# STA302/STA1001, Weeks 11-12

Mark Ebden, 28-30 November

With grateful acknowledgment to Alison Gibbs and Becky Lin

#### Overview

- ▶ Practical example: Analysing house-price data
- ▶ Chapter 7: Variable selection, and going with the flow
- Key MLR aspects



## Exploring a house-price dataset

For 26 houses sold in Chicago a long time ago, we know the selling price Y as well as eight characteristics of each house: square footage, parking information, etc.



Reference: Ashish Sen and Muni Srivastava, Regression Analysis: Theory, Methods and Applications, 2013.

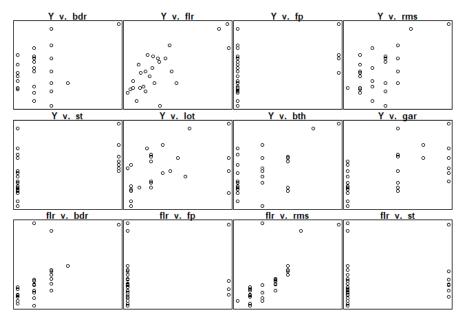
##		Y	bdr	flr	fp	rms	st	lot	bth	gar
##	1	53	2	967	0	5	0	39	1.5	0.0
##	2	55	2	815	1	5	0	33	1.0	2.0
##	3	56	3	900	0	5	1	35	1.5	1.0
##	4	58	3	1007	0	6	1	24	1.5	2.0
##	5	64	3	1100	1	7	0	50	1.5	1.5
##	6	44	4	897	0	7	0	25	2.0	1.0
##	7	49	5	1400	0	8	0	30	1.0	1.0
##	8	70	3	2261	0	6	0	29	1.0	2.0
##	9	72	4	1290	0	8	1	33	1.5	1.5
##	10	82	4	2104	0	9	0	40	2.5	1.0
##	11	85	8	2240	1	12	1	50	3.0	2.0
##	12	45	2	641	0	5	0	25	1.0	0.0
##	13	47	3	862	0	6	0	25	1.0	0.0
##	14	49	4	1043	0	7	0	30	1.5	0.0
##	15	56	4	1325	0	8	0	50	1.5	0.0
##	16	60	2	782	0	5	1	25	1.0	0.0
##	17	62	3	1126	0	7	1	30	2.0	0.0
##	18	64	4	1226	0	8	0	37	2.0	2.0
##	19	66	2	929	1	5	0	30	1.0	1.0
##	20	35	4	1137	0	7	0	25	1.5	0.0
##	21	38	3	743	0	6	0	25	1.0	0.0
##	22	43	3	596	0	5	0	50	1.0	0.0
##	23	46	2	803	0	5	0	27	1.0	0.0

## The response variable and eight predictors

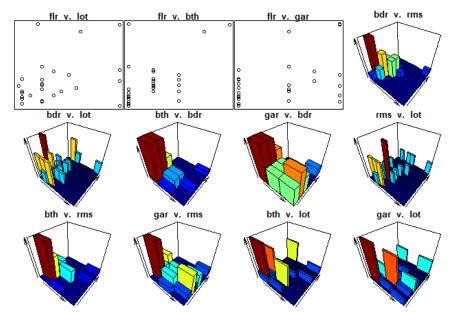
Variable	Meaning
Y	Selling price in thousands of dollars
bdr	Number of bedrooms
flr	Floor space in square feet
fp	Number of fireplaces
rms	Number of rooms
st	Storm windows present (indicator variable)
lot	Frontage in feet
bth	Number of bathrooms
gar	Number of garage parking spaces

We'll use the techniques on last week's slides to analyse the dataset. The data are available on Portal.

