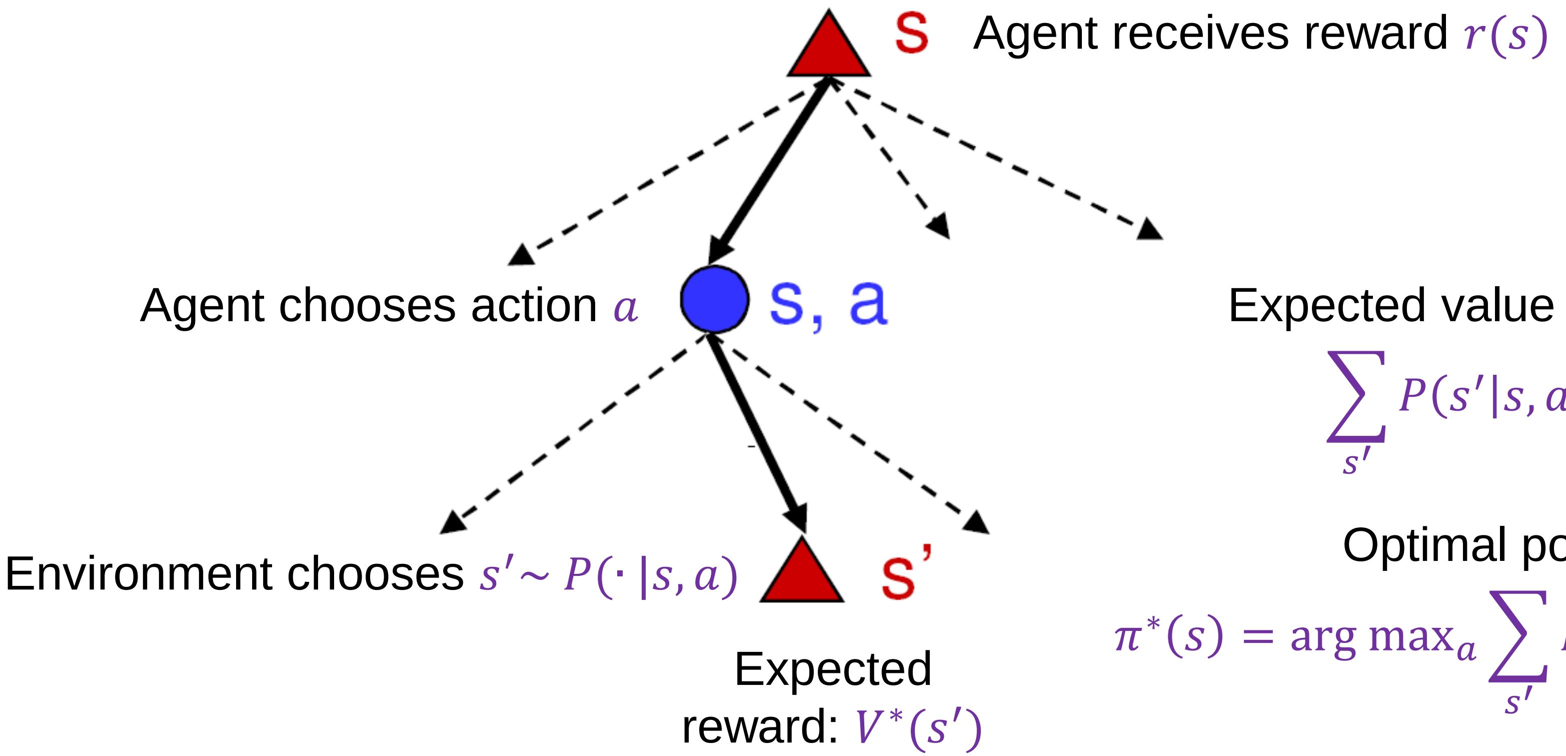
- The *optimal value* of a state is the value achievable by following the best possible policy:

$$V^*(s) = \max_{\pi} \mathbb{E}\left[\sum_{t\geq 0} \gamma^t\, r(s_t) \mid s_0 = s, \pi\right]$$

The Bellman equation

- Recursive relationship between optimal values of successive states:

$$V^*(s) = r(s) + \gamma \max_a \sum_{s'} P(s'|s,a)V^*(s')$$



Expected value for action a:

$$\sum_{s'} P(s'|s,a)V^*(s')$$

Optimal policy:

$$\pi^*(s) = \arg\max_a \sum_{s'} P(s'|s,a)V^*(s')$$

The Bellman equation

- Recursive relationship between optimal values of successive states:

$$V^*(s) = \underset{\text{Reward in current state}}{r(s)} + \underbrace{\gamma \max_a \sum_{s'} P(s'|s,a)V^*(s')}_{\text{Discounted expected future reward assuming agent follows the optimal policy}}$$

