Part I: Semantics (30 marks)

1. When we do a conditional derivation, we get to assume the antecedent in order to show the consequent. With reference to the truth-table of the conditional, explain why this assumption makes sense. (3)

P	Q	P→Q
T	T	T
T	F	F
F	T	T
F	F	T

Every G M's some Generic F

2. Provide an intensional interpretation that shows the following argument is invalid. (3)

T T xMy xM's y

∀x(Gx↔~Fx). ∀x∃y(Gx→Fy∧M(xy)). ∴ ~∃y(Fy∧∀x(Fx→M(xy))).

G → Everything is G or F, not Both

→ ∃F

Some specific F M's all F's

UD: Natural #'s

F¹: Even

G¹: Odd

M²: a ≤ b

3. Provide an English explanation that demonstrates why the following set of sentences is inconsistent. (4)

{∃x∀y(Fx∧~D(xy)), ∀y(Fy→∃xD(yx))}

Inconsistent means no interp where both true
I will assume S1 is T, and show S2 must be F.
S1 says: Some specific F does not stand in the D relation to everything
S2 says: All F's stand in the D relation to some generic thing
SINGLE SENTENCE THAT DRIVES THIS HOME
WRAP UP THE LOGIC

4. Provide a finite extensional interpretation/model that demonstrates the following sentence is not a tautology/logical truth. (4)

T F F T ∀F

∀x(Fx↔Gx)∨(Fa∧∃x(Fx∧Gx∧~H(xx))→~∃x(Gx∧∀y(Fy∧H(yx))))

∃G 0

y H x

All F's H' some specific G.

UD: {0, 1}

F^1: {0, 1} H^2: {(0, 0), (1, 0)}

G^2: {0, 1} a^0: 1

F: {0, 1

G: {0

H^2:

5. a) Provide a truth-functional expansion of the following set of sentences using a
universe of discourse with two members. (4)

{~∀x(D(xx)→~Ax), ∃x∀y(D(xy)∧Gx), ∀x(Ax→∃y~D(xy))}

b) Provide a finite extensional interpretation/model that demonstrates the set of sentences is consistent. (1)

6. What is the difference between the definition of validity in **SENTENTIAL** logic versus in **PREDICATE** logic? Use this difference to explain why we need to use models/interpretations in predicate logic semantics. (3)

7. Provide a shortened truth-table that demonstrates the following argument is invalid. (3)

(W↔Q)→R∨Q. P∧~W. ∴ P↔Q.

8. In three-valued logic, there are three possible truth-values: True (T), False (F), and Unknown (U). Below are the completed three-valued truth-tables for Negation, Conjunction, and Disjunction, as well as an empty table for the Conditional.

P	**Q**	~P	P∧Q	P∨Q	P→Q
T	T	F	T	T	T
T	U	F	U	T	U
T	F	F	F	T	F
U	T	U	U	T	T
U	U	U	U	U	
U	F	U	F	U	
F	T	T	F	T	
F	U	T	F	U	
F	F	T	F	F	

~P v Q

① F v T = T

② F v U = U

③

a) One way to understand the Conditional P→Q in three-valued logic is by making it equivalent to ~P∨Q. Fill in the P→Q column above using this understanding. (2)

b) Given that P is True (T) and Q is Unknown (U), circle the truth-value of the following sentence. (1)

Q∨(~Q∧(Q∨P))

TRUE UNKNOWN FALSE

c) Give one reason for why we would want to use a three-valued logical system.
Briefly justify this reason. (2)

Part II: Symbolization (36 Marks)

Symbolize questions 1-8, and translate question 9 using the provided abbreviation schemes. Read the instructions for question 10 carefully.

1. Unless coffee and cigarettes are good for you, they will be neither cheap nor socially acceptable. (3)

A^1: *a* ~~is cheap.~~ ~~C^1: *a* is c~~offee. D^1: *a* is socially acceptable. F^1: *a* is a cigarette.
G^1: *a* is good for you.

$\forall x((Cx \vee Fx \rightarrow Gx \vee \sim(Ax \vee Dx))$

$\sim Ax \wedge \sim Dx$

2. Not everyone rides bicycles. (3)

A^1: *a* is a person. B^1: *a* is a bicycle. D^2: *a* rides *b*.

$$\exists x(Ax \wedge \sim\exists y(By \wedge D(xy)))$$

$$\sim\forall x(Ax \rightarrow \exists y(By \wedge D(xy)))$$

3. Only if Steven's mother (a(a)) is an engineer will she (Steven's mother) go to graduate school. (4)

a^0: Steven. a^1: The mother of *a*. E^1: *a* is an engineer. G^1: *a* is a graduate school.

