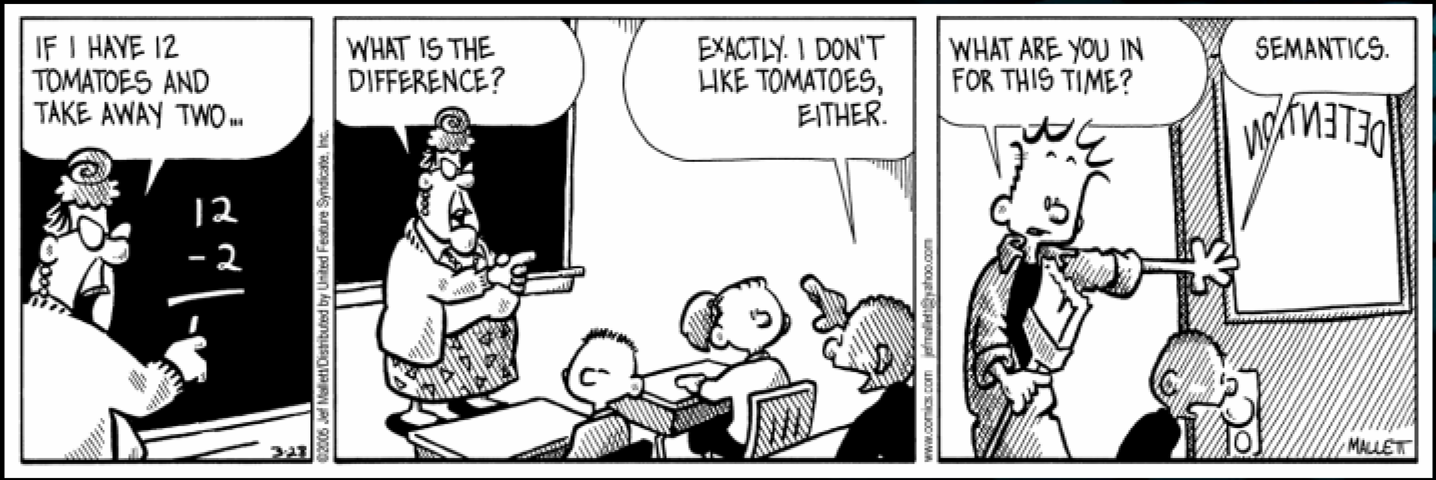


Semantics in Sentential Logic

Unit 2 Part 1: Syntax

Assessing validity requires us
to set statements TRUE or FALSE

For simple statements this is easy

But what about complex statements?

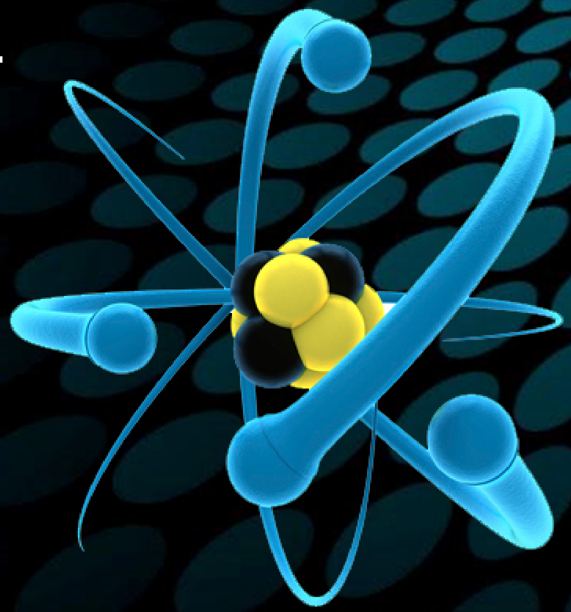
Sentential Logic (SL)

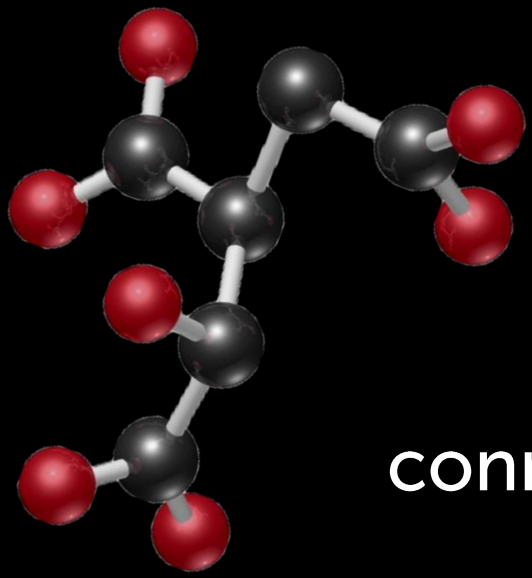
Complex (compound) statements are all built up by joining statements together using LOGICAL CONNECTIVES

AND	$\wedge$	OR	$\vee$
NOT	$\sim$	IF THEN	$\rightarrow$

IF AND ONLY IF $\leftrightarrow$

A statement that has no logical connectives is called ATOMIC





A statement that has logical connectives is called MOLECULAR

$\wedge$, $\vee$, $\rightarrow$, and $\leftrightarrow$, all join two statements

BINARY connectives

$\sim$ is a UNARY connective

All connectives can operate on atomic or molecular statements

Atomic or Molecular?

