1.1 Vector Operations

$u = [-1 \ 0 \ 1]$ and $v = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$

a. Find u^t = transpose(u) $\Rightarrow$ $u^t = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

b. Find v^t = transpose(v) $\Rightarrow$ $v^t = [2 \ 4 \ 6]$

c. Find uv (inner product)

$$uv = \sum_{i=1}^{n} u_i v_i = [-1 \ 0 \ 1] \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} = (-1)(2) + (0)(4) + (1)(6)$$

$$uv = 4$$

d. Find $v \times u$ (outer product)

$$v \times u = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} [-1 \ 0 \ 1] = \begin{bmatrix} -2 & 0 & 2 \\ -4 & 0 & 4 \\ -6 & 0 & 6 \end{bmatrix}$$

1.2 Matrix Operations

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

a. Find B^t = transpose(B)

$$B^t = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

b. Find AB

$$\begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} (1)(1)+(0)(1)+(-1)(1) & (1)(0)+(0)(1)+(-1)(0) \\ (2)(1)+(1)(1)+(0)(1) & (2)(0)+(1)(1)+(0)(0) \\ (0)(1)+(1)(1)+(2)(1) & (0)(0)+(1)(1)+(2)(0) \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 3 & 1 \\ 3 & 1 \end{bmatrix}$$

c. Find A - C

$$A - C = \begin{bmatrix} (1-2) & (0-0) & (-1-0) \\ (2-0) & (1-1) & (0-(-1)) \\ (0-0) & (1-(-1)) & (2-1) \end{bmatrix} = \begin{bmatrix} -1 & 0 & -1 \\ 2 & 0 & 1 \\ 0 & 2 & 1 \end{bmatrix}$$

d. Find CA

$$CA = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} (2)(1)+(0)(2)+(0)(0) & (2)(0)+(0)(1)+(0)(1) & (2)(-1)+(0)(0)+(0)(2) \\ (0)(1)+(1)(2)+(-1)(0) & (0)(0)+(1)(1)+(-1)(1) & (0)(-1)+(1)(0)+(-1)(2 \\ (0)(1)+(-1)(2)+(1)(0) & (0)(0)+(-1)(1)+(1)(1) & (0)(-1)+(-1)(0)+(1)(2) \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & -2 \\ 2 & 0 & -2 \\ -2 & 0 & 2 \end{bmatrix}$$

1.3 Quadratic Form

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \qquad b = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

a. Find $b^* A\, b$

$$b^* A\, b = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} (1)(2) + (0)(0) + (1)(1) & (1)(0) + (0)(1) + (1)(1) & (1)(1) + (0)(1) + (1)(0) \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$= (3)(1) + (1)(0) + (1)(1)$$

$$= 4$$

1.4 Solution of a set of linear equations

$$x + y - z = \varnothing$$
$$x - y + z = 2$$
$$2x - 3y + z = -1$$

a. Solve for x, y and z

L_1: $x + y - z = 0$
L_2: $x - y + z = 2$
L_3: $2x - 3y + z = -1$

$(-2 \times L_1) + L_3 \Rightarrow$
$$-2x - 2y + 2z = 0$$
$$2x - 3y + z = -1$$
$$-5y + 3z = -1$$

$(-1 \times L_1) + L_2 \Rightarrow$
$$-x - y + z = 0$$
$$x - y + z = 2$$
$$0x - 2y + 2z = 2$$

L_1: $x + y - z = 0$
L_2: $-2y + 2z = 2$
L_3: $-5y + 3z = -1$

$(L_2 \div -2) \Rightarrow y - z = -1$

L_1: $x + y - z = 0$
L_2: $y - z = -1$
L_3: $-5y + 3z = -1$

$(5L_2 + L_3) \Rightarrow$
$$5y - 5z = -5$$
$$-5y + 3z = -1$$
$$-2z = -6 \Rightarrow z = \frac{-6}{-2} = \boxed{3 = z}$$

Substituting into L_2 to solve for y

$$y - 3 = -1 \Rightarrow \boxed{y = 2}$$

Solving for x in L_1 $\quad x + 2 - 3 = 0 \Rightarrow \boxed{x = 1}$

Checking initial equation

$1 + 2 - 3 \stackrel{?}{=} 0$
$1 - 2 + 3 = 2$
$2(1) - 3(2) + 3 = -1$

$$\boxed{\begin{array}{l} x = 1 \\ y = 2 \\ z = 3 \end{array}}$$