**Worth:** 2% **Due:** By 8:59pm on Tuesday 10 March

**Remember to write your *full name* and *student number* prominently on your submission.**

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*Please read and understand the policy on Collaboration given on the Course Information Sheet. Then, to protect yourself, list on the front of your submission **every** source of information you used to complete this homework (other than your own lecture and tutorial notes). For example, indicate clearly the **name** of every student with whom you had discussions, the **title and sections** of every textbook you consulted (including the course textbook), the **source** of every web document you used (including documents from the course webpage), etc.*

*For each question, please write up detailed answers carefully. Make sure that you use notation and terminology correctly, and that you explain and justify what you are doing. Marks **will** be deducted for incorrect or ambiguous use of notation and terminology, and for making incorrect, unjustified, ambiguous, or vague claims in your solutions.*

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1. Consider the following decision problem FRUGAL.

   **Input:** A set of *ingredients* $G = \{g_1, g_2, \ldots, g_m\}$, a set of *recipes* $R = \{r_1, r_2, \ldots, r_n\}$, where each recipe is a subset of ingredients ($r_i \subseteq G$), and a positive integer $M$.

   **Output:** Is there some subset of recipes $R' \subseteq R$ with size $|R'| \leqslant M$ such that all together, the recipes in $R'$ use up exactly the ingredients from $G$ ($\bigcup_{r \in R'} r = G$ and $r_1 \cap r_2 = \emptyset$ for all $r_1, r_2 \in R'$)?

   (a) Write down the definition of decision problem $\overline{\text{FRUGAL}}$ (the *complement* of FRUGAL).

   (b) Give a polynomial-time *verifier* for FRUGAL or for $\overline{\text{FRUGAL}}$ (**but not both**). State clearly which one of the two problems you are verifying. Give a detailed justification that your verifier is correct and runs in polytime.

2. Consider the following decision problem SHORTPATHS.

   **Input:** An undirected graph $G = (V, E)$ and a positive integer $k$.

   **Output:** Does every simple path in $G$ contain at most $k$ edges?

   (a) Write down the definition of decision problem $\overline{\text{SHORTPATHS}}$ (the *complement* of SHORTPATHS).

   (b) Give a polynomial-time *verifier* for SHORTPATHS or for $\overline{\text{SHORTPATHS}}$ (**but not both**). State clearly which one of the two problems you are verifying. Give a detailed justification that your verifier is correct and runs in polytime.