

NOTE: I use set notation “ $\{u, v\}$ ” to represent undirected edges.

- (a) There is **no** solution for the input (G, w, L) with $G = (V, E)$ where $V = \{a, b, c\}$, $E = \{\{a, b\}, \{b, c\}\}$, $w(a, b) = w(b, c) = 1$ and $L = \{b\}$.
- (b) *Main idea:* For each required leaf, use a minimum-weight edge to connect it to the rest of the spanning tree, then complete the rest of the spanning tree using any algorithm. (Alternatively, we could begin by finding a MST on the graph without all of the leaves, then connect all of the leaves back using a minimum-weight edge for each one.) **One “trap” to watch out for: each leaf must be connected to some non-leaf.**

MST($G = (V, E), w, L$):

$T \leftarrow \emptyset$

Use a minimum-weight edge for each leaf.

for $v \in L$:

 let $\{u, v\}$ be a minimum-weight edge containing v **with $u \notin L$**

$T \leftarrow T \cup \{\{u, v\}\}$

 remove all edges that contain v (including $\{u, v\}$) from E

Run Kruskal’s algorithm on the remaining graph, starting with the current value of T , to finish the spanning tree.

- (c) We know Kruskal’s algorithm finds a MST on the portion of the graph without the leaves. Moreover, for each leaf, using any edge other than a minimum-weight edge yields a spanning tree whose weight is not minimum.