1. Consider the following "network flow with reduced transmission" problem.

Input: A network $N = (V, E)$ (a directed graph with a single source $s \in V$, a single sink $t \in V$, and positive integer capacities $c(e)$ for every edge $e \in E$). In addition, we are given *real* numbers $t(v) \in [0, 1]$ for every vertex $v \in V$ ($t(v)$ is a *transmission coefficient*).

Output: A maximum flow f (that is, flow values $f(e)$ for every edge $e \in E$ such that $f^{\text{out}}(s)$ is maximum), subject to the following constraints.

- *Capacity constraint*: $0 \leqslant f(e) \leqslant c(e)$ for all edges $e \in E$ (same as the original network flow problem).
- *Modified conservation constraint*: $f^{\text{out}}(v) = t(v) \cdot f^{\text{in}}(v)$ for every vertex $v \in V - \{s, t\}$ (in other words, the flow *out* of every node v is reduced by a factor of $t(v)$ from the flow *into* the node).

Show how to model this problem as a linear program: state explicitly what variables you are using, what your objective function is, and what your constraints are.