NOTE: This file contains sample solutions to the quiz together with the marking scheme and comments for each question. Please read the solutions and the marking schemes and comments carefully. Make sure that you understand why the solutions given here are correct, that you understand the mistakes that you made (if any), and that you understand *why* your mistakes were mistakes.

Remember that although you may not agree completely with the marking scheme given here, it was followed the same way for all students. We will remark your quiz only if you clearly demonstrate that the marking scheme was not followed correctly.

For all remarking requests, please submit your request **in writing** directly to your instructor. For all other questions, please don't hesitate to ask your instructor during office hours or by e-mail.

GENERAL MARKING SCHEME:

- ***A**: All Correct*, except maybe for very few minor errors.
- ***B**: Mostly Correct*, but with a few serious errors, or many small errors.
- ***C**: Mostly Incorrect*, but with a few important elements, or many small elements, done correctly.
- ***10%**: Completely Blank*, or clearly crossed out.
- ***D**: All Incorrect*, except maybe for very few minor elements done correctly.

MARKER'S COMMENTS:

- **common error**: Reduction in the wrong direction (trying to create instandes of SAT from instances of DOUBLESAT).
- **common error**: Transforming $A$ into $A \vee x$ for a new variable $x$: $A \vee x$ is satisfied by at least two settings of its variables no matter what $A$ is (set $x = \mathsf{True}$ and the variables of $A$ can be given any value).
- **common error**: Transforming $A$ into $A \wedge A'$, where $A'$ is a copy of $A$ with all variables renamed: if $A$ can only be satisfied by one assignment of values, then $A'$ has the same property and so does $A \wedge A'$.
- It would work to transform $A$ into $A \vee A'$, where $A'$ is a copy of $A$ with all variables renamed — but nobody actually did this!

**1.** Write a detailed proof that the "DOUBLESAT" problem below is *NP*-complete.

**Input:** A propositional formula $A$.

**Question:** Are there *at least two* different assignments of values to the variables of $A$ that make $A$ True?

HINT: For your reduction, make appropriate use of a new variable.... If you cannot find the idea to make the reduction work, give a detailed explanation of what you are trying to do and what you have to show about it — this will be worth most of the marks.

DOUBLESAT $\in$ *NP*: On input $(A, c)$, it takes polynomial time to verify that $c$ represents two different assignments of values to the variables of $A$ and that both these assignments make $A$ True. Moreover, $A$ is a yes-instance of DOUBLESAT iff this verifier outputs True for some $c$, by definition of DOUBLESAT.

DOUBLESAT is *NP*-hard because SAT $\leqslant_p$ DOUBLESAT:

> On input $A$, output $A' = A \wedge (z \vee \neg z)$, where $z$ is a new variable (one that does not appear in $A$).

Clearly, $A'$ can be computes from $A$ in polytime. Also, if some setting of its variables make $A$ True, then the same setting with $z =$ True makes $A'$ True and the same setting with $z =$ False also makes $A'$ True — so there are at least two settings of its variables that make $A'$ True. Finally, if $A$ is not satisfiable, then no setting of its variables can make $A'$ True — such a setting necessarily also makes $A$ True — so there are fewer than two settings of its variables that make $A'$ True.