Slide 18

$$Var(Y) = E(Y^2) - (E(Y))^2 = E(Y^2) - \mu^2$$

Proof: Let Y be a discrete random variable with pmf $p(y)$ and mean $\mu = E(Y)$

$$Var(Y) = E((Y-\mu)^2) = E(Y^2 - 2\mu Y + \mu^2)$$

$$= E(Y^2) - E(2\mu Y) + E(\mu^2) \quad \text{(From slide 16 Chapter 3, part 1)}$$

$$= E(Y^2) - 2\mu \underbrace{E(Y)}_{\mu} + \mu^2 \quad \text{(From slide 15 chapter 3, part 1)}$$

$$= E(Y^2) - 2\mu^2 + \mu^2$$

$$= E(Y^2) - \mu^2 = E(Y^2) - (E(Y))^2$$

■

Slide 18: $Var(c) = 0$, when c is constant.

proof

$$Var(c) = E(c^2) - (E(c))^2 \quad \text{(From above)}$$

$$= c^2 - c^2 \quad \text{(From slide 15)}$$

$$= 0$$

■

Slide 19

- $Var(ay+b) = \sigma^2_{ay+b} = a^2\, Var(y) = a^2 \sigma^2$, when $Var(y) = \sigma^2$.

Proof:

$$Var(ay+b) = E\Big((ay+b) - E(ay+b)\Big)^2 \qquad \text{(From slide 17)}$$

$$= E\Big(ay+b - E(ay) - E(b)\Big)^2 \qquad \text{(From slide 16)}$$

$$= E\Big(ay+b - aE(y) - b\Big)^2 \qquad \text{(From slide 15)}$$

$$= E\Big(ay - aE(y)\Big)^2 = E\Big(a\big(y - E(y)\big)\Big)^2$$

$$= E\Big(a^2\big(y-E(y)\big)^2\Big) \underset{\text{(slide 15)}}{=} a^2\, E\big(y - E(y)\big)^2$$

$$= a^2\, Var(y) \qquad \text{Based on slide 17 (definition of } Var(y))$$

$$= a^2 \sigma^2$$

$$sd(ay+b) = \sqrt{Var(ay+b)} = \sqrt{a^2\, Var(y)} = |a| \sqrt{Var(y)} = |a|\, \sigma$$

Slide 28

If $Y \sim Bin(n,p)$, then: $E(Y) = np$

Proof: $E(y) = \sum_{y=0}^{n} y p(y) = \sum_{y=0}^{n} y \binom{n}{y} p^{y} (1-p)^{n-y}$

$$= \sum_{y=1}^{n} y \binom{n}{y} p^{y} (1-p)^{n-y} = \sum_{y=1}^{n} y \cdot \frac{n!}{y!\,(n-y)!} p^{y} (1-p)^{n-y}$$

$$= np \sum_{y=1}^{n} \frac{(n-1)!}{(y-1)!\,(n-y)!} p^{y-1} (1-p)^{n-y}$$

define $z = y-1$;

$$= np \sum_{z=0}^{n-1} \frac{(n-1)!}{z!\,(n-(z+1))!} p^{z} (1-p)^{n-(z+1)}$$

$$= np \underbrace{\sum_{z=0}^{n-1} \frac{(n-1)!}{z!\,(n-1-z)!} p^{z} (1-p)^{n-1-z}}_{1} = np$$

Since $z \sim Bin(n-1,p) \Rightarrow \sum_{z=0}^{n-1} \binom{n-1}{z} p^{z} (1-p)^{n-1-z} = 1$