Two Key Concepts

1. Truth-Value Assignment (TVA):
   - An assignment of truth-values to the ATOMIC statements
   - A row in the truth-table

2. Truth-Table
   - A table of ALL possible truth-value assignments

Truth-Tables of Molecular Sentences

Consider $\sim(P \rightarrow Q) \vee (Q \leftrightarrow P)$

P	Q	~	(P	→	Q)	v	(Q	↔	P)

1. Identify the Main Connective

Mark it with an arrow above

P	Q	~	(P	→	Q)	v	(Q	↔	P)

2. Fill in the Atomic Truth-Values

Put in the truth-values for the atomics in a way that makes sense!

There are ALWAYS 2^n rows where n is the number of atomic letters

P	Q	~	(P	→	Q)	v	(Q	↔	P)
T	T								
T	F								
F	T								
F	F								

3. Carry Truth-Values Through

P	Q	~	(P	→	Q)	v	(Q	↔	P)
T	T		T		T		T		T
T	F		T		F		F		T
F	T		F		T		T		F
F	F		F		F		F		F

4. Figure Out Connectives

Do this one at a time

Best to work inside out

P	Q	~	(P	→	Q)	v	(Q	↔	P)
T	T		T	T	T		T	T	T
T	F		T	F	F		F	F	T
F	T		F	T	T		T	F	F
F	F		F	T	F		F	T	F

4. Figure Out Connectives

Then move outwards

P	Q	~	(P	→	Q)	v	(Q	↔	P)
T	T	F	T	T	T		T	T	T
T	F	T	T	F	F		F	F	T
F	T	F	F	T	T		T	F	F
F	F	F	F	T	F		F	T	F

4. Figure Out Connectives

And finally to the main connective

P	Q	~	(P	→	Q)	v	(Q	↔	P)
T	T	F	T	T	T	T	T	T	T
T	F	T	T	F	F	T	F	F	T
F	T	F	F	T	T	F	T	F	F
F	F	F	F	T	F	T	F	T	F

5. Analyze the Sentence

The truth-value of the main connective is all that matters

P	Q	~	(P	→	Q)	v	(Q	↔	P)
T	T	F	T	T	T	T	T	T	T
T	F	T	T	F	F	T	F	F	T
F	T	F	F	T	T	F	T	F	F
F	F	F	F	T	F	T	F	T	F

Make your initial truth-value order make sense

Spend time figuring out the main connective

Sometimes marking the secondary main connectives is helpful

Always go from the parts to the whole, finishing with the main connective

Properties of Sentences

1. Tautology
   - A sentence φ is a tautology iff φ is true on EVERY truth-value assignment

2. Contradiction
   - A sentence φ is a contradiction iff φ is false on EVERY truth-value assignment

3. Contingent Sentence
   - A sentence φ is a contingent sentence iff φ is true on SOME truth-value assignment, and false on SOME truth-value assignments

Sets of Sentences

Use { } to denote the set

Use , to separate the members

$\{\varphi, \psi\}$ is a set of sentences

$\{\varphi\}$ is also a set of sentences

Properties of Sets of Sentences

1. Sentence φ and ψ are LOGICALLY EQUIVALENT iff there is no truth-value assignment where φ and ψ have different truth-values

2. A set of sentences is CONSISTENT iff there is at least one truth-value assignment on which all members of the set are true

3. A set of sentences is INCONSISTENT iff it is not consistent
   - There is no truth-value assignment where all members are true

Single Sentences		Sets of Sentences	
Tautology	All TVAs True	Logically Equivalent	Same truth-value for all TVAs
Contradiction	All TVAs False	Consistent	At least one TVA is true for all sentences
Contingent	Some TVAs True, and some TVAs false	Inconsistent	No TVAs is true for all sentences

Properties of Arguments

An argument is VALID iff:

for EVERY TVA where all the premises are true, then the conclusion is also true

or

there is NO TVA where the premises are true and the conclusion is false

Example: Scharer 4.4 E4 c

Use a full truth-table to determine if this argument is valid

$\sim(P \lor (\sim S \land Q))$. $S \to (P \to Q)$. $\therefore$ $\sim P \leftrightarrow \sim S$

A set of sentences TAUTOLOGICALLY IMPLIES a sentence φ iff there is no truth-value assignment for which all the sentences in the set are true and φ is false