Outline

- Introduction to reinforcement learning
- Markov Decision Process (MDP) formalism
- The Bellman equation
- Q-learning

Q-learning

- Optimal policy in terms of the state value function:

$$\pi^*(s) = \arg\max_a \sum_{s'} P(s'|s,a)V^*(s')$$

  - To use this in practice, we need to know the transition model

- It is more convenient to define the value of a *state-action pair*:

$$Q^\pi(s,a) = \mathbb{E}\left[\sum_{t\geq 0} \gamma^t\, r(s_t)\,|s_0 = s, a_0 = a, \pi\right]$$

Q-value function

- The *optimal* Q-value:

$$Q^*(s,a) = \max_\pi \mathbb{E}\left[\sum_{t\geq 0} \gamma^t \, r(s_t) \mid s_0 = s, a_0 = a, \pi\right]$$

- What is the relationship between $V^*(s)$ and $Q^*(s,a)$?

$$V^*(s) = \max_a Q^*(s,a)$$

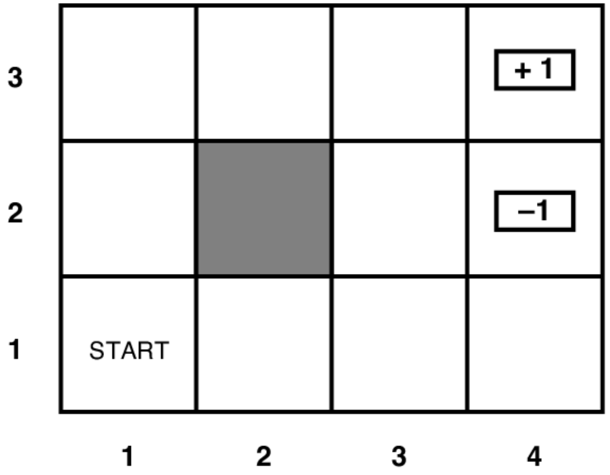
- What is the optimal policy?

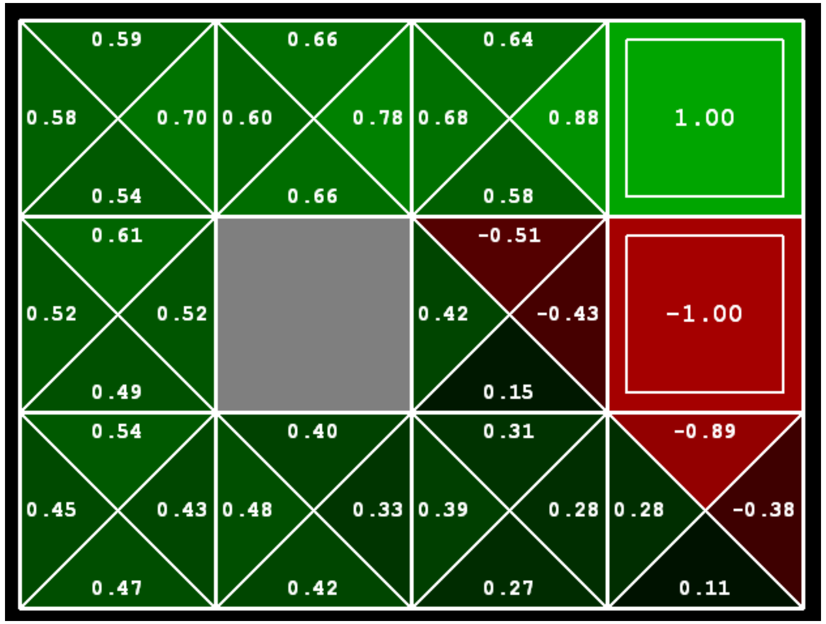
$$\pi^*(s) = \arg\max_a Q^*(s,a)$$

Q-value function

$$Q^*(s,a) = \max_\pi \mathbb{E}\left[\sum_{t\geq 0} \gamma^t\, r(s_t) \mid s_0 = s, a_0 = a, \pi\right]$$

$$\pi^*(s) = \arg\max_a Q^*(s,a)$$





Bellman equation for Q-values

$$V^*(s) = \max_a Q^*(s, a)$$

- Regular Bellman equation:

$$V^*(s) = r(s) + \gamma \max_a \sum_{s'} P(s'|s, a) V^*(s')$$

- Bellman equation for Q-values:

$$Q^*(s, a) = r(s) + \gamma \sum_{s'} P(s'|s, a) \max_{a'} Q^*(s', a')$$

$$= \mathbb{E}_{s' \sim P(\cdot|s,a)} [r(s) + \gamma \max_{a'} Q^*(s', a')|s, a]$$

