MODELING PRIVATE ASSETS

JEN-WEN LIN, PhD, CFA

July 28, 2017

INTRODUCTION

- This presentation aims at introducing a new one-step estimation method for estimating factor loadings of alternative assets using appraisal returns.
- For the purpose of self-containedness, we will also review the existing two-step method and discuss the issues of using this method.

AGENDA

- Review two-step estimation with empirical studies
  1. Regress "unsmoothed" appraisal returns against factors (conventional unsmoothing approach)
  2. Regress appraisal returns against "smoothed" factors (Pedersen, Page and He, 2014)
- Discuss the challenges of one-step estimation and introduce a new estimation method
  - Regress appraisal returns against contemporaneous and lagged factors directly
  - Empirical studies on PE and RE are conduced to demonstrate the effectiveness of the proposed method

4

Econometric model for appraisal formula and two-step approach

WHAT'S WRONG WITH APPRAISAL RETURNS

- Appraisal returns are found to be smoothed and serially correlated. In the literature, we refer to this phenomenon as a stale-pricing bias
- Stale-pricing bias leads to underestimation of volatilities and inaccurate statistical inference on factor exposures

Types	Mean	Volatility	Autocorrelation function			
			1	2	3	4
Public RE	12.7%	22.3%	-0.05	-0.19	0.04	-0.1
Private RE	9.3%	4.5%	**0.63**	**0.31**	**0.29**	**0.33**
De-levered	9.7%	10.4%	0.17	-0.06	-0.06	0.03

Note: NCREIF and REIT data from 1994Q1 to 2013Q4 are used in the above analysis, where for simplicity, the weights 40%, 30%, 20% and 10% are assigned to four sectors, Office, Retail, Apartment, and Industrial, respectively, to construct RE proxies for both public and private real estate investments.

APPRAISAL RETURNS AND ECONOMIC RETURNS

- An appraisal formula mimics how assets are appraised using current and lagged economic returns
  - An appraisal formula connects appraisal returns to economic returns.
  - Economic returns are usually estimated by reversing the appraisal formula.
- Dimson's model is widely seen as a general appraisal formula

$$y_t = w_0 r_t + w_1 r_{t-1} + \cdots + w_m r_{t-m}$$

r's and *w*'s denote the contemporaneous and lagged economic returns and the weights on the corresponding economic returns, respectively.

- Elroy Dimson (1979), "Risk measurement when shares are subject to infrequent trading", *Journal of Financial Economics*, Vol. 7, pp. 197-226.

APPRAISAL RETURNS AND ECONOMIC RETURNS

- Model assumptions

$$y_t = \sum_{i}^{m} w_i r_{t-i} \qquad w_i \geq 0, \sum w_i = 1$$

General appraisal formula

$$r_t - r_f = \alpha + \sum_{j}^{k} \beta_j f_{j,t} + u_t$$

Linear factor model

- Putting them together

$$y_t = r_f + \alpha + \sum_{i}^{m} \sum_{j}^{k} w_i \beta_j f_{j,t-i} + \xi_t$$

Equation (*)

ξ_t is serially correlated

Two-step approach (1)

"Unsmoothing" based on Getmansky et al. (2004, JFE)

- Getmansky, Lo, and Makarov (2004, GLM hereafter) estimate Dimson's model "directly" using a *constrained* moving average process of order m

$$y_t = \theta_0 a_t + \theta_1 a_{t-1} + \cdots + \theta_m a_{t-m}, \qquad \theta_0 = 1, \theta_i \geq 0, i = 1, \dots, m$$

- GLM show that we can retrieve the weight function w's and r's simultaneously by re-parameterizing the above MA process.

$$y_t = \underbrace{\frac{\theta_0}{\sum \theta_i}}_{w_0} \underbrace{\sum \theta_i a_t}_{r_t} + \underbrace{\frac{\theta_1}{\sum \theta_i}}_{w_1} \underbrace{\sum \theta_i a_{t-1}}_{r_{t-1}} + \cdots + \underbrace{\frac{\theta_m}{\sum \theta_i}}_{w_m} \underbrace{\sum \theta_i a_{t-m}}_{r_{t-m}}$$

- The economic returns are calculated as the scaled residuals

$$r_t = a_t \cdot \sum_{i=0}^{m} \theta_i, \qquad w_i = \frac{\theta_i}{\sum_{i=0}^{m} \theta_i}, \qquad i = 0,1, \dots, m$$

Two step approach (2)

Pedersen et al. (2014, FAJ)

$$y_t = r_f + \alpha + \sum_i^m \sum_j^k w_i \beta_j f_{j,t-i} + \xi_t$$

- Rearranging eqn (*), we have

$$y_t = \alpha + \sum_j^k \beta_j \underbrace{\sum_i^m w_i f_{j,t-i}}_{X_{j,t}} + \xi_t$$

1. Estimate $\{w_i\}$ based on GLM;
2. Calculate "smoothed" factors as

$$X_{j,t} = \sum_i^m \widehat{w}_i f_{j,t-i}, i = 1, \dots, m, j = 1, \dots, k$$

   where $\widehat{w}_i$ denotes the weight obtained in step 1;
3. Estimate factor loadings using the following equation

$$y_t = \alpha + \sum_j^k \beta_j X_{j,t} + \xi_t,$$

   where the error terms are serially correlated.

