Department of Mathematics, University of Toronto
MAT224H1S - Linear Algebra II
Winter 2020

Lectures & Administrative Information

Section	Time	Lecture Room	Instructor	Office
L0101	W3, R1-3	PB B250	Z. Qian	HU 1012A
L0201	T4-6, R4	LM 161	V. Dimitrov	tba
L0301	W11-1, F11	LM 159	S. Uppal	PG 112
L0401	M10, W9-11	ES B142	J. Im	tba
L0501	M1, W12-2	BA 1160	Q. Li	tba
L5101	T6-9	SF 1105	D. Butson	tba

Course Coordinator: S. Uppal.

Email: uppal@math.utoronto.ca.

Student-Instructor hours (formerly known as Office Hours): Have a question, or questions? Want to talk about the material in the course or perhaps even next steps? Have a feeling that something isn't sitting right about a particular concept but don't know what question to ask? These are some examples of what Student-Instructor hours are for - they are an opportunity to dialogue with course instructors outside of the classroom in either a small group or an individual setting.

The full schedule and locations of Student-Instructor hours will be posted on the course webpage on Quercus (see below) by the end of the first week of classes. If you would like to book an appointment outside these scheduled hours, please send an email to your instructor indicating the days and times you are available to meet. Also, please try to give at least 24 hours notice for appointments so that there is suitable time to make arrangements.

Email Policy & Etiquette

We will respond to emails as soon as possible, usually within 24-48 hours (except on weekends). Several days before an exam is always a particularly busy time and it may take us longer to respond. If your situation is urgent, it's best to speak with us in person either before or after class or during office hours.

- **Put MAT224 in the subject line, use your UofT email, and always identify yourself**.
- **Be specific**. We're better able to help you the more specific you are. If your question is complex or lengthy and requires multiple back-and-forth emails, we will ask you to to come office hours, or make an appointment, instead.

- **Check the syllabus and Quercus first**. If the answer to your question(s) is available in the syllabus or on the announcements page on Quercus, we will not respond to your email.
- **Be professional**. Please use an appropriate tone, level of formality, and review what you've written before sending your email. Email, in the context of the class and communication with instructors, is professional correspondence and we expect you to treat it as such.

Brief Course Description & Goals

This second course in Linear Algebra expands the breadth and depth of the material from MAT223 - Linear Algebra I, which is a prerequisite for the course. We will analyze particular sets (vector spaces) and special mappings/functions between these sets (linear transformations), and classify all such maps (diagonal and canonical forms). The motivation for much of what we'll do comes from what we covered in MAT223. Always keep in mind that in linear algebra **concepts are as important as computations**.

We recommend you review the material from MAT223 - solving systems of linear equations, subspaces of $\mathbb{R}^n$, span, linear independence and dependence, basis, dimension, rank, column space, null space, projections, and diagonalization - particularly if it's been more than one semester since you took it. We will talk about some of these topics in a more general setting.

It is hoped that by the end of the course you will have

- become fluent in linear algebra and some of its applications.
- become comfortable reading and understanding precise mathematical statements, definitions, and proofs.
- write well written, well explained proofs and solutions to given problems blending full sentences and correction notation.

You will see some superb material in this course. If you run into some trouble along the way, please do not hesitate to contact your instructor or TA for help. See the weekly class schedule below for a full list of topics covered.

Textbook and Reading Material

Required: David B. Damiano & John B. Little: *A Course in Linear Algebra.* ISBN: 978-0-486-46908-9. Dover publications.

This textbook is the best textbook for the course given the content we need to cover and various academic backgrounds of the students enrolled. The textbook has an easy-going, conversational style but doesn't lack rigour. There may be a couple of times we cover material in a more general setting than the textbook (such as Chapter 4) , and others where we don't go in as much depth (such as Chapter 6). Not attending lectures and attempting to learn strictly from the textbook may be problematic. The one aspect where the textbook could be improved is in the number of exercises in each section; there could be more. To compensate, we will give you additional set of problems every week to work on and you will also see examples in class. If you find you need more practice, please ask and we can suggest alternative texts/resources other than those below. There is no solutions manual for this book but there are solutions in the back of the textbook for some exercises.

Recommended: Sheldon Axler: *Linear Algebra Done Right, 3rd ed.* ISBN: 978-3-319-11079-0. Springer.