"$\{\psi, \mu, \lambda\}$ tautologically implies φ"

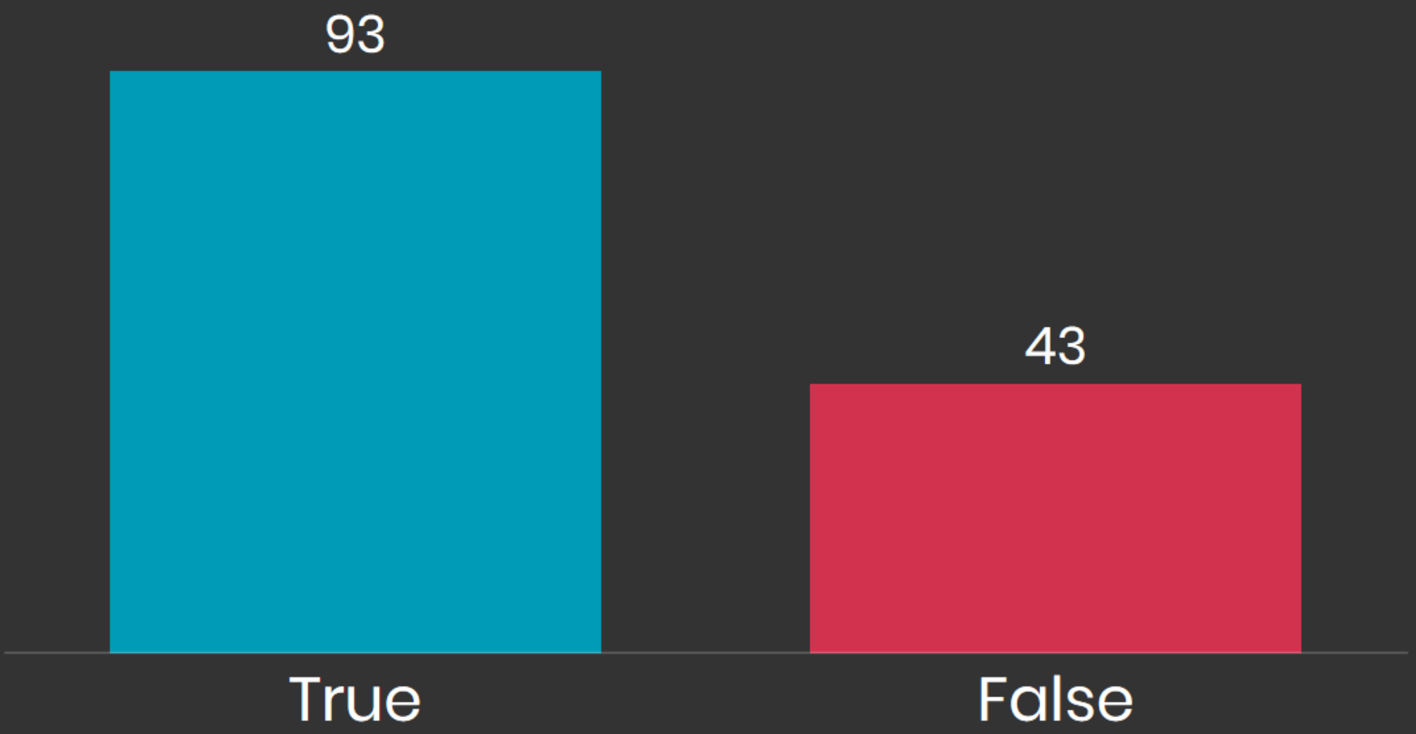
is the same as

"$\psi.\ \mu.\ \lambda.\ \therefore \varphi$ is a valid argument"

