UNIVERSITY OF TORONTO

Faculty of Arts and Science

DECEMBER 2017 EXAMINATIONS

STA302H1S / STA1001H1S

Duration - 3 hours

Examination Aids: A calculator

First Name: ________________ **Surname:** _______________ **Student Number:** ________________

Section: LEC0101/2001 (day) ☐ LEC0501 (evening) ☐

Instructions:

- Print your name on each exam booklet
- Use $\alpha = 0.05$ as a significance level
- **Hand in this Question Paper Please:** Do not enter any answers on this yellow question paper. It will be destroyed after the exam. Answers on this question paper will not be marked. All answers must be provided in the answer booklets included with your examination

Question	Value
1	16
2	13
3	8
4	16
5	6
6	13
7	10
8	12
9	6
Total	100

This exam should have 8 pages including this page

$$S_{xx}=\sum_{i=1}^{n}(x_i-\bar{x})^2, \qquad S_{xy}=\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y}), \qquad \hat{\beta}_0=\bar{y}-\hat{\beta}_1\bar{x}, \qquad \hat{\beta}_1=\frac{S_{xy}}{S_{xx}}=\frac{\sum_{i=1}^{n}x_iy_i-n\bar{x}\,\bar{y}}{\sum_{i=1}^{n}x_i^2-n\bar{x}^2}$$

$$\mathrm{var}(\hat{\beta}_0)=\sigma^2\left[\frac{1}{n}+\frac{\bar{x}^2}{S_{xx}}\right], \qquad \mathrm{var}(\hat{\beta}_1)=\frac{\sigma^2}{S_{xx}}, \qquad \mathrm{cov}(\hat{\beta}_0,\,\hat{\beta}_1)=-\frac{\sigma^2\bar{x}}{S_{xx}}$$

$$\underbrace{\sum_{i=1}^{n}(y_i-\bar{y})^2}_{\text{SST}}=\underbrace{b_1^2\sum_{i=1}^{n}(x_i-\bar{x})^2}_{\text{SSReg}}+\underbrace{\sum_{i=1}^{n}\hat{e}_i^2}_{\text{RSS}}, \qquad \text{SSReg}=\sum_{i=1}^{n}(\hat{y}_i-\bar{y})^2, \qquad F_{\text{obs}}=\frac{\text{MSReg}}{\text{MSE}}$$

$$\mathrm{var}(\hat{y}^*)=\sigma^2\left[\frac{1}{n}+\frac{(x^*-\bar{x})^2}{S_{xx}}\right], \qquad \mathrm{var}(Y^*-\hat{y}^*)=\sigma^2\left[1+\frac{1}{n}+\frac{(x^*-\bar{x})^2}{S_{xx}}\right]$$

$$r=\frac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i-\bar{x})^2\sum_{i=1}^{n}(y_i-\bar{y})^2}}, \qquad \hat{y}_i=\sum_{j=1}^{n}h_{ij}\,y_k \qquad h_{ij}=\frac{1}{n}+\frac{(x_i-\bar{x})(x_j-\bar{x})}{S_{xx}}$$

$$\text{DFBETA}_{ik}=\frac{\hat{\beta}_k-\hat{\beta}_{k(i)}}{\text{s.e. of }\hat{\beta}_{k(i)}}, \qquad \text{DFFITS}_i=\frac{\hat{y}_i-\hat{y}_{i(i)}}{\text{s.e. of }\hat{y}_{i(i)}}, \qquad D_i=\frac{\sum_{j=1}(\hat{y}_{j(i)}-\hat{y}_j)^2}{2S^2}=\frac{r_i^2h_{ii}}{2(1-h_{ii})}$$

$r_i=\frac{\hat{e}_i}{S\sqrt{1-h_{ii}}}$ where $S^2=\text{MSE}=\frac{\text{RSS}}{n-p-1}$ Criteria for ordinary data points on small datasets: $|r_i|<2$, $h_{ii}<2(p+1)/n$, DFBETA $<2/\sqrt{n}$, DFFITS $<2\sqrt{\frac{p+1}{n}}$, $D_i<4/(n-p-1)$

$$\text{Transformations: } f'(\mu)\propto\frac{1}{\sqrt{V(\mu)}} \qquad \text{Covariance matrix: } \mathrm{var}(\mathbf{X})=\mathrm{E}\left[(\mathbf{X}-\mathrm{E}(\mathbf{X})\,(\mathbf{X}-\mathrm{E}(\mathbf{X}))'\right]$$

