Worth: 2% **Due:** By 8:59pm on Tuesday 20 January

Remember to write your *full name* and *student number* prominently on your submission.

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*Please read and understand the policy on Collaboration given on the Course Information Sheet. Then, to protect yourself, list on the front of your submission **every** source of information you used to complete this homework (other than your own lecture and tutorial notes). For example, indicate clearly the **name** of every student with whom you had discussions, the **title and sections** of every textbook you consulted (including the course textbook), the **source** of every web document you used (including documents from the course webpage), etc.*

*For each question, please write up detailed answers carefully. Make sure that you use notation and terminology correctly, and that you explain and justify what you are doing. Marks **will** be deducted for incorrect or ambiguous use of notation and terminology, and for making incorrect, unjustified, ambiguous, or vague claims in your solutions.*

The school attended by your child has just received a large anonymous donation and they've decided to modernize their IT infrastructure. They plan to purchase a number of brand new computers and to set up a school-wide fiber-optic LAN (Local Area Network). They already know the location of each computer and the cost to put in a direct fiber-optic link between any two computers. Now they want to determine how to connect all their computers at the minimum possible cost. However, there is one complication: for various physical and logistic reasons, some of the computers can only be connected with a direct link to **exactly one** other computer on the LAN.

Clearly, this is just the Minimum Spanning Tree problem but with a twist: in addition to the regular input, we are also given a subset of vertices that must become *leaves* in the MST that we construct. Formally, the new problem can be defined as follows.

Input: A connected undirected graph $G = (V, E)$ with positive integer weights $w(e)$ for every edge $e \in E$, and a subset of vertices $L \subseteq V$.

Output: A spanning tree $T \subseteq E$ for G where every node of L is a leaf in T and T has minimum total weight among all such spanning trees.

(a) Does every input (G, w, L) have a solution? Give an explicit counter-example or a general argument to justify your answer.

(b) Irrespective of your answer to the first question, there are at least some inputs for which the problem has one (or more) solution(s). Give an algorithm that finds a solution for any such input. Write your algorithm in pseudocode and provide a clear English description of the main ideas (separately or as detailed comments within your algorithm).

NOTE: Your algorithm must **not** try to verify that the input has a solution—instead, simply treat the statement "the input has a solution" as a precondition and write your algorithm accordingly. In other words, it is fine for your algorithm to behave "badly" (run into error conditions, return incorrect answers, etc.) when it is given an input that does not meet its precondition.

HINT: Please **do** make calls to algorithms from class, where appropriate, *without* copy-and-pasting those algorithms into your answer.

(c) Give a *brief* argument that your algorithm is correct. You do **not** need to write a long proof—in particular, no inductive proof of a loop invariant is required. You may (and should) rely on results proven in class: just quote the results you need and use them as part of your argument.