Two step approaches

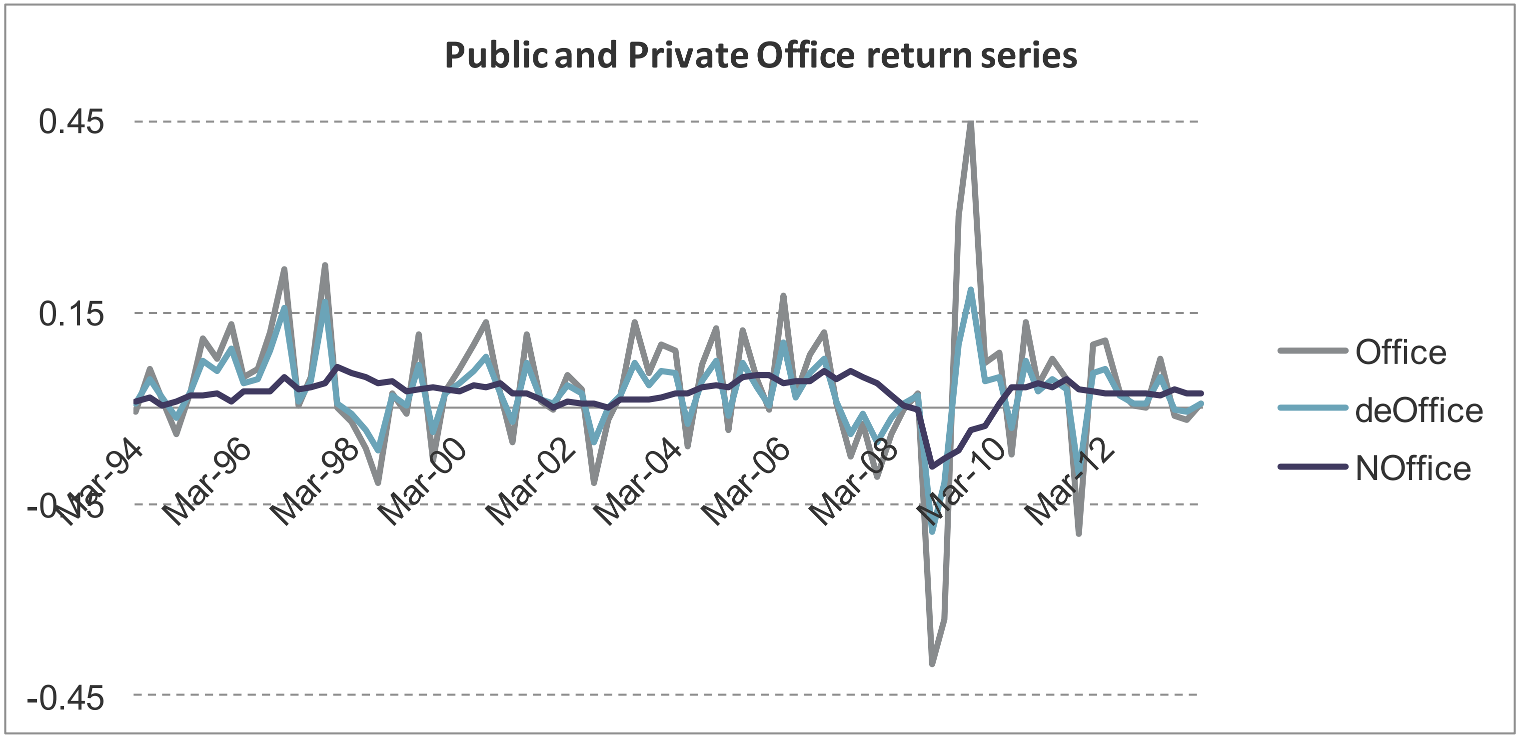
Discussion

- Both (two-step) methods retrieve weights $\{w_i\}$ using only appraisal returns (history)
- Both methods have the problem of Errors In Variable (EIV)
  - The conventional unsmoothing method has the EIV problem at the left-hand side (dependent variable)
  - The approach of Pedersen et al. (2014, PPH hereafter) has the EIV problem at the right-hand-side (independent variable)
- If the effect of EIV is negligible, both methods should yield similar results if the same factors are used

11

The first empirical study on two-step approach and the EIV problem

OFFICE RETURN SERIES



	Office	deOffice	NOffice
Mean	13.24%	10.04%	9.03%
Volatility	22.62%	11.38%	5.32%
1	0.22	0.21	**0.84**
2	-0.22	-0.10	**0.68**
3	-0.11	-0.01	**0.46**
4	0.00	0.03	**0.26**
Cor with Office	100.00%	97.28%	14.86%
Cor with deOffice	97.28%	100.00%	18.53%

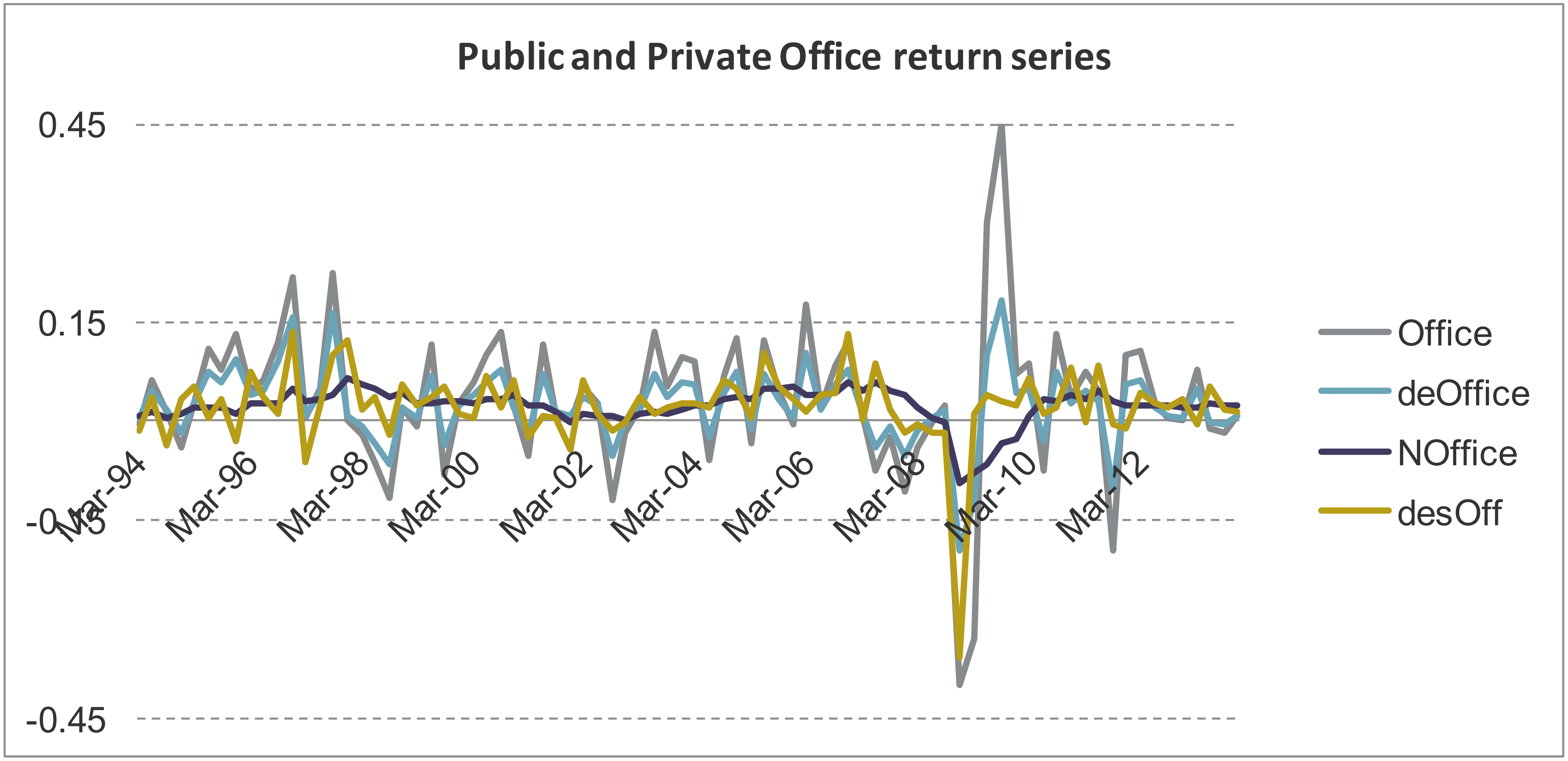
Note: **Office** represents REIT Office return series, **deOffice** represents delivered REIT Office return series, and **NOffice** represents NCREIF Office return series