Finding the optimal policy

- The Bellman equation is a constraint on Q-values of successive states:

$$Q^*(s,a) = \mathbb{E}_{s' \sim P(\cdot|s,a)} \left[r(s) + \gamma \max_{a'} Q^*(s',a') | s,a \right]$$

- We could think of $Q^*(s,a)$ as a table indexed by states and actions, and try to solve the system of Bellman equations to fill in the unknown values of the table
- **Problem:** state spaces for interesting problems are huge
- **Solution:** approximate Q-values using a parametric function:

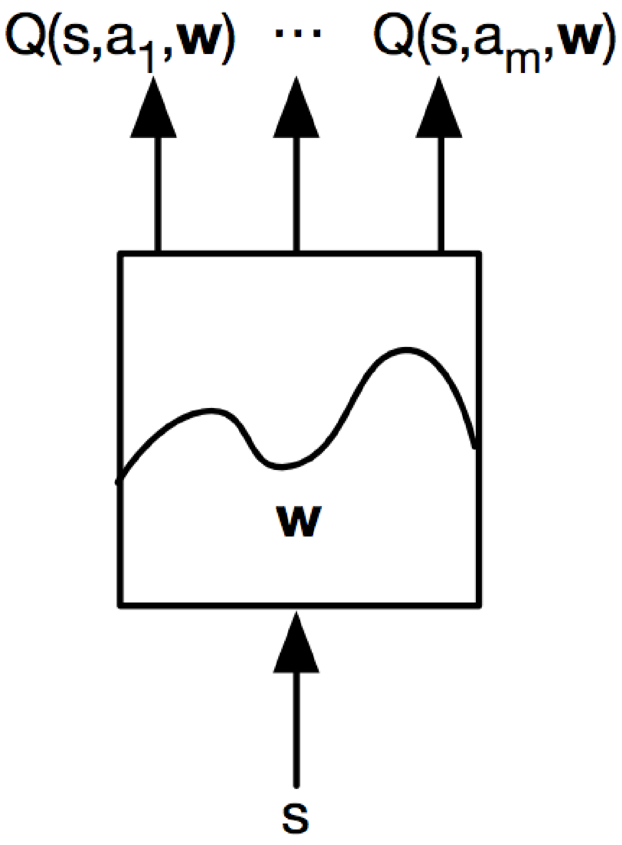
$$Q^*(s,a) \approx Q_w(s,a)$$

Outline

- Introduction to reinforcement learning
- Markov Decision Process (MDP) formalism and classical Bellman equation
- Q-learning
- Deep Q networks

Deep Q-learning

- Train a deep neural network to estimate Q-values:



Source: D. Silver

V. Mnih, K. Kavukcuoglu, D. Silver, A. Graves, I. Antonoglou, D. Wierstra, M. Riedmiller, Human-level control through deep reinforcement learning, *Nature* 2015

Deep Q-learning

$$Q^*(s,a) = \mathbb{E}_{s' \sim P(\cdot|s,a)}\left[r(s) + \gamma \max_{a'} Q^*(s',a')|s,a\right]$$

- Idea: at each iteration i of training, update model parameters w_i to "nudge" the left-hand side toward the right-hand "target":

$$y_i(s,a) = \mathbb{E}_{s' \sim P(\cdot|s,a)}\left[r(s) + \gamma \max_{a'} Q_{w_{i-1}}(s',a')|s,a\right]$$

- Loss function:

$$L_i(w_i) = \mathbb{E}_{s,a \sim \rho}\left[\left(y_i(s,a) - Q_{w_i}(s,a)\right)^2\right]$$

where ρ is a *behavior distribution*

Deep Q-learning

- Target: $y_i(s,a) = \mathbb{E}_{s' \sim P(\cdot|s,a)} \left[r(s) + \gamma \max_{a'} Q_{w_{i-1}}(s',a') | s,a \right]$
- Loss: $L_i(w_i) = \mathbb{E}_{s,a\sim\rho} \left[\left(y_i(s,a) - Q_{w_i}(s,a) \right)^2 \right]$
- Gradient update:

$$\nabla_{w_i} L(w_i) = \mathbb{E}_{s,a\sim\rho} \left[\left(y_i(s,a) - Q_{w_i}(s,a) \right) \nabla_{w_i} Q_{w_i}(s,a) \right]$$
$$= \mathbb{E}_{s,a\sim\rho,s'} \left[\left(r(s) + \gamma \max_{a'} Q_{w_{i-1}}(s',a') - Q_{w_i}(s,a) \right) \nabla_{w_i} Q_{w_i}(s,a) \right]$$