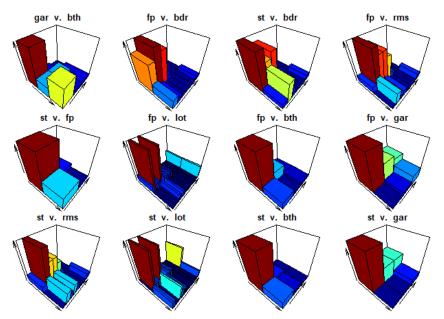
## Pairwise scatter plots



# Pairwise 3D histograms



## Pairwise 3D histograms



# Initial findings from the scatter plots



- ▶ All X's seem to have some relationship with Y
- No obvious nonlinear relationship between Y and any of the predictors; no obvious need for transformations
- ► For later when we do the 7 C's: perhaps there exists an influential point; namely, the very first plot shows a house with eight bedrooms
- ► There are also relationships among some of the X's: for example, between bdr and rms (middle slide of the three: top right)
- But first, let's continue to explore the dataset systematically

## Examining the correlations *r* and their *p*-values

	Υ	bdr	flr	fp	rms	st	lot	bth	gar
Y		0.41	0.74	0.39	0.58	0.46	0.42	0.55	0.54
bdr	1.41		0.68	0.17	0.92	0.23	0.40	0.63	0.24
flr	4.74	3.81		0.16	0.74	0.13	0.34	0.53	0.40
fp	1.32				0.19	-0.02	0.39	0.12	0.41
rms	2.73	10.37	4.81			0.23	0.43	0.69	0.30
st	1.72						-0.04	0.35	0.17
lot	1.50	1.39		1.34	1.55			0.37	0.14
bth	2.47	3.28	2.23		3.98				0.26
gar	2.39		1.40	1.44					

**Upper right:** Pearson's *r*. **Lower left:**  $-\log_{10}(p\text{-value})$ , e.g. 2 means p = 0.01

- ▶ Each correlation *p*-value is equal to an SLR  $\beta_1$  *t*-test's
- ▶ We mainly consider the *p*-values and signs here, and needn't pay as much attention to the magnitude of each *r*
- ▶ All X's have a statistically significant r (or  $R^2$ ) with Y
- ▶ Question: Have we run an appropriate test for st?

## Performing MLR

The next slide describes the MLR. (What can we conclude from the F-test?)

The fitted equation is:

$$\hat{Y}$$
 =18.6  $-$  7.70 bdr  $+$  0.0176 flr  $+$  6.91 fp  $+$  3.90 rms $+$  10.8187 st  $+$  0.2635 lot  $+$  2.37 bth  $+$  1.77 gar

Interpreting the coefficients:

- Recall that b<sub>i</sub> represents the estimated change in mean selling price due to a unit change in X<sub>i</sub> with all other variables held constant
- e.g. A 100-square-foot increase corresponds to a \$1760 increase in mean price, with everything else held constant
- ➤ A 1-bedroom increase corresponds to a \$7700 decrease in mean price (despite the fact that the correlation between price and bdr is positive)

Despite the significant correlation between Y and bth, the t-test on the next slide gives a p-value of 0.366 > 0.05. Why? (Hint: slide 26 from last week)

```
## Call:
## lm(formula = Q)
##
## Residuals:
##
      Min
               10 Median 30
                                      Max
## -10.3058 -2.8417 -0.1511 3.2882 7.9518
##
## Coefficients:
##
             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 18.637664 5.240957 3.556 0.002429 **
## bdr
            -7.697444 1.829426 -4.208 0.000592 ***
## flr 0.017570 0.003235 5.431 4.49e-05 ***
## fp 6.909765 3.083583 2.241 0.038680 *
## rms 3.904374 1.615617 2.417 0.027194 *
            10.818663 2.300203 4.703 0.000205 ***
## st
## lot 0.263522 0.135109 1.950 0.067808 .
## bth
           2.374591 2.557865 0.928 0.366221
## gar
           1.770861 1.404310 1.261 0.224334
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4.717 on 17 degrees of freedom
## Multiple R-squared: 0.9044, Adjusted R-squared: 0.8595
## F-statistic: 20.11 on 8 and 17 DF, p-value: 3.147e-07
```

