Worth: 2% **Due:** By 9:59pm on Monday 3 November

Remember to write your *full name* and *student number* prominently on your submission.

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*Please read and understand the policy on Collaboration given on the Course Information Sheet. Then, to protect yourself, list on the front of your submission **every** source of information you used to complete this homework (other than your own lecture and tutorial notes, and materials available directly on the course webpage). For example, indicate clearly the **name** of every student with whom you had discussions, the **title** of every additional textbook you consulted, the **source** of every additional web document you used, etc.*

*For each question, please write up detailed answers carefully. Make sure that you use notation and terminology correctly, and that you explain and justify what you are doing. Marks **will** be deducted for incorrect or ambiguous use of notation and terminology, and for making incorrect, unjustified, ambiguous, or vague claims in your solutions.*

For each decision problem D below, state whether $D \in P$, $D \in NP$ or $D \in coNP$ and justify each of your claims briefly:

- for decision problems in P, describe an algorithm to solve the problem and briefly argue that your algorithm is correct and runs in polytime;
- for decision problems in NP or $coNP$, describe a suitable verifier for the problem and briefly argue that your verifier is correct and runs in polytime.

Make the strongest claims that you can.

(a) SMALLSUM:

- **Input:** A non-empty finite set of positive integers $S = \{x_1, \ldots, x_n\} \subset \mathbb{Z}^+$, and a positive integer $t \in \mathbb{Z}^+$ (all integers represented in binary).
- **Output:** Does **some** non-empty subset of S have sum **at most** t? ($\exists S' \subseteq S, S' \neq \emptyset \wedge \sum_{x \in S'} x \leqslant t$?)

(b) LARGESUMS:

- **Input:** A non-empty finite set of positive integers $S = \{x_1, \ldots, x_n\} \subset \mathbb{Z}^+$, and a positive integer $t \in \mathbb{Z}^+$ (all integers represented in binary).
- **Output:** Does **every** non-empty subset of S have sum **at least** t? ($\forall S' \subseteq S, S' \neq \emptyset \Rightarrow \sum_{x \in S'} x \geqslant t$?)

(c) EXACTSUM:

- **Input:** A non-empty finite set of positive integers $S = \{x_1, \ldots, x_n\} \subset \mathbb{Z}^+$, and a positive integer $t \in \mathbb{Z}^+$ (all integers represented in binary).
- **Output:** Does **some** non-empty subset of S have sum **exactly** t? ($\exists S' \subseteq S, S' \neq \emptyset \wedge \sum_{x \in S'} x = t$?)