

# Symbolizing in Sentential Logic

Part I

Alex Koo

$$\sim P \rightarrow (Q \wedge (P \rightarrow R) \vee (Q \leftrightarrow S))$$

Negation       $\sim$       Brackets ( ) [ ]

Conditional       $\rightarrow$       Well-Formed Formulas

And       $\wedge$       Informal Notation

Or       $\vee$       Main Connectives

Biconditional       $\leftrightarrow$       Sentence Letters P-Z

“If it’s raining, then I’ll take the TTC  
unless I get a ride from Don.”



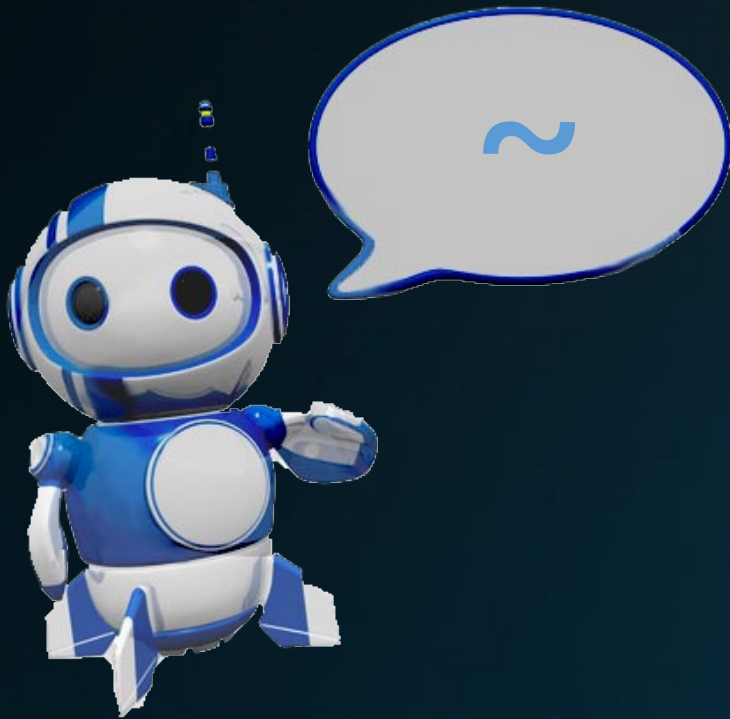
$P \rightarrow (Q \vee R)$

“If it’s raining, then I’ll take the TTC  
unless I get a ride from Don.”

P: It’s raining. Q: I’ll take the TTC. R: I’ll get a ride from Don.

$$P \rightarrow (Q \vee R)$$

# Stylistic Variants



Symbolize: Mary **won't** win the election.

W: Mary will win the election.

**~**W

Symbolize: You are a student **or** a teacher

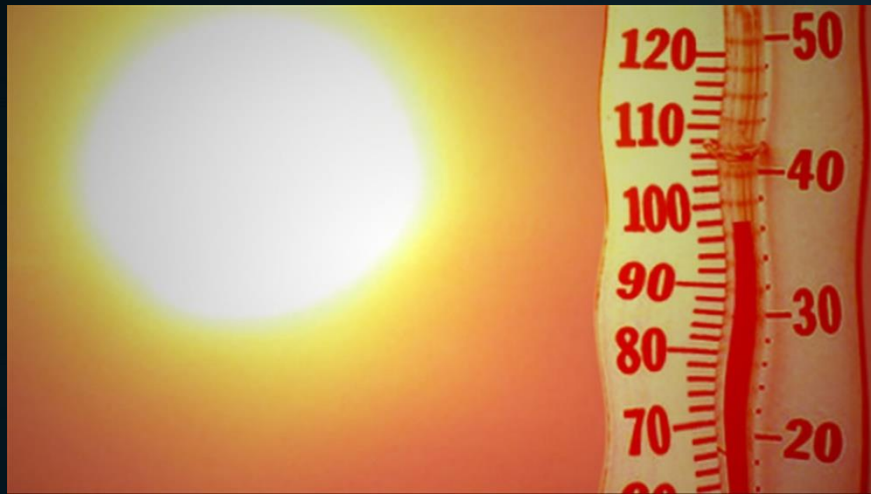
R: You're a student. S: You're a teacher.