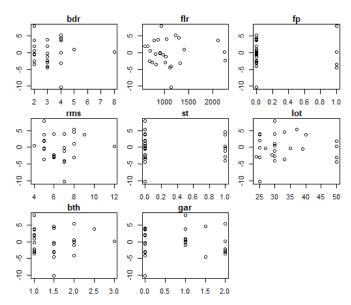
# The Seven C's (7 Checks) for STA302 MLR diagnostics

#### In their proper order this time:

- 1. Plot standardized residuals to help determine whether the proposed regression model is a valid model
  - 2. Identify any leverage points
  - 3. Identify any outliers
  - **4.** Assess the effect of each X on Y
  - **5.** Assess multicollinearity
  - **6.** Assess the assumption of error homoscedasticity
- 7. For time series: examine whether the data are correlated over time

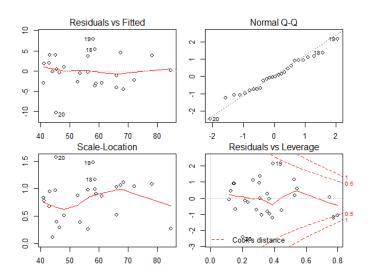
## Residual plots in the house-price example

Recall that plotting residuals versus  $X_j$  for  $j \in \{1, \dots p\}$  helps us to look for curvature, influential points, and outliers.

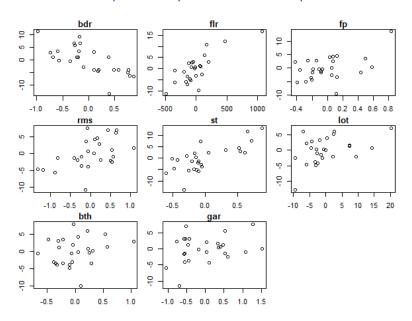


### Output of plot(lm(..))

- One house with large negative residual (perhaps one large positive residual)
- At least one influential point according to the 4/(n-p-1) criterion: large Cook's distance in the bottom-right of the diagnostics panel



## Check 4 in the house-price example: Added variable plots



## Assessing assumptions in the House-price example

We view the added variable plots in context. Consider the plot of Y versus bdr:

- ▶ Not a strong linear relationship; otherwise, no concerns with non-linearity
- ▶ Potential influential point: one house has 8 bedrooms
- SLR may not be a good idea for the price bedroom relationship but may be ok in multiple regression

#### Consider the added variable plot:

- ► The linear relationship is stronger (although it's now negative)
- We want to check if the influential is here. However, the large negative residual is not the house with 8 bedrooms

#### Summary of concerns:

- ► Large negative residual in the added variable plots
- Potential influential point in the added variable plot for flr

## Summary of unusual observations

#### House 8:

- Influential
- Largest Cook's distance
- Very large floor space, everything else including price (Y) was of moderate space

#### House 19:

- Large positive residual
- Small house but middle-ish price; nice house?

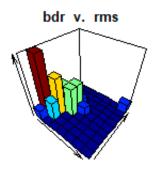
#### House 20:

- Large negative residual
- Smallest price, but all features middle-ish

#### House 11 isn't a concern:

- ▶ It has eight bedrooms but is neither an outlier nor influential
- ▶ It's expensive, but everything is big

# Check 5 in the house-price example: Multicollinearity



- $VIF_{rms} = 8.5$ ,  $VIF_{bdr} = 6.5$ ,  $VIF_{flr} = 2.5$ ,  $VIF_{bth} = 2.1$
- ▶ Indeed we saw what might have been the effects of multicollinearity: the coefficient for bdr wasn't of the expected sign
- ▶ Details of executing the two remedial measures introduced last week (ridge regression, principal component regression) are beyond STA302's scope
- For now, awareness of possible multicollinearity informs our choice of how to simplify the model

## Model simplification

- ▶ If you remove variables during MLR, the  $\beta_j$  values of the remaining predictors become biased, and the *p*-values from *t* and *F* statistics generally shrink below their true values
- ► Therefore, when reducing the dimensionality of your MLR problem, take into account your reduction when reporting CIs and other results



### Model simplification

**Method 1:** If there is multicollinearity in your data set, perform **variable selection**. e.g. choose the subset of predictors that maximizes  $R_{\text{adj}}^2$ .

