

For each topic in the course, I list **primary learning objectives** (skills that you **will** be asked to demonstrate on the final exam) and *secondary learning objectives* (important skills that are part of this course but that may or may not be exercised explicitly on the final exam).

## Greedy Algorithms

- **Write a proof of correctness for a greedy algorithm, following the format from class.**
  - Define *partial solution*.
  - Define what it means for an optimum solution to *extend* a partial solution.
  - Define *promising*.
  - Apply the correct proof structure:
    - \* Proof by induction that every partial solution is promising.
    - \* In the inductive step, top-level cases determined by the conditions in the body of the algorithm's main loop.
    - \* In the inductive step, sub-cases for each case determined by potential differences between the partial solution and an optimum solution.
- *Write an algorithm that uses a greedy strategy.*

## Dynamic Programming

- **Write an algorithm that uses dynamic programming, following the structure from class.**
  - Describe the recursive structure of optimum solutions to a problem.
  - Define an array that stores the optimum value of a solution to the subproblem defined by the array indices.
  - Give a recurrence relation for the array values based on the recursive structure.
  - Write an iterative algorithm to compute array values “bottom-up,” following the recurrence.
  - Use an additional array if needed to reconstruct an optimum solution from the computer array values.
- **Argue the correctness of a dynamic programming algorithm, based on the recursive structure of the problem it solves.**

## Network Flows

- **Solve a problem using network flow techniques, and argue the correctness of your solution.**
  - Describe how to construct a network from a problem input.
  - Describe how to reconstruct a solution from a maximum flow or minimum cut in the network.
  - Argue that every solution to the original problem becomes a valid flow (or cut) in the network, so the maximum flow value (or minimum cut capacity) is at least as good as the optimum solution value.
  - Argue that every valid flow (or cut) in the network becomes a solution to the original problem, so the optimum solution value is at least as good as the maximum flow value (or minimum cut capacity).
- *Prove properties of flows and cuts in networks.*

## Linear Programming

- **Solve a problem using linear programming, and argue the correctness of your solution.**
  - Describe how to construct a linear program from a problem input: define variables, objective function, and constraints explicitly.
  - Describe how to reconstruct a solution to the original problem from a solution to the linear program.
  - Argue that every solution to the original problem becomes a feasible solution in the linear program, so the optimum value of the objective function is at least as good as the optimum solution value.
  - Argue that every feasible solution in the linear program becomes a solution to the original problem, so the optimum solution value is at least as good as the optimum value of the objective function.

## $P$ , $NP$ , $coNP$ and $\leq_p$

- **Prove  $A \leq_p B$  for specific decision problems  $A, B$ .**
  - Describe an explicit construction for inputs  $y_x \in I_B$  given arbitrary inputs  $x \in I_A$ .
  - Argue that the construction can be carried out in polytime and  $x \in A \Leftrightarrow y_x \in B$ .
  - Avoid common mistakes:
    - \* constructions that do **not** run in polytime because they attempt to use a certificate (not part of input  $x$ );
    - \* giving two “half” constructions (constructing  $y_x$  from  $x$  and separately constructing  $x_y$  from  $y$ );
    - \* confusing  $\leq_p$  with  $\xrightarrow{p}$  and trying to describe an algorithm/verifier for  $A$  that makes calls to an algorithm/verifier for  $B$ .
- *Know the definitions of each complexity class and of  $\leq_p$ .*

## NP-Completeness

- **Show that a problem  $A$  is NP-complete.**
  - Give a *verifier* for  $A$  (**not** a “generate-and-verify” algorithm), and argue that it returns TRUE for *some* certificate iff the input is a yes-instance.
  - Find a suitable NP-hard problem  $B$  and prove  $B \leq_p A$ .
  - Avoid the common mistake of trying to show  $A \leq_p B$ .

## Self-Reducibility

- **Show that a problem is polytime self-reducible.**
  - Assume the existence of a polytime algorithm DA for the decision problem.
  - Write an explicit algorithm A for the search or optimization problem, making calls to DA.
  - Argue the correctness of algorithm A (usually through an appropriate loop invariant).
  - Analyse the runtime of algorithm A to show that it runs in polytime.

## Approximation Algorithms

- **Prove bounds on the approximation ratio for both minimization and maximization problems.**
  - Know the definition of *approximation ratio* for both kinds of optimization problems.
  - With some guidance/hints, prove bounds on the approximation ratio.

## In General...

- Pay particular attention to the *marking scheme* for each test and homework: these show you explicitly what aspects of your answers are important!
- Don't forget the tutorial problems: they often contain important clarifications or variations on the ideas presented in lectures.
- *Read the final exam's cover page* (posted on Piazza) and *re-read the course information sheet* (for details of the course policies, particularly the "20% rule" and information about aid sheets).
- Keep in mind the following guidelines for writing the exam.
  - **Plan your time!** Figure out how many minutes you have available for each page/question/mark.
  - **Read the questions carefully!** If something is unclear, please *ask*.
  - **Show what you know!**
    - \* Read **every** question before doing anything else.
    - \* Answer the "easy" questions first.
    - \* For all other questions write down an outline of what you have to do—**this is worth marks**.
    - \* Go back to the questions you're not sure about and work on them, but keep track of your time.
  - **Explain what you're doing!** A correct outline of a solution is worth marks.
  - **Don't ramble!** Write concise, to-the-point answers. Incorrect solution elements will cost you marks.

On the other hand, admit when something does not work: you will get more if we see that you understand your mistakes.
  - (This is the hardest one.) **Relax!** You'll function much better if you are well-rested and relaxed than if you are tired or tense.

It was a pleasure to teach you this term, good luck on all your exams and have a great summer!