D^2: *a* will go to *b*.

$$\exists x(Gx \wedge D(a(a)x)) \rightarrow E\,a(a)$$

$\leftrightarrow$

4. Jenny is the coolest farmer in Dallas exactly on the condition that she doesn't like kale. (4)

b^0: Jenny. d^0: Dallas. F^1: *a* is a farmer. M^1: *a* is kale. C^2: *a* is cooler than *b*.

L^2: *a* likes *b*. N^2: *a* is in *b*.

$$Fb \wedge N(bd) \wedge \forall x(Fx \wedge N(xd) \wedge x \neq b \rightarrow C(bx)) \leftrightarrow \forall x(Mx \rightarrow \sim L(bx))$$

5. Everyone except for Jodie's daughter (d(a)), who is Richard's son's wife (g(b(b))), drinks water. (4)

a^0: Jodie. b^0: Richard. b^1: The son of *a*. d^1: The daughter of *a*. g^1: The wife of *a*.

D^1: *a* is a person. H^1: *a* is water. D^2: *a* drinks *b*.

$$d(a) = g(b(b)) \wedge \forall x(Dx \wedge x \neq d(a) \rightarrow \exists y(Hy \wedge D(xy)))$$

$\exists C_x \quad \forall K_z \quad \exists D_y$

6. Some generic cat is always clawing some specific dog. (4)
   C^1: *a* is a cat. D^1: *a* is a dog. K^1: *a* is a time. C^3: *a* is clawing *b* at time *c*.

$$\exists y(Dy \wedge \forall z(Kz \rightarrow \exists x(Cx \wedge C(xyz))))$$

7. There is only one cheese that Sarah likes, but it's expensive. (4)
   b^0: Sarah. C^1: *a* is a cheese. E^1: *a* is expensive. L^2: *a* likes *b*.

$$Ex \wedge$$

$$\exists x(Cx \wedge L(bx) \wedge \forall y(Cy \wedge L(by) \rightarrow x=y))$$

8. Amongst athletes, being smart is necessary for being good; and in that case they'll get on a team. (4)
   A^1: *a* is an athlete. B^1: *a* will get on a team. G^1: *a* is good. H^1: *a* is smart.

$$\forall x(Ax \rightarrow ((Gx \rightarrow Hx) \wedge (Gx \rightarrow Bx)))$$

9. Translate the following symbolic sentence into an IDIOMATIC English sentence using the abbreviation scheme provided. (3)

$\forall x(Dx \wedge \exists y \exists z(Cy \wedge Cz \wedge y \neq z \wedge H(xy) \wedge H(xz)) \rightarrow \forall y(Dy \wedge \exists w \exists z(Cw \wedge Cz \wedge w \neq z \wedge H(yw) \wedge H(yz)) \rightarrow x=y))$

C^1: *a* is a crime. D^1: *a* is a person. H^2: *a* saw *b*.

At most 1 person saw at least 2 crimes

10. Define a new operator in our system ɿ (called ‘cane’). This operator combines with a variable, and together with a predicate we get a formula. For example, ɿxFx is a formula. We can understand ɿx as saying ‘the thing such that’ – ɿx is the definite descriptor. If we define F to mean F^1: *a* is on my desk, then ɿxFx means ‘*the thing* on my desk’. ɿxϕx, where ϕx is a formula, thus picks out a **term** or **specific individual**, and can be used as a term in our symbolizing.

Using the operator ɿ**,** symbolize the following sentence. (3)

a(h)

The cup on my desk is owned by Harry’s mom.
a^1: The mom of *a*. h^0: Harry. C^1: *a* is a cup. F^1: *a* is on my desk. O^2: *a* owns *b*.

