University of Toronto
CSC**343**

Sample BCNF Problem

Questions

Consider a relation schema R with attributes $ABCDEFGH$ with functional dependencies S:

$$S = \{B \rightarrow CD, \quad BF \rightarrow H, \quad C \rightarrow AG, \quad CEH \rightarrow F, \quad CH \rightarrow B\}$$

1. Which of these functional dependencies violate BCNF?
2. Employ the BCNF decomposition algorithm to obtain a lossless decomposition of R into a collection of relations that are in BCNF. Make sure it is clear which relations are in the final decomposition and project the dependencies onto each relation in that final decomposition.
3. Is your decomposition dependency-preserving?

Explain all your answers and show your rough work.

Solutions

Although one can often skip ahead to some of the conclusions or combine steps, these solutions are very systematic, so that you can see the full pattern.

1. Which of these functional dependencies violate BCNF?

   BCNF requires that the LHS of an FD be a superkey.

   - $\times$ $B^+ = BCDAG$, so B is not a superkey and $B \rightarrow CD$ violates BCNF.
   - $\times$ $BF^+ = BFHCDAG$ (but not E) so $BF \rightarrow H$ also violates BCNF.
   - $\times$ $C^+ = CAG$ so $C \rightarrow AG$ also violates BCNF.
   - $\checkmark$ $CEH^+ = CEHAGFBD$ so CEH is a superkey and $CEH \rightarrow F$ does not violate BCNF.
   - $\times$ $CH^+ = CHAGBD$ (but not EF) so $CH \rightarrow B$ also violates BCNF.

2. Employ the BCNF decomposition algorithm to obtain a lossless decomposition of R into a collection of relations that are in BCNF. Make sure it is clear which relations are in the final decomposition and project the dependencies onto each relation in that final decomposition.

   - Decompose R using FD $B \rightarrow CD$. $B^+ = BCDAG$, so this yields two relations: $R1 = ABCDG$ and $R2 = BEFH$.
   - Project the FDs onto $R1 = ABCDG$.

A	B	C	D	G	closure	FDs
✓					$A^+ = A$	nothing
	✓				$B^+ = BCDAG$	$B \rightarrow ACDG$
		✓			$C^+ = CAG$	$C \rightarrow AG$: violates BCNF; abort the projection

   We must decompose $R1$ further.

   - Decompose $R1$ using FD $C \rightarrow AG$. This yields two relations: $R3 = ACG$ and $R4 = BCD$.
   - Project the FDs onto $R3 = ACG$

A	C	G	closure	FDs
✓			$A^+ = A$	nothing
	✓		$C^+ = CAG$	$C \rightarrow AG$; C is a superkey of $R3$
		✓	$G^+ = G$	nothing
supersets of C			irrelevant	can only generate weaker FDs than what we already have
✓		✓	$AG^+ = AG$	nothing

   This relation satisfies BCNF.

   - Project the FDs onto $R4 = BCD$

B	C	D	closure	FDs
✓			$B^+ = BCDAG$	$B \rightarrow CD$; B is a superkey of $R4$
	✓		$C^+ = CAG$	nothing
		✓	$D^+ = D$	nothing
supersets of B			irrelevant	can only generate weaker FDs than what we already have
	✓	✓	$CD^+ = CDAG$	nothing

   This relation satisfies BCNF.

- Return to $R2 = BEFH$ and project the FDs onto it.

B	E	F	H	closure	FDs
✓				$B^+ = BCDAG$	nothing
	✓			$E^+ = E$	nothing
		✓		$F^+ = F$	nothing
			✓	$H^+ = H$	nothing
✓	✓			$BE^+ = BECDAG$	nothing
✓		✓		$BF^+ = BFCDHAG$	$BF \to H$: violates BCNF; abort the projection

We must decompose $R2$ further.

- Decompose $R2$ using FD $BF \to H$. This yields two relations: $R5 = BFH$ and $R6 = BEF$.
- Project the FDs onto $R5 = BFH$

B	F	H	closure	FDs
✓			$B^+ = BCDAG$	nothing
	✓		$F^+ = F$	nothing
		✓	$H^+ = H$	nothing
✓	✓		$BF^+ = BFCDHAG$	$BF \to H$
✓		✓	$BH^+ = BHCDAG$	nothing
	✓	✓	$FH^+ = FH$	nothing

This relation satisfies BCNF.

- Project the FDs onto $R6 = BEF$

B	E	F	closure	FDs
✓			$B^+ = BCDAG$	nothing
	✓		$E^+ = E$	nothing
		✓	$F^+ = F$	nothing
✓	✓		$BE^+ = BECDAG$	nothing
✓		✓	we can't possibly get E since it's not on a RHS	nothing
	✓	✓	$EF^+ = EF$	nothing

This relation satisfies BCNF.

- Final decomposition:
  (a) $R3 = ACG$ with FD $C \to AG$,
  (b) $R4 = BCD$ with FD $B \to CD$,
  (c) $R5 = BFH$ with FD $BF \to H$,
  (d) $R6 = BEF$ with no FDs.

3. Is your decomposition dependency-preserving?

   No, it is not. For each of the first three of the original FDs in set S, there is a relation that includes all of the FD's attributes. This ensures that they are preserved. However, at least one FD is not: $CEH \to F$. The fact that no relation in the final schema encompasses C, E, H and F doesn't necessarily mean the FD is not preserved. However, one can construct valid instances of the relations in the final schema that, when joined, create a table that violates $CEH \to F$. Here is an example:

B	C	D
b	1	c
f	1	g

C	A	G
1	a	d

E	B	F
2	b	4
2	f	5

B	F	H
b	4	3
f	5	3

The natural join of these four tables is:

A	B	C	D	E	F	G	H
a	b	1	c	2	4	d	3
a	f	1	g	2	5	d	3

This relation violates $CEH \to F$. This demonstrates that our decomposition of R into $R3, R4, R5$ and $R6$ is not a dependency-preserving decomposition.