

STA 303/1002-Methods of Data Analysis II

Sections L0101& L0201, Winter 2018

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LME Example I Worksheet

Example I Exercises

Use the related R Markdown file to achieve the following objectives

- ▶ Learning Objectives
 - ▶ Define fixed and random effects
 - ▶ Write out the models used and the assumptions for inference
 - ▶ Develop a statistical toolbox for analyzing linear mixed models
 - ▶ Interpret the respective R outputs
- ▶ Reference: *SJS, Chapter 10*

Example I: Write out the fitted models (tabulate their parameters)

$y \sim Z_{\text{random}} + \text{fixed}$

Model

of parameters

$\sum \sigma_u^2$ β
2 $\swarrow \searrow$ σ_e^2 4

► 10.6 : $y_{ijk} = \beta_0 + \beta_1 \text{Age} + \beta_2 1_{\text{sex}} + \beta_3 \text{Age} \times 1_{\text{sex}} + u_{ij} + \epsilon_{ijk}$

► 10.5 : $y_{ijk} = \beta_0 + \beta_1 \text{Age} + \beta_2 1_{\text{sex}} + \beta_3 \text{Age} \times 1_{\text{sex}} + u_{ij} + \epsilon_{i, \text{sex}, k}$

► 10.5c : $y_{ijk} = \beta_0 + \beta_1 1_{\text{Age}10} + \beta_2 1_{\text{Age}12} + \beta_3 1_{\text{Age}14} + \beta_4 1_{\text{sex}} + \beta_5 1_{\text{Age}10} 1_{\text{sex}} + \beta_6 1_{\text{Age}12} 1_{\text{sex}} + \beta_7 1_{\text{Age}14} 1_{\text{sex}} + u_{ij} + e_{i, \text{sex}}^{\text{sex}}$

Females only

► Males only : $y_{ik} = \beta_0 + \beta_1 \text{Age} + u_i + \epsilon_{ik}$

► Females : $y_{ik} = \alpha_0 + \alpha_1 \text{Age} + u_i + \epsilon_{ik}$

3 $\swarrow \searrow$ σ_u^2 $\sigma_{e,F}^2$ $\sigma_{e,M}^2$ 4 } Comparable \sum 's.
3 (same as 10.5) 8
2 $\swarrow \searrow$ σ_u^2 σ_e^2 2
2 2

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Example I: Compare models 10.6 and 10.5 (Refer to notes in R Markdown file)

- ▶ Hypotheses:
- ▶ Test statistic
- ▶ Distribution of test statistic under H_0
- ▶ p -value:
- ▶ Conclusion:
- ▶ Other statistics : AIC , BIC

Comparing models

- Can any other pairs of models be compared? No

Models must be based on the same Y & X
varying by Var-Cov. Structure only.

Find the intraclass correlation coefficients by sex

Intraclass Correlation Coefficient:

$$\rho_{IC} = \frac{\text{Cov}(Y_{ijk}, Y_{ijn})}{\sqrt{\text{Var}(Y_{ijk})\text{Var}(Y_{ijn})}} = \frac{\sigma_u^2}{\sigma_u^2 + \sigma_e^2}$$

Model	Intraclass Correlation Coefficient, ρ_{IC}
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10.6

$$\rho_{IC} = \frac{1.816^2}{1.816^2 + 1.386^2} = 0.632$$

10.5

$$\rho_M = \frac{1.848^2}{1.848^2 + 1.6^2} = 0.55$$

$$\rho_F = \frac{1.848^2}{1.848^2 + (1.6 \times 0.468)^2} = 0.85$$

10.5c

Females only

Males only

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$$D = \begin{pmatrix} \sigma_u^2 & T_1 & T_2 & T_3 & T_4 \\ T_1 & T_1^2 & T_1 T_2 & T_1 T_3 & T_1 T_4 \\ T_2 & T_1 T_2 & T_2^2 & T_2 T_3 & T_2 T_4 \\ T_3 & T_1 T_3 & T_2 T_3 & T_3^2 & T_3 T_4 \\ T_4 & T_1 T_4 & T_2 T_4 & T_3 T_4 & T_4^2 \end{pmatrix}$$