- SGD training: replace expectation by sampling *experiences* (s,a,s') using behavior distribution and transition model

Deep Q-learning in practice

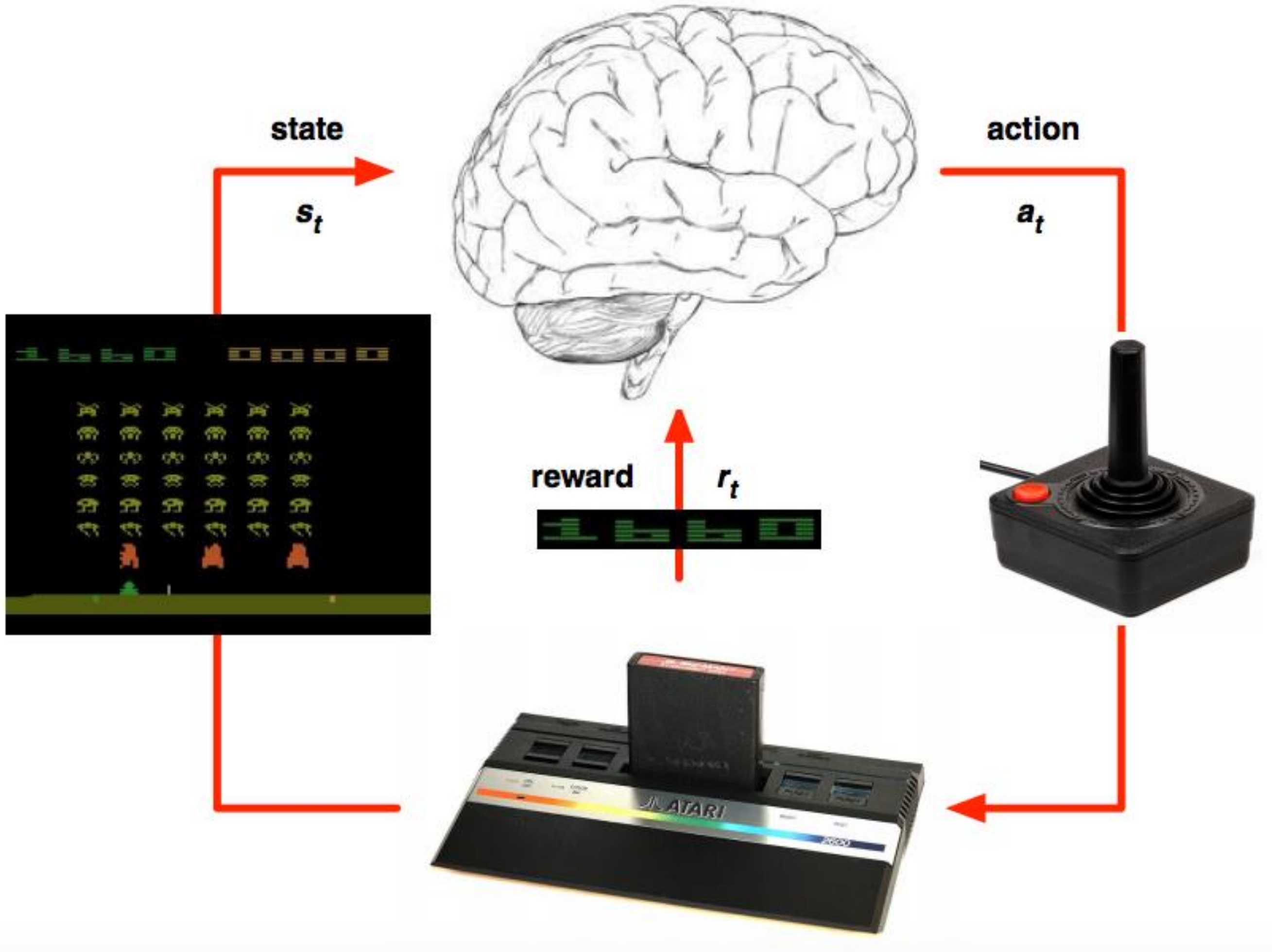
- Training is prone to instability
  - Unlike in supervised learning, the targets themselves are moving!
  - Successive experiences are correlated and dependent on the policy
  - Policy may change rapidly with slight changes to parameters, leading to drastic change in data distribution
- Solutions
  - Freeze target Q network
  - Use *experience replay*

