COMP 9334

Capacity Planning

Assignment, Session 1, 2017

Assignment 1

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Question 1

Answer:

(a)

$$D(j) = V(j) \cdot S(j) = \frac{C(j)}{C(0)} \times \frac{B(j)}{C(j)} = \frac{B(j)}{C(0)}$$

$B(j)$ = Busy time for the device j

$C(0)$ = Number of completions for the system.

$$\therefore D(CPU) = \frac{B(CPU)}{C(0)} = \frac{1102}{1237} \approx 0.895\,s = 895\,ms$$

$$D(Disk1) = \frac{B(Disk1)}{C(0)} = \frac{929}{1231} \approx 0.755\,s = 755\,ms$$

$$D(Disk2) = \frac{B(Disk2)}{C(0)} = \frac{1017}{1231} \approx 0.826\,s = 826\,ms$$

$$D(Disk3) = \frac{B(Disk3)}{C(0)} = \frac{1265}{1231} \approx 1.028\,s = 1028\,ms$$

(b)

$$X(0) \le \min\left[\frac{1}{\max D_i}, \frac{N}{\sum_{i=1}^{k} D_i}\right]$$

$$\text{Bond 1} = \frac{1}{\max D_i} = \frac{1}{D(Disk3)} = \frac{1}{1.028}\ \text{jobs/s} \approx 0.973\ \text{job/s}$$

$$\text{Bond 2} = \frac{N}{\sum_{i=1}^{k} D_i} = \frac{40}{0.895+0.755+0.826+1.028} \approx 11.42\ \text{jobs/s},$$

(c)

$$M_{max} = X_0 \cdot (Z+R) \quad \cdots\cdots$$

$$\therefore \frac{M_{max}}{X_0} - Z = R \quad \cdots\cdots$$

$M_{max} = M$, R is the minimum time bond.

$$\therefore R = \frac{40}{0.973} - 27 = 14.1\,s.$$

Hence, The minimum possible response time is 14.1 s.

Question 2

Answer :

(a) Probability that an arrival is blocked

= Probability that there are m customers in the system.

$$P_m = \frac{\frac{\rho^m}{m!}}{\sum_{k=0}^{m} \frac{\rho^k}{k!}}$$, where $\rho = \frac{\lambda}{\mu}$, according to the question,

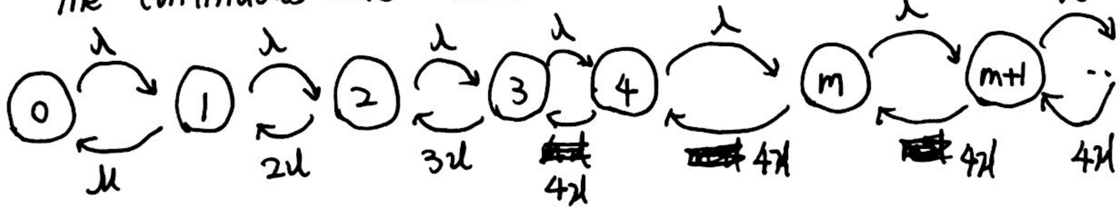
$m = 4$, $\rho = \frac{\lambda}{\mu} = \frac{\frac{1}{3}}{\frac{1}{10}} = \frac{10}{3} \approx 3.33$

$$\therefore P_4 = \frac{\frac{(\frac{10}{3})^4}{4!}}{\frac{(\frac{10}{3})^0}{0!} + \frac{(\frac{10}{3})^1}{1!} + \frac{(\frac{10}{3})^2}{2!} + \frac{(\frac{10}{3})^3}{3!} + \frac{(\frac{10}{3})^4}{4!}} \approx 0.243$$

(b)

(i)

The continuous-time Markov chain :



state 0 = There is no call in the system.

state 1 = There is 1 call in the system (= 1 call at one of the operators, no call in the slots)

state 2 = There is 2 calls in the system (= 2 calls at the two of the operators, no call in the slots.)

state 3 = There is 3 calls in the system (= 3 ,
no)

state 4 = There is 4 calls in the system (= 4
no)

state m = There is m calls in thesystem (= 4 calls at the operators, (m-4) calls in the slots).

state m+1 = There is m+1 calls in the system (= 4 calls at the operators, (m-3) calls in the slots)

...

