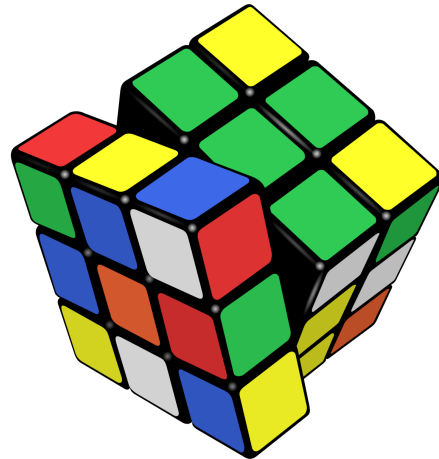


# Quiz 1

Section 1 (LEC0101, LEC2003)



CSC263  
Sep 27 – 2017

# Question 1

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Assume that algorithm A is asymptotically more efficient than B. Which statement is true?

Players correct: 58%

- A) A will be a better choice for all inputs
- B) B will be a better choice for small inputs
- C) A will be a better choice for all inputs except large inputs
- D) A will be a better choice for all inputs except small inputs

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## Question 2

---

Let  $T_W(n)$  be the worst-case and  $T_A(n)$  be the average-case running time of an algorithm. Which one is always true?

Players correct: 40%

A)  $T_W(n) = O(T_A(n))$

B)  $T_W(n) = \Omega(T_A(n))$

C)  $T_W(n) = \Theta(T_A(n))$

D)  $T_W(n) = o(T_A(n))$

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# Question 3

---

There are three possible procedures A, B, C for running an application. Procedure A takes 2 seconds, procedure B takes 3 seconds and procedure C takes 4 seconds to run. This application to be run chooses algorithm A with the probability of 0.30, the algorithm B with the probability of 0.20 and C with probability of 0.50.

What is the average case running time of the application?

Players correct: 84%

- A) 2      B) 4.5      C) 3.2      D) 3

# Question 3

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What is the average case running time of the application?

Players correct: 84%

A) 2      B) 4.5      C) 3.2      D) 3

$$\text{Average-case running time} = \sum x p(x) = 2 * 0.3 + 3 * 0.2 + 4 * 0.5 = 3.2$$

## Question 4

---

What is the minimum number of comparisons required to find a minimum value stored in a max heap?

Players correct: 24%

A)  $n$

B)  $n/2$

C)  $O(\log n)$

D)  $\frac{n}{2} - 1$



# Question 4

---

What is the minimum number of comparisons required to find a minimum value stored in a max heap?

Players correct: 24%

- A)  $n$       **B)  $n/2$**       C)  $O(\log n)$       D)  $\frac{n}{2} - 1$

**Answer:** The minimum element in a max-heap can be anywhere in the leaves of the heap. (why?)

There are up to  $n/2$  elements in the leaves, so finding the minimum needs  $n/2$  times.

# Question 5

---

What is time complexity of increasing and decreasing a value of an element in a max heap?

Players correct: 71%

- A)  $O(n)$ ,  $O(n)$       B)  $O(\log n)$ ,  $O(n)$       C)  $O(\log n)$ ,  $O(\log n)$       D)  $O(n)$ ,  $O(\log n)$

# Question 5

---

What is time complexity of increasing and decreasing a value of an element in a max heap?

Players correct: 71%

A)  $O(n)$ ,  $O(n)$       B)  $O(\log n)$ ,  $O(n)$       C)  $O(\log n)$ ,  $O(\log n)$       D)  $O(n)$ ,  $O(\log n)$

**Answer:**

- Increasing a value need a max-heapify-up operation
- Decreasing a value needs a max-heapify-down operation

Both operations are in  $O(\log n)$  since needs to traverse the height of the tree in the worst case.

# Question 6

---

In a heap of height  $h$ , what is the minimum number of elements?

(Assume the height of the root is zero).

Players correct: 25%

A)  $2^{(h+1)} - 1$

B)  $2^h$

C)  $2^{(h-1)}$

D)  $2^{(h+1)}$

# Question 6

---

In a heap of height  $h$ , what is the minimum number of elements?

(Assume the height of the root is zero).

Players correct: 25%

A)  $2^{(h+1)} - 1$

B)  $2^h$

C)  $2^{(h-1)}$

D)  $2^{(h+1)}$

Heap is a complete binary tree, so it is full at all level bottom level which has at least 1 node. The number of nodes at level 0 is  $1 = 2^0$ , at level 1 is  $2 = 2^1$ , ... , at level  $h - 1$  is  $2^{h-1}$  and at level  $h$  is 1.

