ANSWER TO WEEK 8, SLIDE 13

$$\hat{\underset{\sim}{\beta}} = (\underset{\sim}{X}'\underset{\sim}{X})^{-1} \underbrace{\underset{\sim}{X}'\underset{\sim}{Y}}, \quad \underset{\sim}{X} = \begin{pmatrix} 1 & x_1 \\ \vdots & \vdots \\ 1 & x_n \end{pmatrix}, \quad \underset{\sim}{Y} = \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}$$

$$= \frac{1}{S_{xx}} \begin{bmatrix} \sum x_i^2 & -n\bar{x} \\ -n\bar{x} & n \end{bmatrix} \begin{bmatrix} 1 & \cdots & 1 \\ x_1 & \cdots & x_n \end{bmatrix} \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}$$

$$= \begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix} \begin{pmatrix} \sum_{i=1}^n y_i \\ \sum_{i=1}^n x_i y_i \end{pmatrix}$$

$$= \begin{bmatrix} \frac{\sum (x_i)^2 \bar{y}}{S_{xx}} - \frac{\bar{x} \sum x_i y_i}{S_{xx}} \\ \frac{-n\bar{x}\bar{y}}{S_{xx}} + \frac{\sum x_i y_i}{S_{xx}} \end{bmatrix} \qquad 2 \times 1$$

$$= \begin{bmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \end{bmatrix}$$

ANSWER TO WEEKS 8-9, SLIDE 9, 1 of 2

$$E(\underset{\sim}{\hat{\beta}}) = E\left[(\underset{\sim}{X}'\underset{\sim}{X})^{-1}\underset{\sim}{X}'\underset{\sim}{y}\right], \qquad \underset{\sim}{y} = \underset{\sim}{X}\underset{\sim}{\beta} + \underset{\sim}{e}$$

$$= (\underset{\sim}{X}'\underset{\sim}{X})^{-1}\underset{\sim}{X}'\,\underbrace{E(\underset{\sim}{y})}_{\underset{\sim}{X}\underset{\sim}{\beta}}$$

$$= \underset{\sim}{\beta}$$

$$\text{var}(\underset{\sim}{\hat{\beta}}) = \text{var}\left[\underbrace{(\underset{\sim}{X}'\underset{\sim}{X})^{-1}\underset{\sim}{X}'}_{\text{"}A\text{"}}\underset{\sim}{y}\right]$$

$$= \underbrace{(\underset{\sim}{X}'\underset{\sim}{X})^{-1}X'}_{A}\ \underbrace{\text{var}(\underset{\sim}{y})}_{\sigma^2 I}\ \underbrace{X(X'X)^{-1}}_{A'}$$

$$= \sigma^2 (\underset{\sim}{X}'\underset{\sim}{X})^{-1}\underset{\sim}{X}'\underset{\sim}{X}(\underset{\sim}{X}'\underset{\sim}{X})^{-1}$$

$$= \sigma^2 (\underset{2\times n}{\underset{\sim}{X}'}\underset{n\times 2}{\underset{\sim}{X}})^{-1}$$

$$\mathrm{var}(\hat{\underset{\sim}{\beta}}) = \cancel{\sigma^2}\begin{bmatrix} \dfrac{\Sigma x_i^2}{n S_{xx}} & \dfrac{-\bar{x}}{S_{xx}} \\ \dfrac{-\bar{x}}{S_{xx}} & \dfrac{1}{S_{xx}} \end{bmatrix}$$

RHS

$\mathrm{cov}(\hat{\beta}_0, \hat{\beta}_1)$ ✓ $\mathrm{var}(\hat{\beta}_1)$ ✓

Show =

$$\text{LHS} = \cancel{\sigma^2}\left(\frac{1}{n} + \frac{\bar{x}^2}{S_{xx}}\right)$$

$$\text{RHS} = \frac{\Sigma (x_i - \bar{x})^2}{n S_{xx}} + \frac{\Sigma (2x_i\bar{x} - \bar{x}^2)}{n S_{xx}}$$

$$= \frac{1}{n} + \frac{\Sigma(2x_i\bar{x} - 2\bar{x}^2)}{n S_{xx}} + \frac{\Sigma \bar{x}^2}{n S_{xx}}$$

$$= \frac{1}{n} + \cancel{\frac{2\bar{x}\Sigma(x_i - \bar{x})}{n S_{xx}}} + \frac{\cancel{n}\bar{x}^2}{\cancel{n} S_{xx}}$$

0

$$= \frac{1}{n} + \frac{\bar{x}^2}{S_{xx}}$$

ANSWERS TO WEEKS 8-9, SLIDE 17

$$H = X(X'X)^{-1}X'$$

$$H^2 = HH$$

$$= X(X'X)^{-1}\underbrace{X'X(X'X)^{-1}}_{I}X'$$

$$= X(X'X)^{-1}X'$$

$$= H \qquad \therefore \text{Idempotent}$$

$$H' = \Big(\underbrace{X}_{A}\underbrace{(X'X)^{-1}X'}_{B}\Big)' \qquad \therefore \text{Symm.}$$

$$= \big((X'X)^{-1}X'\big)'X'$$

$$= X\big((X'X)^{-1}\big)'X'$$

$$= X(X'X)^{-1}X' \qquad = H$$

ANSWER TO WEEKS 8-9, SLIDE 20

$$(1-H)(1-H) =$$

$$= 1^2 - 1H - H1 + H^2$$

$$= 1 - H - H + H$$

$$= 1 - H \qquad \therefore \text{ idemp.}$$

$$(1-H)' = 1' - H'$$

$$= 1 - H$$

$\therefore$ symm.

ANSWER TO SLIDE 21, WEEKS 8-9

$$E(\hat{\underset{\sim}{e}}) = E\left[(\underset{\sim}{I} - \underset{\sim}{H})\underset{\sim}{Y}\right]$$

$$= (\underset{\sim}{I} - \underset{\sim}{H}) E(\underset{\sim}{Y})$$

$$= (\underset{\sim}{I} - \underset{\sim}{H}) \underset{\sim}{X} \underset{\sim}{\beta}$$

$$= \underset{\sim}{X}\underset{\sim}{\beta} - \underset{\sim}{X} \underbrace{(\underset{\sim}{X}'\underset{\sim}{X})^{-1}\underset{\sim}{X}'\underset{\sim}{X}}_{I} \beta$$

$$= \underset{\sim}{0}$$