STA305/1004 - Class 3

(adapted from N. Taback)

January 13-14, 2020

Class Outline

- Randomized Experiments vs Observational Studies
- The concepts of:
  1. Randomization
  2. Blocking
  3. Replication
- Summaries of sample populations
- Hypothesis testing via randomization

Quote of the week:
"*Designing an experiment is like gambling with the devil: Only a random strategy can defeat all his betting systems.* (R. A .Fisher)

What is an Observational Study?

- Definition by Imbens and Rubin, 2015
- Assignment mechanism: the process that determines which experimental units receive which treatments
- When the assignment mechanism is unknown then the design is called an OBSERVATIONAL STUDY.

What are Randomized Experiments?

- pg. 20, Imbens and Rubin, 2015
- In RANDOMIZED EXPERIMENTS: "... the assignment mechanism is under the control of the experimenter, and the probability of any assignment of treatments across the units in the experiment is entirely knowable before the experiment begins."

Treatment Assignment Example

Suppose that we have two breast cancer patients and we want to randomly assign these two patients to two treatments (A and B). Then how many ways can this be done?

1. patient 1 receives A and patient 2 receives A
2. patient 1 receives A and patient 2 receives B
3. patient 1 receives B and patient 2 receives A
4. patient 1 receives B and patient 2 receives B

- There are 4 possible treatment assignments.
- The probability of a treatment assignment is $1/4$.
- The probability that an individual patient receives treatment A (or B) is $1/2$.
- In general, if there are N experimental units and two possible treatments for each unit, then there are 2^N possible treatment assignments.

Treatment Assignment Vector Notation

A treatment assignment vector records the treatment that each experimental unit is assigned to receive. If $N = 2$ then the possible treatment assignment vectors are:

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \end{pmatrix},$$

where 1= Treatment A, and 0=Treatment B.

Treatment Assignment Example (cont'd)

- It wouldn't be a very informative experiment if both patients received A or both received B.
- Therefore, it makes sense to rule out this scenario.
- We want to assign treatments to patients such that one patient receives A and the other receives B.
- Then the possible treatment assignments are:

1. patient 1 receives A and patient 2 receives B or (in vector notation) $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$.
2. patient 1 receives B and patient 2 receives A or (in vector notation) $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

- In this case the probability of a treatment assignment is $1/2$, and the probability that an individual patient receives treatment A (or B) is still $1/2$.

Randomized Experiments vs Observational Studies

- RANDOMIZED EXPERIMENTS are currently viewed as the most credible basis for determining cause and effect relationships.
- Health Canada, the U.S. Food and Drug Administration, European Medicines Agency, and other regulatory agencies all rely on randomized experiments in their approval processes for pharmaceutical treatments.

Why Randomization?

- The primary objective in the design of experiments is the avoidance of bias or systematic error (Cox and Reid, 2005).
- One way to avoid bias is to use randomization.

Randomization

- Applied to the allocation of experimental units to treatments
- Provides protection to experimenter against variables unknown to experimenter but may impact the response
- Reduces influence of subjective judgement by the experimenter in treatment allocation

Randomization Experiment Example

- National Supported Work demonstration program (NSW)
- Aim of Experiment: Evaluate the effect of on the job training on unemployment (Rosenbaum, pg. 22- 28)
- Treatment: work experience in the form of subsidized employment then individuals transitioned to unsubsidized employment.
- Control: standard social programs
- Response: earnings ($) in 1978.

Randomization Experiment Example (cont'd)

- Later in course we will compare this with observational studies.
- Participants were matched on pre-treatment covariates.
- Results in 185 treated men matched to 185 treated controls.

Randomization Experiment Example: Data

Covariate	Group	
Age (Mean)	Treated	25.82
	Control	25.70
Years of education (Mean)	Treated	10.35
	Control	10.19
Black (%)	Treated	84%
	Control	85%
Married (%)	Treated	19%
	Control	20%
Earnings in 1974 $ (Mean)	Treated	2096
	Control	2009

What is Blocking?

- Definition: divide the observations into groups called BLOCKS, so that observations in a block are collected under relatively similar conditions.
- Example: Suppose that the yield of a manufacturing process for penicillin varies a lot depending on how much of a certain raw material is used in the process. To compare four variants of the manufacturing process we might randomize within blocks of the raw material.

Blocking: Example

- Refer to the NSW experiment
- Similar men were paired:
  - One member of each pair was randomized to subsidized employment.
- The pair of men would form a block.
- Paired experiments are a form of blocking.

What about Replication?

- One of the main principles of experimental design.
- Replication should be carried out several times.
- Q: Which diet, A or B, results in a greater weight loss?
  - A: Replication means that more than one subject should be assigned to the diets.
- This should be done in such a way that the variation among replicates can provide an accurate measure of errors that affect comparisons between A runs and B runs.

Example: Wheat Yield

Is one fertilizer better than another in terms of yield?

- What is the outcome variable?
- What is the factor of interest?

Example: Wheat Yield

Experimental material?

Plot 1	Plot 2	Plot 3	Plot 4	Plot 5	Plot 6
Plot 7	Plot 8	Plot 9	Plot 10	Plot 11	Plot 12

Example: Wheat Yield

How should we assign treatments/factor levels to plots?

- We want to make sure that we can identify the treatment effect in the presence of other sources of variation.
- What other (besides fertilizer) potential sources could cause variation in wheat yield?

Example: Wheat Yield

- Assigning treatments randomly avoids any pre-experimental bias.
- Assignment mechanism: 12 playing cards, 6 red, 6 black were shuffled (well/ many times) and dealt
  - 1st card black $\rightarrow 1^{st}$ plot gets B
  - 2nd card red $\rightarrow 2^{nd}$ plot gets A
  - 3rd card black $\rightarrow 3^{rd}$ plot gets B
  - and so on
- Completely randomized design

Wheat Yield Example

B 26.9	A 11.4	B 26.6	A 23.7	B 25.3	B 28.5
B 14.2	A 17.9	A 16.5	A 21.1	B 24.3	A 19.6

- Evidence that fertilizer type is different?
- Evidence about differences between two populations is generally measured by comparing summary statistics across two sample populations.
- A statistic is any computable function of the observed data.