Proof

① Show x_1, x_2, x_3, y_1, y_2 is linearly independent \Rightarrow span $\{x_1, x_2, x_3\} \cap \text{span}\{y_1, y_2\} = \{0\}$ suppose x1, x2, x3, y1, y2 are linearly independent

=) only 4= (2= 6= 6= 0 such that (1x1+ (3x3+ (45)+ (5)=0 (x)

WTS spandki, dz, lb3 (span {5,, 23} = {0} 0 = 0 xit 0 x2 t 0 x3 E span {x1, Nb, Ns} 0 = 091+ 092 E Spon (51, 42) > 0 € Span{x1, Ne, Ne} (Span{y1, Yz}

assume span (11, 12, 12) 1 span (5, 52) \$ {0}

=> there are not all zero scalar (1*, (2*, -- 6x*

such that citalit (xxxt (xxx) = (4491+ (xxx) (1 + 1) + (3 + 1) + (3 + 1) + (3 + 1) - (3 + 2) = 0which is contradiction to (*)

thus span (1/1, 1/2, 1/2) (span (5, 1/2) = {0}

(2) Show span $\{X_1, X_2, X_3\} \cap \text{span} \{y_1, y_2\} = \{0\} \Rightarrow X_1, X_2, X_3, Y_4, Y_4, Y_5 \text{ ore linearly independent}$ Suppose span($x_1, \lambda_2, \lambda_3$) \wedge span $\{y_1, y_2\} = \{0\}$

=) only (1= 6= 6= 6= 0 such that Cillit (2)/2+ (3)/3= (451+ 65/2 (1)(1 + (2)(2 + (3)(2 - (4)) - (5)(2 = 0)

=) N1, N2, N3, Y1, Y2 core linearly independent.

Thus list X1, X2, X3, Y1, Y2 is linearly independent (=> span{11, 12, 12} (1 span{14, 12}= {0})