NOTE: This file contains sample solutions to the quiz together with the marking scheme and comments for each question. Please read the solutions and the marking schemes and comments carefully. Make sure that you understand why the solutions given here are correct, that you understand the mistakes that you made (if any), and that you understand *why* your mistakes were mistakes.

Remember that although you may not agree completely with the marking scheme given here, it was followed the same way for all students. We will remark your quiz only if you clearly demonstrate that the marking scheme was not followed correctly.

For all remarking requests, please submit your request **in writing** directly to your instructor. For all other questions, please don't hesitate to ask your instructor during office hours or by e-mail.

GENERAL MARKING SCHEME:

- ***A**: All Correct*, except maybe for very few minor errors.
- ***B**: Mostly Correct*, but with a few serious errors, or many small errors.
- ***C**: Mostly Incorrect*, but with a few important elements, or many small elements, done correctly.
- ***10%**: Completely Blank*, or clearly crossed out.
- ***D**: All Incorrect*, except maybe for very few minor elements done correctly.

MARKER'S COMMENTS:

- **common error**: there is an important difference between a complement **subset** of vertices (as used in the reduction from VERTEXCOVER to INDEPENDENTSET) and a complement **graph** (as required here)

1. Recall the CLIQUE decision problem from last week's tutorial.

Input: An undirected graph $G = (V, E)$ and a positive integer k.

Question: Does G contain a *clique* of size at least k, *i.e.*, a subset of k or more vertices such that G contains **every** possible edge between the vertices in the clique?

In last week's tutorial, you showed that CLIQUE $\in$ *NP*. Now, show that CLIQUE is *NP*-hard. Give a detailed reduction and argument of correctness. (HINT: You can use any of the problems you know to be *NP*-hard from lectures, tutorials, or the textbook — except for CLIQUE itself, of course!)

We show that CLIQUE is *NP*-hard by proving IS $\leqslant_p$ CLIQUE (where IS is the INDEPENDENTSET problem).

On input (G, k) (for IS), where $G = (V, E)$, construct (G', k') (for CLIQUE) as follows:

Set $k' = k$ and $G' = (V, \overline{E})$, where $\overline{E}$ is the *complement* of E, *i.e.*, for all $x, y \in V$, $(x, y) \in \overline{E} \Leftrightarrow (x, y) \notin E$.

Clearly, (G', k') can be computed from (G, k) in polytime (in linear time, in fact).

Also, if G contains an independent set I of size k or more, then I forms a clique in G': since G contains no edge between any two vertices of I, G' contains every edge between any two vertices of I.

Finally, if G' contains a clique C of size k or more, then C forms an independent set in G: since G' contains every edge between any two vertices of C, G contains no edge between any two vertices of C.