GLM IMPLEMENTATION

$$y_t = \underbrace{\frac{\theta_0}{\sum \theta_i}}_{w_0} \underbrace{\sum \theta_i a_t}_{r_t} + \underbrace{\frac{\theta_1}{\sum \theta_i}}_{w_1} \underbrace{\sum \theta_i a_{t-1}}_{r_{t-1}} + \cdots + \underbrace{\frac{\theta_m}{\sum \theta_i}}_{w_m} \underbrace{\sum \theta_i a_{t-m}}_{r_{t-m}}$$

Coefficient	Estimate	s.e	t-stat	Weight
theta 0	1	--	--	30.28%
theta 1	0.90	0.12	7.83	27.38%
theta 2	0.89	0.15	5.78	26.94%
theta 3	0.59	0.15	3.93	17.95%
theta 4	0.59	0.11	5.25	17.99%
theta 5	0.32	0.12	2.65	9.74%

OFFICE RETURN SERIES REVISITED



Note: **Office** represents REIT Office return series, **deOffice** represents delivered REIT Office return series, **NOffice** represents NCREIF Office return series, and desOff represents the de-smoothed returns

RETURN COMPARISON

	Office	deOffice	NOffice	desOff
Mean	13.24%	10.04%	9.03%	9.03%
Volatility	22.62%	11.38%	5.32%	11.47%
1	0.22	0.21	**0.84**	0.02
2	-0.22	-0.10	**0.68**	0.08
3	-0.11	-0.01	**0.46**	0.07
4	0.00	0.03	**0.26**	0.02
Correlation with Office	100%	97%	15%	50%
Correlation with deOffice	97%	100%	19%	53%

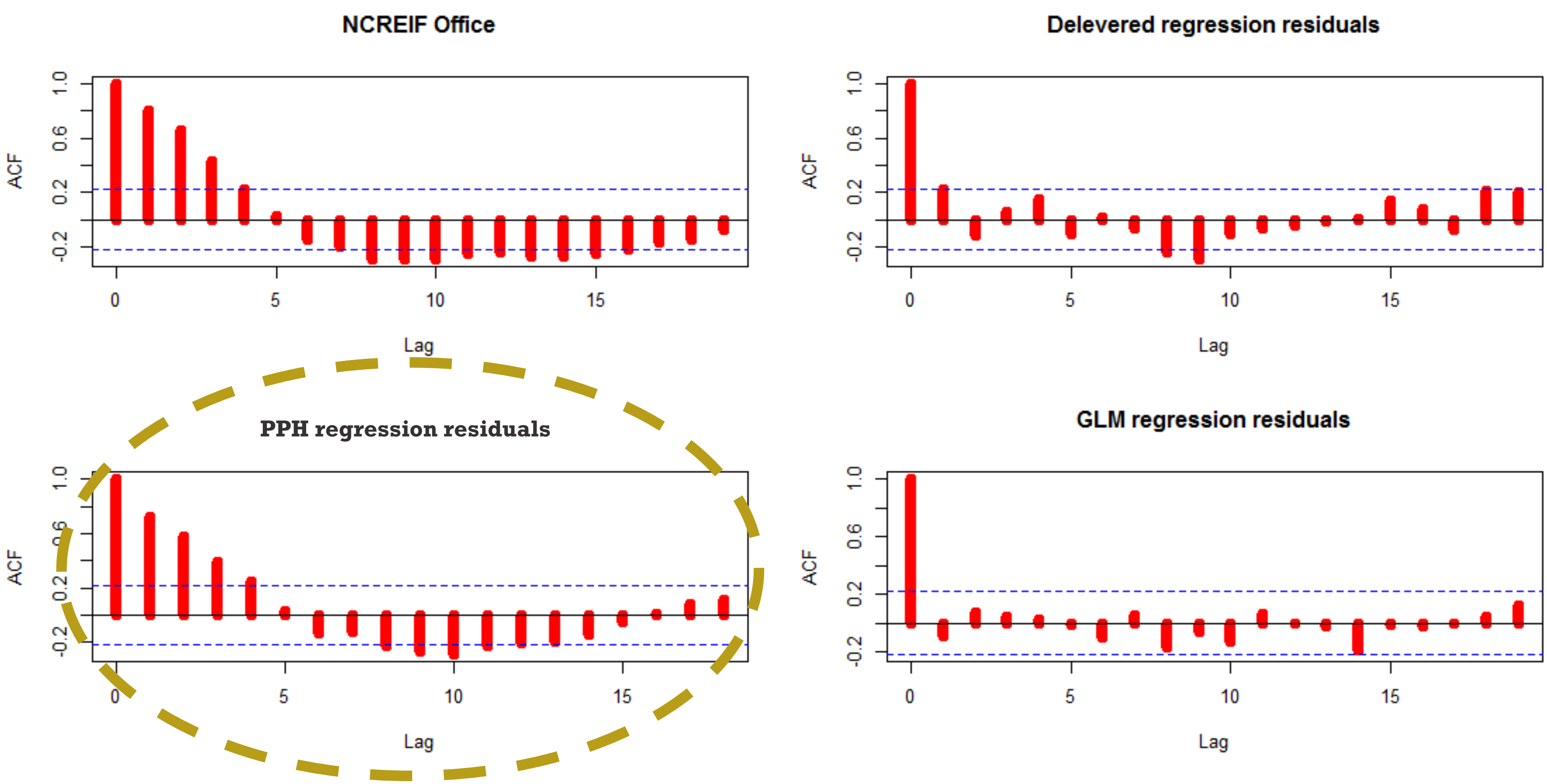
Note: **Office** represents REIT Office return series, **deOffice** represents delivered REIT Office return series, **NOffice** represents NCREIF Office return series, and **desOff** stands for the de-smoothed Office return series.

ESTIMATES OF FACTOR EXPOSURES

Parameter	NCREIF	DeOffice	GLM	PPH
alpha	0.02	0.01	0.02	0.01
	0.00	0.08	0.00	0.44
Mkt	0.07	0.42	0.19	0.34
	0.48	0.00	0.06	0.02
Bond	0.14	0.26	-0.32	0.12
	0.69	0.16	0.37	0.66
R-squared	5%	38%	14%	42%

Note: The Newey West p-values are reported in the above table. The parameters are in bold if significant at 10% level.

OOPS! WE HAVE A PROBLEM!



WHAT'S WRONG?

