NOTE: This file contains sample solutions to the quiz together with the marking scheme and comments for each question. Please read the solutions and the marking schemes and comments carefully. Make sure that you understand why the solutions given here are correct, that you understand the mistakes that you made (if any), and that you understand *why* your mistakes were mistakes.

Remember that although you may not agree completely with the marking scheme given here, it was followed the same way for all students. We will remark your quiz only if you clearly demonstrate that the marking scheme was not followed correctly.

For all remarking requests, please submit your request **in writing** directly to your instructor. For all other questions, please don't hesitate to ask your instructor during office hours or by e-mail.

GENERAL MARKING SCHEME:

- ***A****: All Correct*, except maybe for very few minor errors.
- ***B****: Mostly Correct*, but with a few serious errors, or many small errors.
- ***C****: Mostly Incorrect*, but with a few important elements, or many small elements, done correctly.
- ***10%****: Completely Blank*, or clearly crossed out.
- ***D****: All Incorrect*, except maybe for very few minor elements done correctly.

MARKER'S COMMENTS:

- By definition, an edge in a graph is either present or not — there *cannot* be multiple edges between two nodes. [*Instructor's Note*: Such a structure is called a *multigraph*, but that is **not** the standard definition of a graph, as given in CSC 263 and in the course textbook.]
- It was possible to argue that there is a unique MST in G (because all of its edge weights are distinct), and to use this fact to reach the conclusion.

1. TRUE OR FALSE? For every graph G whose edge weights are all distinct, every MST of G contains the two edges e_1, e_2 with the two smallest weights. (If true, give a brief general argument; if false, give a counter-example.)

> TRUE. Suppose G is a graph whose edge weights are all distinct. Let e_1, e_2 be the two edges with smallest weights ($c(e_1) < c(e_2) <$ cost of every other edge).
>
> For a contradiction, suppose T is a MST that does not contain both e_1 and e_2. W.l.o.g., suppose T does not contain e_2. Consider the endpoints (u, v) of e_2. They are connected by a path P in T. This path contains at least two edges (it cannot contain just one as this would just be e_2 itself). Since e_1, e_2 have the two smallest edge costs, there is at least one edge e' on P with $c(e') > c(e_2)$. But then, $T' = T \cup \{e_2\} - \{e'\}$ is a spanning tree and $c(T') < c(T)$.
>
> This contradicts the fact that T is a MST. Hence, every MST of G contains both e_1 and e_2.