STA304 A2 Q2

Ruijie Sun

November 2019

a)

$a_1 = \frac{35000}{90000} = 0.389$

$a_2 = \frac{45000}{90000} = 0.5$

$a_3 = \frac{10000}{90000} = 0.111$

$D = \frac{B^2}{4} = \frac{0.01}{4} = 0.0025$

$N^2D = 90000^2 * 0.0025 = 20250000$

$\sum_{i=1}^{3}(N_i)^2 * p_i * q_i/a_i = 35000^2 * 0.47 * 0.53/0.389 + 45000^2 * 0.23 * 0.77/0.5 + 10000^2 * 0.03 * 0.97/0.111 = 1527912090$

$\sum_{i=1}^{3} N_i * p_i * q_i = 35000 * 0.47 * 0.53 + 45000 * 0.23 * 0.77 + 10000 * 0.03 * 0.97 = 16979$

$n = \frac{\sum_{i=1}^{3}(N_i)^2 * p_i * q_i/a_i}{N^2D + \sum_{i=1}^{3} N_i * p_i * q_i} = \frac{1527912090}{20250000+16979} = 75.39$ or 76

$n_1 = n * 0.389 = 29.6$ or 30

$n_2 = n * 0.5 = 38$

$n_3 = n * 0.111 = 8.4$ or 8

b)

$\sum_{i=1}^{3} N_i * \sqrt{\frac{\hat{p_i} * \hat{q_i}}{c_i}} = 35000 * \sqrt{\frac{0.47 * 0.53}{9}} + 45000 * \sqrt{\frac{0.23 * 0.77}{10}} + 10000 * \sqrt{\frac{0.03 * 0.97}{16}} = 5822.824 + 5988.552 + 426.468 = 12237.844$

$n_1 = n * (\frac{N_1 * \sqrt{\hat{p_i} * \hat{q_i}/c_1}}{\sum_{i=1}^{3} N_i * \sqrt{\frac{\hat{p_i} * \hat{q_i}}{c_i}}}) = n(\frac{5822.824}{12237.844}) = n(0.476)$

Similarly,

$n_2 = n(\frac{5988.552}{12237.844}) = n(0.489)$

$n_3 = n(\frac{426.468}{12237.844}) = n(0.035)$

Thus, $a_1 = 0.476, a_2 = 0.489, a_3 = 0.035$

$\sum_{i=1}^{3}(N_i)^2 * \hat{p_i} * \hat{q_i}/a_i = \frac{(35000)^2(0.47)(0.53)}{0.476} + \frac{(45000)^2(0.23)(0.77)}{0.489} + \frac{(10000)^2(0.03)(0.97)}{0.035} = 1457598604$

$\sum_{i=1}^{3}(N_i) * \hat{p_i} * \hat{q_i} = 35000 * 0.47 * 0.53 + 45000 * 0.23 * 0.77 + 10000 * 0.03 * 0.97 = 16979$

$D = \frac{B^2}{4} = \frac{0.01}{4} = 0.0025$

$N^2D = 90000^2 * 0.0025 = 20250000$

$n = \frac{\sum_{i=1}^{3}(N_i)^2 * \hat{p_i} * \hat{q_i}/a_i}{N^2D + \sum_{i=1}^{3} N_i * \hat{p_i} * \hat{q_i}} = \frac{1457598604}{20250000+16979} = 71.9$ or 72

Hence,

$n_1 = na_1 = (72))(0.476) = 34.27 \approx= 34$

$n_2 = na_2 = (72))(0.489) = 35.2 \approx= 35$

$n_3 = na_3 = (72))(0.035) = 2.5 \approx= 3$

c)
all c_i can be replaced by 1 now.

$\sum_{i=1}^{3} N_i * \sqrt{\hat{p_i} * \hat{q_i}} = 35000\sqrt{(0.47)(0.53)} + 45000\sqrt{(0.23)(0.77)} + 10000\sqrt{(0.03)(0.97)} = 38111.807$

$n_1 = n(\frac{N_1\sqrt{\hat{p_1}\hat{q_1}}}{\sum_{i=1}^{3} N_i * \sqrt{\hat{p_i} * \hat{q_i}}}) = n(\frac{17468.472}{38111.807}) = n(0.458)$

Similarly,

$n_2 = n(\frac{18937.463}{38111.807}) = n(0.497)$

$n_3 = n(\frac{1705.872}{38111.807}) = n(0.045)$

Thus, $a_1 = 0.458, a_2 = 0.497, a_3 = 0.045$

$\sum_{i=1}^{3} N_i * \hat{p_i} * \hat{q_i} = 16979$

$D = \frac{B^2}{4} = \frac{0.01}{4} = 0.0025$

$N^2D = 90000^2 * 0.0025 = 20250000$

$n = \frac{(\sum_{i=1}^{3} N_i * \sqrt{\hat{p_i} * \hat{q_i}})^2}{N^2D + \sum_{i=1}^{3} N_i * \hat{p_i} * \hat{q_i}} = \frac{(38111.807)^2}{20250000+16979} = 71.67 \approx 72$

$n_1 = na_1 = 72(0.458) = 32.98 \approx 33$

$n_2 = na_2 = 72(0.497) = 35.78 \approx 36$

$n_3 = na_3 = 72(0.045) = 3.24 \approx 3$