Experience replay

- At each time step:
  - Take action a_t according to *epsilon-greedy policy*
  - Store experience $(s_t, a_t, r_{t+1}, s_{t+1})$ in *replay memory buffer*
  - Randomly sample *mini-batch* of experiences from the buffer

s_1, a_1, r_2, s_2
s_2, a_2, r_3, s_3
s_3, a_3, r_4, s_4
$\ldots$
$s_t, a_t, r_{t+1}, s_{t+1}$

Deep Q-learning in Atari

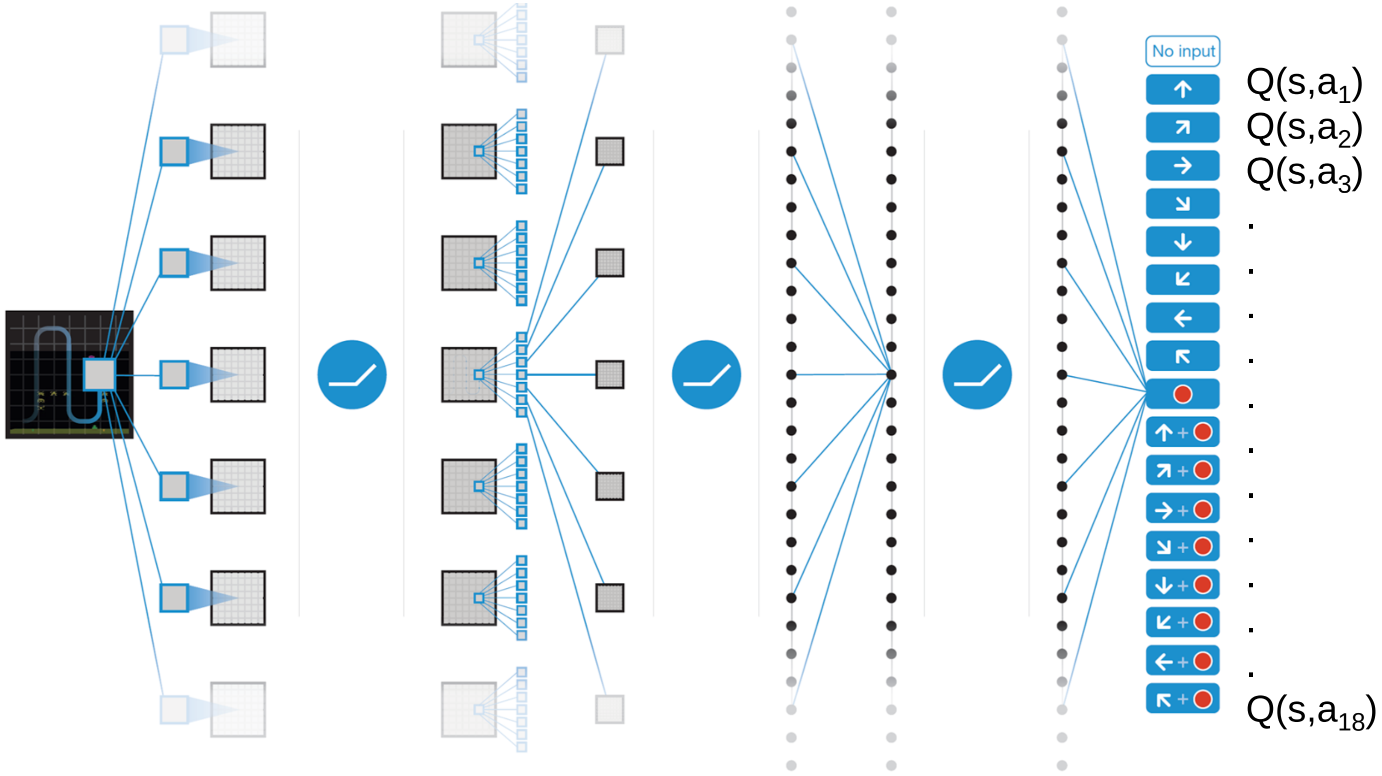


V. Mnih, K. Kavukcuoglu, D. Silver, A. Graves, I. Antonoglou, D. Wierstra, M. Riedmiller,
Human-level control through deep reinforcement learning, *Nature* 2015

Deep Q-learning in Atari

- End-to-end learning of $Q(s,a)$ from pixels s
- Output is $Q(s,a)$ for 18 joystick/button configurations
- Reward is change in score for that step

