STA 304H1F-1003H Fall 2019
Assignment 1-Question 1-Solution

Question 1. (15 marks)

We consider a population of $N = 5$ service-stations, labeled 1,2,3,4,5 with respective price of a litre of high-grade petrol in a certain month, $y_1 = 5.82, \quad y_2 = 5.33, \quad y_3 = 5.76, \quad y_4 = 6.20, \quad y_5 = 5.89$. Consider a simple random sample without replacement design with sample size $n = 3$. For your convenience, several parts of the following may be combined into a single table.

(a) Find the values of the population parameters μ, the median, and the σ. List every possible sample of size $n = 3$. For each sample, what is the probability that it is the one selected?

Solution:

Population: Size N=5 $\quad Y = 5.82,\ 5.33,\ 5.76,\ 6.20,\ 5.89$

Population Mean: $\mu = \frac{\sum_{i=1}^{5} y_i}{5} = 5.8$ **(1 mark)**

Population Median: middle obeservation: Median = 5.82 **(1 mark)**

Population standard deviation: $\sigma = \sqrt{\frac{\sum_{i=1}^{5}(y_i-\mu)^2}{5}} = 0.27964$ **(1 mark)**

Simple random sample without replacement (SRS) of size n=3:

Number of possible sample: $\binom{N}{n} = \binom{5}{3} = 10$

Table 1: List of all possible samples, and their probabilities of selection

Sample	Y_s	**Prob**
(1,2,3)	(5.82, 5.33, 5.76)	$\frac{1}{10}$
(1,2,4)	(5.82, 5.33, 6.20)	$\frac{1}{10}$
(1,2,5)	(5.82, 5.33, 5.89)	$\frac{1}{10}$
(1,3,4)	(5.82, 5.76, 6.20)	$\frac{1}{10}$
(1,3,5)	(5.82, 5.76, 5.89)	$\frac{1}{10}$
(1,4,5)	(5.82, 6.20, 5.89)	$\frac{1}{10}$
(2,3,4)	(5.33, 5.76, 6.20)	$\frac{1}{10}$
(2,3,5)	(5.33, 5.76, 5.89)	$\frac{1}{10}$
(2,4,5)	(5.33, 6.20, 5.89)	$\frac{1}{10}$
(3,4,5)	(5.76, 6.20, 5.89)	$\frac{1}{10}$

(3 marks)

(b) What is the sampling distribution of $\bar{y}$?

Solution:

The sampling distribution of $\bar{y}$ is given by the following table

```
   ybar Prob
1  5.64  0.1
2  5.78  0.1
3  5.68  0.1
4  5.93  0.1
5  5.82  0.1
6  5.97  0.1
7  5.76  0.1
8  5.66  0.1
9  5.81  0.1
10 5.95  0.1
```

(5 marks)

OR use the Table 1 above and add the values of $\bar{y}$

Sample	Y_s	**Prob**	$\bar{y}$
(1,2,3)	(5.82, 5.33, 5.76)	$\frac{1}{10}$	5.637
(1,2,4)	(5.82, 5.33, 6.20)	$\frac{1}{10}$	5.783
(1,2,5)	(5.82, 5.33, 5.89)	$\frac{1}{10}$	5.68
(1,3,4)	(5.82, 5.76, 6.20)	$\frac{1}{10}$	5.927
(1,3,5)	(5.82, 5.76, 5.89)	$\frac{1}{10}$	5.823
(1,4,5)	(5.82, 6.20, 5.89)	$\frac{1}{10}$	5.97
(2,3,4)	(5.33, 5.76, 6.20)	$\frac{1}{10}$	5.763
(2,3,5)	(5.33, 5.76, 5.89)	$\frac{1}{10}$	5.66
(2,4,5)	(5.33, 6.20, 5.89)	$\frac{1}{10}$	5.807
(3,4,5)	(5.76, 6.20, 5.89)	$\frac{1}{10}$	5.95

(c) Find $\mathbf{E}(\bar{y})$, $\mathbf{V}(\bar{y})$, $\mathbf{Bias}(\bar{y})$ and $\mathbf{MSE}(\bar{y})$

Solution:

1.The expected value of $\hat{\mu} = \bar{y}$:

$\mathbf{E}(\bar{y}) = 5.64\frac{1}{10} + 5.78\frac{1}{10} + 5.68\frac{1}{10} + 5.93\frac{1}{10} + 5.82\frac{1}{10} + 5.97\frac{1}{10} + 5.76\frac{1}{10} + 5.66\frac{1}{10} + 5.81\frac{1}{10} + 5.95\frac{1}{10}$

$\boxed{\mathbf{E}(\bar{y}) = 5.8}$

(1 mark)

2.The variance of $\hat{\mu} = \bar{y}$:

$\mathbf{V}(\bar{y}) = (5.64 - 5.8)^2 \times \left(\frac{1}{10}\right) + (5.78 - 5.8)^2 \times \left(\frac{1}{10}\right) + (5.68 - 5.8)^2 \times \left(\frac{1}{10}\right) + (5.93 - 5.8)^2 \times \left(\frac{1}{10}\right) + (5.82 - 5.8)^2 \times \left(\frac{1}{10}\right) + (5.97 - 5.8)^2 \times \left(\frac{1}{10}\right) + (5.76 - 5.8)^2 \times \left(\frac{1}{10}\right) + (5.66 - 5.8)^2 \times \left(\frac{1}{10}\right) + (5.81 - 5.8)^2 \times \left(\frac{1}{10}\right) + (5.95 - 5.8)^2 \times \left(\frac{1}{10}\right) = \boxed{0.01303}$

(1 mark)

We can also use

$\mathbf{V}(\bar{y}) = \mathbf{E}(\bar{y}^2) - \mathbf{E}(\bar{y})^2 = (5.64)^2\frac{1}{10} + 5.78^2\frac{1}{10} + 5.68^2\frac{1}{10} + 5.93^2\frac{1}{10} + 5.82^2\frac{1}{10} + 5.97^2\frac{1}{10} + 5.76^2\frac{1}{10} + 5.66^2\frac{1}{10} + 5.81^2\frac{1}{10} + 5.95^2\frac{1}{10} - (5.8)^2 = \boxed{0.01303}$

Or this one too

$\mathbf{V}(\bar{y}) = \frac{N-n}{N-1}\frac{\sigma^2}{n} = 0.013$

3.The Bias of $\hat{\mu} = \bar{y}$:

$\mathbf{Bias}(\bar{y}) = \mathbf{E}(\bar{y}) - \mu = \boxed{5.8 - 5.8 = 0}$

(1 mark)

4.The MSE of $\hat{\mu} = \bar{y}$:

$\mathbf{MSE}(\bar{y}) = \mathbf{Var}(\bar{y}) + \mathbf{Bias}(\bar{y})^2 = \mathbf{Var}(\bar{y}) = \boxed{0.13033}$

(1 mark)

End of Question 1

Sample

```r
Y=c(5.82,5.33,5.76,6.20,5.89)
y1bar<-mean(c(5.82, 5.33, 5.76))
y2bar<-mean(c(5.82, 5.33, 6.20))
y3bar<-mean(c(5.82, 5.33, 5.89) )
y4bar<-mean(c(5.82, 5.76, 6.20))
y5bar<-mean(c(5.82, 5.76, 5.89))
y6bar<-mean(c(5.82, 6.20, 5.89))
y7bar<-mean(c(5.33, 5.76, 6.20))
y8bar<-mean(c(5.33, 5.76, 5.89))
y9bar<-mean(c(5.33, 6.20, 5.89))
y10bar<-mean(c(5.76, 6.20, 5.89))
```