**Method 2:** If there isn't multicollinearity, you can conduct a **partial** *F***-test** on predictors you suspect of being unhelpful. Remove them if the result is non-significant.

Applying the two methods to the house-price data:

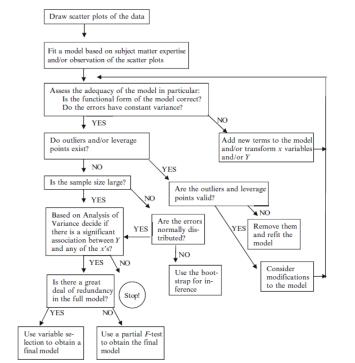
- 1. The subset turns out to be everything but bth. Therefore, bth would removed by Method 1.
- Among the choices of removing gar and/or bth, the effect on  $R_{\rm adj}^2$  is small in all cases (well under 1% change). Method 1 naively maximizes  $R_{\rm adj}^2$  without regard to this; the method is known for its timidity in removing predictors.
- 2. Removing bth and gar leads to a non-significant partial F-test (p > 0.05). Therefore, bth and gar would be removed by Method 2. Removing any other predictor(s) instead of or in addition to these leads to a significant p-value for the partial F-test.

## The MLR approach

- 1. Plot standardized residuals to help determine whether the proposed regression model is a valid model
  - 2. Identify any leverage points
  - **3.** Identify any *outliers*
- **4.** Assess the effect of each *X* on *Y*
- **5.** Assess multicollinearity
- **6.** Assess the assumption of error homoscedasticity
- 7. For time series: examine whether the data are correlated over time

The Seven C's can also be viewed as fitting into a broader **flowchart approach** to **MLR** provided on the next slide.

- ▶ The flowchart mentions the "bootstrap", which you're not responsible for
- ► The bootstrap uses data resampling to numerically approximate the sampling distribution of the test statistic under H<sub>0</sub> (rather than using theoretical results, which we did assuming normally distributed errors)



#### Overview

- ▶ Practical example: Analysing house-price data
- ▶ Chapter 7: Variable selection, and going with the flow
- Key MLR aspects



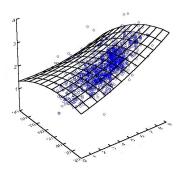
## Why do regression? Two main reasons

- 1. Predict the value of a variable given the value(s) of 1+ others
  - ▶ For interpolation or extrapolation, for example in house prices
  - It doesn't matter which variables are in the model or how they were chosen (model building/variable selection OK)
  - However, we must consider the level of smoothing that's appropriate. If we overfit, predictions may not work well on new data
- 2. Understanding an underlying mechanism
  - Describe the relationships among variables
  - ▶ What are the effects of predictor(s) on the response?
  - Describe differences in relationships between different sets of data
  - For example, data sets for the Montreal Protocol (CFC data), Meadowfoam, etc
  - This is inferential modelling. The form of the model is assumed to be known but there is uncertainty about the value(s) of the coefficients

It can be risky doing inferential modelling on the same data that you used to build a model (to describe how Y might originate in general).

# Why do multiple regression?

- ightharpoonup Multiple X's give a better prediction of Y, e.g. house prices
- ► MLR allows polynomial fits
- MLR allows comparison of regression line for two groups, e.g. meadowfoam data
- ➤ You can control for some X's, e.g. a baseline value in a study, such as mammals and brain size

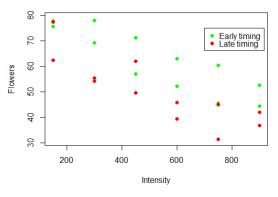


### Experiments vs Observational Studies

#### **Experiment** (e.g. for the meadowfoam dataset):

A treatment is imposed on experimental units. Advantages:

- Strength of conclusion
- ▶ If properly randomized, you can make cause-and-effect conclusions
- e.g. you can say that increasing light intensity causes a decrease in the number of flowers per plant on average

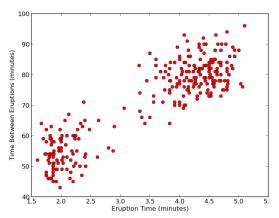


### Experiments vs Observational Studies

Observational study (e.g. for the CFC data or house-price data):

Data are measured without an intervention.

