Note: The following are only for your academic curiosity. There will be **no** exercise, quiz, exam that it will require you to know the bellow techniques. The bellow techniques are not in your notes and we do not want to spend time on the details of finding closed form solutions for function defined by induction.

This note is to explain the details of finding a closed form solution for the following function defined by induction:

$$f(n) = \begin{cases} 3 & n = 0 \\ 6 & n = 1 \\ 5f(n-1) - 6f(n-2) + 2 & n > 1 \end{cases}$$

The closed form solution of the above function is not easy to be found by substituting terms. The technique to find such solutions is by using a generating function. This is to define the function:

$$g(x) := \sum_{n=0}^{\infty} f(n) \cdot x^n$$

The idea behind this technique is that the function g is defined for all x. After calculations if we manage to transform $g(x)$ into $\sum_{n=0}^{\infty} h(n) \cdot x^n$, we would have that for all n, $f(n) = h(n)$. More details on the above technique can be found here: `http://www.wikihow.com/Solve-Recurrence-Relations` (another example using this technique can be found in the second answer to the math exchange question: `https://math.stackexchange.com/questions/355963/how-to-derive-a-closed-form-of-a-simple-recursion`)

For linear functions defined by induction, i.e. they are of the form $a_n = c_1 a_{n-1} + \ldots + c_k a_{n-k}$, there is another technique to find closed form solutions. The technique is to use the characteristic polynomial of the recurrence. You can read more about it here; `https://math.berkeley.edu/~arash/55/8_2.pdf`, or here; `http://www.math.northwestern.edu/~mlerma/courses/cs310-04w/notes/dm-recurrences.pdf` (also the it is the forth method of the first link of this document).