PROBLEM 1: RANK

$$\mathbf{A} \quad = \quad \begin{bmatrix} 2 & 2 & 6 \\ -1 & 1 & -1 \end{bmatrix}$$

a. **A** has 2 rows, so the row rank of $\mathbf{A} \leq 2$.

b. **A** has 3 rows, so the column rank of $\mathbf{A} \leq 3$.

c. Rank($\mathbf{A}$) $\leq$ min(2, 3).

d. Row rank of **A**:

$$\begin{bmatrix} 2 & 2 & 6 \\ -1 & 1 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 3 \\ -1 & 1 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 3 \\ 0 & 2 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} \Rightarrow \text{row rank} = 2$$

e. Column rank of **A**:

$$\begin{bmatrix} 2 & 2 & 6 \\ -1 & 1 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 6 \\ -1/2 & 1 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 2 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 1 & 2 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 1 & 0 \end{bmatrix} \Rightarrow \text{column rank} = 2$$

f. rank(**A**) = row rank = column rank = 2.