- ▶ You can say that there's an association, but you can't say that X causes Y
- ▶ Perhaps they are both related to a third variable
- e.g. Old Faithful data: you can't say that a longer duration causes a large interval: perhaps both are related to some other geological phenomenon



## Multiple Regression

#### Comparison of:

- t-tests  $(H_0: \beta_j = 0$ , all other X's in the model)
- ▶ The ANOVA *F*-test ( $H_0: \beta_1 = \cdots = \beta_p = 0$ , a test for the overall model)

It's possible to have:

*F*-test significant (p < 0.05), and all or some *t*-tests significant:

The model has useful variables for explaining Y

F-test non-significant, and all t-tests non-significant:

► There are no explanatory variables useful for explaining Y

F-test significant, and all t-tests non-significant:

▶ Multicollinearity; individual X's don't predict Y over and above the others

*F*-test non-significant, and some *t*-tests significant:

▶ There are two possible ways this can happen (next slide)

# Significant *t*-test results accompanying a non-significant *F*-test result

1. The model has no predictive value, but in your t-test analysis there were some Type I errors (falsely rejecting  $H_0$ ). This is always a risk with multiple tests



2. The predictors were chosen poorly. e.g. if there was one useful predictor among many not useful predictors, the contribution from the useful one may not be enough for the *F*-test to be significant



# Problems with Regression Models and their Consequences

You might have the wrong model for several reasons:

- 1. The true relationship is nonlinear:
- ▶ Biased  $\hat{\beta}$ 's
- ► Meaningless *t* and *F*-tests / Cls
- 2. You omitted an important predictor variable:
- ▶ Biased  $\hat{\beta}$ 's
- ► Meaningless *t* and *F*-tests / Cls

Actual model: 
$$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + e$$

Fitted model: 
$$\hat{Y} = b_0 + b_1 x_1$$
 and thus  $\mathrm{E}(b_1) \neq \beta_1$ 

- 3. You included an unimportant predictor variable:
- $\triangleright$   $\hat{\beta}$ 's aren't biased
- ▶ The variance is unnecessarily large (inefficient)

## Problems with Regression Models and their Consequences

The Gauss-Markov conditions might not be met:

#### 1. Non-constant variance:

- $\triangleright$   $\hat{\beta}$ 's aren't biased
- $\triangleright$   $S^2$  is a biased estimate
- ▶ Invalid *t* and *F*-tests / CIs

#### 2. Correlated errors:

- $\triangleright$   $\hat{\beta}$ 's aren't biased
- ► S<sup>2</sup> isn't estimating the right thing
- ▶ Invalid t- and F-tests / Cls

#### 3. Non-normal errors:

- $\triangleright$   $\hat{\beta}$ 's aren't biased
- $S^2$  is an unbiased estimate for  $\sigma^2$
- ▶ Invalid t- and F-tests / Cls, except for large sample sizes
- ► Invalid Pls

# Other problems with Regression Models and their Consequences

- 1. Multicollinearity:
- ightharpoonup var( $\hat{\beta}$ ) will be high inefficient
- 2. The predictor variables were measured with error:
- ▶ Biased  $\hat{\beta}$  and  $S^2$
- ▶ Invalid *t* and *F*-tests / CIs
- ▶ Generally ok provided the variance in X's is small
- ▶ To fix: take multiple measurements, or use Structural Equation Modelling



"Ask the realtor if we can list the litter box as a third bathroom."

