NOTE: This file contains sample solutions to the quiz together with the marking scheme and comments for each question. Please read the solutions and the marking schemes and comments carefully. Make sure that you understand why the solutions given here are correct, that you understand the mistakes that you made (if any), and that you understand *why* your mistakes were mistakes.

Remember that although you may not agree completely with the marking scheme given here, it was followed the same way for all students. We will remark your quiz only if you clearly demonstrate that the marking scheme was not followed correctly.

For all remarking requests, please submit your request **in writing** directly to your instructor. For all other questions, please don't hesitate to ask your instructor during office hours or by e-mail.

GENERAL MARKING SCHEME:

- ***A****: All Correct*, except maybe for very few minor errors.
- ***B****: Mostly Correct*, but with a few serious errors, or many small errors.
- ***C****: Mostly Incorrect*, but with a few important elements, or many small elements, done correctly.
- ***10%****: Completely Blank*, or clearly crossed out.
- ***D****: All Incorrect*, except maybe for very few minor elements done correctly.

MARKER'S COMMENTS: *Mostly well done.*

1. Show that the following INDEPENDENTSET (IS) decision problem belongs to *NP*.

Input: An undirected graph $G = (V, E)$ and a positive integer k.

Question: Does G contain an *independent set* of size at least k, *i.e.*, a subset of vertices $I \subseteq V$ such that $|I| \geqslant k$ and G contains **no** edge between any two vertices in I?

Do you think it is likely that IS $\in P$? Why or why not?

Verifier for IS:

On input (G, k, c), where c is a subset of $\boldsymbol{k}$ vertices of G:
Return True if G does not contain any one of the edges between vertices in c;
return False otherwise.

This takes time $\mathcal{O}(k^2 m)$: there are $\mathcal{O}(k^2)$ pairs of vertices in c and $\mathcal{O}(m)$ edges to check for each one.

Also, if there is some value of c such that the verifier returns True for (G, k, c), then G contains an independent set of size k or more (c is such an independent set), and if G contains an independent set of size k or more, then there is some value of c such that the verifier returns True for (G, k, c) (let c be the independent set).

It does not appear likely that IS $\in$ *P*, because checking every subset of k vertices takes more than polynomial time **(time $\boldsymbol{\Omega(n^k)}$ where $\boldsymbol{k}$ can depend on $\boldsymbol{n}$)**, and there is no obvious way to speed this up.