

(a) c_1, c_2, \dots, c_k is linearly independent.

proof:

if c_1, c_2, \dots, c_k is linearly dependent.

then there are not all zero scalars d_1, d_2, \dots, d_k , such that

$$d_1 c_1 + d_2 c_2 + \dots + d_k c_k = 0$$

$\Rightarrow d_1 a_1 + d_2 a_2 + \dots + d_k a_k = 0 \rightarrow$ contradiction to a_1, a_2, \dots, a_k is linearly independent.

(b) It does not follow c_1, c_2, \dots, c_k is linearly dependent.

proof if c_1, c_2, \dots, c_k is linearly dependent

\Rightarrow there are not all zeros scalar d_1, d_2, \dots, d_k , such that

$$d_1 c_1 + d_2 c_2 + \dots + d_k c_k = 0$$

$$\Rightarrow d_1 b_1 + d_2 b_2 + \dots + d_k b_k = 0$$

$\Rightarrow b_1, b_2, \dots, b_k$ are linearly dependent

which is contradiction to no assumption of b_1, b_2, \dots, b_k .