It begins slightly in more general setting than Damiano & Little but this could, and may be in the future, the textbook for this course.

Recommended (advanced): Stephen H. Friedberg, Arnold J. Insel, Lawrence E. Spence: *Linear Algebra, 4th ed.* ISBN:978-0-130-08451-4. Pearson Education

This is the textbook used for the specialist linear algebra MAT240/MAT247. A solid, well written text, with good problems but a little advanced for this course. For those of you looking to delve a little deeper, this is the text to read.

Course Webpage

The website for this course is accessible through Quercus.

Please check the website frequently for course announcements and materials. All announcements posted are considered to have been announced to the class and not having read or seen an announcement **is not** an accepted reason for not following guidelines or missing deadlines. You may configure your preferences on Quercus to receive email notification as soon as an announcement has been posted.

Marking Scheme

Your final grade will be calculated by the following formula:

- Writing Assignments - 10% of your final grade.
- Term Test I & II - 50% of you final grade (combined). The higher of Term Test I/II will count for 30% of your final grade; the lower 20%.
- Final Exam - 40% of your final grade.

Your raw scores for each piece of term work will be recorded on Quercus. Please check regularly that your marks have been recorded accurately. If there are any discrepancies, please email the course coordinator within two weeks of the grade having been posted at uppal@math.utoronto.ca. You will, of course, need evidence that your grade is not recorded correctly.

Course Components & Assignments

Lectures

You will get the most out of lectures if you come to really engage with the material as opposed to just taking notes (or not). Try to make sense of individual topics and their connections to other topics and how to translate seemingly abstract concepts into simple terms. If you do choose to take notes, I suggested re-writing and revising your notes the same day, while concepts are still fresh in your mind.

While cell phones are not prohibited in lecture, **recording or taking pictures in lecture is strictly prohibited without the consent of your instructor**. Please ask before doing.

Discussion Groups (formerly known as Tutorials)

Every student should be registered in one tutorial (TUT) section through ACORN. If you've not already done so, please do so as soon as possible as some time slots will fill up quickly and no additional sections will be added to those that do. Enrolling in a tutorial on ACORN will enroll you in a Discussion Group. Discussion Groups begin the week of Monday September 16.

Discussion Groups are an integral part of the course and should be regarded as just as important as lectures. During your Discussion Groups your TA will lead a discussion focusing on (but perhaps not exclusively on) 'Tutorial Problems' which will be posted on the course website each Friday and will be discussed in your tutorial the following week. The discussion may be about a particular question or the concepts underlining a question. This process may include working in small groups or individually on the tutorial problems, asking questions as they arise, or even a presentation your worked solution with your entire Discussion Group for peer feedback. The intention of the Discussion Groups is to help develop your problem solving skills, and improve your fluency with course content. It's important then that you practice early and often, identify what you're having the most trouble with and ask questions. Discussion Groups will focus primarily on engagement and practice, not answers.

As with lectures, **recording or taking pictures in tutorials is strictly prohibited without the consent of your TA**. Please ask before doing.

Term Tests & Final Exam

There will be two 1hr 50min minute term tests and one 3hr final exam common to all sections of the course. Each term test will emphasize material not already tested but may build on previous material. The final exam will be cumulative.

The dates and times of the term tests are:

- Term Test I - Friday February 14, 2:10-4:00pm. An early sitting is available from 12:10am-2:00pm for those with a legitimate conflict*.
- Term Test II - Friday March 20, 2:10-4:00pm. An early sitting is available from 12:10am-2:00pm for those with a legitimate conflict*.

The date of the final exam is to be determined by the Faculty of Arts & Science but will be scheduled sometime April 6-25.

*A legitimate conflict is if you are registered in another course that has a lecture or tutorial that meets during the scheduled time of the exam. If you have a legitimate conflict and need to register for the early sitting of the exams, please read and follow the instructions that will be posted on Quercus roughly one weeks before the date of each term test.

Each term test and the final exam may contain multiple choice questions, short answer questions, theory questions, precise definitions and statements of theorems. Exact details about exam content and format will be posted on the course website roughly one week before the date of each term test. The term tests and the final exam are closed book and no calculators or other aids are allowed.

There will be no make-up exams.

Writing Assignments

The purpose of the writing assignments is to improve your writing skills and understanding of what constitutes good mathematical exposition.

Each assignment will consist of one or two problems, possibly with multiple parts. The problems may include topics we've covered in lecture; may include topics we've not covered but you have the skills to solve; or may be a combination of the two.