## Exam results by Section

After the exams are marked, I'll check for any significant differences between the grades of Sections 1 and 2. If one exists then the marks may be tweaked to eliminate this.

If you're in Section 1 but have been attending Section 2's lectures and wish to be viewed within Section 2 for the purposes of exam marks adjustment (or vice versa), please let me know directly by Friday 8 December and give your reasons.



The expectation is that Sections 1 and 2 will perform similarly on the shared exam, having been taught the same. Therefore, hopefully this slide is pointless!

#### Course Evaluations



#### FAS Fall 2017 Undergrad for Meth Data Analysis STA302H1-F-LEC0101

Medium	Online
Timing	Scheduled
	Start Date 2017-11-23 00:00
	End Date 2017-12-08 23:59

Response R	ate			
	Responded	Invited	% Rate	
Students	39	341	11.44%	

Please visit https://courseevaluations.utoronto.ca

### Next steps

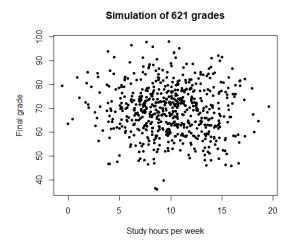
- ▶ Solutions to Chapter 6 have been posted on Portal under 'Homework'
- ► Chapter 7: You may wish to attempt the  $R_{\rm adj}^2$  portion of 1(a), 2(a), or 3(a), but not all three
- ▶ No more homework!



# Appendix



## Detailed example of partial correlation



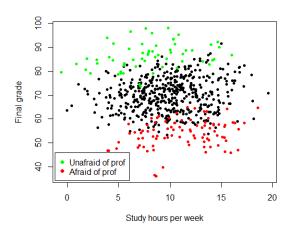
It may appear that studying more doesn't improve your final grade:

- ▶ No pattern is distinguishable
- $r \approx -0.06$ , with a *p*-value of  $\sim 0.14$

## Detailed example of partial correlation

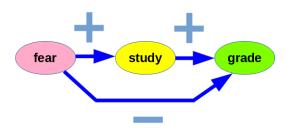
Students answer a question from 1 to 10: "Were you afraid of the prof?"

- ▶ Green points: Score of 2.5 or lower
- ▶ Red points: Score of 6 or higher



### What happened?

As a graphical model (which you aren't responsible for in this course):



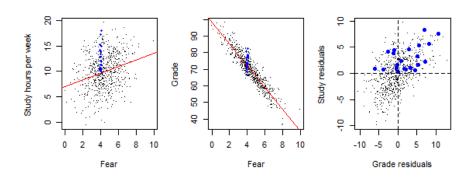
#### Partial correlations:

- ▶ Between Fear and Study: Significantly positive
- ▶ Between Fear and Grade: Significantly negative
- ▶ Between Study and Grade: Significantly positive

### Regressing to produce the two sets of residuals

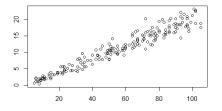
The partial correlation between Study and Grade given Fear is the r between:

- ightharpoonup The residuals  $\hat{e}_{Study}$  resulting from the regression of Study versus Fear
- ightharpoonup The residuals  $\hat{e}_{Grade}$  resulting from the regression of Grade versus Fear



## Chapter 4: Weighted Least Squares

Why do WLS? Heteroscedasticity, i.e. nonconstant variance When to use it? Example:



- A linear relationship is apparent
- ▶ The distribution of residuals isn't skewed
- ► There is nonconstant variance

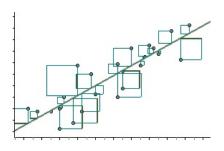
The idea: Downweight points which have more variability

### Weighted Least Squares

We find the least squares estimates by minimizing the Weighted Residual Sum of Squares, WRSS:

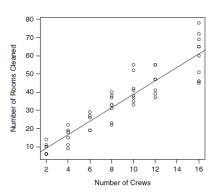
WRSS = 
$$\sum_{i=1}^{n} w_i (y_i - \hat{y}_i)^2$$

where  $w_i \propto 1/\sigma_i^2$  are weights, in which  $\sigma_i^2$  is the error variance for the *i*th observation, i.e.  $var(e_i) = \sigma_i^2$ .