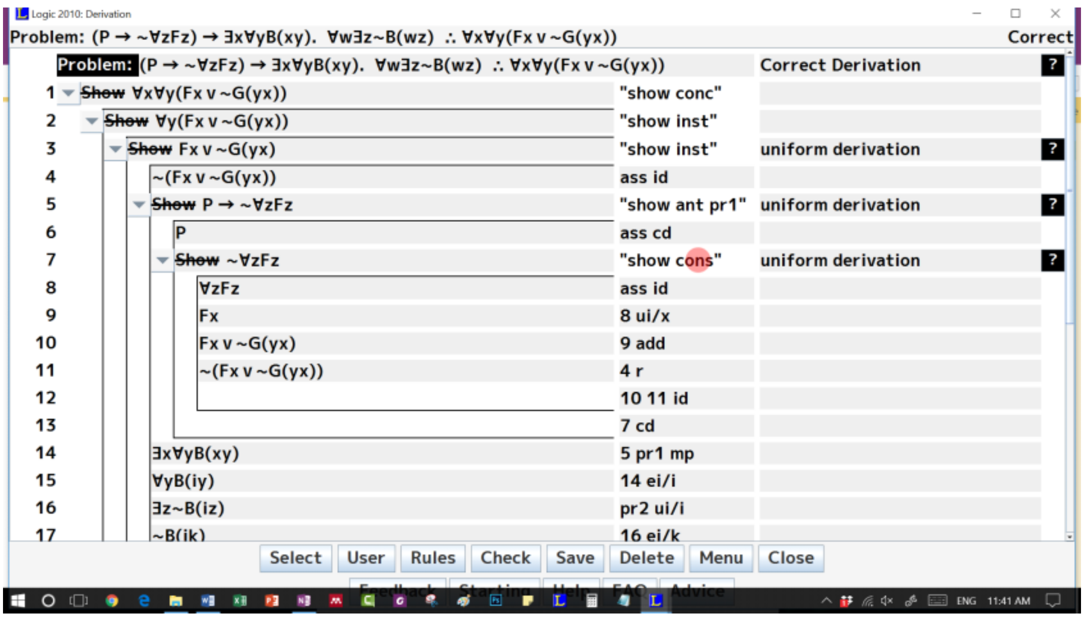
O(a(h) ɿx(Cx ∧ Fx))

~~∃x(Cx ∃xFx ∧ O(a(h)x))~~

Part III: Derivations (34 marks)

1. Show the following argument is valid using a derivation. Use only the **basic** rules: MP, MT, ADD, MTP, ADJ, S, R, DN, CB, BC, EI, EG, and UI. (8)

(P→~∀zFz)→∃x∀yB(xy). ∀w∃z~B(wz). ∴ ∀x∀y(Fx∨~G(yx))


Logic 2010: Derivation
Problem: (P → ~∀zFz) → ∃x∀yB(xy). ∀w∃z~B(wz) ∴ ∀x∀y(Fx v ~G(yx)) Correct

Problem: (P → ~∀zFz) → ∃x∀yB(xy). ∀w∃z~B(wz) ∴ ∀x∀y(Fx v ~G(yx)) Correct Derivation

1	~~Show~~ ∀x∀y(Fx v ~G(yx))	"show conc"	
2	~~Show~~ ∀y(Fx v ~G(yx))	"show inst"	
3	~~Show~~ Fx v ~G(yx)	"show inst"	uniform derivation
4	~(Fx v ~G(yx))	ass id	
5	~~Show~~ P → ~∀zFz	"show ant pr1"	uniform derivation
6	P	ass cd	
7	~~Show~~ ~∀zFz	"show cons"	uniform derivation
8	∀zFz	ass id	
9	Fx	8 ui/x	
10	Fx v ~G(yx)	9 add	
11	~(Fx v ~G(yx))	4 r	
12		10 11 id	
13		7 cd	
14	∃x∀yB(xy)	5 pr1 mp	
15	∀yB(iy)	14 ei/i	
16	∃z~B(iz)	pr2 ui/i	
17	~B(ik)	16 ei/k	

Select User Rules Check Save Delete Menu Close

2. Show the following statement is a theorem of logic using a derivation. Use only the **basic** rules: MP, MT, ADD, MTP, ADJ, S, R, DN, CB, BC, EI, EG, and UI. (8)

∴ (∀x∀yG(yx)∧(G(ab)→∃xF(a(x)a)))→∃x(F(xa)∧∃yG(b(y)x))

∴ (∀x∀yG(yx)∧(G(ab)→∃xF(a(x)a)))→∃x(F(xa)∧∃yG(b(y)x))