R**v**S



Symbolize: It's hot **and** sunny.

W: It's hot. Z: It's sunny.



W<sup>^</sup>Z



But

Moreover

Although

Even though

However

Despite the fact that

In addition to

Yet

Also

Symbolize: I run **if and only if** a cat is chasing me.

R: I run. Z: A cat is chasing me.

$$R \leftrightarrow Z$$



# Logical Equivalence

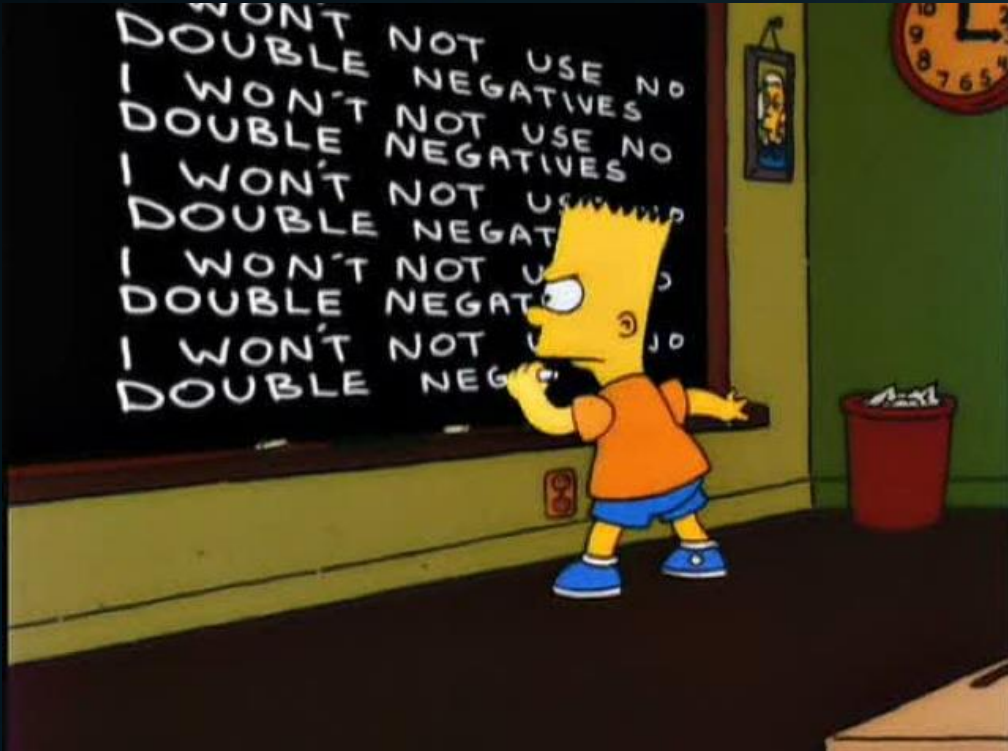


# Logical Equivalence



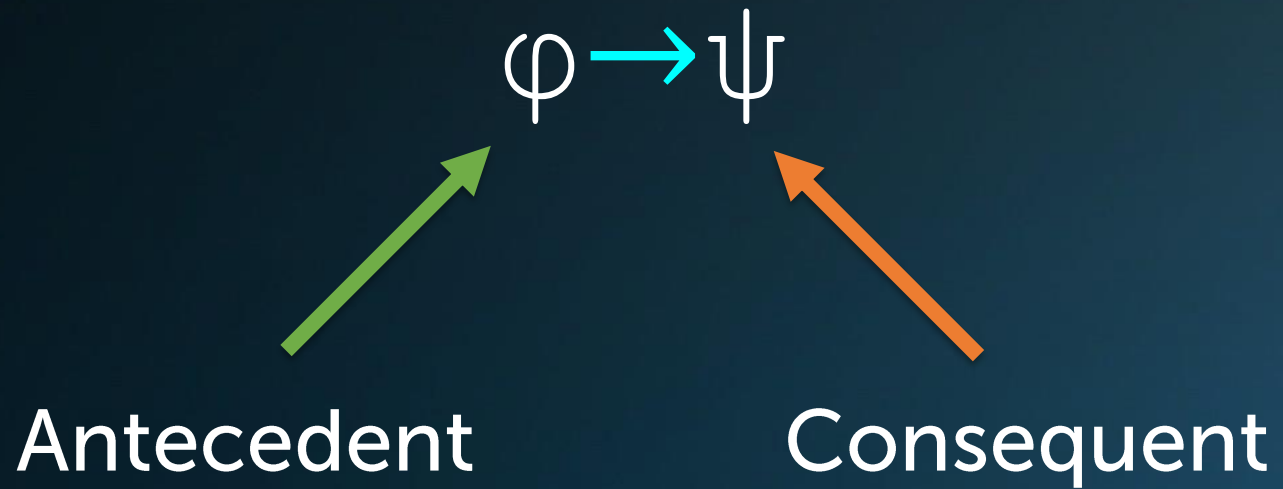


# Double Negation



P	$\sim P$	$\sim\sim P$
T	F	T
F	T	F

# Conditionals



S: I study. P: I pass.

If I study, then I pass

$S \rightarrow P$

I'll pass if I study

$S \rightarrow P$

Whenever I study, I pass

$S \rightarrow P$

I'll study provided that I pass

$P \rightarrow S$



Oxygen is **NECESSARY** for fire

Consequent

Antecedent





Rain is SUFFICIENT for wet sidewalks



Antecedent



Consequent



Sleep and exercise are necessary  
conditions for a well balanced life

P: Sleep. Q: Exercise. R: Well balanced life.



Sleep and exercise are necessary conditions for a well balanced life

P: Sleep. Q: Exercise. R: Well balanced life.



Sleep **and** exercise are **necessary conditions** for a well balanced life

P: Sleep. Q: Exercise. R: Well balanced life.

**→** **^**



Sleep and exercise are necessary conditions for a well balanced life

P: Sleep. Q: Exercise. R: Well balanced life.

$$R \rightarrow P \wedge Q$$



