NOTE: This file contains sample solutions to the quiz together with the marking scheme and comments for each question. Please read the solutions and the marking schemes and comments carefully. Make sure that you understand why the solutions given here are correct, that you understand the mistakes that you made (if any), and that you understand *why* your mistakes were mistakes.

Remember that although you may not agree completely with the marking scheme given here, it was followed the same way for all students. We will remark your quiz only if you clearly demonstrate that the marking scheme was not followed correctly.

For all remarking requests, please submit your request **in writing** directly to your instructor. For all other questions, please don't hesitate to ask your instructor during office hours or by e-mail.

GENERAL MARKING SCHEME:

- ***A***: *All Correct*, except maybe for very few minor errors.
- ***B***: *Mostly Correct*, but with a few serious errors, or many small errors.
- ***C***: *Mostly Incorrect*, but with a few important elements, or many small elements, done correctly.
- ***10%***: *Completely Blank*, or clearly crossed out.
- ***D***: *All Incorrect*, except maybe for very few minor elements done correctly.

MARKER'S COMMENTS:

- **common error**: vagueness/ambiguity about what the variables represent (variables must be *values*)

1. Consider the following "network flow with reduced transmission" problem.

Input: A network $N = (V, E)$ (a directed graph with a single source $s \in V$, a single sink $t \in V$, and positive integer capacities $c(e)$ for every edge $e \in E$). In addition, we are given *real* numbers $t(v) \in [0, 1]$ for every vertex $v \in V$ ($t(v)$ is a *transmission coefficient*).

Output: A maximum flow f (that is, flow values $f(e)$ for every edge $e \in E$ such that $f^{\text{out}}(s)$ is maximum), subject to the following constraints.

- *Capacity constraint*: $0 \leqslant f(e) \leqslant c(e)$ for all edges $e \in E$ (same as the original network flow problem).
- *Modified conservation constraint*: $f^{\text{out}}(v) = t(v) \cdot f^{\text{in}}(v)$ for every vertex $v \in V - \{s, t\}$ (in other words, the flow *out* of every node v is reduced by a factor of $t(v)$ from the flow *into* the node).

Show how to model this problem as a linear program: state explicitly what variables you are using, what your objective function is, and what your constraints are.

Variables: f_e for every edge e

Objective: maximize $\displaystyle\sum_{(s,u)\in E} f_{(s,u)}$

Constraints:

$$0 \leqslant f_e \leqslant c(e) \quad \forall e \in E$$

$$\sum_{(u,v)\in E} t(v) f_{(u,v)} = \sum_{(v,u)\in E} f_{(v,u)} \quad \forall v \in V - \{s, t\}$$