

### PROBLEM 3.1: EIGENSET OF A 2 x 2 MATRIX

$$\mathbf{A} = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}$$

a.  $\det(\mathbf{A}) = (1)(1) - (3)(3) = 1 - 9 = -8$ .

b.  $\text{tr}(\mathbf{A}) = 1 + 1 = 2$ .

c. Eigenvalues:

$$\det(\mathbf{A} - \lambda \mathbf{I}) = (1-\lambda)(1-\lambda) - 9 = 0$$

$$\lambda^2 - 2\lambda + 1 - 9 = 0$$

$$\lambda^2 - 2\lambda + 8 = 0 = (\lambda - 4)(\lambda + 2) \Rightarrow \text{eigenvalues } \lambda = 4, -2$$

d.  $\det(\mathbf{A}) = -8 = (4)(-2) = \text{product of eigenvalues}$ .

e.  $\text{tr}(\mathbf{A}) = 2 = (4) + (-2) = \text{sum of eigenvalues}$ .

f. Eigenvectors:

$$\begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 4 \begin{bmatrix} x \\ y \end{bmatrix} \Rightarrow x + 3y = 4x \Rightarrow 3x - 3y = 0 \Rightarrow x = y$$

$$\mathbf{v}_1 = \begin{bmatrix} x \\ x \end{bmatrix} \Rightarrow \mathbf{e}_1 = \begin{bmatrix} 0.70711 \\ 0.70711 \end{bmatrix} \text{ when normalized}$$

$$\begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = -2 \begin{bmatrix} x \\ y \end{bmatrix} \Rightarrow x + 3y = -2x \Rightarrow 3x + 3y = 0 \Rightarrow x = -y$$

$$\mathbf{v}_2 = \begin{bmatrix} x \\ -x \end{bmatrix} \Rightarrow \mathbf{e}_2 = \begin{bmatrix} 0.70711 \\ -0.70711 \end{bmatrix} \text{ when normalized}$$