CSC236 Summer 2017
Assignment #3: Formal Languages and Finite Automata
Due August 10th, by 6:00 pm

The aim of this assignment is to give you some practice in formal languages and finite automata.

For each question, please write up detailed answers carefully. Make sure that you use notation and terminology correctly, and that you explain and justify what you are doing. Marks will be deducted for incorrect or ambiguous use of notation and terminology, and for making incorrect, unjustified, ambiguous, or vague claims in your solutions. You should clearly cite any sources or people you consult, other than the course notes, lecture materials, and tutorial exercises.

Your assignment must be typed to produce a PDF document **A3.pdf** (hand-written submissions are not acceptable). You may work on the assignment in groups of 1, 2, or 3, and submit a single assignment for the entire group on MarkUs.

1. Give incomplete DFA's (not including the dead states) accepting the following languages over the alphabet $\{0,1\}$:

    (a) The set of all strings with three consecutive 1's (these instances of 1's can appear at any part of the string).

    (b) The set of all strings with the 101 as a substring.

    (c) The set of strings ending in 11.

2. Let A be a DFA and q a particular state of A, such that $\delta(q,a) = q$ for all input symbols a. Show by induction on the length of the input that for all input strings w, we have $(q,w) \vdash^* (q,\varepsilon)$.

3. Draw the state diagram of the following NFA and convert it to a DFA. Informally describe the language it accepts.

	0	1
$\to p$	$\{p,q\}$	$\{p\}$
q	$\{r,s\}$	$\{s\}$
r	$\{p,r\}$	$\{s\}$
$*s$	$\emptyset$	$\emptyset$

4. Write regular expressions for the following languages:

    (a) The set of strings of 0's and 1's whose seventh symbol from the right end is 1.

    (b) The set of 0's and 1's with at most one pair of consecutive 1's.

    (c) The set of all strings of 0's and 1's not containing 110 as a substring.

5. Prove or disprove each of the following statements about regular expressions.

    (a) $(R+S)^* = R^* + S^*$

    (b) $(RS+R)^* RS = (RR^*S)^*$

    (c) $(R+S)^* = (R^*S)^*$

    (d) $S(RS+S)^* R = RR^*S(RR^*S)^*$

6. The finite automaton if Figure 1 accepts no words of length zero or length one, it accepts only two words of length two (01 and 10). There is a recurrence equation for the number $N(k)$ of words of length k that this automaton accepts (i.e. $N(0) = 0$, $N(1) = 0$, $N(2) = 2$, $N(3) = 0$, $N(4) = 2$ etc.). Discover this recurrence and demonstrate your understanding by identifying the number $N(14)$ (give a short explanation not just a random number).
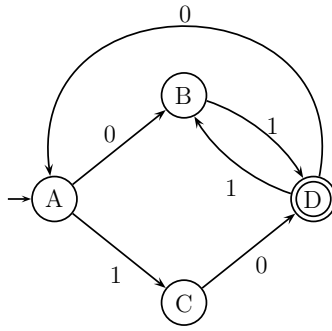
Figure 1: A DFA accepting a language over the alphabet $\{0, 1\}$.

7. Prove that the following languages are not regular. You may use the pumping lemma and the closure properties of regular languages under union, intersection and complement.

   (a) $L = \{0^n 1^m 0^n \mid m, n \geq 0\}$.

   (b) $L = \{w \in \{0, 1\}^* \mid w \text{ is palindrome}\}$.
       (A palindrome is a string that reads the same forward and backward. e.g.: racecar)

8. Show that the regular languages are closed under the following operation.

$$\frac{1}{2}(L) = \{x \in \Sigma^* \mid \text{there exists } y \in \Sigma^* \text{ with } |y| = |x| \text{ such that } xy \in L\}.$$