STA457 Final Practice Questions (2017 Summer)

1. Vector autoregression and cointegration

Consider a bivariate VAR(p) model

$$\begin{bmatrix} X_{1t} \\ X_{2t} \end{bmatrix} = \sum_{i}^{p} \begin{bmatrix} \phi_{11}^{(i)} & \phi_{12}^{(i)} \\ \phi_{21}^{(i)} & \phi_{22}^{(i)} \end{bmatrix} \begin{bmatrix} X_{1,t-i} \\ X_{2,t-i} \end{bmatrix} + \begin{bmatrix} a_{1t} \\ a_{2t} \end{bmatrix}. \quad (1)$$

Answer the following questions:

1. State how to check the stationarity of Equation (1);
2. Describe the methods to select the order for Equation (1), i.e. the value of p, taught in class.
3. State how to how to test Granger causality for the case that X_{1t} grander causes X_{2t} but not the other way around. Based on the same condition, express X_{2t} as the transfer function noise model of X_{1t}.
4. Suppose that $\phi_{kl}^{(i)} = 0$ for $i = 2, \ldots, p$, $k, l = 1,2$. Derive the implied model for X_{2t};
5. Suppose that

   $$\phi_1(B)X_{1t} = \theta_1(B)u_{1t}$$

   and

   $$\phi_2(B)X_{2t} = \theta_2(B)u_{2t},$$

   where $\phi_k(B) = 1 - \phi_1^{(k)}B - \cdots - \phi_{p_k}^{(k)}B^{p_k}$ and $\theta_k(B) = 1 + \theta_1^{(k)}B - \cdots - \theta_{q_k}^{(k)}B^{q_k}$ for $k = 1,2$.

   Describe how to test Granger causality using univariate approach.
6. Suppose that X_{1t} and X_{2t} are not weakly stationary. How do you model the joint dynamics of $\{X_{1t}, X_{2t}\}$? Discuss your decisions based on whether these two series are cointegrated or not.
7. Discuss the reasons why we have to choose different models based the condition of cointegration.
8. Discuss the Engle-Granger approach for modeling cointegrated X_{1t} and X_{2t}.
9. Discuss the implication of Granger representation theorem.

2. Bootstrap time series

Consider an AR(2) model

$$y_t = \mu + \phi_1 y_{t-1} + \phi_2 y_{t-2} + a_t, \qquad a_t \sim NID(0,1).$$

1. Describe the steps of (unconditional) parametric bootstrap for the above AR(2) process.
2. Describe the steps of carrying out the Sieve bootstrap for the above AR(2) process.
3. Describe the steps of carrying out the block bootstrap method for the above AR(2) process.
4. Discuss the pros and cons for the above methods.

3. Modeling seasonality

1. Define the seasonal autoregressive integrated moving average (SARIMA) model;
2. Define the periodic autoregressive (PAR) model;
3. Define the periodic moving average (PMA) model.

4. State space model

Express a given time series model as a state-space representation. For example, consider

1. $y_t - \phi_1 y_{t-1} - \phi_2 y_{t-2} = a_t + \theta_1 a_{t-1} + \theta_2 a_{t-2},\ a_t \sim NID(0,1)$;
2. $y_t = \alpha + \sum_{i=1}^{p} \beta f_{it} + a_t,\ a_t \sim NID(0,1)$;
3. Review the structural time series model in the course note.

5. ARCH/GARCH process

1. Define the autoregressive conditional heteroskedasticity (ARCH) process;
2. Define the generalized autoregressive conditional heteroskedasticity (GARCH) process.