Each assignment will be assessed with individual feedback intended to help you understand the gap between your solution (present position) and a perfect solution (desired goal); and how to bridge the two. You will receive either a score of 1 for a neatly written, clear attempt to solve the problem(s); or 0 for no clear attempt to solve the problem(s) or a messy, scribbled solution. Presentation counts, as does content, though you don't need to have a perfect solution to get a perfect score. The idea is to give you some freedom to make mistakes; to be creative, and express yourself with the course content. Working on the assignments in groups is encouraged though each assignment should be written independently.

There will be seven writing assignments throughout the term with your "best" six counting toward your writing assignment grade. Each assignment is worth an equal value toward your writing assignment grade. The schedule for the writing assignments is as follows:

Writing Assignment 1: posted Friday January 17, due Friday January 24
Writing Assignment 2: posted Friday January 24, due Friday January 31
Writing Assignment 3: posted Friday January 31, due Friday February 7
Writing Assignment 4: posted Friday February 21, due Friday February 28
Writing Assignment 5: posted Friday February 28, due Friday March 6
Writing Assignment 6: posted Friday March 6, due Friday March 13
Writing Assignment 7: posted Friday March 20, due Friday March 27

Warning: The assignments are generally on the short side and low stakes. If you do nothing else every week but the assignments, you will surely struggle with this course. You'll need to practice writing solutions to more than one or two problems per week to be successful in this course.

Tips to do well

- Attend every lecture and Discussion Group.
- Come to lecture and your Discussion Groups prepared. For lectures, this means reviewing the material from the previous week, and reading the relevant sections in the textbook beforehand. Be active while reading - write definitions and statements of theorems and note any concepts that are unclear and any questions you may have. You can then either bring them up in lecture or see if the lecture has answered your questions. For Discussion Groups, this means attempting the Tutorial Problems in advance. The key is to discover what you do and don't know and where there are gaps in your understanding. Once you look up a solution, or have someone show you a solution, you lose out on this valuable insight.
- Practice, practice, practice. Learning linear algebra is like learning a new language, to master it requires consistent practice. Once you've read the textbook and reviewed your notes, you won't gain much by re-reading them ad-nauseum. Practice problems as much as you can. Practice early and often rather than cramming in short bursts.
- Learn, don't memorize. Learning is an active process; memorizing is passive.
- Form study groups. You will learn from one another, through both your expertise and your mistakes.

- Ask questions. Lots of them. If you're stuck on a problem and don't know where to begin, a good starting point is to identify the keywords and ask yourself "what does this mean?".
- Complete **all** the term work. Consistently, the top marks for the course are earned by students who don't defer any term tests and complete all the assignments, even though we drop your lowest assignment score.
- Average 8 hours (480 minutes) of study a week for this course - 1/5th of a full time job. Being engaged in lectures and tutorials is 200 minutes and gets you almost halfway there. The remaining time should be spent mainly practicing problems.

Course Policies

Missed Term Tests

- You will be assigned a grade of 0 for any term test you do not write unless you submit a University of Toronto Verification of Student Illness or Injury form http://www.illnessverification.utoronto.ca/index.php within one week after the date of the exam.
- The form must have all the required fields properly filled out and it must list the doctor's OHIP number.
- The form must clearly state that on the date of the term test you were unable to write. Accordingly, it's expected that you will have met your doctor on the date of the term test. Illness before the term test is not sufficient grounds for not writing the term test nor is the claim that you would have performed "sub-optimally". The form **cannot just report that you told the doctor after-the-fact that you were ill previously**.
- The form must be original and completed by your Dentist, Nurse/Nurse Practitioner, Physician/Surgeon, Psychologist, Psychotherapist or Social Worker registered and licensed in the Province of Ontario for completion.
- Once you submit your form, it will be reviewed before it will be accepted. Part of the review process may include following up with your doctor, your college registrar, or the undergraduate chair of the math department. It is an academic offence to feign illness to miss a term test.
- If you do miss either Term Test I or II for a legitimate reason that you can document, and your documentation is accepted, then your final exam will account for 65% of your final grade, and the term test you do write will account for 25% of your final grade.

