

CSC373 Winter 2015 Problem Set # 3

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January 25, 2015

- (a) Subproblems: Determine whether or not you can purchase exactly $N - 3$ or $N - 10$ or $N - 25$ Tofu Nuggets given that you can only purchase boxes of 3, 10 or 25.

Solution to the problem: If one of three subproblems is true, then the solution to this problem is true. Otherwise, it is false.

Suppose you can purchase exactly N Tofu Nuggets. Then "last" box of Tofu Nuggets purchased is of size 3 or 10 or 25. So the problem can be reduced to whether or not you can purchase exactly $N - 3$ or $N - 10$ or $N - 25$ Tofu Nuggets.

- (b) Define array T such that:

The indices of T are from -24 to N .

Each element in T is the boolean value TRUE or FALSE

$T[0] = \text{TRUE}$

$T[i] = \text{FALSE}$ for $-24 \leq i \leq -1$

$T[i] = T[i - 3]$ or $T[i - 10]$ or $T[i - 25]$ for $i > 0$

- (c) Base cases: $T[0] = \text{TRUE}$ and $T[i] = \text{FALSE}$ for $-24 \leq i \leq -1$.

General cases: $T[i] = T[i - 3]$ or $T[i - 10]$ or $T[i - 25]$ for $i > 0$

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(d) 1  for  $i$  in  $-24, \dots, -1$ 
      2       $T[i] = \text{FALSE}$ 
      3   $T[0] = \text{TRUE}$ 
      4  for  $i$  in  $1, \dots, N$ 
      5       $T[i] = T[i - 3]$  or  $T[i - 10]$  or  $T[i - 25]$ 
      6  return  $T[N]$ 
```

Worst case complexity: $\Theta(N)$

It is in polynomial time.

Explanation: The first loop (line 1 to line 2) takes 24 steps which is constant time. Line 3 takes constant time to run. The second loop (line 4 to line 5) takes $(3 + 1)N = 4N$ (3 for accessing array indices and 1 for getting the boolean value of "or"). Therefore in total, the algorithm takes $\Theta(N)$ time to run. It is in polynomial time.