PROBLEM 3: GENERALIZED INVERSE

$$\mathbf{A} = \begin{bmatrix} 2 & 2 & 6 \\ -1 & 1 & -1 \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

a. Find generalized inverse **G**:

$$\mathbf{W} = \begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix}, \qquad \mathbf{W}^{-1} = \begin{bmatrix} 1 & -2 \\ 1 & 2 \end{bmatrix} / (2+2) = \begin{bmatrix} 1/4 & -1/2 \\ 1/4 & 1/2 \end{bmatrix}$$

$$(\mathbf{W}^{-1})^t = \begin{bmatrix} 1/4 & 1/4 \\ -1/2 & 1/2 \end{bmatrix}$$

$$\mathbf{G} = \begin{bmatrix} 1/4 & 1/4 & 0 \\ -1/2 & 1/2 & 0 \end{bmatrix}^t = \begin{bmatrix} 1/4 & -1/2 \\ 1/4 & 1/2 \\ 0 & 0 \end{bmatrix}$$

b. General solution to $\mathbf{A}\,\mathbf{x} = \mathbf{0}$:

$$\mathbf{C} = \mathbf{G}\,\mathbf{A} - \mathbf{I}$$

$$= \begin{bmatrix} 1/4 & -1/2 \\ 1/4 & 1/2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 2 & 2 & 6 \\ -1 & 1 & -1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\mathbf{x}_h = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2z \\ z \\ -z \end{bmatrix} = z \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$$

c. Find the general solution to $\mathbf{A}\,\mathbf{x} = \mathbf{b}$:

$$\mathbf{x} = \mathbf{G}\,\mathbf{b} + \mathbf{C}\,\boldsymbol{\theta} = \mathbf{G}\,\mathbf{b} + \mathbf{x}_h$$

$$= \begin{bmatrix} 1/4 & -1/2 \\ 1/4 & 1/2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 2z \\ z \\ -z \end{bmatrix}$$

$$= \begin{bmatrix} 1/4 \\ 1/4 \\ 0 \end{bmatrix} + \begin{bmatrix} 2z \\ z \\ -z \end{bmatrix} = \begin{bmatrix} 1/4 + 2z \\ 1/4 + z \\ -z \end{bmatrix}$$