### To find the weights:

- 1. If you have multiple observations for each value of x, you can estimate  $\sigma_i^2$
- 2. Example choices in SLR might be  $w_i = 1/x_i$  or  $w_i = 1/x_i^2$ . That is, the variance increases in proportion to the mean or mean<sup>2</sup>
- 3. Often, the weights are chosen in an iterative fitting process with  $\hat{y}_i$ . Iteratively reweighted least squares (IRLS) is popular in logistic regression STA303
- 4. If you know that  $y_i$  is the average (or median) of  $n_i$  observations, then  $\text{var}(y_i) \propto 1/n_i$ . This is a frequent source of unequal variance in datasets, and naturally  $w_i = n_i$  is a good weight



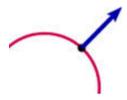
## Minimizing WRSS

Solving the normal equations that result from the equation on the previous slide, we obtain the weighted (W) least squares estimates:

$$\hat{\beta}_{1W} = \frac{\sum_{i=1}^{n} w_i (x_i - \bar{x}_W) (y_i - \bar{y}_W)}{\sum_{i=1}^{n} w_i (x_i - \bar{x}_W)^2} \qquad \hat{\beta}_{0W} = \bar{y}_W - \hat{\beta}_{1W} \bar{x}_W$$

$$\hat{\beta}_{0W} = \bar{y}_W - \hat{\beta}_{1W} \bar{x}_W$$

where  $\bar{y}_W = \sum w_i y_i / \sum w_i$  and  $\bar{x}_W = \sum w_i x_i / \sum x_i$  are weighted averages.



#### In matrix terms

Suppose  $var(\mathbf{e}) = \sigma^2 \mathbf{V}$  where  $\mathbf{V}$  is a diagonal matrix in which the *i*th entry is  $1/w_i$ . Then,

$$\begin{pmatrix} \hat{\beta}_{0W} \\ \hat{\beta}_{1W} \end{pmatrix} = \hat{\boldsymbol{\beta}}_W = \left( \mathbf{X}' \mathbf{V}^{-1} \mathbf{X} \right)^{-1} \mathbf{X}' \mathbf{V}^{-1} \mathbf{Y}$$

and  $\operatorname{var}(\boldsymbol{\hat{\beta}}_{W}) = \sigma^{2} \left( \mathbf{X}' \mathbf{V}^{-1} \mathbf{X} \right)^{-1}$ .

Compare these to what we derived earlier:

- On Week 8, slide 9:  $\hat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}$
- ▶ In Weeks 8–9, docucam/board work:  $var(\hat{\beta}) = \sigma^2(\mathbf{X}'\mathbf{X})^{-1}$

## Example: Question 1 from page 122

"A full professor of statistics at a major US university is interested in estimating the third quartile of salaries for full professors with 6 years of experience in that rank. [Data provided below or on Portal.] Using weighted least squares, estimate the 2005-2006 third quartile for salary of full professors with 6 years of experience."

Table 4.3 Data on salaries

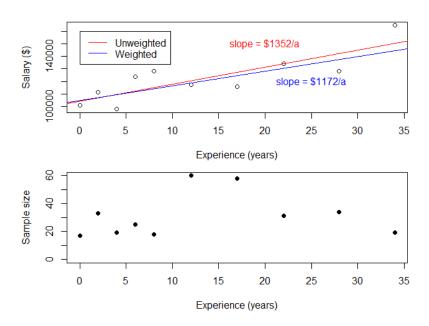
Years of experience as a full professor	Sample size, $n_i$	Third quartile (\$)
0	17	101,300
2	33	111,303
4	19	98,000
6	25	124,000
8	18	128,475
12	60	117,410
17	58	115,825
22	31	134,300
28	34	128,066
34	19	164,700