- Recall that PPH regress the following regression

$$y_t = \alpha + \sum_{j}^{k} \beta_j X_{j,t} + \xi_t,$$

where the error terms are serially correlated

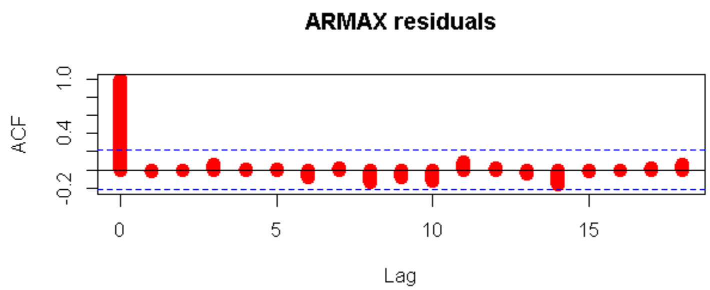
$$\xi_t = \sum_{i} w_i u_{t-i}$$

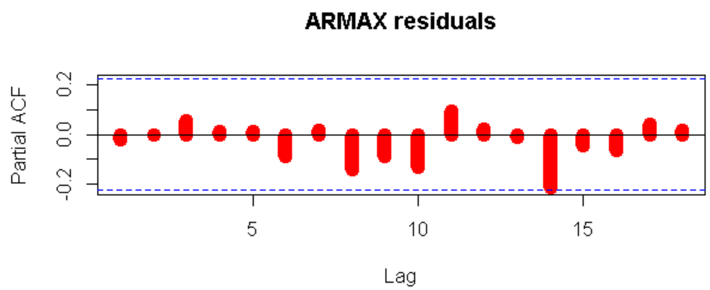
- The OLS estimators are consistent but not efficient (maybe okay for a large sample but not in our case)
- More advanced approaches are needed for estimating regression with serially correlated errors

ARMAX/TFN MODEL

- Fit an ARAMX/TFN model

$$y_t = \alpha + \beta_m X_{mkt} + \beta_b X_{bond} + \sum_{i=0}^{5} \theta_i u_{t-i}$$





Residuals u_t look okay!

Parameter	GLM	PPH	PPH-adj
alpha	0.02	0.01	0.02
	0.00	0.44	0.01
Mkt	0.19	0.34	0.15
	0.06	0.02	0.02
Bond	-0.32	0.12	-0.26
	0.37	0.66	0.89
R-squared	14%	42%	NA

Adjusted PPH estimates are smaller than those using GLM !

STATISTICAL JUSTIFICATION

- **GLM:** The effect of EIV on the Left-hand side

$$\beta_{GLM} = \frac{cov(y_t + \varepsilon_t, x_t)}{var(x_t)} = \frac{cov(y_t, x_t)}{var(x_t)} + \frac{cov(\varepsilon_t, x_t)}{var(x_t)}$$

$$= \beta + \frac{cov(\varepsilon_t, \boxed{x_t})}{var(x_t)}$$

We usually assume that measurement error is noise

- **PPH:** The effect of EIV on the Right-hand side

$$\beta_{PIMCO} = \frac{cov(y_t, x_t)}{var(x_t + \varepsilon_t)} = \frac{cov(y_t, x_t)}{var(x_t) + var(\varepsilon_t)}$$

$$= \frac{\beta}{1 + var(\varepsilon_t)/var(x_t)} \leq \beta$$

21

DIRECT ESTIMATION OF FACTOR LOADINGS FROM APPRAISAL

The 29th Northfield research conference on March 21st , 2017

WORKING PAPER AVAILABLE

- SSRN: https://ssrn.com/abstract=2935424 or http://dx.doi.org/10.2139/ssrn.2935424

APPRAISAL FORMULA WITH LINEAR FACTOR MODEL

- Eqn. (*) has the form of a multiple regression

$$y_t = \alpha + \sum_{i}^{m} \sum_{j}^{k} \beta_{ij} f_{j,t-i} + \xi_t$$

$$\beta_j = \sum_{i=0}^{m} \beta_{ij}, \quad j = 1, \dots, k$$

$$\beta_{ij} = w_i \beta_j, \qquad i = 0, \dots, m, j = 1, \dots, k$$

$$w_i = \frac{\beta_{ij}}{\sum^i \beta_{ij}} = \frac{\beta_{ij}}{\beta_j}, \; j = 1, \dots, k, \; i = 0,1, \dots, m.$$

- Challenges arise when we try to estimate *w's* and beta's from the above multiple regression using OLS

FIVE CHALLENGES IN THE LITERATURE

1. An ad-hoc one-step regression usually requires a large number of parameters
2. The number of statistically significant lagged factors tends to be different across factors, i.e. different m for different factors
3. Regression coefficients of the same factor must be of the same sign, i.e.

$$sgn(\beta_{1j}) = \cdots = sgn(\beta_{mj}), \quad j = 1, \ldots, k,$$

   where sgn(x) returns the sign of x.

4. In eqn. (*), all factors are smoothed by the same appraisal weights. This observation implies

$$\frac{\hat{\beta}_{i1}}{\hat{\beta}_1} = \frac{\hat{\beta}_{i2}}{\hat{\beta}_2} = \cdots = \frac{\hat{\beta}_{ik}}{\hat{\beta}_k}, \quad i = 1, \ldots, m.$$

Table 3: Dynamic regression approach with three risk factors (Barber and Wang 2013; Fan, Fleming and Warren, 2013)

Papers	Barber and Wang (BW)		Fan, Fleming and Warren (FFW)	
Variables	Coefficient	Summed Coefficient	Coefficient	Summed Coefficient
Rm-Rf	**0.38**	1.00	**0.41**	0.85
L1	**0.10**		**0.13**	
L2	**0.12**		**0.11**	
L3	0.06		**0.09**	
L4	**0.15**		**0.11**	
L5	0.03			
L6	0.06			
L7	0.02			
L8	0.07			
SMB	0.11	0.64	**0.30**	0.46
L1	**0.17**		0.16	
L2	0.10			
L3	0.07			
L4	-0.12			
L5	**0.19**			
L6	-0.02			
L7	**0.16**			
L8	-0.03			
HML	-0.02	0.01	**-0.10**	-0.10
L1	**-0.15**			
L2	0.05			
L3	-0.01			
L4	-0.03			
L5	0.07			
L6	0.00			
L7	0.11			
L8	**-0.02**			
Annualized alpha	**1.55%**		**5.6%**	
Adj. R-squares	62.9%		75.8%	

Note: The dependent variables in both BW and FFW are quarterly Cambridge Associate Private Equity Index returns less risk free rate. The sample period for BW is from the second quarter of 1986 to the third quarter of 2011, and the sample period for FFW is from the first quarter of 1993 to the fourth quarter of 2011. The independent variables in BW are the contemporaneous and 8 lagged Fama-French market, size, and book-to-market factors. The independent variables in FFW are the contemporaneous and lagged Wilshire 5000 index returns (market), Wilshire 4500 index minus S&P 500 index returns (size), and S&P 500 Value index minus S&P 500 Growth index returns (book-to-market). The regression coefficients are bold font if significant at 5% level.

