Week 1, slide 32:

$$\sum_{i=1}^{n} \hat{e}_i x_i = 0$$

$$\sum (\hat{e}_i - \bar{\hat{e}})(x_i - \bar{x}) + \underbrace{\sum (\hat{e}_i - \bar{\hat{e}})}_{0} \bar{x} = 0$$

$$\sum_{i=1}^{n} (\hat{e}_i - \bar{\hat{e}})(x_i - \bar{x}) = 0$$

$$\therefore \operatorname{cov}(\hat{e}_i, x_i) = 0$$

and $r_{\hat{e}_i, x_i} = 0$

Weeks 4-5, slide 23:

$$\operatorname{var}(\hat{y}_i) = \operatorname{var}\left(\sum_{j=1}^{n} h_{ij} y_j\right)$$

$$= \sum_{j=1}^{n} h_{ij}^2 \operatorname{var}(y_j)$$

$$= \boxed{\sum_{j=1}^{n} h_{ij}^2} \sigma^2$$

slide 14

$$\operatorname{var}(\hat{y}_i) = \sigma^2 \boxed{h_{ii}}$$

WEEKS 4-5, SLIDE 14

Show $h_{ij} = h_{ji}$

$$LS = \frac{1}{n} + \frac{(x_i - \bar{x})(x_j - \bar{x})}{S_{xx}}$$

$$= \frac{1}{n} + \frac{(x_j - \bar{x})(x_i - \bar{x})}{S_{xx}}$$ b/c mult. is commutative

$$= RS$$

$$\begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1n} \\ \vdots & & & \\ \vdots & & & \\ h_{n1} & \cdots & & h_{nn} \end{bmatrix}$$

Show $\sum_{j=1}^{n} h_{ij}^2 = h_{ii}$

$$LS = \sum_{j=1}^{n} \left(\frac{1}{n} + \frac{(x_i - \bar{x})(x_j - \bar{x})}{S_{xx}} \right)^2$$

$$= \sum_{j=1}^{n} \left[\frac{1}{n^2} + \underbrace{2 \frac{1}{n} \frac{(x_i - \bar{x})(x_j - \bar{x})}{S_{xx}}}_{0} + \frac{(\ )^2(\ )^2}{S_{xx}^2} \right]$$

$$= \frac{n}{n^2} + \frac{(x_i - \bar{x})^2}{S_{xx}^2} \underbrace{\sum_{j=1}^{n} (x_j - \bar{x})^2}_{S_{xx}}$$

$$= \frac{1}{n} + \frac{(x_i - \bar{x})^2 \cancel{(x_i - \bar{x})}}{S_{xx}}$$

$$= h_{ii}$$

WEEKS 4-5, SLIDE 14

Show avg. h_{ii} is $2/n$:

$$h_{ii} = \frac{1}{n} + \frac{(x_i - \bar{x})^2}{S_{xx}}$$

$$\frac{1}{n}\sum_{i=1}^{n} h_{ii} = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{n} + \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2}{S_{xx}}$$

$$= \frac{1}{n} + \frac{1}{n}$$

Show $\sum_{j=1}^{n} h_{ij} = 1$

$$\sum_{j=1}^{n}\left[\frac{1}{n} + \frac{(x_i - \bar{x})(x_j - \bar{x})}{S_{xx}}\right]$$

$$= \sum \frac{1}{n} + \frac{x_i - \bar{x}}{S_{xx}} \underbrace{\sum (x_j - \bar{x})}_{0}$$

$$= 1$$