CSC373 Winter 2015 Problem Set # 6

Name: Weidong An
Student Number: 1000385095
UTOR email: weidong.an@mail.utoronto.ca
March 1, 2015

Construct linear program:

- Variables: f_e for each $e \in E$
- Constraints:
  - Capacity constraint: $0 \leq f_e \leq c(e)$ for all $e \in E$
  - Conservation: $\alpha_u \sum_{(u,v)\in E} f_{(u,v)} \leq \sum_{(v,w)\in E} f_{(v,w)} \leq \beta_u \sum_{(u,v)\in E} f_{(v,w)}$ for all $v \in V - \{s,t\}$
- Objective function: maximize $\sum_{(s,u)\in E} f_{(s,u)}$

Correctness:

- Any valid flow f yields a feasible solution because the representation of this linear program is exactly same as the representation of Partially Conserved Maximum Flow problem. Therefore, maximum objective value $\geq max\{f^{out}(s)\}$
- Any feasible solution yield a valid flow f because the representation of Partially Conserved Maximum Flow problem is exactly same as the representation of this linear program. Therefore, $max\{f^{out}(s)\} \geq$ maximum objective value

Hence, maximum objective value $= max\{f^{out}(s)\}$