Explain why a vector space has either one or infinitely many vectors?

If a set V is a vector space, then we know

① Zero Vector $\vec{0} \in V$

② V is closed to AC and SC.

Thus, in term of the number of vectors in V, there are 2 cases.

Case 1: $V = \{\vec{0}\}$

then $\forall x, y \in V$ and $c \in \mathbb{R}$, $cx + y = \vec{0} \in V$

Case 2: $V \neq \{\vec{0}\}$ and $\{\vec{0}\} \subseteq V$

then $\forall x, y \in V$ and $c \in \mathbb{R}$, $cx + y \in V$

Thus there are infinitely many vectors in V

since there are infinitely combination of $cx + y$.