Total number of nodes:  $(1 + 2 + 2^2 \dots + 2^{h-1}) + 1 = (2^h - 1) + 1 = 2^h$ .

# Question 7

---

Given four numeric values  $a < b < c < d$  as the elements' priorities, how many possible max heaps would be possible to build?

Players correct: 40%

- A) 2      B) 3      C) 4      D) 5

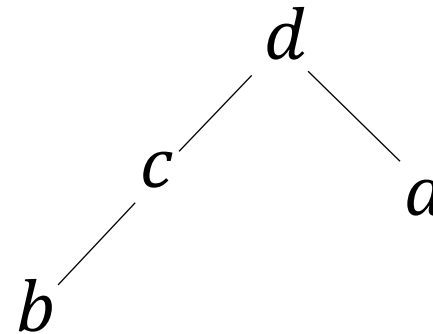
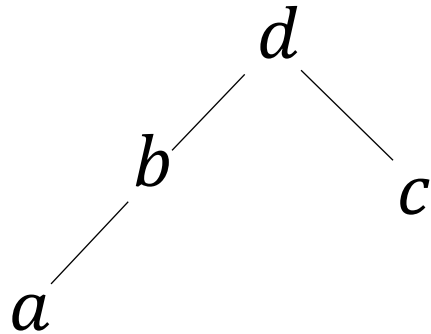
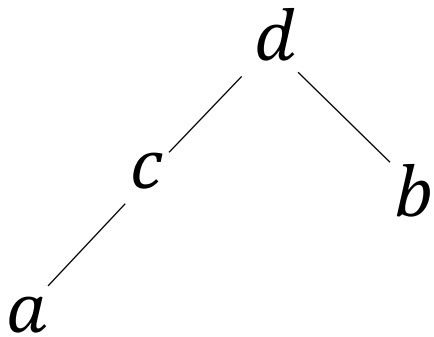
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Players correct: 40%

- A) 2    **B) 3**    C) 4    D) 5



## Question 8

---

Assume that the root of a heap lies on level 1, the immediate children of the root lies on level 2. Then,  $k^{th}$  largest element in the max-heap lies always on the  $k$  level of the heap.

A) True

B) False

Players correct: 79%



# Question 8

---

Assume that the root of a heap lies on level 1, the immediate children of the root lies on level 2. Then,  $k^{th}$  largest element in the max-heap lies always on the  $k$  level of the heap.

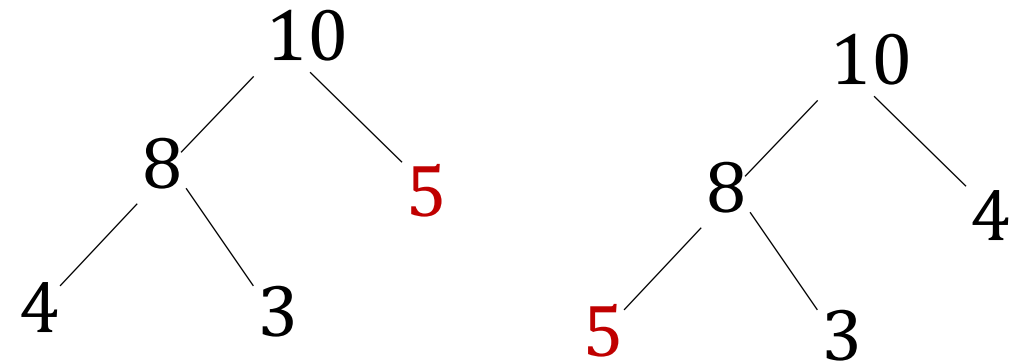
A) True

B) False

Players correct: 79%

For values of  $k > 2$  it is not always true.

Counter example:



## Question 9

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A **ternary** max heap is defined similar to a **binary** max heap but each node has at most **3 children** instead of **2 children**. Let  $A = \{15, 6, 8, 14, 3, 2\}$  be a ternary max heap with starting index at zero. We insert a new element 11. What is the index of 11 after being inserted to  $A$ . (A must remain a ternary max heap.)

- A) 1      B) 2      C) 3      D) 4

Players correct: 44%

# Question 9

A **ternary** max heap is defined similar to a **binary** max heap but each node has at most **3 children** instead of **2 children**.

Let  $A = \{20, 16, 17, 19, 8, 2, 14\}$  be a ternary max heap with starting index at zero. We insert a new element 18. What is the index of 18 after being inserted to  $A$ . (A must remain a ternary max heap.)

- A) 1    **B) 2**    C) 3    D) 4

Players correct: 44%

