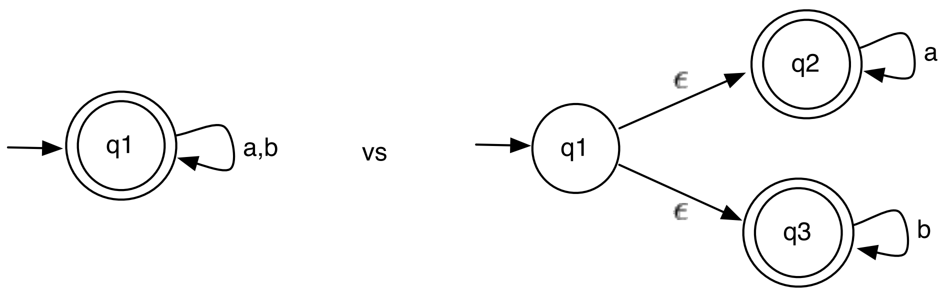
CSC236 Tutorial Exercises, July 26

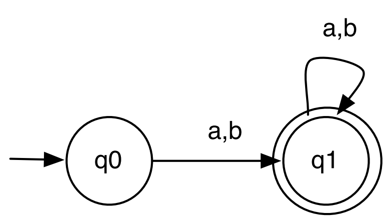
Sample Solutions

Let the alphabet be $\Sigma = \{a, b\}$

1. Are regular expressions $(a+b)^*$ and $a^* + b^*$ equivalent? Explain.
   Solution: Let $R_1 = (a+b)^*$ and $R_2 = a^* + b^*$. $R1 \neq R2$, because R_1 includes all strings in the alphabet Σ while R_2 includes repetitions of a (including 0 repetitions) or repetitions of b, but no strings that include both a and b. The corresponding NFA are different as well:



2. Draw a DFA corresponding to the regular expression $(a+b)(a+b)^*(a^*+b^*)$. Write down the corresponding state invariant that you could use to prove the equality of your DFA to the regular language represented by the provided regexp. You don't need to provide the proof.
   Solution: First, note that $L(a^*+b^*) \subseteq L(a+b)^*$ and not only that, but any string that can be generated by $(a+b)^*(a^*+b^*)$ can be generated by $(a+b)^*$ due to the definition of the Kleene's star. Hence, we can simplify $(a+b)(a+b)^*(a^*+b^*) = (a+b)(a+b)^*$. The corresponding DFA is



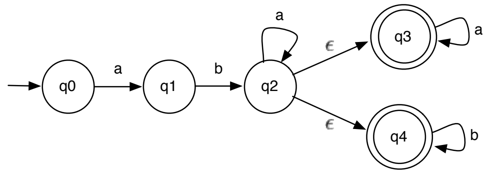
   State invariants are then as follows:

$$\delta^*(q_0, s) = \begin{cases} q_1 & \texttt{if s starts with an } a \texttt{ or a } b \\ q_0 & \texttt{otherwise} \end{cases}$$

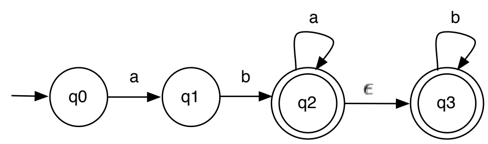
3. Consider a regexp R_1: $a(ba^*)(a^* + b^*)$

   (a) Draw an NFA M_2 corresponding to the R_1 above

   Solution: There are many answers to this question, including

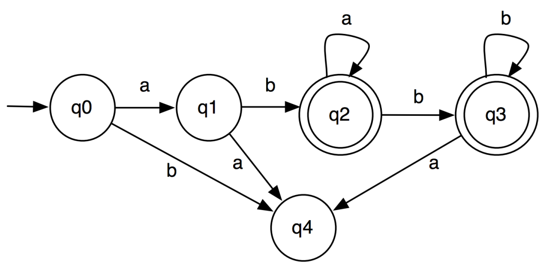
   and

   The second solution follows from $a(ba^*)(a^* + b^*) = ab(a^*a^* + a^*b^*) = ab(a^* + a^*b^*) = aba^*b^*$

   (b) Write down the language $\mathcal{L}$ that it represents (a sentence describing all strings)
   Answer: $\mathcal{L}$ contains all strings that start with ab, concatenated with repetitions of a (including zero repetitions of a) followed by repetitions of b (including zero repetitions of b).

   (c) Draw a corresponding DFA

4. Consider the NFA A with transition relation $\delta = \{(q_0, a, q_1), (q_1, b, q_0), (q_1, b, q_2), (q_2, a, q_0)\}$, with initial state q_0 and final states $F = \{q_0\}$. Use the subset construction to find an equivalent DFA.
   Answer: Since the initial state of the NFA A is q_0 the initial state of the equivalent DFA that we get from the subset construction, call it B, is $\{q_0\}$. The state q_0 has only one transition (q_0, a, q_1), then we have the transition for B; $(\{q_0\}, a, \{q_1\})$. The state q_1 has two transitions $(q_1, b, q_0), (q_1, b, q_2)$, the transition from the state $\{q_1\}$ will be $(\{q_1\}, b, \{q_0, q_2\})$. Similarly, we have the transition relation for the DFA B:

$$\{(\{q_0\}, a, \{q_1\}), (\{q_1\}, b, \{q_0, q_2\}), (\{q_0, q_2\}, a, \{q_0, q_1\}), (\{q_0, q_1\}, a, \{q_1\}), (\{q_0, q_1\}, b, \{q_0, q_2\})\}$$

   The set of accepting states of the DFA B is:

$$F = \{\{q_0\}, \{q_0, q_2\}, \{q_0, q_1\}\}$$