

PROBLEM 4: DETERMINANTS

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \\ 1 & 2 & 1 \end{bmatrix}$$

a. Find $\det(\mathbf{A})$:

$$|\mathbf{A}| = 0 - 1 \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} + 0 = -1(2 - 1) = -1$$

b. Is \mathbf{A} invertible?

$$|\mathbf{A}| \neq 0, \text{ so } \mathbf{A} \text{ is nonsingular and invertible.}$$

c. What is $\text{rank}(\mathbf{A})$?

$$|\mathbf{A}| \neq 0, \text{ so } \mathbf{A} \text{ is of full rank} = 3$$

d. What is $\det(\mathbf{A}^t)$?

$$|\mathbf{A}^t| = |\mathbf{A}| = -1$$

Please turn over to the next page...

R SCRIPT USEFUL FOR ASSIGNMENT 2:

```
#PROBLEM 2: RANK
A<- matrix(c(2,2,6, -1,1,-1), ncol=3, byrow=TRUE)
A

require('Matrix') #make sure package 'Matrix' is loaded
as.numeric(rankMatrix(A)) #find rank(A)

#PROBLEM 3: GENERALIZED INVERSE
A<- matrix(c(2,2,6, -1,1,-1), ncol=3, byrow=TRUE)
A
b<- c(1,0)
b

require('MASS') #make sure package 'MASS' is loaded
G<- ginv(A) #find a generalized inverse
G
A %*% G %*% A #check is really generalized inverse

G2<- matrix(c(1/4,-1/2, 1/4,1/2, 0,0), ncol=2, byrow=TRUE) #G by hand calculation
A %*% G2 %*% A #check is really generalized inverse

#general solution to  $Ax = b$ 
require('Matrix')
I<- as.matrix(Diagonal(3)) #3x3 identity matrix
I

C<- G %*% A - I #matrix to find general solution to homogeneous eqtns  $Ax = b$ 
C

C2<- G2 %*% A - I #matrix for hand G
C2

#particular solution to  $Ax = b$ 
xp<- G %*% b #particular solution
xp
A %*% xp #check = b

xp2<- G2 %*% b #particular solution for by hand G
xp2
A %*% xp2 #check = b

#PROBLEM 4: DETERMINANTS
A<- matrix(c(0,1,0, 2,0,1, 1,2,1), ncol=3, byrow=TRUE)
A

det(A) #determinant
as.numeric(rankMatrix(A)) #check rank. det(A) != 0 means full rank = 3
det(t(A)) #determinant of transpose
```