### R code for Question 1

```
Q <- read.csv("ch4q1.txt",sep="")</pre>
x <- Q[,1] # years of experience
y <- Q[,3] # salary
w \leftarrow Q[,2] \# weights
m \leftarrow lm(y\sim x); mW \leftarrow lm(y\sim x, weights=w)
print(mW$coefficients[1] + 6*mW$coefficients[2])
x1 = c(-2.40)
y1 = m$coefficients[1] + x1*m$coefficients[2]
y1w = mW$coefficients[1] + x1*mW$coefficients[2]
plot(x,y,xlab="Experience (years)",ylab="Salary ($)")
lines(x1,y1,type="l",col="red")
lines(x1,y1w,type="l",col="blue")
```

```
## (Intercept)
## 111793.8
```

### R output for Question 1



#### Tests and confidence intervals

If our usual assumptions of normality and independence hold, the same general procedures for tests and CIs apply as in unweighted least squares. For example, recall from Week 3 slides 17–19 that our test statistic was

$$\frac{\hat{\beta}_1}{\operatorname{se}\left(\hat{\beta}_1\right)} = \frac{\hat{\beta}_1}{S/\sqrt{Sxx}}$$

Now in weighted least squares:

$$t_{\rm obs} = \frac{\hat{\beta}_{1W}}{S_W/\sqrt{{\sf WS}xx}}$$

where WS $xx = \sum_{i=1}^{n} w_i (x_i - \bar{x}_W)^2$  is the "weighted Sxx", and the weighted residual standard error  $S_W$  is such that

$$S_W^2 = \frac{\sum_{i=1}^n w_i (y_i - \hat{\beta}_{0W} - \hat{\beta}_{1W} x_i)^2}{n-2}$$

Under  $H_0: \beta_1 = 0$ , the test statistic  $t_{\text{obs}}$  is an observation from the  $t_{n-2}$  distribution.

## Chapter 4: What do you need to know?

STA302 gives an introduction to this method, but as per the syllabus, we skip  $\S4.1.1$  (PIs for weighted least squares),  $\S4.1.2$  (leverage for WLS), and  $\S4.1.3$  (using LS to calculate WLS).

Try homework questions 2 and 3 — they are quick, and solutions are already online.



## Reminder about Datacamp: until 22 March

Name \$	Enrolled	Completed
Introduction to R	109	49
Intermediate R	26	11
Intro to Python for Data Science	27	9
Correlation and Regression	17	2
Intro to SQL for Data Science	10	5
Intermediate Python for Data Science	11	3
Intro to Statistics with R: Correlation and Linear Regression	8	0



#### Some suggestions:

- Practise all proofs in slides and extra assigned questions
- Know how to read and interpret R output
- Summary, ANOVA output
- Diagnostics plots
- Other output that you have seen from slides.
- Review the non-credit homework assignments and the midterm papers (both sections)
- ▶ Doing some old exams might help you to see which topics are important

#### Exam format

- ► A cover page and formula page will appear on Portal
- ▶ Question types: multiple choice, short answer, proofs, data analysis
- ► The coverage is on the entire course with emphasis (75%) on material taught from §3.3 (mid-October+) onwards





- This Friday, Monday, Tuesday and Wednesday there will be TA office hours and instructor office hours as per normal
- ▶ On Friday 8 December, there's the Exam Jam
- Limited TA office hours will occur after Wednesday 6 December, at days and times to be announced over Portal. The usual times will no longer apply by default
- On Wednesday 13 December there will be a bonus instructor office hours at the usual Wednesday time (bookable online as usual)
- ► TAs and the instructor won't generally be available to meet at other times but we'll continue to check Piazza

### Final Steps

#### On Tuesday **5 December** in OI G162:

- ▶ We won't have a lecture for Section 1
- ► From 10 am to 12 pm, there will be an optional **drop-in help session**, open to all
- You may like to meet among your Recognized Study Groups, work individually, etc

#### Wishing you all the best...

- ▶ On the exam
- For the Christmas holidays
- ▶ In 2018+