FIVE CHALLENGES IN THE LITERATURE (CONT'D)

5. Empirical studies show that the estimates of factor exposures for private equity based on appraisal returns tend to be smaller than practitioners' expectation or those estimated using the cash flow approach

Authors	Data period	β_{Market}	β_{SMB}	β_{HML}
Driessen, Lin and Phalippou (2012)	1980-2003	1.71	-0.92	1.43
Franzoni, Nowak and Phalippou (2012)	1975-2007	1.40	-0.12	0.72
Ang, Chen, Goetzmann, and Phalippou (2013)	1992-2008	1.39	-0.07	0.74

PROPOSED METHOD

- Appraisal with *k* factors and *m* appraisal lags

$$y_t = \alpha + \sum_{i=0}^{m} \sum_{j=1}^{k} w_i \beta_j f_{j,t-i} + \sum_{i=0}^{m} w_i u_{t-i}$$

1. (Constrained) linear regression model with moving average errors
2. Multiple-input distributed lag model
3. The above model can be formulated as a state-space model and estimated using the Kalman Filter. See Appendix for the state-space representation of the above model.

EXAMPLE

- Appraisal with a *single* factor and *two* appraisal lags

$$y_t = \alpha + \beta \sum_{i=0}^{2} w_i f_{t-i} + e_t,$$

$$e_t = w_0\xi_t + w_1\xi_{t-1} + w_2\xi_{t-2},\ \xi_t \sim N(0, Q).$$

where f_t denotes the factor return, and the error term e_t is given by

Two appraisal lags may be used for modeling private equity!!

EXAMPLE (CON'TD)

– *State equation:*

$$\begin{bmatrix} \xi_t \\ \xi_{t-1} \\ \xi_{t-2} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \xi_{t-1} \\ \xi_{t-2} \\ \xi_{t-3} \end{bmatrix} + \begin{bmatrix} \xi_t \\ 0 \\ 0 \end{bmatrix},$$

– *Observation equation:*

$$y_t = [w_0, w_1, w_2] \begin{bmatrix} \xi_t \\ \xi_{t-1} \\ \xi_{t-2} \end{bmatrix} + [\alpha, \beta w_0, \beta w_1, \beta w_2] \begin{bmatrix} 1 \\ f_t \\ f_{t-1} \\ f_{t-2} \end{bmatrix} + \varphi_t,$$

where $\xi_t \sim N(0, Q)$, $\varphi_t \sim N(0, R)$ and $R = 0$.

PARAMETRIC WEIGHT FUNCTION

- For alternative assets with long appraisal lags (such as real estate), the estimation of m appraisal weights may not be feasible.

- The explicitly estimated appraisal weights tend to be sensitive to extreme observations.

- In view of the mixed data sampling (MIDAS) regression literature, we could use a parametric weight function, such as normalized exponential Almon lag polynomial function, to avoid the above issues.

PARAMETRIC WEIGHT FUNCTION

- Two of the most popular parametric weight functions

– Normalized beta probability density function

$$c_i^{u,nz} = c(i;\theta = [\theta_1, \theta_2, \theta_3]) = \frac{x_i^{\theta_1 - 1}(1 - x_i)^{\theta_2 - 1}}{\sum_{i=1}^{N} x_i^{\theta_i - 1}(1 - x_i)^{\theta_2 - 1}},$$

– Normalized exponential Almon lag polynomial function

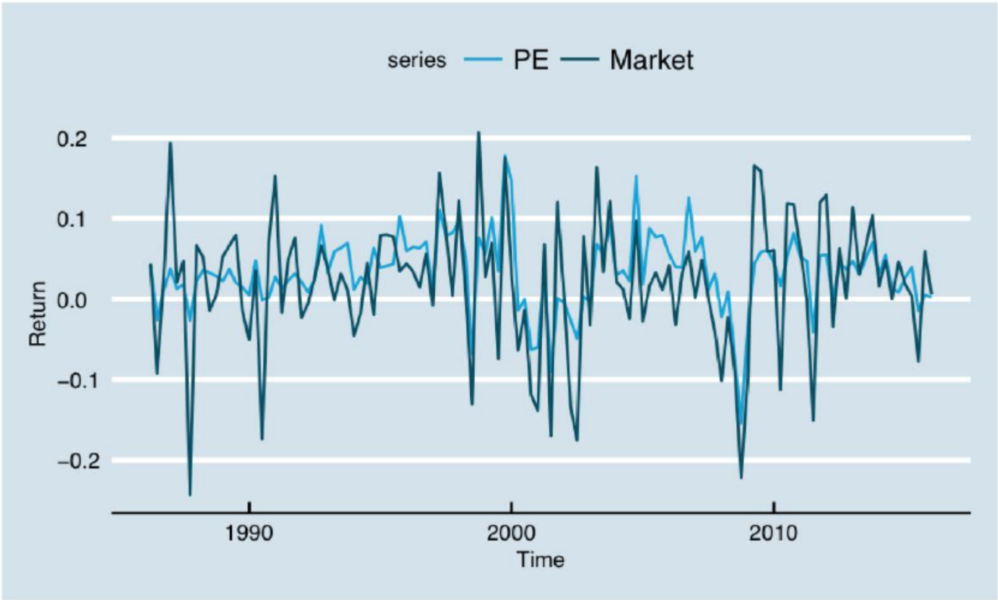
$$c_i^{u} = c(i;\theta = [\theta_1, \theta_2]) = \frac{e^{\theta_1 i + \theta_2 i^2}}{\sum_{i=1}^{N} e^{\theta_1 i + \theta_2 i^2}},$$

where $N = m + 1$ and $x_i = i/(N + 1)$.

Empirical studies on private equity

US PRIVATE EQUITY INDEX

- The US private equity (PE) index returns from Cambridge Associates and the quarterly Fama-French (benchmark) excess market factor returns from the second quarter of 1986 to the first quarter of 2016 are used.

