(a) $U = Y_1 + Y_2$ (Ex 6.52)

$Y_1 \sim poi(\lambda_1)$; $Y_2 \sim poi(\lambda_2)$

$m_{Y_1}(t) = e^{\lambda_1(e^t - 1)}$; $m_{Y_2}(t) = e^{\lambda_2(e^t-1)}$

$\therefore$ $m_U(t) = m_{Y_1+Y_2}(t) = m_{Y_1}(t) \cdot m_{Y_2}(t)$

$= e^{\lambda_1 \cdot (e^t - 1)} \cdot e^{\lambda_2(e^t-1)}$

$= e^{(\lambda_1+\lambda_2)(e^t-1)}$

$U \sim poi(\lambda')$; $\lambda' = \lambda_1 + \lambda_2$

(b) $P(Y_1 | (Y_1+Y_2)) = \dfrac{P(Y_1 = y_1, Y_1+Y_2 = u)}{P(Y_1+Y_2=u)} = \dfrac{P(Y_1 = y_1, Y_2 = u - y_1)}{P(Y_1+Y_2 = u)}$

$$= \frac{\dfrac{e^{-\lambda_1}(\lambda_1)^{y_1}}{y_1!} \cdot \dfrac{e^{-\lambda_2}(\lambda_2)^{u-y_1}}{(u-y_1)!}}{\dfrac{e^{-(\lambda_1+\lambda_2)}(\lambda_1+\lambda_2)^u}{u!}}$$

$$= \frac{u!}{y_1! \cdot (u-y_1)!} \cdot \frac{\cancel{e^{-(\lambda_1+\lambda_2)}}}{\cancel{e^{-(\lambda_1+\lambda_2)}}} \cdot \left[\frac{\lambda_1^{y_1} \cdot \lambda_2^{u-y_1}}{(\lambda_1+\lambda_2)^u}\right]$$

$$= \binom{u}{y_1} \cdot \left[\frac{\lambda_1^{y_1} \cdot \lambda_2^{u-y_1}}{(\lambda_1+\lambda_2)^{y_1+u-y_1}}\right]$$

$$= \binom{u}{y_1} \cdot \left(\frac{\lambda_1}{\lambda_1+\lambda_2}\right)^{y_1} \cdot \Bigg(\underbrace{\frac{\lambda_2}{\lambda_1+\lambda_2}}_{= 1 - \frac{\lambda_1}{\lambda_1+\lambda_2}}\Bigg)^{u-y_1}$$

$$= \binom{u}{y_1}\Bigg(\underbrace{\frac{\lambda_1}{\lambda_1+\lambda_2}}_{=p}\Bigg)^{y_1}\Bigg(\underbrace{1 - \frac{\lambda_1}{\lambda_1+\lambda_2}}_{1-p}\Bigg)^{u-y_1}$$

Thus $[Y_1 | (Y_1+Y_2)] \sim Bin(u, p)$; $p = \dfrac{\lambda_1}{\lambda_1+\lambda_2}$