Go to **www.menti.com** and use the code **45 13 15**

Mentimeter

An argument can be true

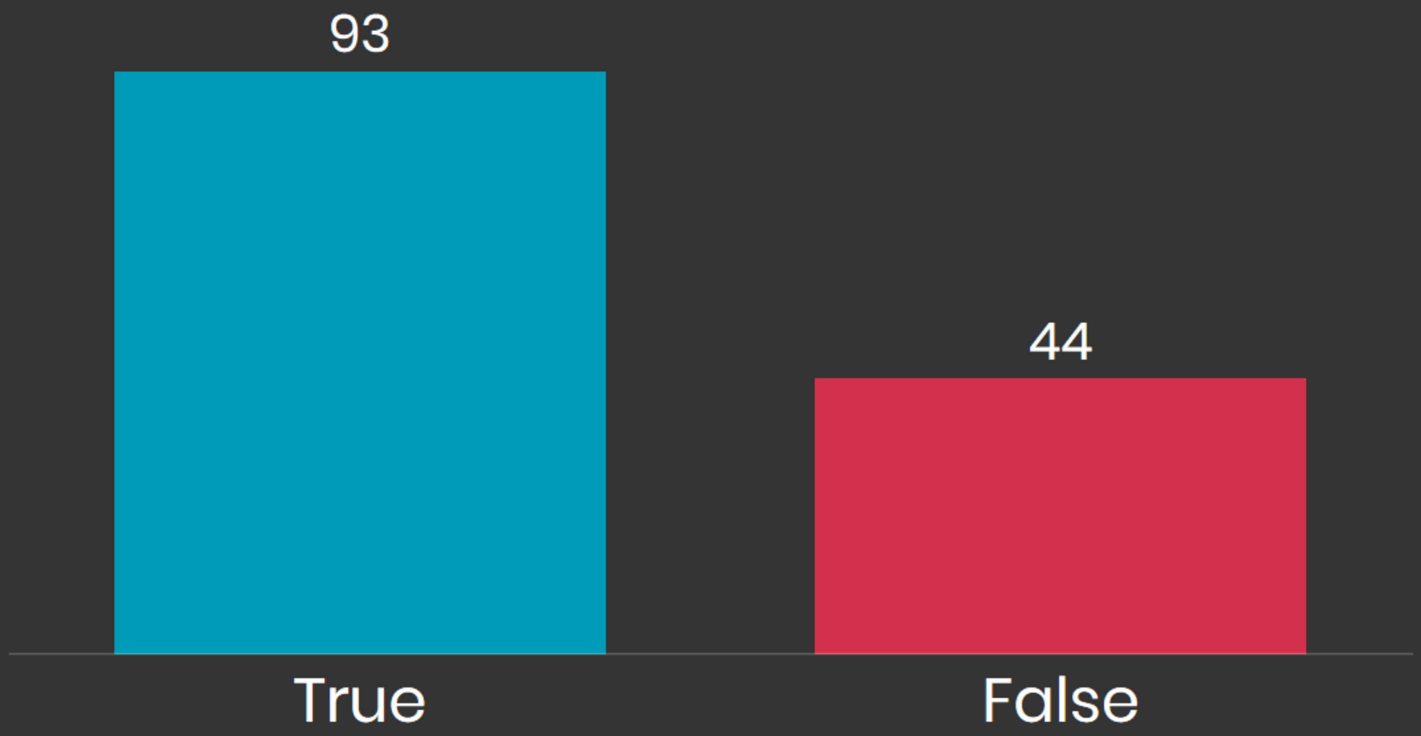


Slide is not active Activate

136

Mentimeter

A set of false sentences is consistent



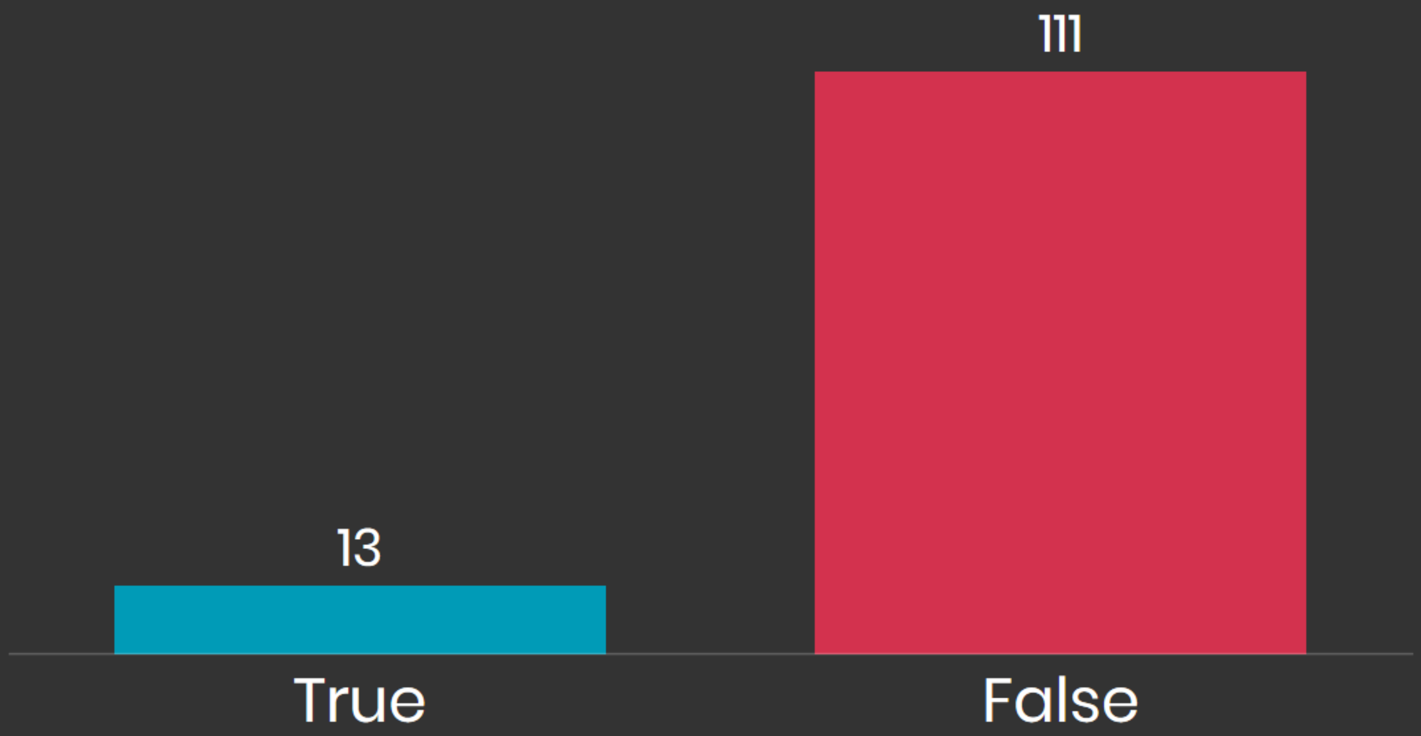
Slide is not active Activate

137

Go to **www.menti.com** and use the code **45 13 15**

