CSC236: Assignment 3

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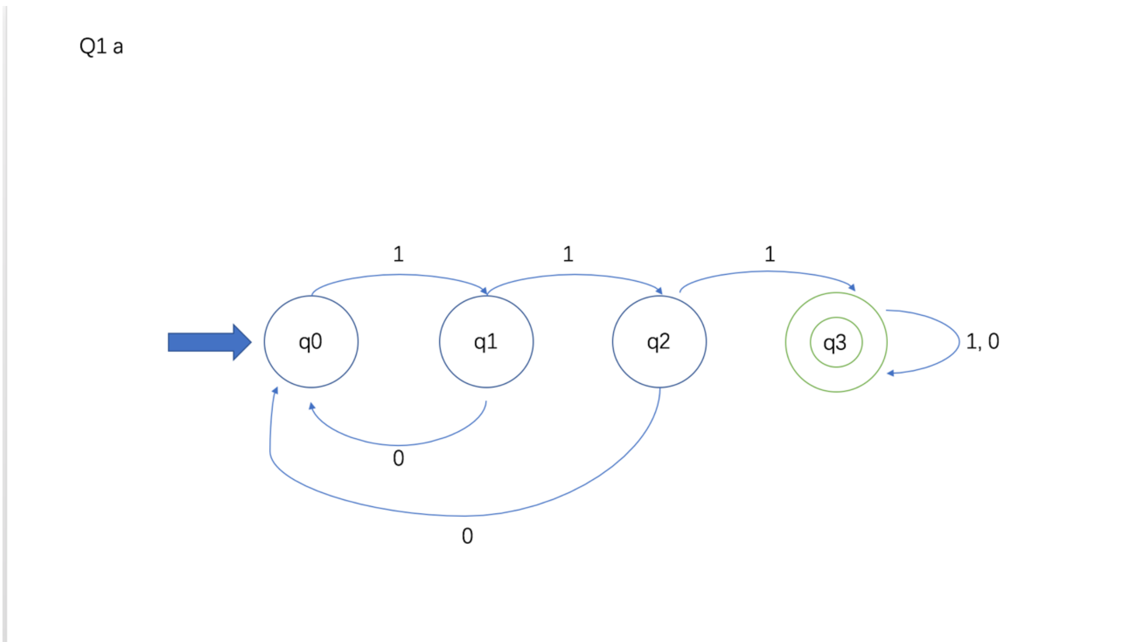
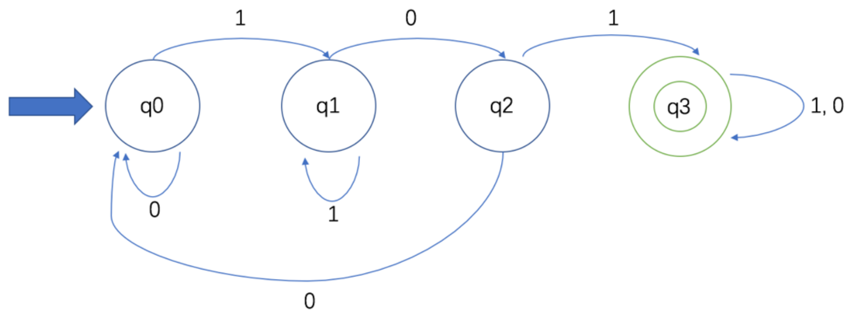
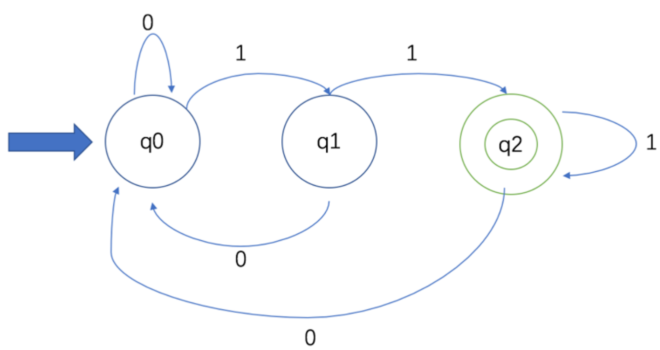
1.



Q1b



Q1c



2.

