Worth: 2% **Due:** By 9:59pm on Monday 22 September

Remember to write your *full name* and *student number* prominently on your submission.

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*Please read and understand the policy on Collaboration given on the Course Information Sheet. Then, to protect yourself, list on the front of your submission **every** source of information you used to complete this homework (other than your own lecture and tutorial notes, and materials available directly on the course webpage). For example, indicate clearly the **name** of every student with whom you had discussions, the **title** of every additional textbook you consulted, the **source** of every additional web document you used, etc.*

*For each question, please write up detailed answers carefully. Make sure that you use notation and terminology correctly, and that you explain and justify what you are doing. Marks **will** be deducted for incorrect or ambiguous use of notation and terminology, and for making incorrect, unjustified, ambiguous, or vague claims in your solutions.*

Suppose that we have a connected graph $G = (V, E)$ (representing a network of mobile clients), along with edge weights $w(e) \in \mathbb{N}, \forall e \in E$ (representing the cost of establishing a direct connection between the endpoints of e). A minimum spanning tree T in G corresponds to a least expensive selection of direct connections that ensure all mobile clients are connected to each other.

Now suppose that the costs of establishing direct connections change over time: some pairs of clients become more expensive to connect, while others become less expensive. We would like to make adjustments to T to account for these changes, without having to recompute T every time. In this problem set, we consider only one possible case: when the weight of an edge outside of T becomes smaller.

Formally, you are given a connected graph $G = (V, E)$ with edge weights $w(e) \in \mathbb{N}, \forall e \in E$, along with a MST $T \subseteq E$ in G. You are also given one edge $e_0 \in E - T$ and a new weight $w_0 < w(e_0)$ for e_0. Your task is to compute a MST T_0 for the updated graph G_0 that is the same as G except that w_0 is the new weight of e_0.

Write a detailed algorithm that takes as inputs (G, w, T, e_0, w_0) and that outputs T_0. Analyse the running time of your algorithm and prove its correctness *carefully*.