STA304 A2 Q1

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a)
$\mu_1 = (1+4+4+5)/4 = 3.5$
$\mu_2 = (5+6+9+11)/4 = 7.75$

b)
$\sigma_1^2 = ((1-3.5)^2 + (4-3.5)^2 + (4-3.5)^2 + (5-3.5)^2)/4 = 2.25$
$\sigma_2^2 = ((5-7.75)^2 + (6-7.75)^2 + (9-7.75)^2 + (11-7.75)^2)/4 = 5.6875$

c)

Stratum 1

possible SRS	$\bar{y}_1$	$\hat{\hat{\tau}}_1$
1, 4, 4	3	12
1, 4, 5	$\frac{10}{3}$	$\frac{40}{3}$
1, 4, 5	$\frac{10}{3}$	$\frac{40}{3}$
4, 4, 5	$\frac{13}{3}$	$\frac{52}{3}$

$\mu_{\hat{\tau}_1} = (12 + \frac{40}{3} + \frac{40}{3} + \frac{52}{3})/4 = 14$
$V(\hat{\tau}_1) = ((12-14)^2 + (\frac{40}{3}-14)^2 + (\frac{40}{3}-14)^2 + (\frac{52}{3}-14)^2)/4 = 4$

Stratum 2

possible SRS	$\bar{y}_2$	$\hat{\tau}_2$
5, 6, 9	$\frac{20}{3}$	$\frac{80}{3}$
5, 6, 11	$\frac{22}{3}$	$\frac{88}{3}$
5, 9, 11	$\frac{25}{3}$	$\frac{100}{3}$
6, 9, 11	$\frac{26}{3}$	$\frac{104}{3}$

$\mu_{\hat{\tau}_2} = (\frac{80}{3} + \frac{88}{3} + \frac{100}{3} + \frac{104}{3})/4 = 31$
$V(\hat{\tau}_2) = ((\frac{80}{3} - 31)^2 + (\frac{88}{3} - 31)^2 + (\frac{100}{3} - 31)^2 + (\frac{104}{3} - 31)^2)/4 = 10.11$

d)

Stratum 1 SRS	Stratum 2 SRS	$\hat{T}_{str}$	$\hat{\mu}_{str}$
1, 4, 4	5, 6, 9	38.67	4.83
1, 4, 4	5, 6, 11	41.33	5.17
1, 4, 4	5, 9, 11	45.33	5.67
1, 4, 4	6, 9, 11	46.67	5.83
1, 4, 5	5, 6, 9	40	5
1, 4, 5	5, 6, 11	42.67	5.33
1, 4, 5	5, 9, 11	46.67	5.83
1, 4, 5	6, 9, 11	48	6
1, 4, 5	5, 6, 9	40	5
1, 4, 5	5, 6, 11	42.67	5.33
1, 4, 5	5, 9, 11	46.67	5.83
1, 4, 5	6, 9, 11	48	6
4, 4, 5	5, 6, 9	44	5.5
4, 4, 5	5, 6, 11	46.67	5.83
4, 4, 5	5, 9, 11	50.67	6.33
4, 4, 5	6, 9, 11	52	6.5

$V(\hat{\tau_{str}}) = ((38.67 - 45)^2 + ... + (52 - 45)^2)/16 = 14.11306$
$V(\hat{\mu_{str}}) = ((4.83 - 5.625)^2 + ... + (6.5 - 5.625)^2)/16 = 0.2202484$

e) and f)

Stratum 1 SRS	Stratum 2 SRS	$\hat{\tau}_{str}$	$\hat{\mu}_{str}$
1, 4, 4	5, 6, 9	38.67	4.83
1, 4, 4	5, 6, 11	41.33	5.17
1, 4, 4	5, 9, 11	45.33	5.67
1, 4, 4	6, 9, 11	46.67	5.83
1, 4, 5	5, 6, 9	40	5
1, 4, 5	5, 6, 11	42.67	5.33
1, 4, 5	5, 9, 11	46.67	5.83
1, 4, 5	6, 9, 11	48	6
1, 4, 5	5, 6, 9	40	5
1, 4, 5	5, 6, 11	42.67	5.33
1, 4, 5	5, 9, 11	46.67	5.83
1, 4, 5	6, 9, 11	48	6
4, 4, 5	5, 6, 9	44	5.5
4, 4, 5	5, 6, 11	46.67	5.83
4, 4, 5	5, 9, 11	50.67	6.33
4, 4, 5	6, 9, 11	52	6.5

$\mu = (1 + 4 + 4 + 5 + 5 + 6 + 9 + 11)/8 = 5.625$
$\tau = 1 + 4 + 4 + 5 + 5 + 6 + 9 + 11 = 45$

$E(\hat{\mu}) = 5.625 = \mu$
$E(\hat{\tau}) = 45 = \tau$