NOTE: This file contains sample solutions to the quiz together with the marking scheme and comments for each question. Please read the solutions and the marking schemes and comments carefully. Make sure that you understand why the solutions given here are correct, that you understand the mistakes that you made (if any), and that you understand *why* your mistakes were mistakes.

Remember that although you may not agree completely with the marking scheme given here, it was followed the same way for all students. We will remark your quiz only if you clearly demonstrate that the marking scheme was not followed correctly.

For all remarking requests, please submit your request **in writing** directly to your instructor. For all other questions, please don't hesitate to ask your instructor during office hours or by e-mail.

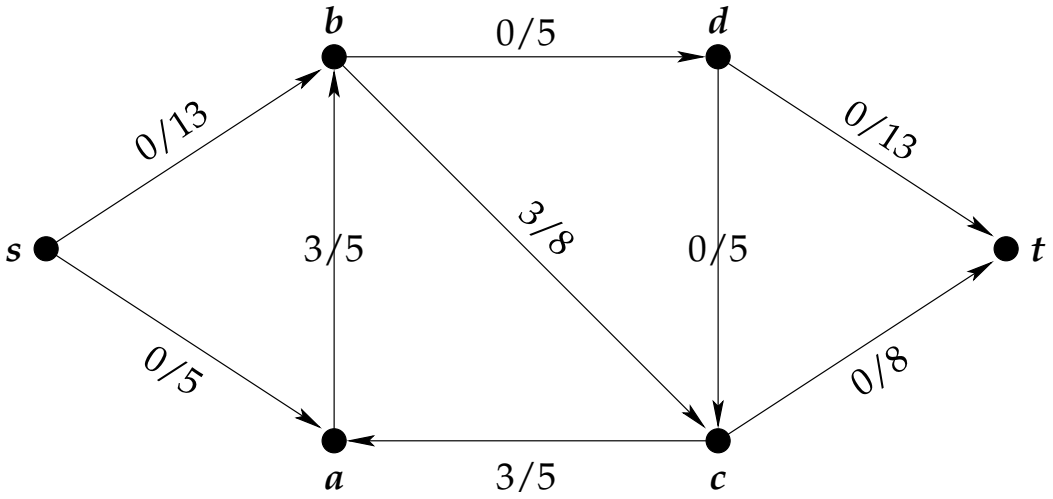
GENERAL MARKING SCHEME:

- ***A****: All Correct*, except maybe for very few minor errors.
- ***B****: Mostly Correct*, but with a few serious errors, or many small errors.
- ***C****: Mostly Incorrect*, but with a few important elements, or many small elements, done correctly.
- ***10%****: Completely Blank*, or clearly crossed out.
- ***D****: All Incorrect*, except maybe for very few minor elements done correctly.

MARKER'S COMMENTS: *Generally well done.*

- **error code E1**: Using only forward edges on augmenting paths does not yield a maximum flow.

1. Find a maximum flow in the following network, then give convincing evidence that your flow is maximum. Explain what you are doing and show your work (in particular, show explicitly all augmenting paths used).



- First, augment along path: $s \xrightarrow{0/13} b \xrightarrow{0/5} d \xrightarrow{0/13} t$. (Add 5 to each forward edge's flow.)
- Next, augment along path: $s \xrightarrow{5/13} b \xrightarrow{3/8} c \xrightarrow{0/8} t$. (Add 5 to each forward edge's flow.)
- Finally, augment along path: $s \xrightarrow{0/5} a \xleftarrow{3/5} c \xrightarrow{5/8} t$. (Add 3 to each forward edge's flow and subtract 3 from each backward edge's flow.)

The final flow value is $|f| = f(s,b) + f(s,a) = 10 + 3 = 13$.

This flow is maximum because the cut $X = \big(\{s,a,b\},\{c,d,t\}\big)$ has capacity $c(X) = c(b,d) + c(b,c) = 5 + 8 = 13$, same as the flow value.