1. **Incorrect.**

   It is possible for G to contains a Hamiltonian path *without* containing a Hamiltonian cycle (e.g., $G = a—b—c$). So the answers for both decision problems do not always match.

2. **Incorrect.**

   Although the reduction itself is correct, it *cannot* be used to conclude that HP is NP-hard because it is in the wrong direction.

3. **Incorrect.**

   The reduction function does **not** have access to a certificate for its input. Put another way: C cannot be computed from G in polytime (unless $P = NP$) so the reduction as described is not computable in polytime.

4. **Correct!**

   Well... almost! The reduction actually works fine **except** for **one** particular input: when G is an undirected graph with exactly two vertices and one edge. Then, G' contains a Hamiltonian path even though G does not contain any Hamiltonian cycle.

   For all other inputs, the reduction is correct. This means that it could easily be fixed by checking its input for that one particular case and outputting something different in that case.

5. **Incorrect.**

   G' may contain a Hamiltonian path that does not start at v_1 and end at v_1', even when G has no Hamiltonian cycle. (For example, $G = v_2—v_1—v_3$ does not contain a Hamiltonian cycle, but G' contains a Hamiltonian path: (v_2, v_1), (v_1, v_3), (v_3, v_1').) So the answers for both decision problems do not always match.