NOTE: This file contains sample solutions to the quiz together with the marking scheme and comments for each question. Please read the solutions and the marking schemes and comments carefully. Make sure that you understand why the solutions given here are correct, that you understand the mistakes that you made (if any), and that you understand *why* your mistakes were mistakes.

Remember that although you may not agree completely with the marking scheme given here, it was followed the same way for all students. We will remark your quiz only if you clearly demonstrate that the marking scheme was not followed correctly.

For all remarking requests, please submit your request **in writing** directly to your instructor. For all other questions, please don't hesitate to ask your instructor during office hours or by e-mail.

GENERAL MARKING SCHEME:

- ***A**: All Correct*, except maybe for very few minor errors.
- ***B**: Mostly Correct*, but with a few serious errors, or many small errors.
- ***C**: Mostly Incorrect*, but with a few important elements, or many small elements, done correctly.
- ***10%**: Completely Blank*, or clearly crossed out.
- ***D**: All Incorrect*, except maybe for very few minor elements done correctly.

MARKER'S COMMENTS:

- **common error**: Missing or vague/ambiguous specification of the "middle part" of the network (the connections between s_i's and t_j's).
- **common error**: Missing or incorrect capacities specified for some of the edges.

Consider the problem of creating a weekly schedule of TA office hours. You are given a list of TA's $t_1, t_2, \ldots, t_n$ and a list of time slots $s_1, s_2, \ldots, s_m$ for office hours. Each TA is available for some of the time slots and unavailable for others. Each time slot s_j must be assigned at most one TA, and every week, each TA t_i is responsible for some positive integer number of office hours h_i.

We want to know if there is a feasible schedule of office hours, *i.e.*, if it is possible to assign time slots to TA's to satisfy all of the problem constraints (each TA gets exactly h_i time slots and each time slot gets at most one TA — some time slots may remain unfilled).

1. Describe precisely how to model this problem as a network flow problem. (Don't forget to specify all edge directions and capacities in your network.)

Create a network N with

- vertices $V = \{s, s_1, \ldots, s_m, t_1, \ldots, t_n, t\}$,
- edges $E = \{(s, s_i) : 1 \leqslant i \leqslant m\} \cup \{(s_i, t_j) : 1 \leqslant i \leqslant m, 1 \leqslant j \leqslant n$, and TA t_j is available at time $s_i\} \cup \{(t_j, t) : 1 \leqslant j \leqslant n\}$, where $c(s, s_i) = 1$ and $c(s_i, t_j) = 1$ and $c(t_j, t) = h_j$ for $1 \leqslant i \leqslant m, 1 \leqslant j \leqslant n$.
  (Note: It is also correct to do this with all edges directed in the opposite direction.)

2. Explain clearly the correspondence between valid assignments of TAs to office hour time slots and valid integer flows in your network above.

- Every valid assignment of TAs to time slots generates a valid flow in N by setting $f(s_i, t_j) = 1$ iff t_j is assigned to s_i, $f(s, s_i) = 1$ iff someone is assigned to time s_i, $f(t_j, t) =$ the number of hours assigned to t_j.
- Every valid integer flow in N corresponds to a valid assignment of TAs to time slots by assigning t_j to s_i for all edges with $f(s_i, t_j) = 1$, because no time can have more than one TA assigned and no TA t_i can be assigned to more than h_i times, by the capacity and conservation constraints.