Draw the Variance-Covariance Structure of Model 10.5

$$D_M = \begin{pmatrix} \hat{\sigma}_{e,M}^2 + \hat{\sigma}_u^2 & \hat{\sigma}_u^2 & \hat{\sigma}_u^2 & \hat{\sigma}_u^2 \\ \hat{\sigma}_u^2 & 1.67^2 + 1.85^2 & 1.85^2 & 1.85^2 \\ \hat{\sigma}_u^2 & 1.85^2 & 1.67^2 + 1.85^2 & 1.85^2 \\ \hat{\sigma}_u^2 & 1.85^2 & 1.85^2 & 1.67^2 + 1.85^2 \end{pmatrix}$$

$$\text{Var}(y)$$

$$nk \times nk$$

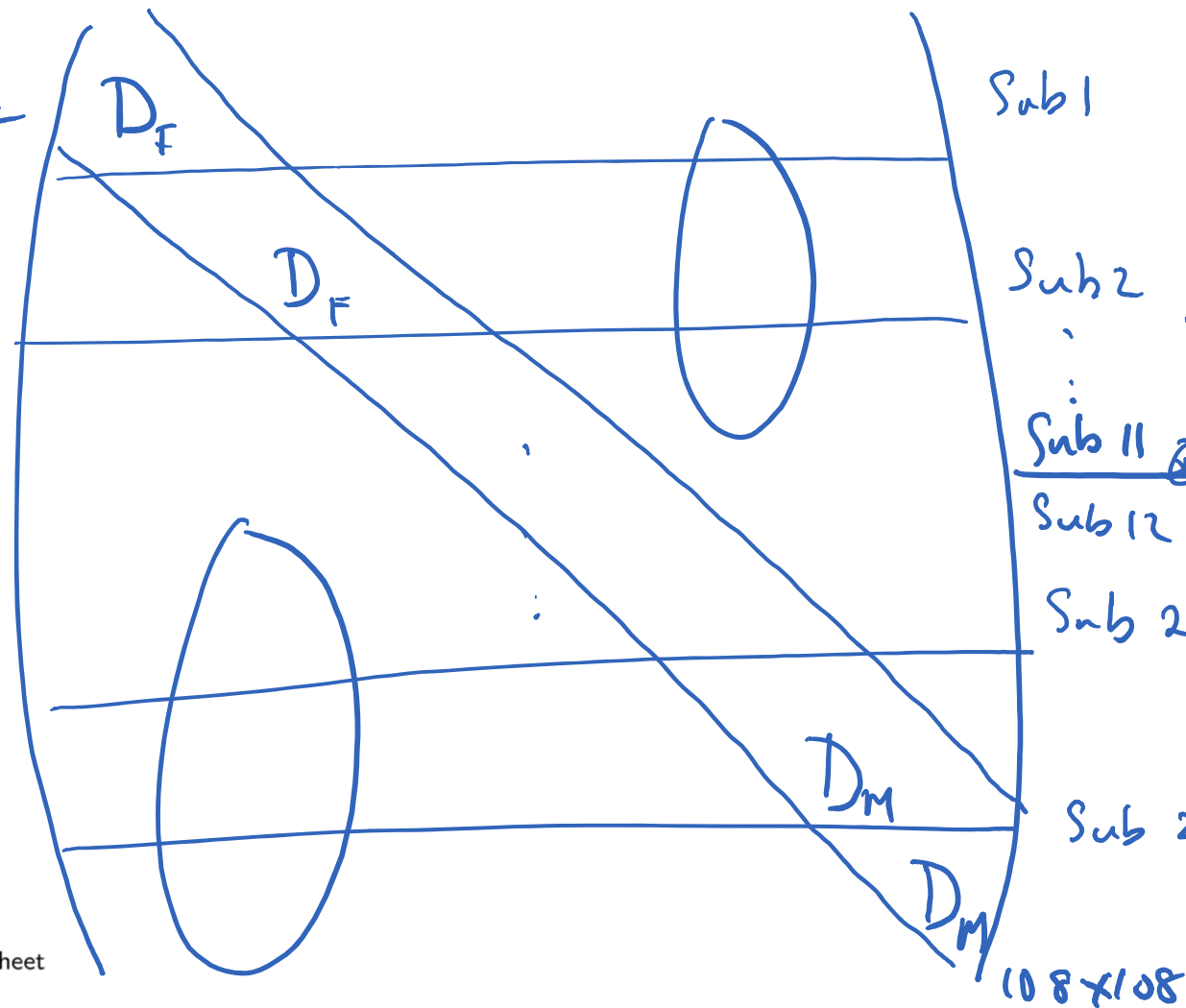
$$108 \times 108$$

$$108 = 27 \times 4$$

$$n = 27$$

$$k = 4$$

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Sub 1

Sub 2

Sub 11

Sub 12

Sub 26

Sub 27

108x108

$$D_F = \begin{pmatrix} 1.85^2 & 1.67^2 + 1.85^2 & 1.85^2 & 1.85^2 \\ 1.85^2 & 1.85^2 & 1.67^2 + 1.85^2 & 1.85^2 \\ 1.85^2 & 1.85^2 & 1.85^2 & 1.67^2 + 1.85^2 \\ 1.85^2 & 1.85^2 & 1.85^2 & 1.85^2 + (1.67 \times 0.47)^2 \end{pmatrix}$$

↓

$$4 \times 4$$

⊗ First 11 subjects are Females and last 16 are Males