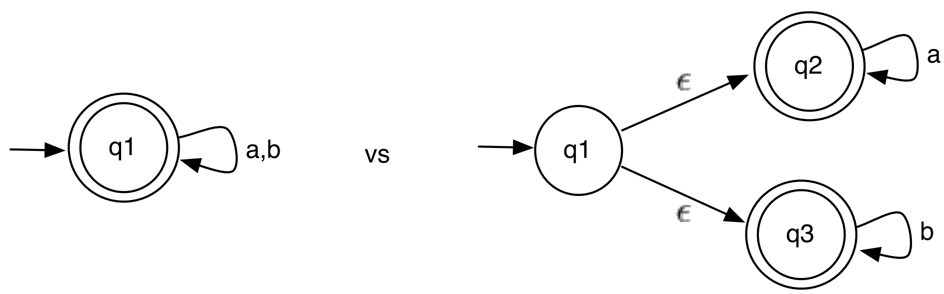
CSC236 Tutorial Exercises, March 30/31

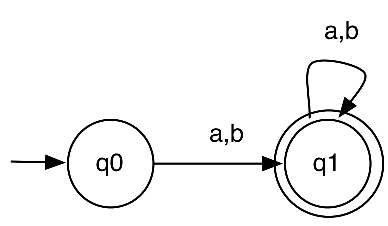
Sample Solutions

Let the alphabet be $\Sigma = \{a, b\}$

1. Are regular expressions $(a+b)^*$ and $a^* + b^*$ equivalent? Explain.
   Solution: Let $R_1 = (a+b)^*$ and $R_2 = a^* + b^*$. $R1 \neq R2$, because R_1 includes all strings in the alphabet Σ while R_2 includes repetitions of a (including 0 repetitions) or repetitions of b, but no strings that include both a and b. The corresponding NFA are different as well:



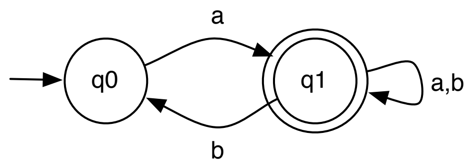
2. Draw a DFA corresponding to the regular expression $(a+b)(a+b)^*(a^* + b^*)$. Write down the corresponding state invariant that you could use to prove the equality of your DFA to the regular language represented by the provided regexp. You don't need to provide the proof.
   Solution: First, note that $L(a^*+b^*) \subseteq L(a+b)^*$ and not only that, but any string that can be generated by $(a+b)^*(a^*+b^*)$ can be generated by $(a+b)^*$ due to the definition of the Kleene's star. Hence, we can simplify $(a+b)(a+b)^*(a^*+b^*) = (a+b)(a+b)^*$. The corresponding DFA is



   State invariants are then as follows:

$$\delta^*(q_0, s) = \begin{cases} q_1 & \texttt{if s starts with an } a \texttt{ or a } b \\ q_0 & \texttt{otherwise} \end{cases}$$

3. Consider an FSA M_1:

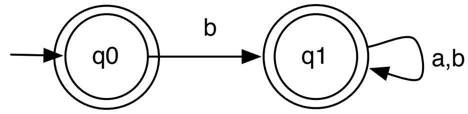


(a) Is this a DFA or an NFA? Why?
Answer: This is an NFA, because $\delta(q_1, b) = \{q_1, q_0\}$, i.e. there is more than one path to take upon observing b in state q_1. Also, the transition $\delta(q_0, b)$ is not determined but sometimes dead states are omitted even in drawing DFSA, so you have to be careful calling FSA an NFA just because dead states are missing.

(b) Write down the language $\mathcal{L}$ that it represents (a sentence describing all strings included in the language $\mathcal{L}$)
Answer: This language contains all strings that start with an a

(c) Write down the complement $\overline{\mathcal{L}}$ of $\mathcal{L}$, i.e. $\overline{\mathcal{L}} = \Sigma^* - \mathcal{L}$ in one sentence
Answer: Language $\overline{\mathcal{L}}$ contains all strings in Σ^* that do not start with an a (including empty string).

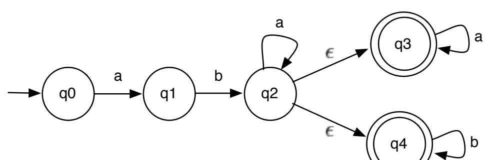
(d) Draw an FSA for $\overline{\mathcal{L}}$



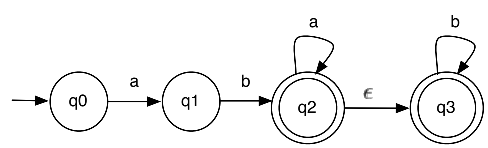
4. Consider a regexp R_1: $a(ba^*)(a^* + b^*)$

(a) Draw an NFA M_2 corresponding to the R_1 above

Solution: There are many answers to this question, including



and



The second solution follows from $a(ba^*)(a^* + b^*) = ab(a^*a^* + a^*b^*) = ab(a^* + a^*b^*) = aba^*b^*$

(b) Write down the language $\mathcal{L}$ that it represents (a sentence describing all strings)
Answer: $\mathcal{L}$ contains all strings that start with ab, concatenated with repetitions of a (including zero repetitions of a) followed by repetitions of b (including zero repetitions of b).

(c) Draw a corresponding DFA