

Given a network $N = (V, E)$ with capacities $c(u, v)$ for all edges $(u, v) \in E$ and conservation factors α_u, β_u for all vertices $u \in V - \{s, t\}$, construct the following linear program.

variables: $f_{u,v}$ for each edge $(u, v) \in E$

objective function: maximize $\sum_{(s,u) \in E} f_{s,u}$

constraints:

- $0 \leq f_{u,v} \leq c(u, v)$ for every edge $(u, v) \in E$
- $\sum_{(v,w) \in E} f_{v,w} \geq \alpha_v \sum_{(u,v) \in E} f_{u,v}$ for every vertex $v \in V - \{s, t\}$
- $\sum_{(v,w) \in E} f_{v,w} \leq \beta_v \sum_{(u,v) \in E} f_{u,v}$ for every vertex $v \in V - \{s, t\}$

Then, every valid flow in the network yields a feasible solution to the linear program because flow values satisfy each constraint in the linear program. Hence, the maximum value of the objective function is at least as large as the maximum flow.

Conversely, every feasible solution to the linear program yields a valid flow in the network because every constraint on the flow is represented by some linear inequality. Hence, the maximum value of the objective function is no larger than the maximum flow.