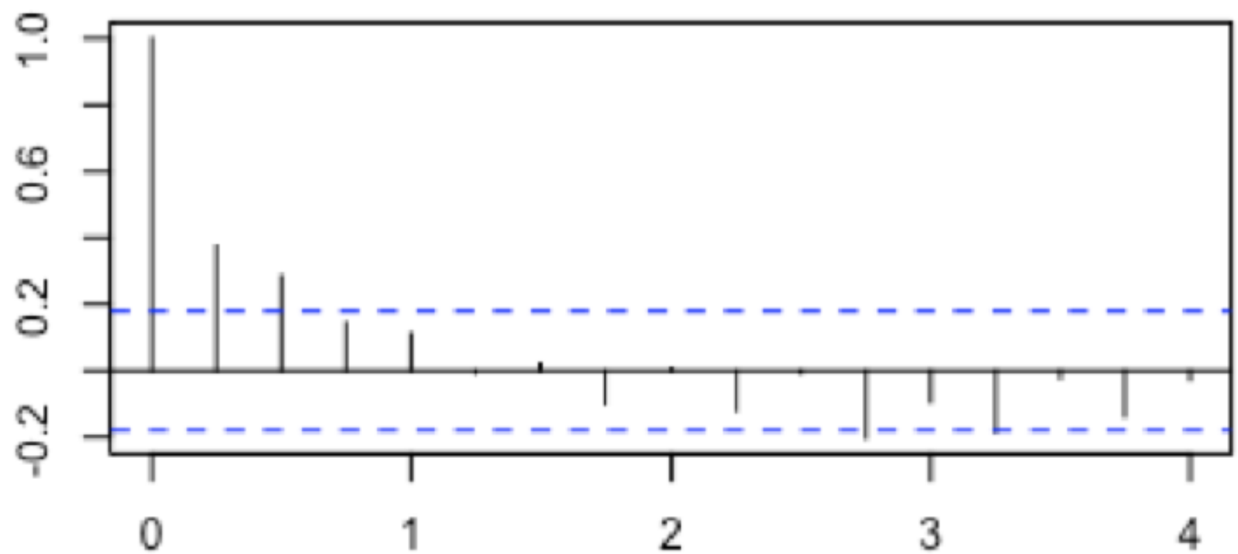


	US PE	FF market
mean	0.033	0.019
volatility	0.047	0.084
ACF(1)	0.376	-0.003

IDENTIFY THE NUMBER OF APPRAISAL LAGS

- We can identify the number of appraisal lags (*m*) using the sample autocorrelation function plot.

Figure 1. Sample autocorrelation function of Cambridge Associates US PE index returns from 1986 Q2 to 2016 Q1.



INITIAL ASSESSMENT USING AN UNCONSTRAINED REGRESSION

$$y_t = \alpha + \sum_{i=0}^{2} \beta_{m,i} f_{m,t-i} + \sum_{i=0}^{2} \beta_{smb,i} f_{smb,t-i} + \sum_{i=0}^{2} \beta_{hml,i} f_{hml,t-i} + e_t$$

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	0.021	0.003	7.057	0.000
L(mktQ, 0:2)0	0.376	0.039	9.582	0.000
L(mktQ, 0:2)1	0.105	0.040	2.626	0.010
L(mktQ, 0:2)2	0.125	0.040	3.158	0.002
L(smbQ, 0:2)0	0.053	0.065	0.815	0.417
L(smbQ, 0:2)1	0.063	0.066	0.950	0.344
L(smbQ, 0:2)2	-0.039	0.065	-0.603	0.547
L(hmlQ, 0:2)0	-0.077	0.043	-1.797	0.075
L(hmlQ, 0:2)1	-0.079	0.042	-1.853	0.067
L(hmlQ, 0:2)2	0.050	0.042	1.192	0.236

$$\hat{\beta}_m = 0.376 + 0.105 + 0.125 = 0.606$$

$$\hat{\beta}_{hml} = -0.077 - 0.079 = -0.156$$

SSM ESTIMATION RESULTS

$\hat{\beta}_m = 0.376 + 0.105 + 0.125 = 0.606$

	Estimate	Std error	t value
w_0	0.653	0.038	17.035
w_1	0.216	0.029	7.408
alpha	0.022	0.004	5.522
market	**0.582**	**0.057**	**10.172**
hml	-0.150	0.059	-2.526
sqrtExp(Q)	-6.322	0.175	-36.051

$\hat{\beta}_{hml} = -0.077 - 0.079 = -0.156$

	Estimate	Std error	t value
θ_1	-2.094	0.740	-2.829
θ_2	0.319	0.194	1.641
alpha	0.023	0.004	5.606
market	**0.578**	**0.057**	**10.154**
hml	-0.149	0.059	-2.528
sqrtExp(Q)	-6.344	0.176	-36.115

Normalized exponential Almon lag polynomial

COMPARISON OF APPRAISAL WEIGHTS

	Ad-hoc (market)	Ad-hoc (hml)	SSM (explicit)	SSM (parametric)
w_0	0.62	0.49	0.65	0.66
w_1	0.17	0.51	0.22	0.21
w_2	0.21	0.00	0.13	0.13

For the ad-hoc regression, the appraisal weight estimate at lag i is given by

$$w_i = \frac{\beta_{ij}}{\sum_{i=0}^{m} \beta_{ij}}, \; j = \; market, \; hml.$$

38

Appraisal with high frequency information

APPRAISAL FREQUENCY AND THE ESTIMATION OF FACTOR EXPOSURES

- Appraisal returns are usually reported on a quarterly basis (low frequency).
- Previous research assumes that assets are appraised using information at the same frequency.
- We argue that the misspecification on appraisal frequency would lead to biased estimates of the factor exposures.
- Due to time constraints, we will only show the empirical study on this topic today. See Section 4 of the working paper for more detail.

ECONOMETRIC MODEL FOR APPRAISAL USING MONTHLY DATA

Mixed data sampling (MIDAS) single input distributed lag model

$$y_t = a + \sum_{l=0}^{8} \beta_l f_{m,t-l/3} + \sum_{i=0}^{2} \vartheta_i u_{t-i},$$

where a is an intercept, $\beta_l = \theta_l \beta$, θ_l denotes the monthly appraisal weight, and $f_{m,t-l/3}$ denotes the monthly market factor, and u_t denotes the quarterly error term.

UNRESTRICTED MIDAS REGRESSION

	Estimate	Std error	t value
ma1	0.314	0.094	3.337
ma2	0.232	0.090	2.580
intercept	0.022	0.004	5.460
X.0/m	0.479	0.065	7.421
X.1/m	0.434	0.060	7.266
X.2/m	0.325	0.053	6.100
X.3/m	0.242	0.063	3.853
X.4/m	0.135	0.061	2.210
X.5/m	0.022	0.053	0.413
X.6/m	0.127	0.064	1.975
X.7/m	0.102	0.058	1.742

Remark: Our initial estimation shows that the estimate of β_8 is insignificant. This implies that the "high frequency" appraisal weight θ_8 is zero.

MARKET BETA FROM UNRESTRICTED MIDAS REGRESSION

- The sum of regression coefficients against contemporaneous and lagged market factors is the estimate of (long-run) ...el

X.0/m	0.479
X.1/m	0.434
X.2/m	0.325
X.3/m	0.242
X.4/m	0.135
X.5/m	0.022
X.6/m	0.127
X.7/m	0.102

$$\sum_{l=0}^{7} \beta_l \approx 1.866$$

SSM (EXPLICIT WEIGHT)

