(a) None of the "proofs" have shown that DS $\in$ *NP*. Consider the following verifier.

```
V(G, k, C):
    if C ⊆ V and |C| = k and ∀u ∈ V − C, ∃v ∈ C, (u, v) ∈ E:
        return True
    return False
```

$V(G,k,C)$ runs in polynomial time: there are fewer than n^2 pairs of vertices to check (and no more than m edges to verify for each pair of vertices) to confirm or deny that C forms a dominating set in G. Also, $V(G,k,C) = \textsc{True}$ for some C iff G contains some dominating set of size k—because $V(G,k,C)$ returns True exactly when C forms a dominating set of size k in G.

(b) 1. Incorrect.
- Wrong direction: showing DS $\leqslant_p$ VertexCover does *not* imply that DS is *NP*-hard.
- The reduction function does not depend on input (G,k) alone: it is described in terms of a dominating set D, even though D is *not* part of the input.
- The last part of the correctness argument is vague (representing "wishful thinking"): removing "the" ℓ extra vertices from C is not something trivial to do and should be explained in detail—where the writer would realize that it is not actually possible.

2. Incorrect.
- The reduction function does not preserve answers: consider the input $(G = (V,E),k)$ with $V = \{a,b,c\}$, $E = \{\{a,b\},\{b,c\},\{c,a\}\}$, and $k = 1$, then G does *not* contain any vertex cover of size k but G *does* contain a dominating set of size k.
- The argument of correctness is not an argument at all but just a statement—another case of "wishful thinking."

3. Correct—almost...
- The reduction is correct for all inputs (G,k) where $k \leqslant n = |V|$, but it fails when $k > n$: then, G does not contain a vertex cover of size k but G' will contain a dominating set of size $n+m$ ($D = V'$).
  Easy fix: output $k' = k+m$ when $k > n$. Then both (G,k) and (G',k') are no-instances.

4. Incorrect.
- The reduction function does not depend on input (G,k) alone: it is described in terms of a vertex cover C, even though C is *not* part of the input.
- The argument of correctness is mixed in with the description of the reduction function, and more importantly, the "if" direction of the argument is missing.