Ex. 3.155, slide

$$m(t) = \frac{1}{6}e^{t} + \frac{2}{6}e^{2t} + \frac{3}{6}e^{3t}$$

(a) $\frac{d}{dt}m(t) = \frac{1}{6}e^{t} + \frac{4}{6}e^{2t} + \frac{9}{6}e^{3t} \Rightarrow m^{(1)}(0) = \frac{d}{dt}m(t)\Big|_{t=0} = \frac{1}{6} + \frac{1}{4} + \frac{9}{6} = 7/3$

$E(y) = m^{(1)}(0) = 7/3$

~~(b) Var~~

(b) $Var(y) = E(y^2) - (EY)^2$ -

$E(y^2) = m^{(2)}(0)$;

$$\frac{d}{dt}\left(\frac{d}{dt}m(t)\right) = \frac{d}{dt}\left(\frac{1}{6}e^{t} + \frac{4}{6}e^{2t} + \frac{9}{6}e^{3t}\right) = \frac{1}{6} + \frac{8}{6}e^{2t} + \frac{27}{6}e^{3t}$$

$$m^{(2)}(0) = \frac{1}{6} + \frac{8}{6} + \frac{27}{6} = 6$$

$E(y^2) = 6$

$Var(y) = E(y^2) - (EY)^2 = 6 - \left(7/3\right)^2 = 6 - 49/9 = 0.5555$

c) Since $m(t) = E\left(e^{tY}\right) = \sum_y e^{ty}\cdot p(y) = e^{ty_1}p(y_1) + e^{ty_2}p(y_2) + e^{ty_3}p(y_3)$

Y	1	2	3
p(y=y)	$\frac{1}{6}$	$2/6$	$3/6$

$E(Y) = 1\cdot 1/6 + 2\cdot 2/6 + 3\cdot 3/6 = 7/3$