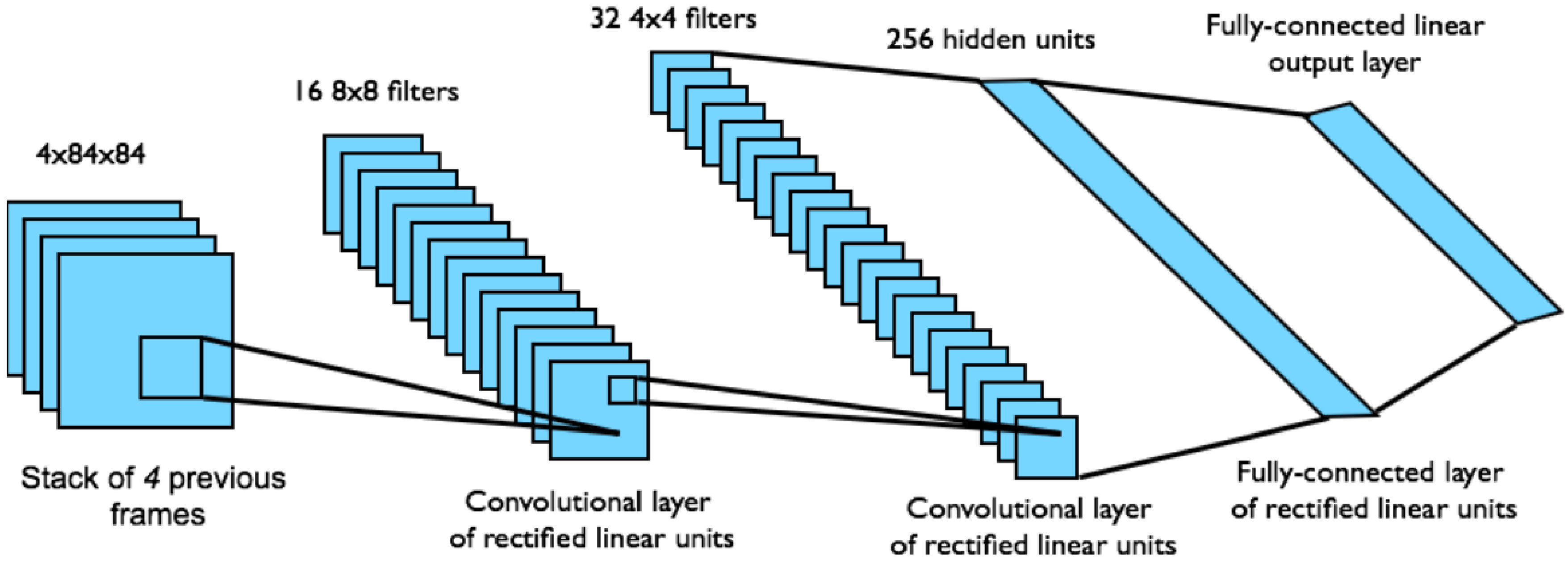
Deep Q-Network (DQN)



Deep Q-learning in Atari

- Input state is stack of raw pixels (grayscale) from last 4 frames
- Network architecture and hyperparameters fixed for all games

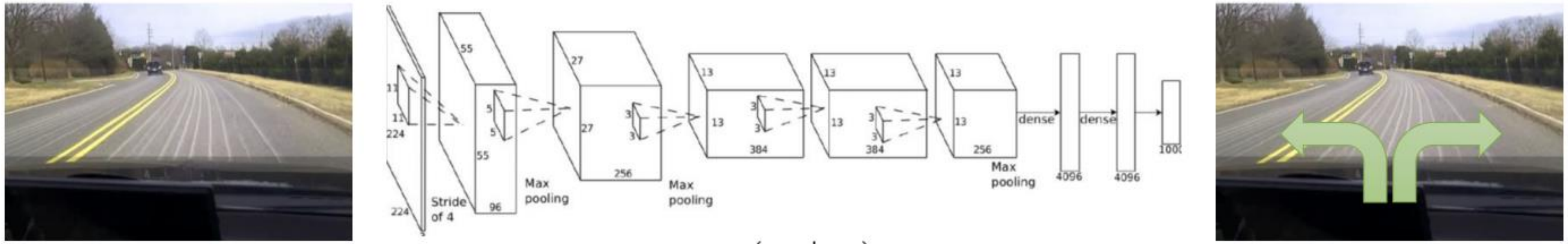
Deep Q-Network (DQN)



