QUESTION 1.   [8 MARKS]

Prove that for all natural numbers n greater than 1, the set of the first n positive integers $\{1, \ldots, n\}$ has $3 \cdot 2^{n-2}$ subsets that omit either the element 1, or the element 2, or both the elements 1 and 2.

QUESTION 2.   [8 MARKS]

Recall the Fibonacci sequence:

$$\forall n \in \mathbb{N} \qquad F(n) = \begin{cases} n, & \text{if } n < 2 \\ F(n-2) + F(n-1), & \text{if } n \geq 2 \end{cases}$$

Prove that for all natural numbers n, $F(n) < 2^n$.

Question 3. [8 marks]

Find some natural number k such that for all natural numbers n greater than k, $3^n > 3n^3$. Prove your claim. You may use the binomial expansion, $(n+1)^3 = n^3 + 3n^2 + 3n + 1$.

Total Marks $=$ 24