$$\text{For } f(\boldsymbol{\theta})=\mathbf{c}'\boldsymbol{\theta}, \quad \frac{\partial f(\boldsymbol{\theta})}{\partial\boldsymbol{\theta}}=\mathbf{c} \qquad \text{For } f(\boldsymbol{\theta})=\boldsymbol{\theta}'\mathbf{A}\boldsymbol{\theta}, \quad \frac{\partial f(\boldsymbol{\theta})}{\partial\boldsymbol{\theta}}=2\mathbf{A}\boldsymbol{\theta}$$

$$\text{RSS}(\boldsymbol{\beta})=\sum_{i=1}^{n}(y_i-\beta_0-\beta_1x_i)^2=(\mathbf{Y}-\mathbf{X}\boldsymbol{\beta})'(\mathbf{Y}-\mathbf{X}\boldsymbol{\beta}) \qquad \hat{\boldsymbol{\beta}}=(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y} \quad \text{and} \quad \mathrm{var}\left(\hat{\boldsymbol{\beta}}\right)=\sigma^2(\mathbf{X}'\mathbf{X})^{-1}$$

$$\mathbf{X}'\mathbf{X}=\begin{pmatrix} n & n\bar{x} \\ n\bar{x} & \sum x_i^2 \end{pmatrix} \qquad \text{and} \qquad (\mathbf{X}'\mathbf{X})^{-1}=\frac{1}{S_{xx}}\begin{pmatrix} \frac{1}{n}\sum_{i=1}^{n}x_i^2 & -\bar{x} \\ -\bar{x} & 1 \end{pmatrix}$$

$$\hat{\mathbf{e}}=(\mathbf{I}-\mathbf{H})\,\mathbf{Y} \text{ where } \mathbf{H}=\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}' \qquad \mathrm{var}(\hat{\mathbf{e}})=(\mathbf{I}-\mathbf{H})\,\mathrm{E}(\mathbf{Y}\mathbf{Y}')\,(\mathbf{I}-\mathbf{H})=\sigma^2(\mathbf{I}-\mathbf{H})$$

$$\underbrace{\mathbf{Y}'\,(\mathbf{I}-\frac{1}{n}\mathbf{J})\,\mathbf{Y}}_{\text{SST}}=\underbrace{\mathbf{Y}'\left(\mathbf{H}-\frac{1}{n}\mathbf{J}\right)\mathbf{Y}}_{\text{SSReg}}+\underbrace{\mathbf{Y}'\,(\mathbf{I}-\mathbf{H})\mathbf{Y}}_{\text{RSS}} \qquad \text{Adj } R^2=1-\frac{n-1}{n-p-1}\frac{\text{RSS}}{\text{SST}}$$

$$\text{Partial } F\text{-test}: F=\frac{(\text{RSS(reduced)}-\text{RSS(full)})\,/\,(\text{df}_{\text{reduced}}-\text{df}_{\text{full}})}{\text{RSS(full)}\,/\,\text{df}_{\text{full}}}\sim F_{\text{df}_{\text{reduced}}-\text{df}_{\text{full}},\,\text{df}_{\text{full}}} \text{ under } H_0$$

$$\text{VIF}_j=\frac{1}{1-R_j^2} \qquad r_{\text{partial}}(X_1,X_2)=r(\hat{e}_{X_1\text{ vs }Z},\,\hat{e}_{X_2\text{ vs }Z}) \qquad \text{WRSS}=\sum_{i=1}^{n}w_i\,(y_i-\hat{y}_i)^2 \text{ for } w_i\propto 1/\sigma_i^2$$

$$\hat{\beta}_{1W}=\frac{\text{WS}xy}{\text{WS}xx}=\frac{\sum_{i=1}^{n}w_i\,(x_i-\bar{x}_W)\,(y_i-\bar{y}_W)}{\sum_{i=1}^{n}w_i\,(x_i-\bar{x}_W)^2} \qquad \hat{\beta}_{0W}=\bar{y}_W-\hat{\beta}_{1W}\,\bar{x}_W$$

$$\begin{pmatrix}\hat{\beta}_{0W}\\ \hat{\beta}_{1W}\end{pmatrix}=\hat{\boldsymbol{\beta}}_W=\left(\mathbf{X}'\mathbf{V}^{-1}\mathbf{X}\right)^{-1}\mathbf{X}'\mathbf{V}^{-1}\mathbf{Y} \quad \text{and } \mathrm{var}\left(\hat{\boldsymbol{\beta}}_W\right)=\sigma^2\left(\mathbf{X}'\mathbf{V}^{-1}\mathbf{X}\right)^{-1}$$