# CSC236 Summer 2017
# Assignment #1: Induction
# **Due June 8th, by 6:00 pm**

The aim of this assignment is to give you some practice with various forms of induction. For each question below you will present a proof by induction, using the type of induction specified. For full marks on your proofs, you will need to make it clear to the reader that the base case(s) is/are verified, that the inductive step follows for each element of the domain (typically the natural numbers), where the inductive hypothesis is used and that it is used in a valid case.

Your assignment must be typed to produce a PDF document **a1.pdf** (hand-written submissions are not acceptable). You may work on the assignment in groups of 1, 2, or 3, and submit a single assignment for the entire group on MarkUs.

1. Consider the Fibonacci function $f$:

   $$f(n) = \begin{cases} 1, & \text{if } n = 0 \text{ or } n = 1 \\ f(n-2) + f(n-1) & \text{if } n > 1 \end{cases}$$

   Use simple induction to prove that if $n$ is a natural number, then $f(0) + f(2) + \cdots + f(2n) = f(2n+1)$.

   You may **not** derive or use a closed-form for $f(n)$ in your proof.

2. Use simple induction to show that $x^2 - 1$ is divisible by 8 for any odd natural number $x$.

3. Can we represent any amount with coins of denominations 3 and 5? If yes, prove your answer, if not, can we find a number that any amount greater or equal to it we can represented it with the above coins? Prove your answer.

4. Use the Well-Ordering Principle to show that given any natural number $n \geq 1$, there exists an odd integer $m$ and a natural number $k$ such that $n = 2^k * m$.

5. Define a set $M \subseteq \mathbb{Z}^2$ as follows:

   (a) $(3, 2) \in M$,

   (b) for all $(x, y) \in M$, $(3x - 2y, x) \in M$,

(c) nothing else belongs to $M$.

Use structural induction to prove that $\forall (x, y) \in M, \exists k \in \mathbb{N}, (x, y) = (2^{k+1} + 1, 2^k + 1)$.

6. Suppose $n$ people are positioned such that each person has a unique nearest neighbour. Each person has a single water balloon that they throw at their nearest neighbour. (We'll assume every throw hits its target.) A dry person is one who is not hit by a water balloon.

   (a) Describe an example that demonstrates than if $n$ is even, there may be no dry person.

   (b) Use simple induction to show that if $n$ is odd, then there is always at least one dry person.

7. Let $P$ be a convex polygon with consecutive vertices $v_1, v_2, ..., v_n$. Use complete induction to show that when $P$ is triangulated into $n-2$ triangles, the $n-2$ triangles can be numbered $1, 2, ..., n-2$ so that $v_i$ is a vertex of triangle $i$ for $i = 1, 2, ..., n-2$.