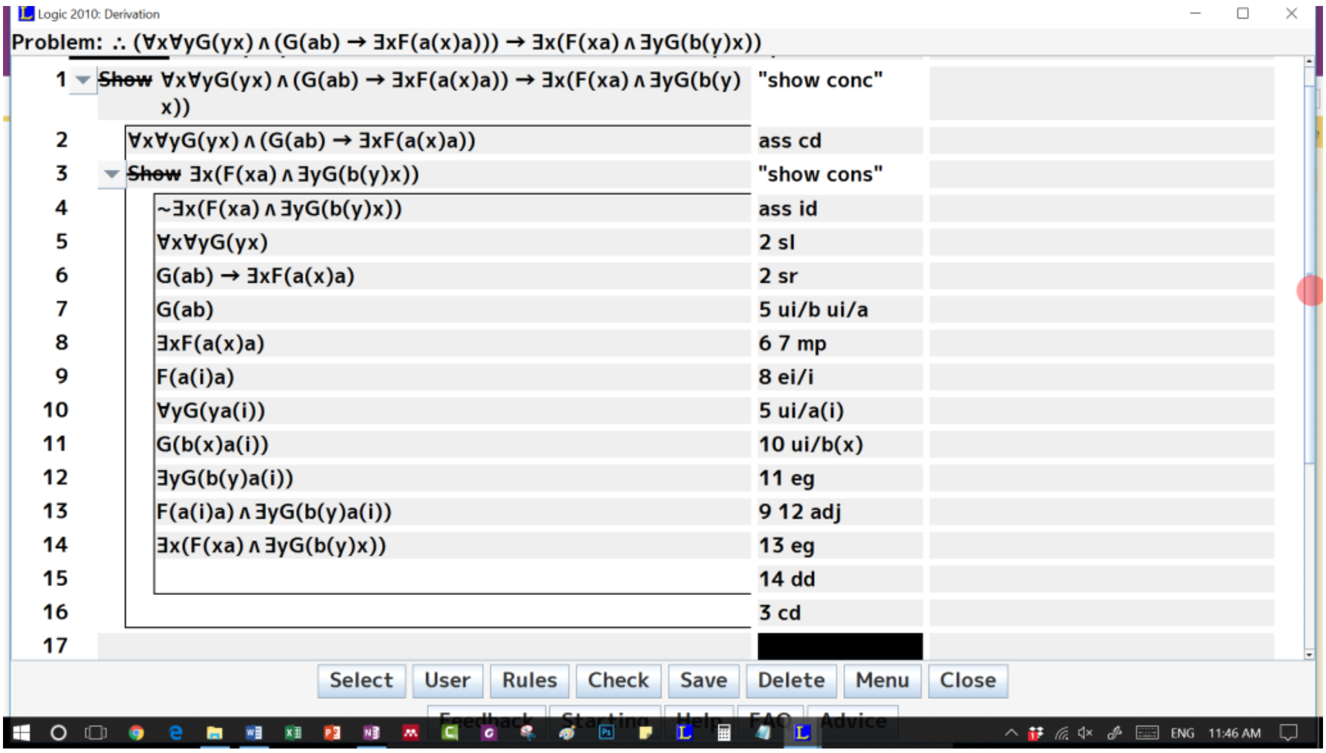
F(α a) ∧ ∃y G(b(y) α)

α = a(i)

∃y

G(b(β)α)

G(b(β) a(i))


Logic 2010: Derivation

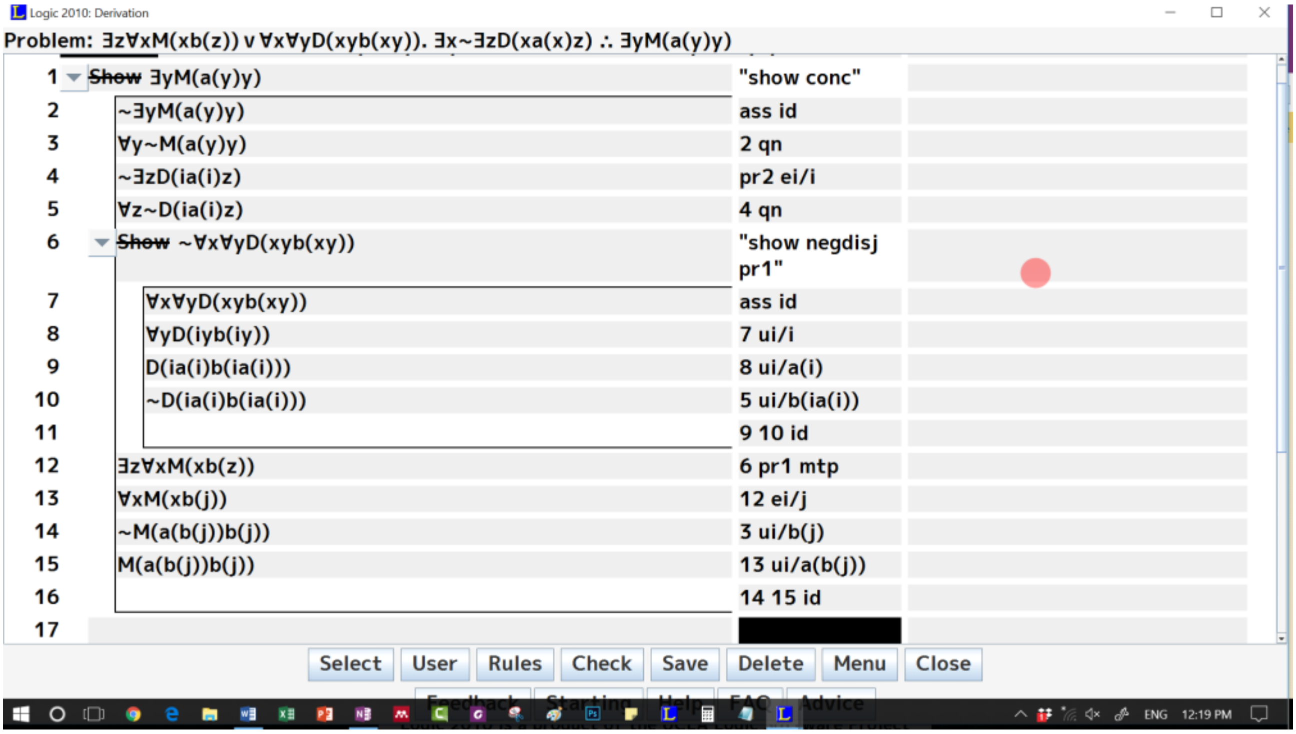
Problem: ∴ (∀x∀yG(yx) ∧ (G(ab) → ∃xF(a(x)a))) → ∃x(F(xa) ∧ ∃yG(b(y)x))

