CSC236 Tutorial Exercises, June 7, 2017

These exercises are intended to give you practice with recurrences.

1. Consider the recurrence relation

$$T(n) = \begin{cases} 1, & \text{if } n = 1 \\ 1 + T(\lceil \frac{n}{3} \rceil), & \text{if } n > 1 \end{cases}$$

   Use complete induction to prove that for every positive natural number n, $T(n) \geq c * lg(n)$, for some positive real constant c.

2. Consider the recurrence relation

$$T(n) = \begin{cases} 1, & \text{if } n = 0 \\ 3, & \text{if } n = 1 \\ 3T(n-1) - 2T(n-2), & \text{if } n > 1 \end{cases}$$

   Find a closed form for $T(n)$, and prove that it is correct using induction.

3. Consider the recurrence relation

$$T(n) = \begin{cases} 1, & \text{if } n = 0 \\ T(n-1) + n - 2, & \text{if } n > 0 \end{cases}$$

   Unwind the recurrence carefully, following the pattern below, for some n that is comfortably greater than 1:

$$\begin{aligned} T(n) &= T(n-1) + n - 2 \\ &= T(n-2) + n - 1 - 2 + n - 2 = T(n-2) + n - 3 + n - 2 \\ &= T(n-3) + n - 2 - 2 + n - 3 + n - 2 = T(n-3) + n - 4 + n - 3 + n - 2 \end{aligned}$$

   Continue to see a pattern that leads to a guess at a closed form for $T(n)$.