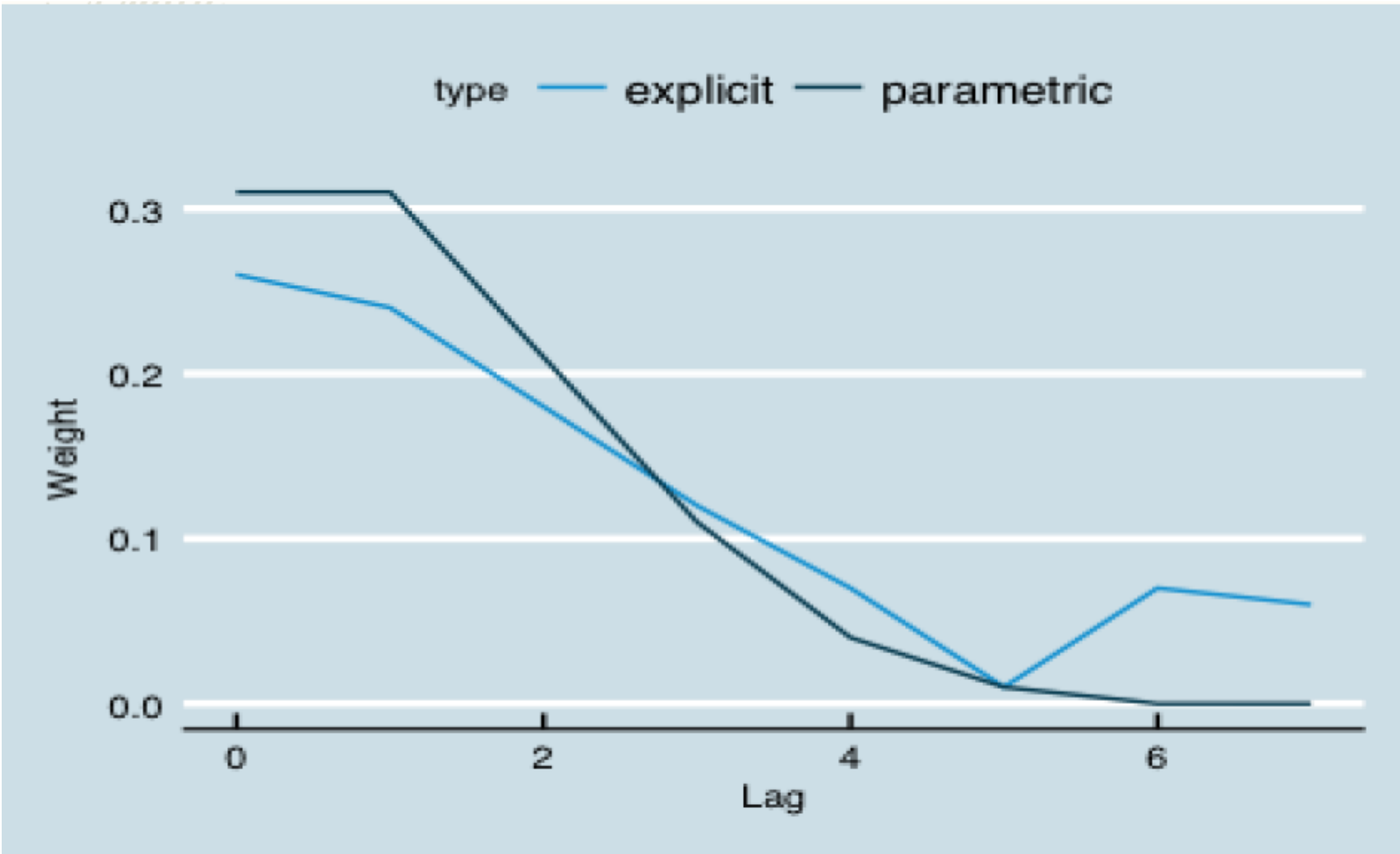
	Estimate	Std error	t value
w0	0.260	0.030	8.619
w1	0.238	0.031	7.689
w2	0.177	0.028	6.359
w3	0.117	0.025	4.666
w4	0.068	0.028	2.406
w5	0.014	0.027	0.522
w6	0.068	0.026	2.584
alpha	0.022	0.004	5.413
beta	**1.847**	**0.189**	**9.772**
sqrtExp(Q)	-5.350	0.176	-30.397

SSM (NORMALIZED BETA DENSITY FUNCTION

	Estimate	Std error	t value
θ_1	2.000	0.491	4.072
θ_2	6.187	1.451	4.264
alpha	0.023	0.004	6.617
beta	**1.427**	**0.132**	**10.792**
sqrtExp(Q)	-5.603	0.162	-34.688

- The estimates of market beta using high frequency data are significantly larger than those estimated based on quarterly information and align with those from the cash flow approach.

MONTHLY APPRAISAL WEIGHTS OF US PE (CAMBRIDGE ASSOCIATES)



Empirical studies on private real estates

PRIVATE REAL ESTATE

- Study the relationship between the quarterly NCREIF property index (NPI) returns and the monthly NAREIT composite index returns from 1986 Q3 to 2016 Q1.

	NPI	NAREIT*
mean	0.019	0.026
volatility	0.022	0.086
ACF(1)	0.806	0.075

Remark: The squared root of time rule is used to calculate the (quarterly) statistics for NAREIT returns.

INITIAL ASSESSMENT

- Identify (quarterly) appraisal lags *m*=4 based on the sample autocorrelation functions of appraisal returns
- Preliminary identification using an unrestricted MIDAS distributed lag model
- Final model:

$$y_t = a + \beta \sum_{l=1}^{14} \theta_l f_{t-\frac{l}{3}} + \sum_{k=0}^{4} \vartheta_k \xi_{t-k}.$$

ESTIMATION RESULT (SSM WITH EXPLICIT WEIGHT)

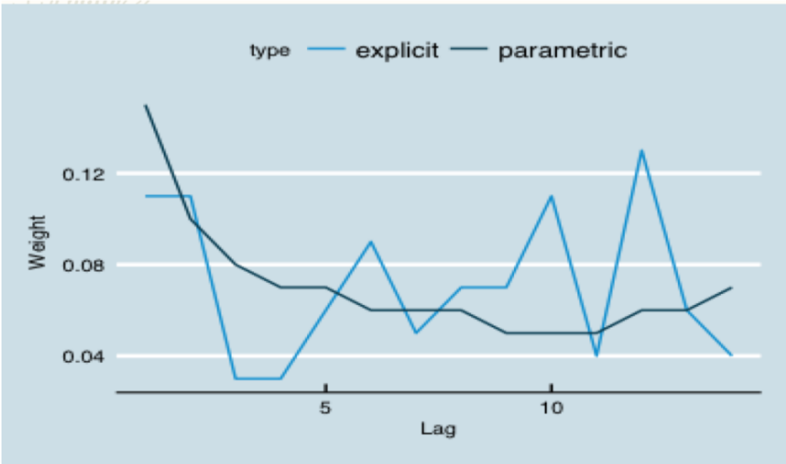
	Estimate	Std error	t value
w_1	0.107	0.025	4.295
w_2	0.111	0.023	4.792
w_3	0.035	0.024	1.429
w_4	0.030	0.030	1.012
w_5	0.056	0.019	2.933
w_6	0.092	0.023	4.055
w_7	0.049	0.031	1.608
w_8	0.075	0.023	3.217
w_9	0.073	0.029	2.483
w_{10}	0.106	0.028	3.792
w_{11}	0.037	0.022	1.664
w_{12}	0.129	0.024	5.304
w_{13}	0.061	0.045	1.349
alpha	0.014	0.003	4.298
beta	0.606	0.103	5.864
sqrExp(Q)	-5.835	0.185	-31.473

