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1. Regularized linear regression.

(a) $$\frac{\partial \mathcal{E}_{reg}}{\partial w_j} = \frac{1}{N}\sum_{i=1}^{N} x_j^{(i)}(y^{(i)} - t^{(i)}) + \lambda w_j$$

$$\frac{\partial \mathcal{E}_{reg}}{\partial b} = \frac{1}{N}\sum_{i=1}^{N} y^{(i)} - t^{(i)}$$

$$w_j \leftarrow w_j - \alpha \frac{\partial \mathcal{E}_{reg}}{\partial w_j} = w_j - \alpha\left(\frac{1}{N}\sum_{i=1}^{N} x_j^{(i)}(y^{(i)} - t^{(i)}) + \lambda w_j\right)$$

$$b \leftarrow b - \alpha \frac{1}{N}\sum_{i=1}^{N} y^{(i)} - t^{(i)}$$

Compared to original update rule, $w_j$ will be smaller if we take proper $\lambda$. So, it will improve generalization eventually.

(b) Let $$\frac{\partial \mathcal{E}_{reg}}{\partial w_j} = \frac{1}{N}\sum_{i=1}^{N} x_j^{(i)}(y^{(i)} - t^{(i)}) + \lambda w_j = 0$$

$$\Rightarrow \frac{1}{N}\sum_{j'=1}^{D}\left(\sum_{i=1}^{N} x_j^{(i)} x_{j'}^{(i)}\right) w_{j'} + \lambda w_j - \frac{1}{N}\sum_{i=1}^{N} x_j^{(i)} t^{(i)} = 0$$

$$\Rightarrow A_{jj'} = \begin{cases} \frac{1}{N}\sum_{i=1}^{N} x_j^{(i)} x_{j'}^{(i)} & \text{if } j \neq j' \\ \lambda + \frac{1}{N}\sum_{i=1}^{N} x_j^{(i)} x_{j'}^{(i)} & \text{if } j = j' \end{cases}$$

$$c_j = \frac{1}{N}\sum_{i=1}^{N} x_j^{(i)} t^{(i)}$$

2. Visualizing the cost function

(a) $$\mathcal{E}(w_1, w_2) = \frac{1}{3}\left[\frac{1}{2}(2w_1-1)^2 + \frac{1}{2}(w_2-2)^2 + \frac{1}{2}(w_2-0)^2\right]$$

$$= \frac{1}{6}(2w_1-1)^2 + \frac{1}{6}\left[(w_2-2)^2 + w_2^2\right]$$

$$= \frac{4}{6}(w_1-\tfrac{1}{2})^2 + \frac{1}{6}\left[w_2^2 - 4w_2 + 4 + w_2^2\right]$$

$$= \frac{2}{3}(w_1-\tfrac{1}{2})^2 + \frac{2}{6}(w_2^2 - 2w_2 + 2)$$

$$= \frac{2}{3}(w_1-\tfrac{1}{2})^2 + \frac{1}{3}(w_2-1)^2 + \frac{1}{3}$$

$$= c_1(w_1-d_1)^2 + c_2(w_2-d_2)^2 + \mathcal{E}_0$$

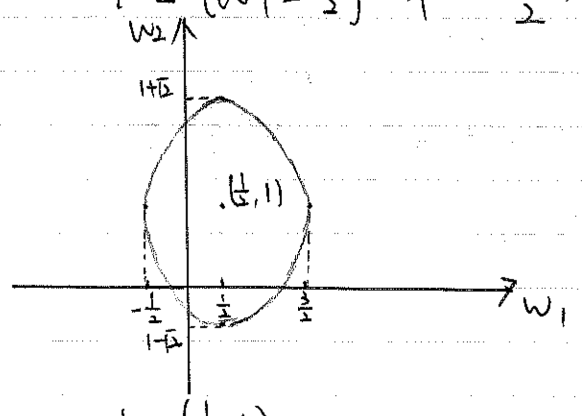
where $c_1 = \frac{2}{3}$, $d_1 = \frac{1}{2}$, $c_2 = \frac{1}{3}$, $d_2 = 1$, $\mathcal{E}_0 = \frac{1}{3}$

(b) $\mathcal{E} = 1$

$$1 = \frac{2}{3}(w_1-\tfrac{1}{2})^2 + \frac{1}{3}(w_2-1)^2 + \frac{1}{3}$$

$$\frac{2}{3} = \frac{2}{3}(w_1-\tfrac{1}{2})^2 + \frac{1}{3}(w_2-1)^2$$

$$1 = (w_1-\tfrac{1}{2})^2 + \frac{(w_2-1)^2}{2}$$




center $(\frac{1}{2}, 1)$

$a = 1$

$b = \sqrt{2}$