(b) (i) The transition rates are shown ~~on~~ at the chain, where

$\lambda = \frac{1}{3} \approx 0.333$ calls/min

$\mu = \frac{1}{\text{mean service rate}} = \frac{1}{10} = 0.1$ service/min.

(ii)

$$\lambda P_0 = \mu P_1$$
$$2\mu P_2 = \lambda P_1$$
$$3\mu P_3 = \lambda P_2$$
$$4\mu P_4 = \lambda P_3$$
$$\vdots$$
$$4\mu P_m = \lambda P_{m-1}$$
$$4\mu P_{m+1} = \lambda P_m$$
$$\vdots$$
$$P_0 + P_1 + P_2 + P_3 + P_4 + \cdots + P_m + P_{m+1} \cdots = 1$$

where $\lambda = 0.333$ calls/min

$\mu = 0.1$ service/min.

(iii) The steady state probabilities:

I don't make expressions for them, because I made the calculation jobs by using a python program (which I submitted as "solve_equations.py". Therefore, I could calculate $P_0, P_1, P_2 \ldots P_m, P_{m+1}$ without express them by using only λ and μ.

Please check my codes attached, thank you.

(iv) From the result of Part (a)

the rejection rate with no slots is 0.243, which 50% is about 0.121, hence we need to fetch a $P(M+4) < 0.121$, where we have $m-4$ slots.

According to the calculation with "solve_equation.py" I made, the smallest M could be 3, i.e. there are 4 operators and 3 holding slots. In the situation,

$P(M+4) = P(7) = 0.0929$, which is the ~~smallest M~~ first time (the smallest M), $P(M+4) < 0.121$.

Therefor, the smallest M is 3.

(v) According to the Little's Law:

$$N_{avg} = X(0) \times R_{avg}, \text{ where.}$$

$$X(0) = \lambda$$

$$R_{avg} = T = \frac{C(\rho, m)}{m\mu(1-\rho)} + \frac{1}{\mu}. \quad (\lambda = \frac{1}{3}, \mu = \frac{1}{10}, \therefore \rho = \frac{5}{6})$$

$$= \frac{0.0514}{4 \times \frac{1}{10} \times (1-\frac{5}{6})} + \frac{1}{\frac{1}{10}}.$$

$$= 10.772 \text{ mins.}$$

$$\Rightarrow N_{avg} = \frac{1}{3} \times 10.772 = 3.591 \text{ calls.}$$

$$\therefore \text{The waiting time} = \frac{N_{avg} \times P(\text{slots})}{X(0)}$$

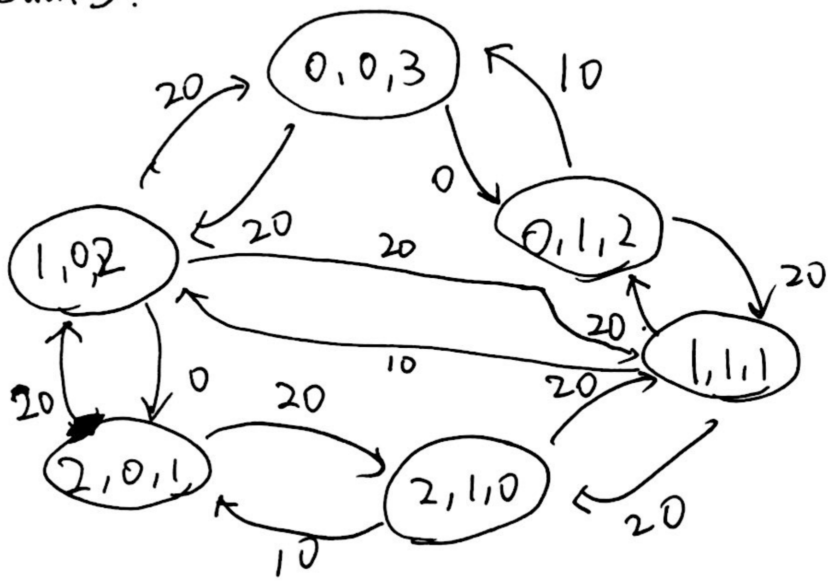
$$= \frac{N_{avg} \times [P(5) + P(6) + P(7)]}{X(0)}$$

$$= \frac{3.591 \times [0.1338 + 0.1115 + 0.0929]}{\frac{1}{3}}$$

$$= 3.6429 \text{ (mins)}$$

Question 3.