Breakout demo

https://www.youtube.com/watch?v=TmPfTpjtdgg

Policy Gradient Methods



$\mathbf{s}_t$ $\pi_\theta(\mathbf{a}_t|\mathbf{s}_t)$ $\mathbf{a}_t$

Sources: Stanford CS 231n, Berkeley Deep RL course, David Silver's RL course

Image source

Policy Gradient Methods

- Instead of indirectly representing the policy using Q-values, it can be more efficient to parameterize and learn it directly
  - Especially in large or continuous action spaces

Image source: OpenAI Gym

Stochastic policy representation

- Learn a function giving the probability distribution over actions from the current state:

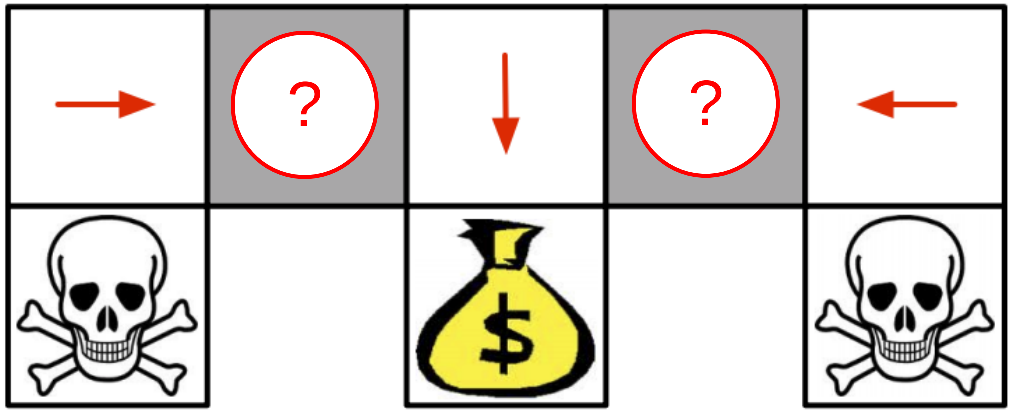
$$\pi_\theta(a|s) \approx P(a|s)$$

Stochastic policy representation

- Learn a function giving the probability distribution over actions from the current state:

$$\pi_\theta(a|s) \approx P(a|s)$$

- Why stochastic policies?
  - There are examples even of grid world scenarios where only a stochastic policy can reach optimality



The agent can't tell the difference between the gray cells

Source: [D. Silver]

Stochastic policy representation

- Learn a function giving the probability distribution over actions from the current state:

$$\pi_\theta(a|s) \approx P(a|s)$$

- Why stochastic policies?
  - It's mathematically convenient!
    - Softmax policy:

$$\pi_\theta(a|s) = \frac{\exp(f_\theta(s,a))}{\sum_{a'} \exp(f_\theta(s,a'))}$$

    - Gaussian policy (for continuous action spaces):

$$\pi_\theta(a|s) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(a - f_\theta(s))^2}{2\sigma^2}\right)$$

Expected value of a policy

$$J(\theta) = \mathbb{E}\left[\sum_{t\geq 0} \gamma^t \, r_t \mid \pi_\theta\right]$$

$$= \mathbb{E}_\tau[r(\tau)]$$

Expectation of return over *trajectories* $\tau = (s_0, a_0, r_0, s_1, a_1, r_1, \dots)$

$$= \int_\tau r(\tau)\underbrace{p(\tau;\theta)}_{\text{Probability of trajectory } \tau \text{ under policy with parameters } \theta} d\tau$$

Probability of trajectory τ
under policy with
parameters θ

Finding the policy gradient

$$J(\theta) = \int_{\tau} r(\tau)p(\tau;\theta)d\tau$$

$$\nabla_\theta J(\theta) = \int_{\tau} r(\tau)\nabla_\theta p(\tau;\theta)d\tau$$

$$= \int_{\tau} r(\tau)p(\tau;\theta)\boxed{\frac{\nabla_\theta p(\tau;\theta)}{p(\tau;\theta)}}d\tau$$

$$= \int_{\tau} r(\tau)p(\tau;\theta)\nabla_\theta \log p(\tau;\theta)\, d\tau$$

$$= \mathbb{E}_\tau[r(\tau)\nabla_\theta \log p(\tau;\theta)]$$

Finding the policy gradient

$$\nabla_\theta J(\theta) = \mathbb{E}_\tau\left[r(\tau)\underbrace{\nabla_\theta \log p(\tau;\theta)}_{\substack{\text{Probability of trajectory} \\ \tau = (s_0, a_0, s_1, a_1, \ldots)}}\right]$$

$$p(\tau;\theta) = \prod_{t\geq 0} \pi_\theta(a_t|s_t)P(s_{t+1}|s_t, a_t)$$

$$\log p(\tau;\theta) = \sum_{t\geq 0}\left[\log \pi_\theta(a_t|s_t) + \log P(s_{t+1}|s_t, a_t)\right]$$

$$\nabla_\theta \log p(\tau;\theta) = \sum_{t\geq 0} \underbrace{\nabla_\theta \log \pi_\theta(a_t|s_t)}_{\text{The } \textit{score function}}$$

