CSC 236H1 TERM TEST 1 October 2008

QUESTION 1. [8 MARKS]

Prove that for all natural numbers n greater than 1, the set of the first n positive integers $\{1, \ldots, n\}$ has $3 \cdot 2^{n-2}$ subsets that omit either the element 1, or the element 2, or both the elements 1 and 2.

QUESTION 2. [8 MARKS]

Recall the Fibonacci sequence:

$$orall n \in \mathbb{N} \qquad F(n) = egin{cases} n, & ext{if } n < 2 \ F(n-2) + F(n-1), & ext{if } n \geq 2 \end{cases}$$

Prove that for all natural numbers n, $F(n) < 2^n$.

QUESTION 3. [8 MARKS]

Find some natural number k such that for all natural numbers n greater than k, $3^n > 3n^3$. Prove your claim. You may use the binomial expansion, $(n+1)^3 = n^3 + 3n^2 + 3n + 1$.

Student #: _______

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CONT'D...

Total Marks = 24

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