Mentimeter

A sentence that is not a tautology is a contradiction



Slide is not active

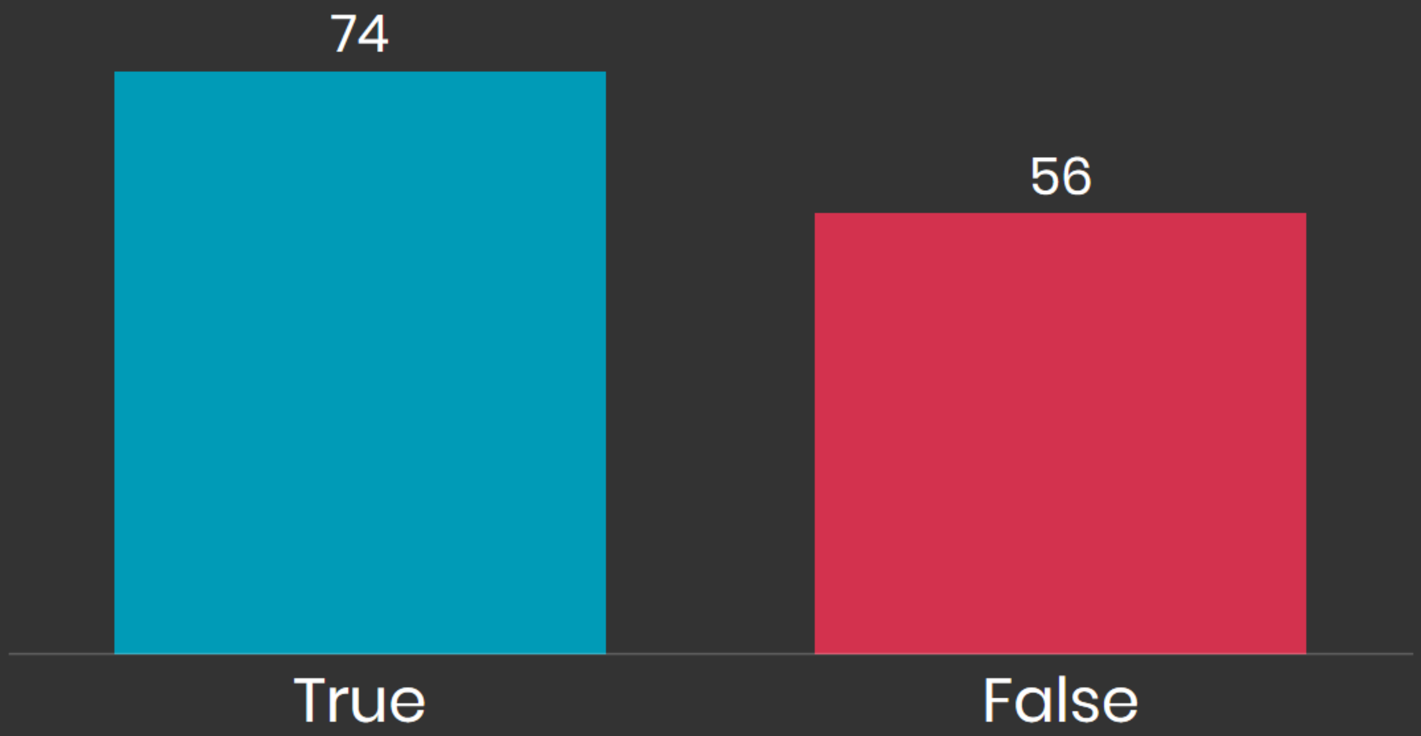
Activate

124

Go to **www.menti.com** and use the code **45 13 15**

Mentimeter

An invalid argument requires that whenever the premises are true the conclusion is false



