

PROBLEM 2: EIGENSET OF A 3 x 3 EXCHANGEABLE CORRELATION MATRIX

$$A = \begin{bmatrix} 1 & 0.2 & 0.2 \\ 0.2 & 1 & 0.2 \\ 0.2 & 0.2 & 1 \end{bmatrix}$$

a. $\det(A) = 1 [(1)(1) - (0.2)(0.2)] - 0.2 [(0.2)(1) - (0.2)(0.2)] + 0.2$

$$[(0.2)(0.2) - (0.2)(1)] = 1(0.96) - 0.2(0.16) + 0.2(0.16)$$

$$= 0.896$$

b. Characteristic equation for A:

$$\det(A - \lambda \mathbf{I}) = (1-\lambda) [(1-\lambda)(1-\lambda) - 0.04] - 0.2 [0.2(1-\lambda) - 0.04] + 0.2 [0.04 - 0.2(1-\lambda)] = 0$$

$$\lambda^3 - 3\lambda^2 + 2.88\lambda - 0.896 = 0$$

For $\lambda = 1.4$, this becomes $(1.4)^3 - 3(1.4)^2 + 2.88(1.4) - 0.896 = 0$. \checkmark

For either $\lambda = 0.8$, this becomes $(0.8)^3 - 3(0.8)^2 + 2.88(0.8) - 0.896 = 0$. \checkmark

c. $\det(A) = 0.896 = (1.4)(0.8)(0.8)$, the product of the eigenvalues of A. Also note that $1.4 + 0.8 + 0.8 = 3.0 = \text{tr}(A)$.

d. 1) Compute $\|\mathbf{v}\|^2 = \mathbf{v}^t \mathbf{v} = 0.57735^2 + 0.57735^2 + 0.57735^2 = 1.00000$. \checkmark

Compute $A\mathbf{v} - 1.4\mathbf{v}$ and see if it is 0.

$$\begin{bmatrix} 1 & 0.2 & 0.2 \\ 0.2 & 1 & 0.2 \\ 0.2 & 0.2 & 1 \end{bmatrix} \begin{bmatrix} 0.57735 \\ 0.57735 \\ 0.57735 \end{bmatrix} - 1.4 \begin{bmatrix} 0.57735 \\ 0.57735 \\ 0.57735 \end{bmatrix}$$

$$= \begin{bmatrix} 0.80829 \\ 0.80829 \\ 0.80829 \end{bmatrix} - \begin{bmatrix} 0.80829 \\ 0.80829 \\ 0.80829 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \checkmark$$

R SCRIPT USEFUL FOR ASSIGNMENT 3.2:

```
A<- matrix(c(1,.2,.2, .2,1,.2, .2,.2,1), ncol=3, byrow=TRUE)
A
det(A) #determinant

eA<- eigen(A) #eigenset
eA
prod(eA$values) #product of eigenvalues = determinant?

v<- c(0.57735,0.57735,0.57735)
v%*%v #norm of v

A%*%v - 1.4*v
```