- Grass is green
- I like fried chicken
- I'll have fries or salad
- Clean your room!
- It's not nice out
- If it snows, then either John or Sally will take the bus, unless it's a snow day in which case John and Sally will walk

- Atomic
- Atomic
- Molecular
- Neither
- Molecular or Atomic
- Molecular

Symbols for Sentential Logic

1. Symbols for atomic statements
   - Capital letters P-Z

2. Symbols for the logical connectives
   - $\sim \rightarrow \leftrightarrow \wedge \vee$

3. Symbols for organization
   - ( ), [ ]

You can have fries or salad — $P \vee Q$

If it rains, then the sidewalks are wet — $P \rightarrow R$

I don't like cats — $\sim T$ or T

It's Saturday and we have logic — $X \wedge Y$

Official Notation Rules

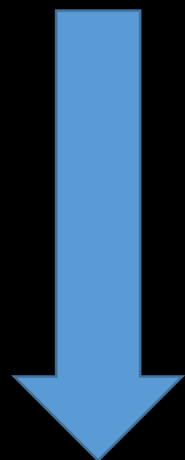
1. Use round brackets, ( ), around every binary connective
2. Never use brackets around unary connectives or atomic statements

Official notation is unambiguous

Informal Notation Rules

1. Use round brackets ( ), around binary connectives that otherwise would be ambiguous
2. Never use round brackets around unary connectives or atomic statements
3. Know the hierarchy
4. For strings of all $\wedge$ or all $\vee$, use the right-most rule

Informal Notation Hierarchy



$\rightarrow \;\leftrightarrow$

$\wedge \;\vee$

$\sim$

Use ( ) to disambiguate connectives
on the same level

Good Examples

Official

- $(P \vee Q)$
- $(P \vee (Q \rightarrow (P \wedge R)))$
- $((P \rightarrow Q) \rightarrow R)$
- $(\sim(P \leftrightarrow Q) \wedge (S \rightarrow \sim T))$
- $(((P \wedge Q) \wedge R) \wedge S)$
- $\sim(P \leftrightarrow \sim(Q \wedge \sim S))$

Informal

- $P \vee Q$
- $P \vee (Q \rightarrow P \wedge R)$
- $(P \rightarrow Q) \rightarrow R$
- $\sim(P \leftrightarrow Q) \wedge (S \rightarrow \sim T)$
- $P \wedge Q \wedge R \wedge S$
- $\sim(P \leftrightarrow \sim(Q \wedge \sim S))$

Bad Examples

Not Well-Formed

- $\sim(P)$
- $(\sim Q)$
- $P \vee Q \wedge R \wedge S$

Well-Formed

- $\sim P$
- $\sim Q$
- $(P \vee Q) \wedge R \wedge S \quad (((P \vee Q) \wedge R) \wedge S)$
  $P \vee (Q \wedge R \wedge S) \quad (P \vee ((Q \wedge R) \wedge S))$
  $(P \vee Q) \wedge (R \wedge S) \quad ((P \vee Q) \wedge (R \wedge S))$

Not Well-Formed:

- $\sim(P \rightarrow (Q \vee R \rightarrow \sim(S \wedge (T \leftrightarrow \sim P)))$

Well-Formed:

- $\sim(P \rightarrow (Q \vee R \rightarrow \sim(S \wedge (T \leftrightarrow \sim P))))$

Let ϕ and ψ represent any sentence

A well-formed formula (WFF) in SL is generated by the following three steps:

1. Sentence letters (P-Z) are symbolic sentences.
2. If ϕ is a sentence then $\sim\phi$ is a symbolic sentence.
3. If ϕ and ψ are symbolic sentences, then so are $(\phi\rightarrow\psi)$, $(\phi\leftrightarrow\psi)$, $(\phi\wedge\psi)$, and $(\phi\vee\psi)$.

A WFF is strictly speaking a sentence that is in official notation

For our purposes, a well-formed formula is one that is perfectly unambiguous

MAIN CONNECTIVE

IMPORTANT

Every molecular sentence is in one of the 5 following forms:

$\sim\phi$, $(\phi \rightarrow \psi)$, $(\phi \leftrightarrow \psi)$, $(\phi \wedge \psi)$, and $(\phi \vee \psi)$

The main connective is the logical connective in the forms above

It's what the sentence 'is'

Find the Main Connective

Official	Informal	Abstract
• (P∨Q)	• P∨Q	• $\phi \vee \psi$
• (P∨(Q→(P∧R)))	• P∨(Q→P∧R)	• $\phi \vee \psi$
• (~(P↔Q)∧(S→~T))	• ~(P↔Q)∧(S→~T)	• $\phi \wedge \psi$

$\sim P \rightarrow Q \wedge R$

Go to **www.menti.com** and use the code **79 38 45**

Mentimeter

What is the Main Connective?

$\sim$	$\rightarrow$	$\wedge$
0	0	0

Slide is not active Activate

0

$((P \leftrightarrow Q) \vee (\sim R \vee S \rightarrow T))$

Go to **www.menti.com** and use the code **79 38 45**

Mentimeter

What is the Main Connective?

↔	The First v	~	The Second v	→
0	0	0	0	0

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0

P∧Q∧(R∧S)

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Mentimeter

What is the Main Connective?

0	0	0
The First ∧	The Second ∧	The Third ∧

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0

$\sim(P \leftrightarrow (Q \rightarrow R \wedge T))$

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Mentimeter

What is the Main Connective?

0	0	0	0
$\sim$	$\leftrightarrow$	$\rightarrow$	$\wedge$

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0

$(\sim P \rightarrow Q \wedge S) \leftrightarrow T \vee W$

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Mentimeter

What is the Main Connective?

0	0	0	0	0
~	→	∧	↔	∨

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0

You will also have to identify SECONDARY CONNECTIVES, and so on.

These are the main connectives of the molecular parts of a connective

$$((P \leftrightarrow Q) \vee (\sim R \vee S \rightarrow T))$$