Sleep and exercise are necessary conditions for a well balanced life

P: Sleep. Q: Exercise. R: Well balanced life.

$$R \rightarrow (P \wedge Q)$$



Sleep and exercise are necessary conditions for a well balanced life

P: Sleep. Q: Exercise. R: Well balanced life

$$(R \rightarrow (P \wedge Q))$$





For coffee to be necessary for being alert,  
it is sufficient that one be depressed or old.

P: Having coffee. Q: Being alert.  
R: One is depressed. S: One is old.





For coffee to be **necessary** for being alert,  
it is **sufficient** that one be depressed **or** old.

P: Having coffee. Q: Being alert.  
R: One is depressed. S: One is old.

For coffee to be **necessary** for being alert, it is **sufficient** that one be depressed **or** old.



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→ → V



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( **V** )  $\rightarrow$  (  $\rightarrow$  )



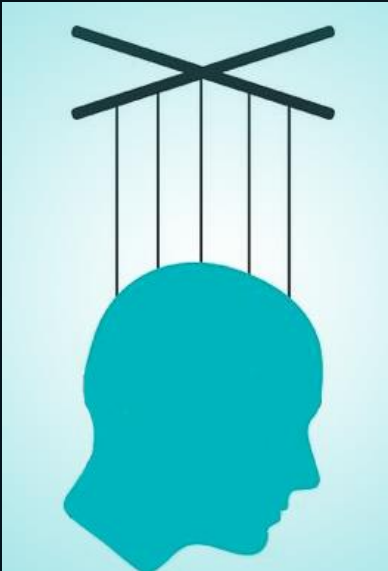
For coffee to be **necessary** for being alert, it is **sufficient** that one be depressed **or** old.

P: Having coffee. Q: Being alert.  
R: One is depressed. S: One is old.

$$(R \vee S) \rightarrow (Q \rightarrow P)$$

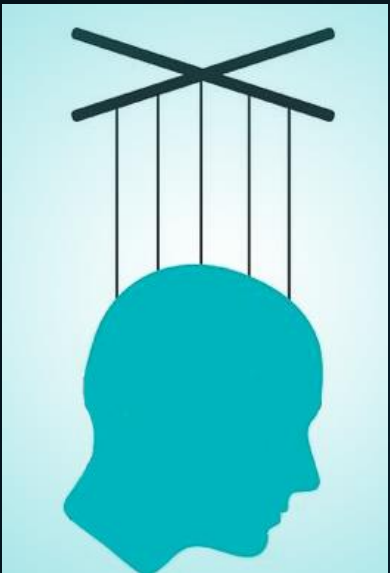
Free will is **necessary** **and** **sufficient**  
for moral responsibility

P: Having free will. Q: Having moral responsibility.



