Let U be a subset of a vector space V.

a) Show that span$U$ is the intersection of all the subspaces of V that contain U

b) what does this say if $U = \phi$?

a) WTS $\operatorname{span} U = \bigcap \{W, \text{a subspace of } V \mid U \subseteq W\}$

pf:

Step 1: Show $\operatorname{Span} U \subseteq \bigcap \{W, \text{a subspace of } V \mid U \subseteq W\}$

Span$U$ only has linear combinations of vectors in U, so every vector in Span U has to be in every vector space W that contains U.

$\Rightarrow \operatorname{Span} U \subseteq \bigcap \{W, \text{a subspace of } V \mid U \subseteq W\}$

Step 2: Show $\cap\{W, \text{a subspace of } V \mid U \subseteq W\} \subseteq \operatorname{Span} U$

And we know $\operatorname{Span} U$ is a subspace W^* of V that contains U which means

$$\cap\{W, \text{a subspace of } V \mid U \subseteq W\} \subseteq W^* = \operatorname{Span} U$$

Since we know:

1. $\operatorname{Span} U \subseteq \cap\{W, \text{a subspace of } V \mid U \subseteq W\}$
2. $\cap\{W, \text{a subspace of } V \mid U \subseteq W\} \subseteq \operatorname{Span} U$

Thus $\operatorname{span} U = \cap\{W, \text{a subspace of } V \mid U \subseteq W\}$

b) $\operatorname{Span} \phi = \{\vec{0}\}$

Since $\operatorname{Span} \phi = \cap\{W, \text{a subspace of } V \mid \phi \subseteq W\} = \{\vec{0}\}$