

**PROBLEM 4.2: SPECTRAL DECOMPOSITION OF A 3 x 3 EXCHANGEABLE CORRELATION MATRIX**

$$A = \begin{bmatrix} 1 & 0.2 & 0.2 \\ 0.2 & 1 & 0.2 \\ 0.2 & 0.2 & 1 \end{bmatrix}$$

a. All eigenvalues (1.4, 0.8, 0.8) are greater than zero, so A is positive definite.

b. Spectral decomposition:

$$\mathbf{v}_1 = \begin{bmatrix} 0.57735 \\ 0.57735 \\ 0.57735 \end{bmatrix} \quad \mathbf{v}_2 = \begin{bmatrix} 0.66057 \\ 0.08532 \\ -0.74590 \end{bmatrix} \quad \mathbf{v}_3 = \begin{bmatrix} 0.47991 \\ -0.81203 \\ 0.33212 \end{bmatrix}$$

$$1.4 \mathbf{v}_1 \mathbf{v}_1^t + 0.8 \mathbf{v}_2 \mathbf{v}_2^t + 0.8 \mathbf{v}_3 \mathbf{v}_3^t =$$

$$1.4 \begin{bmatrix} 0.57735 \\ 0.57735 \\ 0.57735 \end{bmatrix} \begin{bmatrix} 0.57735 & 0.57735 & 0.57735 \end{bmatrix} +$$

$$0.8 \begin{bmatrix} 0.66057 \\ 0.08532 \\ -0.74590 \end{bmatrix} \begin{bmatrix} 0.66057 & 0.08532 & -0.74590 \end{bmatrix} +$$

$$0.8 \begin{bmatrix} 0.47991 \\ -0.81203 \\ 0.33212 \end{bmatrix} \begin{bmatrix} 0.47991 & -0.81203 & 0.33213 \end{bmatrix}$$

$$= 1.4 \begin{bmatrix} 0.33333 & 0.33333 & 0.33333 \\ 0.33333 & 0.33333 & 0.33333 \\ 0.33333 & 0.33333 & 0.33333 \end{bmatrix} +$$

$$0.8 \begin{bmatrix} 0.43635 & 0.05636 & -0.49272 \\ 0.05636 & 0.00728 & -0.06364 \\ -0.49272 & -0.06364 & 0.55637 \end{bmatrix} +$$

$$\begin{aligned}
& 0.8 \begin{bmatrix} 0.23031 & -0.38970 & 0.15939 \\ -0.38970 & 0.65939 & -0.26969 \\ 0.15939 & -0.26969 & 0.11030 \end{bmatrix} \\
= & \begin{bmatrix} 1.00000 & 0.20000 & 0.20000 \\ 0.20000 & 1.00000 & 0.20000 \\ 0.20000 & 0.20000 & 1.00000 \end{bmatrix} \quad \sqrt{\phantom{x}}
\end{aligned}$$