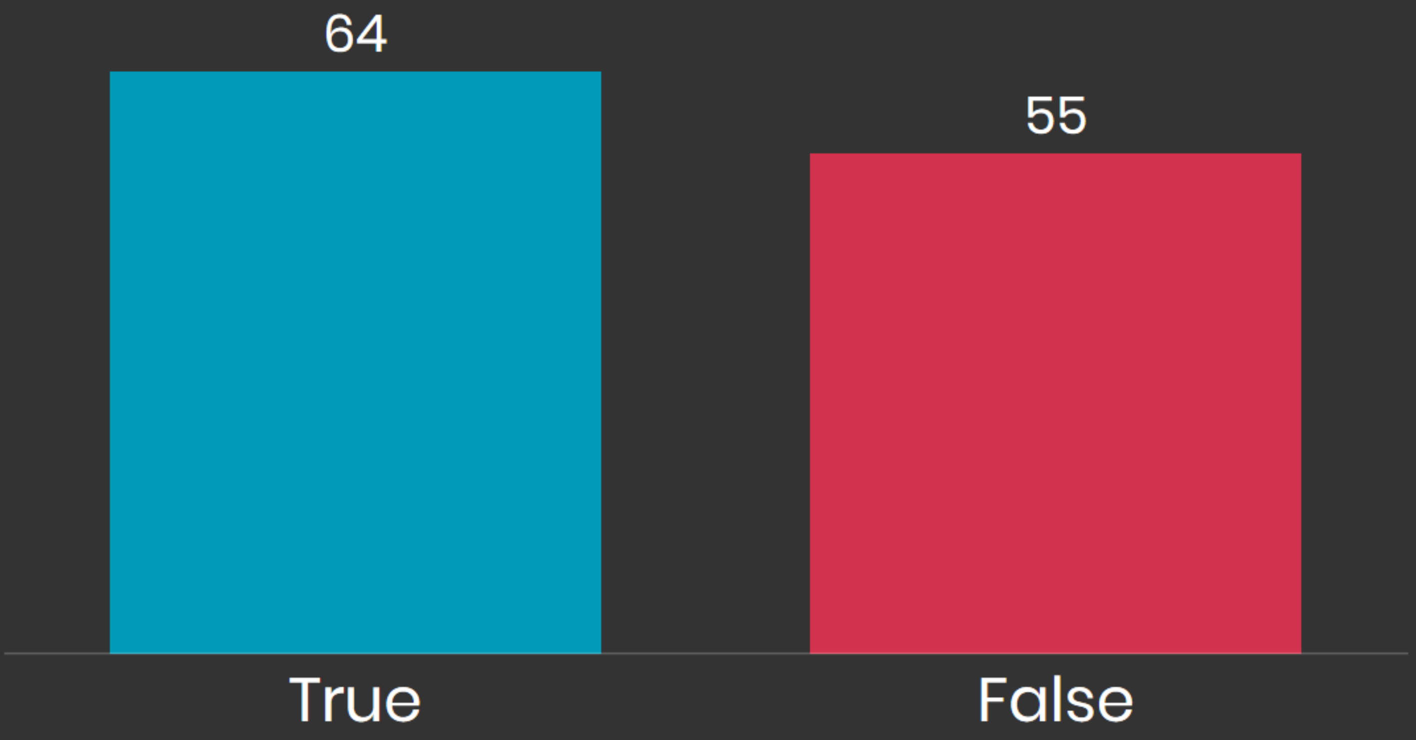
Slide is not active Activate

130

Go to **www.menti.com** and use the code **45 13 15**

Mentimeter

The conclusion to a sound argument is true and valid



Slide is not active

Activate

119

Shortened Truth-Tables

We don't need full truth-tables
to demonstrate certain properties

We can shortcut to the
only TVA we actually need

Single Sentences		Sets of Sentences	
Tautology	All TVAs True	Logically Equivalent	Same truth-value for all TVAs
Contradiction	All TVAs False	Consistent	At least one TVA is true for all sentences
Contingent	Some TVAs True, and some TVAs false	Inconsistent	No TVAs is true for all sentences

We can't show anything that requires us to know information about ALL TVAs

But we can show that they do NOT have these properties

What about VALIDITY?

Not in general, but we can show invalidity

(or that a set of sentences does NOT tautologically imply a sentence)

If you want to show…	Then get a TVA that makes the MAIN CONNECTIVE…
Sentence not a tautology	F
Sentence not a contradiction	T
Sentence is contingent	Make 2: one T and one F
Set of sentences not logically equivalent	One sentence T, one sentence F
Set of sentences consistent	All sentences T
