STA304 A2 Q2

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a)

$\mu_x = \frac{\sum_{h=1}^{H} \mu_{x,h} * N_h}{\sum_{h=1}^{H} N_h}$

b)
$N = \sum_{h=1}^{H} N_h$
$\hat{\mu_D} = \bar{y_{st}} + \mu_x - \bar{x_{st}}$
$= \sum_{h=1}^{H} (\frac{N_h}{N})(\frac{\sum_{i=1}^{n_h} y_{h,i}}{n_h}) + \mu_x - \sum_{h=1}^{H} (\frac{N_h}{N})(\frac{\sum_{i=1}^{n_h} x_{h,i}}{n_h})$

$E(\hat{\mu_D}) = \sum_{h=1}^{H} \frac{N_h}{N} * \frac{1}{n_h} * n_h * E(y_h) + \mu_x - (\sum_{h=1}^{H} \frac{N_h}{N} * \frac{1}{n_h} * n_h * E(x_h))$
$= \mu_y + \mu_x - \mu_x$
$= \mu_y$

c)
$N = \sum_{h=1}^{H} N_h$
$V(\hat{\mu_D}) = \sum_{h=1}^{H} (\frac{N_h}{N * n_h})^2 * n_h * V(y_h) + \sum_{h=1}^{H} (\frac{N_h}{N * n_h})^2 * n_h * V(x_h)$
$= \sum_{h=1}^{H} (\frac{N_h}{N})^2 * \frac{1}{n_h} * (\frac{N_h - 1}{N_h} {S_{y,h}}^2) + \sum_{h=1}^{H} (\frac{N_h}{N})^2 * \frac{1}{n_h} * (\frac{N_h - 1}{N_h} {S_{x,h}}^2)$
$= \sum_{h=1}^{H} \frac{(N_h)(N_h - 1)}{N^2 * n_h} * ({S_{y,h}}^2 + {S_{x,h}}^2)$

d)
$n_i = n/H$ for any i

e)
$\bar{d} = \mu_x - \bar{x_{st}} = 0$