1	~~Show~~ ∀x∀yG(yx) ∧ (G(ab) → ∃xF(a(x)a)) → ∃x(F(xa) ∧ ∃yG(b(y) x))	"show conc"
2	∀x∀yG(yx) ∧ (G(ab) → ∃xF(a(x)a))	ass cd
3	~~Show~~ ∃x(F(xa) ∧ ∃yG(b(y)x))	"show cons"
4	~∃x(F(xa) ∧ ∃yG(b(y)x))	ass id
5	∀x∀yG(yx)	2 sl
6	G(ab) → ∃xF(a(x)a)	2 sr
7	G(ab)	5 ui/b ui/a
8	∃xF(a(x)a)	6 7 mp
9	F(a(i)a)	8 ei/i
10	∀yG(ya(i))	5 ui/a(i)
11	G(b(x)a(i))	10 ui/b(x)
12	∃yG(b(y)a(i))	11 eg
13	F(a(i)a) ∧ ∃yG(b(y)a(i))	9 12 adj
14	∃x(F(xa) ∧ ∃yG(b(y)x))	13 eg
15		14 dd
16		3 cd
17		

Select User Rules Check Save Delete Menu Close

3. Show the following argument is valid using a derivation. You may use the basic rules as well as the **derived** rules: CDJ, DM, NC, NB, SC, QN, and AV. (9)

∃z∀xM(xb(z))∨∀x∀yD(xyb(xy)). ∃x~∃zD(xa(x)z). ∴ ∃yM(a(y)y)



Logic 2010: Derivation

Problem: ∃z∀xM(xb(z)) v ∀x∀yD(xyb(xy)). ∃x~∃zD(xa(x)z) ∴ ∃yM(a(y)y)

1	~~Show~~ ∃yM(a(y)y)	"show conc"
2	~∃yM(a(y)y)	ass id
3	∀y~M(a(y)y)	2 qn
4	~∃zD(ia(i)z)	pr2 ei/i
5	∀z~D(ia(i)z)	4 qn
6	~~Show~~ ~∀x∀yD(xyb(xy))	"show negdisj pr1"
7	∀x∀yD(xyb(xy))	ass id
8	∀yD(iyb(iy))	7 ui/i
9	D(ia(i)b(ia(i)))	8 ui/a(i)
10	~D(ia(i)b(ia(i)))	5 ui/b(ia(i))
11		9 10 id
12	∃z∀xM(xb(z))	6 pr1 mtp
13	∀xM(xb(j))	12 ei/j
14	~M(a(b(j))b(j))	3 ui/b(j)
15	M(a(b(j))b(j))	13 ui/a(b(j))
16		14 15 id
17		

Select User Rules Check Save Delete Menu Close

4. Show the following argument is valid using a derivation. You may use the basic rules as well as the **derived** rules: CDJ, DM, NC, NB, SC, QN, and AV. (9)

~(∀xF(xx)→∃z(Bz∧Gz)). ∴ ∃xF(xa(b))↔~∀x∃y(Gx∧By)