University of Toronto
Faculty of Arts and Science

**Midterm, Summer 2017**

# CSC263: Data Structure and Analysis

**Duration: 60 minutes**

First Name: _____ Last Name: _____ Student number: _____

**Do not turn this page until you have received the signal to start.**
**Answer all questions. WRITE LEGIBLY! Total Mark: 100**

| Question | Points | Score |
|---|---|---|
| 1 | 40 | |
| 2 | 12 | |
| 3 | 12 | |
| 4 | 16 | |
| 5 | 10 | |
| 6 | 10 | |
| Total: | 100 | |

1. (40 points) **SHORT ANSWER QUESTIONS:**

(I) **The depths of any two leaves in a max heap differ by at most 1.**
◯ True ◯ False

(II) **A tree with n nodes and the property that the heights of the two children of any node differ by at most 1 has $O(\log n)$ height.** ◯ True ◯ False

(III) **For any constants $x, y > 1$, we have $n^x = O(y^n)$.** ◯ True ◯ False

(IV) **Suppose we have a hash table with $m$ buckets, currently containing $n$ items. Suppose that instead of a linked list, each bucket is implemented as a binary search tree. Give the best asymptotic (big-O) characterization of the worst and best case time complexity of adding an entry to this hash table.**
◯ Best case $O(1)$, worst case $O(n)$
◯ Best case $O(1)$, worst case $O(\log n)$
◯ Best case $O(logn)$, worst case $O(n)$
◯ Best case $O(n)$, worst case $O(n)$
◯ Best case $O(logn)$, worst case $O(n^2)$

(V) **Inserting an element into an AVL tree with $n$ nodes requires $\Theta(\log n)$ rotations.**
◯ True ◯ False

(VI) **5. For which data structure the order of insertion does not matter, i.e. the resulting data structure is identical regardless of the oder the elements were inserted?**
◯ Unsorted sequence
◯ Heap
◯ AVL tree
◯ Hash table with chaining
◯ None of the above

(VII) **Which of the following sorting algorithms in its typical implementation gives best performance when applied on an array which is sorted or almost sorted (maximum 1 or two elements are misplaced).**
◯ Quick Sort ◯ Insertion Sort ◯ Heap Sort

(VIII) **Let T be an AVL tree of height 4. What is the smallest number of entries it can store? Note that a tree with one node (only the root) has the height of zero and stores one element.**
◯ 9 ◯ 10 ◯ 11 ◯ 12 ◯ 13

(IX) **Given a hash table $T$ with 25 slots that stores 2000 elements, the load factor $\alpha$ for $T$ is:**
◯ 80 ◯ 0.0125 ◯ 8000 ◯ 1.25

(X) **In a binary max heap containing n numbers, the smallest element can be found in time**
◯ $O(n)$ ◯ $O(logn)$ ◯ $O(\log \log n)$ ◯ $O(1)$

2. (12 points) **GROWTH OF FUNCTIONS:**

(I) Give the best asymptotic (big-O) and (big-Ω) characterization of the worst case and the best case time complexities of the algorithm DoAgain$(A, n)$

**Algorithm DoAgain**$(A, n)$
# Input: Array $A$ of size $n > 1$ storing integers.
sum $\leftarrow 0$
for $c = 0$ to $n^2$ do
  if $A[0] < 0$ then
    for $k = 0$ to $n - 1$ do
      $sum \leftarrow sum + c * A[k]$

3. (12 points) **HEAP**

For a max-heap of size $n$, you will be given the state of the max-heap before an operation and you have to show the state of the max-heap after the operation. Fill in the tables and draw the heap trees before and after the operations

(I) Insert(16)

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | |
|---|---|---|---|---|---|---|---|---|
| before: | 32 | 15 | 7 | 6 | 2 | 4 | 3 | |

after:

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | |
|---|---|---|---|---|---|---|---|
| | | | | | | | |

(II) ExtractMax()

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | |
|---|---|---|---|---|---|---|---|---|
| before: | 32 | 15 | 7 | 6 | 2 | 4 | 3 | |

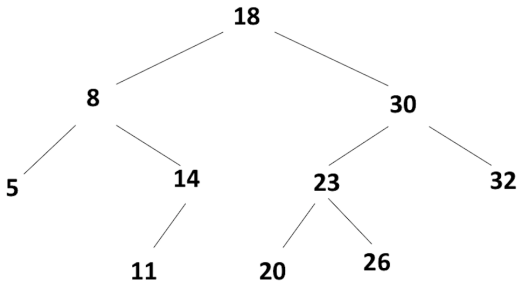
after:

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | |
|---|---|---|---|---|---|---|---|
| | | | | | | | |

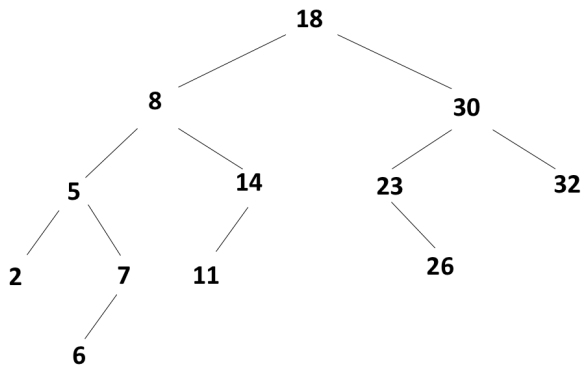
4. (16 points) **AVL TREE**

**Perform the following insertions into the given AVL trees and perform the appropriate rotations to rebalance the tree.**

(I) Draw the result when 25 is added to the AVL tree below. Show your work, in particular, identify clearly any rotation performed.




(II) Draw the result when 18 is deleted from the AVL tree below. Show your work, in particular, identify clearly any rotation performed.



5. (10 points) **HASH TABLE**

**Consider a hash table of size 11 storing entries with integer keys. Suppose the hash function is** $h(k) = k \mod 11$**. Insert, in the given order, entries with keys** $0, 1, 6, 7, 10, 22, 21$ **into the hash table using linear probing to resolve collisions. Show all the work.**

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | |

6. (10 points) **QUICKSORT Randomized quicksort compares individual pairs of elements but it does not necessarily compare every element to every other element. When the input is the array** $[2, 8, 5, 3]$**, what is the probability that randomized quicksort compares 2 and 8 directly to each other? Explain your reasoning.**