Duration: **50 minutes**
Aids Allowed: **One *single*-sided *hand*written *8.5"×11"* aid sheet.**

**Student Number:** ___________

**Last (Family) Name(s):** ___________

**First (Given) Name(s):** ___________

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*Do **not** turn this page until you have received the signal to start.*
In the meantime, please read the instructions below *carefully*.

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This term test consists of 3 questions on 8 pages (including this one), printed on both sides of the paper. *When you receive the signal to start, please make sure that your copy of the test is complete, fill in the identification section above, and write your name on the back of the last page.*

Answer each question directly on the test paper, in the space provided, and use one of the "blank" pages for rough work. If you need more space for one of your solutions, use a "blank" page and *indicate clearly the part of your work that should be marked*.

In your answers, you may use without proof any theorem or result covered in lectures, tutorials, problem sets, assignments, or the textbook, as long as you give a clear statement of the result(s)/ theorem(s) you are using. You must justify all other facts required for your solutions.

Write up your solutions carefully! In particular, use notation and terminology correctly and explain what you are trying to do —part marks *will* be given for showing that you know the general structure of an answer, even if your solution is incomplete.

If you are unable to answer a question (or part of a question), remember that you will get 10% of the marks for any solution that you leave *entirely blank* (or where you cross off everything you wrote to make it clear that it should not be marked).

MARKING GUIDE

| | |
|---|---|
| Nº 1: | ____/12 |
| Nº 2: | ____/10 |
| Nº 3: | ____/ 8 |
| TOTAL: | ____/30 |

*Good Luck!*

## Question 1. [12 MARKS]

Consider the following *Art Gallery Guarding* problem. We are given a set $\{p_1, p_2, \ldots, p_n\}$ of positive real numbers that specify the positions of paintings in a long hallway in an art gallery. We want to position guards in the hallway to protect every painting, using as few guards as possible. Suppose that a single guard can protect all the paintings within distance 1 or less of his or her position (on both sides).

We propose to solve this problem using the following greedy algorithm.

```
Sort the positions so p_1 ⩽ p_2 ⩽ ··· ⩽ p_n.
G ← ∅   # current set of guards' locations
g ← −∞   # position of rightmost guard in G = maximum number in G
for i ← 1,...,n:   # start at the leftmost painting and move to the right
    if p_i > g + 1:   # p_i is unprotected by the guards currently in G
        # Place a guard 1 unit to the right of p_i.
        g ← p_i + 1
        G = G ∪ {g}
return G
```

Write a detailed proof that this algorithm always finds a minimum number of guards. We recommend that you follow the proof format outlined in class: start with a clear definition of "promising partial solution" for the algorithm, then use this as part of your argument.

HINT: You may want to start with an outline and fill in the details later—this is likely to be a bit long.

*Use the space on this "blank" page for scratch work, or for any solution that did not fit elsewhere.*
***Clearly label each such solution with the appropriate question and part number.***

## Question 2. [10 MARKS]

A new "healthy fast-food" restaurant sells *Tofu Nuggets* in boxes that contain either 3, 10 or 25 nuggets. You visit the restaurant with a group of your friends. Can you purchase exactly the right amount of nuggets so that everyone gets to try one but nobody has to try more than one?

We formalize the "Tofu Nugget" problem as follows.

- **Input:** An integer $N \geqslant 0$ (the size of your group).
- **Question:** Determine whether or not you can purchase exactly $N$ Tofu Nuggets (given that you can only purchase boxes of 3, 10 or 25).

Write an algorithm that solves this problem by using dymamic programming, and analyse its running time. Follow the steps from the "dynamic programming paradigm" covered in class. In particular, describe carefully the recursive structure of the problem in order to justify the correctness of your recurrence.

HINT: There is nothing to optimize—use an array that simply stores whether or not a solution exists.

*Use the space on this "blank" page for scratch work, or for any solution that did not fit elsewhere.*
***Clearly label each such solution with the appropriate question and part number.***

## Question 3. [8 MARKS]

Let $N = (V, E)$ be any network with integer capacities. Let $F$ be the maximum flow value in $N$. Let $P$ be a simple path in $N$ from $s$ to $t$ and let $N'$ be the network obtained from $N$ by adding 1 to the capacity of every edge on $P$.

### Part (a) [4 MARKS]

True or False? The maximum flow value in $N'$ is *at least* $F + 1$.
Justify your answer (give a short proof or a counterexample).

### Part (b) [4 MARKS]

True or False? The maximum flow value in $N'$ is *exactly* $F + 1$.
Justify your answer (give a short proof or a counterexample).

*Use the space on this "blank" page for scratch work, or for any solution that did not fit elsewhere.*
***Clearly label each such solution with the appropriate question and part number.***

*On this page, please write nothing except your name.*

**Last (Family) Name(s):** ____________________

**First (Given) Name(s):** ____________________

Total Marks = 30