Free will is **necessary** and **sufficient**  
for moral responsibility

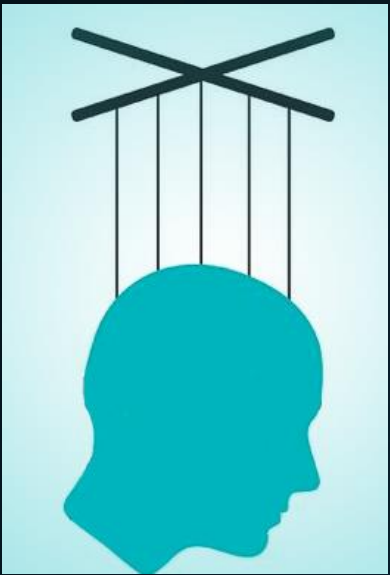
P: Having free will. Q: Having moral responsibility.



$(Q \rightarrow P)$

Free will is **necessary** and **sufficient**  
for moral responsibility

P: Having free will. Q: Having moral responsibility.

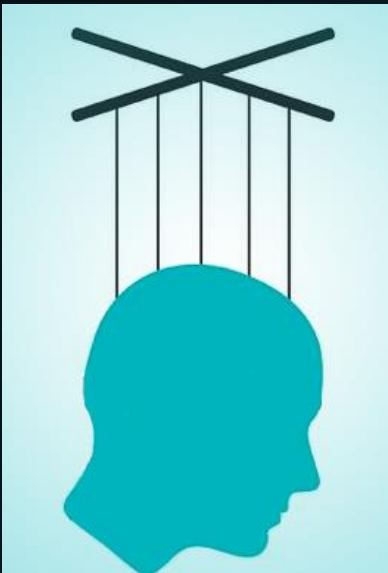


$$(Q \rightarrow P) \wedge (P \rightarrow Q)$$



Free will is **necessary and sufficient**  
for moral responsibility

P: Having free will. Q: Having moral responsibility.



$(P \leftrightarrow Q)$

or

$(Q \leftrightarrow P)$





$P \rightarrow Q$

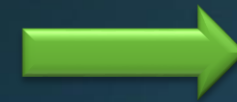
$Q \rightarrow P$

If I see a puppy, then I will be happy.

P: I see a puppy. Q: I will be happy.



$P \rightarrow Q$



$Q \rightarrow P$

**ONLY** if I see a puppy will I be happy.

P: I see a puppy. Q: I will be happy.

'Only' swaps the antecedent  
and consequent of a conditional

Tip: Symbolize without the 'only' then make the swap

Assuming that I'm not in a hurry, only if  
I haven't had coffee do I bump into things.

R: I'm in a hurry. S: I had coffee. T: I bump into things.



Assuming that I'm not in a hurry, only if  
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Assuming that I'm not in a hurry, only if  
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$\sim R \rightarrow ( \rightarrow )$



Assuming that I'm not in a hurry, ~~only~~ if  
I **haven't** had coffee **do** I bump into things.

R: I'm in a hurry. S: I had coffee. T: I bump into things.



$\sim R \rightarrow ( \rightarrow )$

Assuming that I'm not in a hurry, ~~only~~ if  
I **haven't** had coffee **do** I bump into things.

R: I'm in a hurry. S: I had coffee. T: I bump into things.



$$\sim R \rightarrow (\sim S \rightarrow T)$$



Assuming that I'm not in a hurry, **only if**  
I **haven't** had coffee **do** I bump into things.

R: I'm in a hurry. S: I had coffee. T: I bump into things.



$$\sim R \rightarrow (\sim S \rightarrow T)$$

Assuming that I'm not in a hurry, **only if**  
I **haven't** had coffee **do** I bump into things.

R: I'm in a hurry. S: I had coffee. T: I bump into things.



$$\sim R \rightarrow (T \rightarrow \sim S)$$

Identify connectives

Look for main breaks

Paraphrase

Read the logic back