Score function $\nabla_\theta \log \pi_\theta(a|s)$

- For softmax policy:

$$\pi_\theta(a|s) = \frac{\exp(f_\theta(s,a))}{\sum_{a'} \exp(f_\theta(s,a'))}$$

$$\nabla_\theta \log \pi_\theta(a_t|s_t) = \nabla_\theta f_\theta(s,a) - \sum_{a'} \pi_\theta(a'|s)\, \nabla_\theta f_\theta(s,a')$$

- For Gaussian policy:

$$\pi_\theta(a|s) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(a - f_\theta(s))^2}{2\sigma^2}\right)$$

$$\nabla_\theta \log \pi_\theta(a_t|s_t) = \frac{\left(a - f_\theta(s)\right)}{\sigma^2} \nabla_\theta f_\theta(s) - \text{const.}$$

Finding the policy gradient

$$\nabla_\theta J(\theta) = \mathbb{E}_\tau[r(\tau)\nabla_\theta \log p(\tau;\theta)]$$

$$\nabla_\theta \log p(\tau;\theta) = \sum_{t\geq 0} \nabla_\theta \log \pi_\theta(a_t|s_t)$$

$$\nabla_\theta J(\theta) = \mathbb{E}_\tau\left[\underbrace{\left(\sum_{t\geq 0}\gamma^t r_t\right)}_{\text{Return of trajectory } \tau}\underbrace{\left(\sum_{t\geq 0}\nabla_\theta \log \pi_\theta(a_t|s_t)\right)}_{\text{Gradient of log-likelihood of actions under current policy}}\right]$$

- How do we estimate the gradient in practice?

Finding the policy gradient

$$\nabla_\theta J(\theta) = \mathbb{E}_\tau [r(\tau) \nabla_\theta \log p(\tau; \theta)]$$

$$\nabla_\theta \log p(\tau; \theta) = \sum_{t \geq 0} \nabla_\theta \log \pi_\theta (a_t | s_t)$$

$$\nabla_\theta J(\theta) = \mathbb{E}_\tau \left[\left(\sum_{t \geq 0} \gamma^t r_t \right) \left(\sum_{t \geq 0} \nabla_\theta \log \pi_\theta (a_t | s_t) \right) \right]$$

- Stochastic approximation: sample N trajectories $\tau_1, \ldots, \tau_N$

$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \left(\sum_{t=0}^{T_i} \gamma^t r_{i,t} \right) \left(\sum_{t=0}^{T_i} \nabla_\theta \log \pi_\theta (a_{i,t} | s_{i,t}) \right)$$

REINFORCE algorithm

1. Sample N trajectories τ_i using current policy π_θ

2. Estimate the policy gradient:

$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} r(\tau_i) \left(\sum_{t=0}^{T_i} \nabla_\theta \log \pi_\theta(a_{i,t} | s_{i,t}) \right)$$

3. Update parameters by gradient ascent:

$$\theta \leftarrow \theta + \eta \nabla_\theta J(\theta)$$

Williams et al. Simple statistical gradient-following algorithms for connectionist reinforcement learning. Machine Learning, 8(3):229-256, 1992

REINFORCE: Single-step version

1. In state s, sample action a using current policy π_θ, observe reward r

2. Estimate the policy gradient:
$$\nabla_\theta J(\theta) \approx r \, \nabla_\theta \log \pi_\theta(a|s)$$

3. Update parameters by gradient ascent:
$$\theta \leftarrow \theta + \eta \nabla_\theta J(\theta)$$

- What effect does this update have?
  - Push up the probability of good actions, push down probability of bad actions

Acknowledgement

Thanks to the following courses and corresponding researchers for making their teaching/research material online

- Deep Learning, Stanford University
- Introduction to Deep Learning, University of Illinois at Urbana-Champaign
- Introduction to Deep Learning, Carnegie Mellon University
- Convolutional Neural Networks for Visual Recognition, Stanford University
- Natural Language Processing with Deep Learning, Stanford University
- And Many More ......