Define P（n）: “(q, w) $\vdash^*$ (q, ε)”, where |w| = n

Base Case： n = 0

w = ε

(q, w) $\vdash^0$ (q, ε) $\vdash^*$ (q, ε)

Inductive Step：

Let n = k, Assume P(k), i.e., (q, w) $\vdash^*$ (q, ε), where |w| = k

Let w’ = wa. We want to show P(k+1), i.e., (q, w’) $\vdash^*$ (q, ε)

(q, w’) $\vdash^0$ (q, wa) $\vdash^k$(q, a) (By inductive hypothesis)

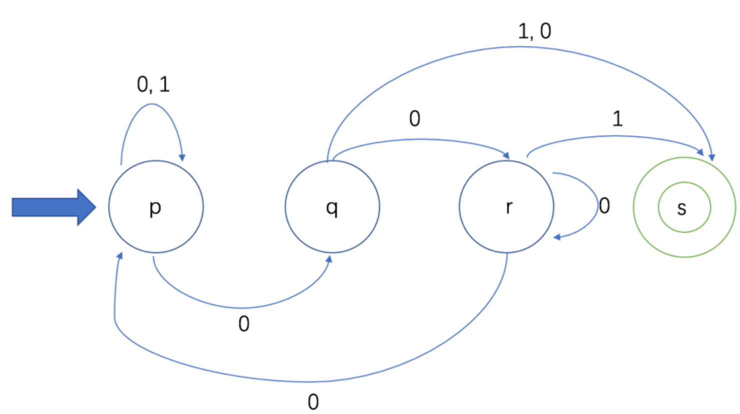
Then, (q, a) $\vdash$ (q, ε)

So P(k+1) holds.

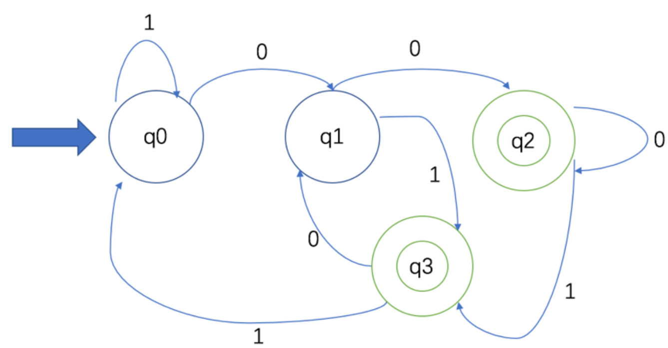
3.

The automation accepts the language that ends with “01” or “00”.

Q3. NFA



Q3. DFA



4.

a)

(0+1)* 1 (0+1) (0+1) (0+1) (0+1) (0+1) (0+1)

b)

(0+10)* (11+ ε) (0+10)*

c)

(0+10)* (1)*

5.

a)

The statement is False. Disprove by giving counterexample.

Let $\sum = \{a, b\}$, R = a, S = b

Proof for ab $\in$ L((a+b)*):

a $\in$ L((a+b)), b $\in$ L((a+b)), so ab$\in$ L((a+b)*)

Proof for ab $\notin$ L(a* + b*):

ab $\notin$ L(a*) and ab $\notin$ L(b*), so ab $\notin$ L(a* + b*)

Since ab $\in$ L((a+b)*) but ab $\notin$ L(a* + b*), the statement is false.

b)

The statement is False. Disprove by giving counterexample.

Let $\sum = \{a, b\}$, R = a, S = b

Firstly, we know that ε $\in$ L((aa*b)*), because there could be zero replication of aa*b.

However, $\varepsilon \notin$ L((ab + a)*ab) because the language must contain at least "ab". (since ε $\in$ (ab + a)*)

Since ε $\in$ L((ab + a)*)), but $\varepsilon \notin$ L((ab + a)*ab), the statement is false.

c)

The statement is False. Disprove by giving counterexample.

Let $\sum = \{a, b\}$, R = a, S = b

Firstly, we know that a $\in$ L(a), then a $\in$ L(a + b). Therefore, a $\in$ L((a + b)*).

Secondly, a $\notin$ L((a*b)), then a $\notin$ L((a*b)*).

All string in the language, L((a*b)*) must contain either at least one b or is ε.

Since a $\in$ L((a + b) $*$), but $a \notin$ L((a*b)*), the statement is false.

d)

The statement is False. Disprove by giving counterexample.