(a)



STATE	Job amount in CPU1	Job amount in CPU2	Job amount in disk
state (0,0,3)	0	0	3
state (1,0,2)	1	0	2
state (0,1,2)	0	1	2
state (2,0,1)	2	0	1
state (1,1,1)	1	1	1
State (2,1,0)	2	1	0

The transition rates are shown in the graph, for:

① Jobs complete at CPU1 at a rate of $\frac{1000}{50} = 20$ jobs/s.

② Jobs complete at CPU2 at a rate of $\frac{1000}{100} = 10$ jobs/s.

③ Jobs complete at Disk at a rate of $\frac{1000}{50} = 20$ jobs/s.

Question 3

(b) Equations could be fetched:

$$① \quad 20P(0,0,3) - 20P(1,0,2) - 10P(0,1,2) + 0P(2,0,1) + 0P(1,1,1) + 0P(2,1,0) = 0$$

$$② \quad 30P(0,1,2) - 20P(1,1,1) = 0$$

$$③ \quad -20P(2,0,1) - 20P(1,1,1) + 30P(2,1,0) = 0$$

$$④ \quad 40P(2,1,0) - 10P(2,1,0) = 0$$

$$⑤ \quad -20P(1,0,2) - 20P(0,1,2) + 50P(1,1,1) - 20P(2,1,0) = 0$$

$$⑥ \quad P(0,0,3) + P(1,0,2) + P(0,1,2) + P(2,0,1) + P(1,1,1) + P(2,1,0) = 1$$

(c) By using "q3.py" (attached in the zip file):

$$P(0,0,3) = 0.2697$$

$$P(1,0,2) = 0.2039$$

$$P(0,1,2) = 0.1316$$

$$P(2,0,1) = 0.0395$$

$$P(1,1,1) = 0.1974$$

$$P(2,1,0) = 0.1579$$

Question 3:

(d) For this system.

The system throughput = CPU1 throughput + CPU2 throughput.

$$= \text{Utilization of CPU1} \times \text{Service rate of CPU1} + U(CPU2) \times S(CPU2)$$

$$= [P(1,0,2) + P(2,0,1) + P(1,1,1) + P(2,1,0)] \times 20 \text{ jobs/s} + [P(0,1,2) + P(1,1,1) + P(2,1,0)] \times 10 \text{ jobs/s} = 11.974 + 4.869$$

$$= 16.843 \text{ jobs/s}$$

(e).

$$\text{Throughput of CPU1} = U(CPU1) \times S(CPU1)$$

$$= [P(1,0,2) + P(2,0,1) + P(1,1,1) + P(2,1,0)] \times 20 \text{ jobs/s}$$

$$= (0.2039 + 0.0395 + 0.1974 + 0.1579) \times 20$$

$$= 11.974 \text{ jobs/s}.$$

(f) The throughput of the disk $= U(Disk) \times S(Disk)$

$$= [1 - P(2,1,0)] \times 20 \text{ jobs/s}$$

$$= 16.843 \text{ jobs/s}.$$

$$\text{Response time of the disk} = \frac{\text{user amount in the system}}{\text{the throughput of the disk}}$$

$$= \frac{3}{16.843} \text{ s}.$$

$$= 0.178 \text{ s} \simeq 178 \text{ ms}$$

$$\text{Service time of the disk} = \frac{\text{Utilization (disk)}}{\text{throughput (disk)}} = \frac{0.8421}{16.843} = 0.05 \text{ s} = 50 \text{ ms}$$

$\therefore$ the waiting time = Response time of the disk − the service time of the disk

$$= 178 - 50 \text{ ms} = 128 \text{ ms}.$$