Writing Assignments

- Late assignments will not be accepted nor will any extensions be given for any reason including, but not limited to, not having an internet connection, technical issues (problems with your browser, power outages, problems with your laptop, software issues, etc). Start and finish assignments early and don't wait until the last minute to submit them - you do so at your own risk.
- Medical notes will not be accepted for missing an assignment.
- Under no circumstances will the weight of any assignment(s) be transferred to the final exam.

Academic Resources

Accessibility Needs

The University of Toronto is committed to accessibility. If you require accommodations for a disability, or have any accessibility concerns about the course, the classroom or course materials, please contact Accessibility Services - http://www.studentlife.utoronto.ca/ - as soon as possible.

Writing and English Language Instruction

For information on campus writing centres and writing courses, please visit http://www.writing.utoronto.ca/.

Other Resources

Student Life Programs and Services: http://www.studentlife.utoronto.ca

Academic Success Centre: http://www.studentlife.utoronto.ca/asc

Health and Wellness Centre: http://www.studentlife.utoronto.ca/hwc

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Academic Integrity

Academic integrity is fundamental to learning and scholarship at the University of Toronto. Participating honestly, respectfully, responsibly, and fairly in this academic community ensures that the U of T degree that you earn will be valued as a true indication of your individual academic achievement, and will continue to receive the respect and recognition it deserves.

Familiarize yourself with the University of Toronto's Code of Behaviour on Academic Matters

http://www.governingcouncil.utoronto.ca/policies/behaveac.htm.

It is the rule book for academic behaviour at the U of T, and you are expected to know the rules.

The University of Toronto treats cases of academic misconduct very seriously. All suspected cases of academic dishonesty will be investigated following the procedures outlined in the Code. The consequences for academic misconduct can be severe, including a failure in the course and a notation on your transcript. If you have any questions about what is or is not permitted in this course, please do not hesitate to contact me. If you have questions about appropriate research and citation methods, seek out additional information from me, or from other available campus resources like the U of T Writing Website. If you are experiencing personal challenges that are having an impact on your academic work, please speak to me or seek the advice of your college registrar.

Weekly Class Schedule

The first day of classes is Monday January 6, and the last day of classes is Friday April 3. Each lecturer will cover the material in his or her own style. Some lecturers may include in-class problem solving in groups or individually. All term tests, assignments, and final exam will cover material common to all sections of the course.

Your instructor may be slightly ahead or behind this schedule. This schedule is subject to change.

Week 1 beginning Monday January 6.
Lecture: Vector Spaces, Subspaces.
Section 1.1
Section 1.2

Week 2 beginning Monday January 13.
Lecture: Linear Combinations, Linear Dependence and Linear Independence.
Section 1.3
Section 1.4

Note: Section 1.5, Solving Systems of Linear Equations, is material from MAT223 that you should review on your own.

Week 3 beginning Monday January 20. **Discussion Groups begin**
Lecture: Bases and Dimension. Linear Transformations.
Section 1.6
Section 2.1

Week 4 beginning Monday January 27.
Lecture: Linear Transformations between Finite Dimensional Vector Spaces, Kernel & Image.
Section 2.2
Section 2.3

Week 5 beginning Monday February 3.
Lecture: Dimension Theorem, Applications of Dimension Theorem, Composition of Linear Transformations.
Section 2.4
Section 2.5

Week 6 beginning Monday February 10. **Term Test I Friday February 14**.
Lecture: Inverse of Linear Transformation, Isomorphism, Change of Basis.
Section 2.6
Section 2.7

Week 7 beginning Monday February 24.
Lecture: Determinant Function, Determinant as Area, Determinant of an $n \times n$ matrix, further properties of the Determinant.
Section 3.1
Section 3.2
Section 3.3

Week 8 beginning Monday March 2.
Lecture: Eigenvalues & Eigenvectors, Diagonalizability.
Section 4.1
Section 4.2

Week 9 beginning Monday March 9.
Lecture: Fields, Vector Space over a Field.
Section 5.1
Section 5.2

Week 10 beginning Monday March 16. **Term Test II Friday March 20**
Lecture: Triangular Form, Canonical Form for Nilpotent Mappings.
Section 6.1
Section 6.2

Week 11 beginning Monday March 23.
Lecture: Jordan Canonical Form, Computing Jordan Form.
Section 6.3
Section 6.4

Week 12 beginning Monday March 30. **Classes end April 3**
Lecture: Characteristic and Minimal Polynomial. Catch-up/Review for Final Exam.
Section 6.5