Let $\sum = \{a, b\}$, R = a, S = b

Firstly, we know that ε $\in$ L(ab + b), then ba $\in$ L(b(ab + b)*a).

Secondly, we know that $\varepsilon \in L$((aa*b)*) and $\varepsilon \in L$((a)*), then the only possible string of the language, L(aa*b(aa*b)*), of length two is ab, so ba $\notin$ L(aa*b(aa*b)*).

Since $ba \in L$(b(ab + b)*a), but $ba \notin$ L(aa*b(aa*b)*), the statement is false.

6.

$$N(k) = \begin{cases} 0 & if\ k = 0 \\ 0 & if\ k = 1 \\ 2 & if\ k = 2 \\ 0 & if\ k = 3 \\ 2 & if\ k = 4 \\ N(k-2) + 2N(k-3) & if\ k > 4 \end{cases}$$

Explanation: If k is greater than 4, it is clear that the string that is accepted by the automaton must end with 11 or 010 or 001.

Case 1: if the string that is accepted by the automaton ends with 11. It means that we can get the target string of length k by concatenating each accepted string of length (k-2) with 11.

Case 2: if the string that is accepted by the automaton ends with 010 or 001. It means that we can get the target string of length k by concatenating each accepted string of length (k-3) with 010 or 001.

Therefore, if k is greater than 4, N(k) = N(n-2) + 2N(k-3).

Then N(14) = N(12) + 2N(11)

= N(10) + 2N(9) + 2(N(9)+2N(8))

= N(10) + 4N(9) + 4N(8)

= N(8) + 2N(7) + 4(N(7)+2N(6)) + 4(N(6)+2N(5))

…

$= 114$

7.

a)

1. Suppose L is regular.
2. Let n be the pumping constant.
3. Choose $w = 0^n 1^m 0^n$ (Note that $w \in L$ and $m, n \geq 0$)
4. By PL, w can be factored into xyz such that $|xy| \leq n$, $|y| > 0$ and, for all $i \geq 0$, $xy^i z \in L$
5. Since $|xy| \leq n$ and $|y| > 0$, $x = 0^p$, $y = 0^q$, $z = 1^m 0^n$ where $p + q = n$, $p \geq 0$, $q > 0$, then $p < n$.
6. Consider $i = 0$, then $xy^i z = xz = 0^p 1^m 0^n \in L$. We have obtained a contradiction of the assumed regularity of L. (Since $p < n \longrightarrow p \neq n$)
7. Hence, L is not regular

b)

1. Suppose L is regular.
2. Let n be the pumping constant.
3. Since $L = \{w \in \{0, 1\}^* | w \text{ is palindrome}\}$, we choose $w = 0^n 1^m 0^n$ (Note that $w \in L$ and $m, n \geq 0$)
4. By PL, w can be factored into xyz such that $|xy| \leq n$, $|y| > 0$ and, for all $i \geq 0$, $xy^i z \in L$
5. Since $|xy| \leq n$ and $|y| > 0$, $x = 0^p$, $y = 0^q$, $z = 1^m 0^n$ where $p + q = n$, $p \geq 0$, $q > 0$, then $p < n$.
6. Consider $i = 0$, then $xy^i z = xz = 0^p 1^m 0^n \in L$. We have obtained a contradiction of the assumed regularity of L. (Since $p < n \longrightarrow p \neq n$)
7. Hence, L is not regular

8.

Define automata which accepts language L:

M = {Q, S, $\sum$, δ, F}

Define reversal automata which accepts L[R.]

M' = {Q, S', $\sum$, δ^R, $\{S\}$}

- $\forall p, q \in$ Q, (q, σ, p) $\in \delta^R \Leftrightarrow$ (p, σ, q) $\in \delta$.
- $\forall f \in$ F, (S', ε, f) $\in \delta^R$
- $Now\ S\ is\ the\ new\ single\ final\ states.$

Define M'' which accepts language $\frac{1}{2}$(L).

M'' = (Q, S, $\sum$, δ, F')

F' = $\bigcup_{w \in \text{L}} \boldsymbol{q_i}$, $(where\ n = |w|,\ w = xy,\ (S,$

$w) \vdash_{\delta}^{\frac{n}{2}} (q_i, y)\ and\ (S',\ w^R) \vdash_{\delta^R}^{\frac{n}{2}+1} (q_i, x))$