

- (a) The solution for some input N is either TRUE or FALSE.
- If the solution for N is TRUE, then it is possible to purchase $N - 3$ nuggets or $N - 10$ nuggets or $N - 25$ nuggets because we must purchase at least one box (where each “or” is inclusive, in other words, more than one possibility could hold).
 - If the solution for N is FALSE, then it is impossible to purchase $N - 3$ nuggets and $N - 10$ nuggets and $N - 25$ nuggets (none of the possibilities can hold).
- (b) Let $A[n] = \text{TRUE}$ if it is possible to purchase exactly n nuggets (FALSE otherwise), for $-24 \leq n \leq N$.
(The negative values are not necessary but they remove the need for multiple conditions in the recurrence relation—and corresponding if-statements in the algorithm.)
- (c)
- $A[-24] = A[-23] = \dots = A[-1] = \text{FALSE}$ (impossible to purchase a negative number of nuggets);
 - $A[0] = \text{TRUE}$ (degenerate case: buy nothing, get zero nuggets);
 - $A[n] = A[n - 3] \vee A[n - 10] \vee A[n - 25]$, for $n = 1, 2, \dots, N$ (purchasing n nuggets requires purchasing a box of size 3, 10 or 25).
- (d) **Algorithm:**

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TOFUNGUGGETS( $N$ ):
    for  $n \leftarrow -24, -23, \dots, -1$ :
         $A[n] \leftarrow \text{FALSE}$ 
     $A[0] \leftarrow \text{TRUE}$ 
    for  $n \leftarrow 1, 2, \dots, N$ :
         $A[n] \leftarrow A[n - 3] \vee A[n - 10] \vee A[n - 25]$ 
    return  $A[N]$ 

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Runtime: $\Theta(N + 25)$.

This is **not** polytime: it is an exponential function of $\log_2 N$, the *size* of input N .