

In-class Exercises: Projection and Minimal Basis

1. Suppose we have these FDs: $S = \{ABE \rightarrow CF, DF \rightarrow BD, C \rightarrow DF, E \rightarrow A, AF \rightarrow B\}$

Project the FDs onto: $L = CDEF$

Attributes to take all subsets X of:				Closure of the subset X^+	Functional dependencies inferred
C	D	E	F		

Solution:

C	D	E	F	closure	FDs
✓				$C^+ = CDFBD$	$C \rightarrow DF$
	✓			$D^+ = D$	nothing
		✓		$E^+ = EA$	nothing
			✓	$F^+ = F$	nothing
✓	✓			$CD^+ = CDFB$	nothing, since $CD \rightarrow DF$ is weaker than $C \rightarrow DF$ which we have already
✓		✓		$CE^+ = CEDFAB$	nothing, since $CE \rightarrow DF$ is weaker than $C \rightarrow DF$ which we have already
✓			✓	$CF^+ = CFDB$	nothing, since $CF \rightarrow D$ is weaker than $C \rightarrow DF$ which we have already
	✓	✓		$DE^+ = DEA$	nothing
	✓		✓	$DF^+ = DFB$	nothing
		✓	✓	$EF^+ = EFABCD$	$EF \rightarrow CD$
✓	✓	✓		$CDE^+ = CDEF$	nothing, since $CDE \rightarrow F$ is weaker than $C \rightarrow DF$ which we have already
✓	✓		✓	$CDF^+ = CDFB$	nothing
✓		✓	✓	since EF is a key, supersets of EF can only yield FDs that are weaker than ones we have.	
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Final answer: The projection of S onto L is $C \rightarrow DF, EF \rightarrow CD$.

2. Find a minimal basis for this set of FDs: $S = \{ABF \rightarrow G, BC \rightarrow H, BCH \rightarrow EG, BE \rightarrow GH\}$.

Solution:

Step 1: Split the RHSs to get our initial set of FDs, $S1$:

- (a) $ABF \rightarrow G$
- (b) $BC \rightarrow H$
- (c) $BCH \rightarrow E$
- (d) $BCH \rightarrow G$
- (e) $BE \rightarrow G$
- (f) $BE \rightarrow H$.

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Step 2: For each FD, try to reduce the LHS:

- (a) $A^+ = A, B^+ = B, F^+ = F$. In fact, no singleton LHS yields anything. $AB^+ = AB, AF^+ = AF$, and $BF^+ = BF$, so none of them yields G either. We cannot reduce the LHS of this FD.
- (b) Since this FD has only two attributes on the LHS, and no singleton LHS yields anything, we cannot reduce the LHS of this FD.
- (c) Since no singleton LHS yields anything, we need only consider LHSs with two or more attributes. We only have three to begin with, so that leaves LHSs with two attributes. $BC^+ = BCHEG$. So we can reduce the LHS of this FD, yielding the new FD: $BC \rightarrow E$.
- (d) By the same argument, we can reduce this FD to: $BC \rightarrow G$.
- (e) Since no singleton LHS yields anything, we cannot reduce the LHS of this FD.
- (f) Since no singleton LHS yields anything, we cannot reduce the LHS of this FD.

Our new set of FDs, let's call it $S2$, is

- (a) $ABF \rightarrow G$
- (b) $BC \rightarrow H$
- (c) $BC \rightarrow E$
- (d) $BC \rightarrow G$
- (e) $BE \rightarrow G$
- (f) $BE \rightarrow H$.

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Step 3: Try to eliminate each FD.

从step2的结果，一个一个遮挡，看哪个是没用的，删！

- (a) $ABF_{S2-(a)}^+ = ABF$. We need this FD.
- (b) $BC_{S2-(b)}^+ = BCEGH$. We can remove this FD.
- (c) $BC_{S2-\{(b),(c)\}}^+ = BCG$. We need this FD.
- (d) $BC_{S2-\{(b),(d)\}}^+ = BCEGH$. We can remove this FD.
- (e) $BE_{S2-\{(b),(d),(e)\}}^+ = BEH$. We need this FD.
- (f) $BE_{S2-\{(b),(d),(f)\}}^+ = BEG$. We need this FD.

Our final set of FDs is:

- (a) $ABF \rightarrow G$
- (b) $BC \rightarrow E$
- (c) $BE \rightarrow G$
- (d) $BE \rightarrow H$.