Set of sentences not inconsistent	All sentences T
Argument is invalid	Premises all T and Conclusion F

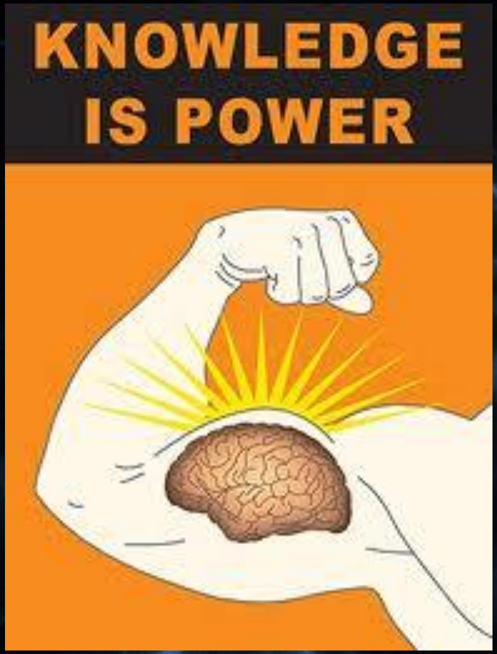
Procedure

1. Identify the MAIN CONNECTIVE ⬇ and set the truth value to what you need

2. Work backwards from the easiest thing to show to the hardest

3. Whenever you establish something for an atomic, write it in and follow it through

Know Your Connectives!



What makes a connective T or F?

How many ways can a connective be made T or F?

Start from with the connective that is the easiest to analyze

- Has the least number of cases

Connectives and Cases

Connective	Ways True	Ways False
~	1	1
→	3	1
∧	1	3
∨	3	1
↔	2	2

Example: Scharer 4.5 EG2
Show that the following sentence is not a tautology:

$$(\sim(S \vee P) \vee (Q \leftrightarrow R)) \rightarrow (\sim(R \leftrightarrow P) \vee (S \rightarrow Q))$$

Example: Scharer 4.5 EG2 c
Show that this argument is invalid:

$$S \rightarrow P. \quad Q \rightarrow (R \vee S). \quad \therefore \sim P \rightarrow (Q \wedge R)$$

Example: Scharer 4.5 E1n
Show that the following set is consistent:

$$\sim(S \vee T) \leftrightarrow (U \wedge W). \quad U \leftrightarrow T. \quad \sim S \leftrightarrow \sim W$$