MORE ESTIMATION RESULT AND MONTHLY APPRAISAL WEIGHTS

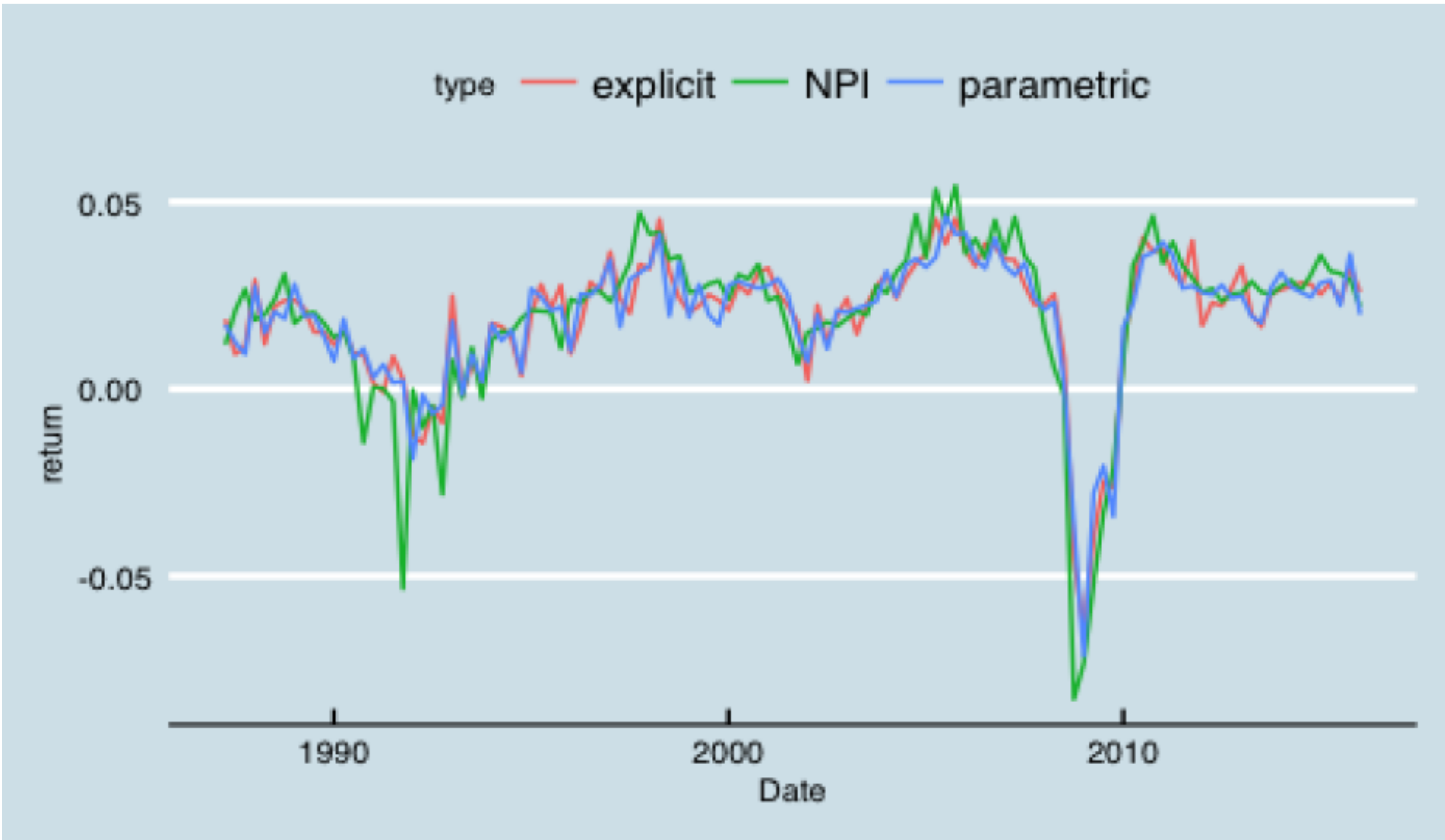
- SSM with normalized beta density function

	Estimate	Std error	t value
θ_1	0.426	0.092	4.637
θ_2	0.735	0.114	6.452
alpha	0.015	0.003	4.475
beta	0.578	0.094	6.134
sqrtExp(Q)	-5.719	0.181	-31.562

- Monthly appraisal weights



FORECAST NPI RETURNS USING MONTHLY REITS RETURNS



POSSIBLE EXTENSIONS
PERIODIC APPRAISAL SCHEME

- Some appraisal returns show strong seasonality in the fourth quarter.
- A periodic appraisal scheme may be given by

$$y_{4,n} = \widetilde{w}_0 r_{4,n} + \widetilde{w}_1 r_{3,n} + \widetilde{w}_2 r_{2,n}$$
$$y_{s,n} = w_0 r_{s,n} + w_1 r_{s-1,n} + w_2 r_{s-2,n}, s = 1,2,3,$$

where $x_{s,n}$ denotes the returns at the s quarter of year n with $x_{l,n} = x_{s+l,n-1}$ for $l \leq 0$, w_i denotes the i-th appraisal weights for the first three quarters of the year, and $\widetilde{w}_i$ denotes the i-th appraisal weights at the fourth quarter of the year for $i = 0,1,2$.

POSSIBLE EXTENSIONS
MIXED FREQUENCY APPRAISAL

- We have considered the case that assets are appraised using monthly (high frequency) data.
- In reality, assets may be appraised using mixed frequency information, such as a low frequency private factor (may be unobservable) and a high frequency public counterparty.
- This extension may be done by adding an additional state variable to Dimson's model using high frequency information.

CONCLUDING REMARKS

- Future work of this project includes

1. More work and empirical studies on two extensions
2. Statistical tests for evaluating models with different appraisal schemes
3. An R package implementing the proposed method and tests

- Questions or suggestions?

APPENDIX

Appraisal with k factors and m lags

1) *State equation:*

$$\begin{bmatrix} \xi_t \\ \xi_{t-1} \\ \xi_{t-2} \\ \vdots \\ \xi_{t-m} \end{bmatrix} = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & 0 & \ddots & 0 & 0 \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix} \begin{bmatrix} \xi_{t-1} \\ \xi_{t-2} \\ \xi_{t-3} \\ \vdots \\ \xi_{t-m-1} \end{bmatrix} + \begin{bmatrix} \xi_t \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

2) *Observation equation:*

$$y_t = [w_0 \quad w_1 \quad w_2 \quad \dots \quad w_m] \begin{bmatrix} \xi_t \\ \xi_{t-1} \\ \xi_{t-2} \\ \vdots \\ \xi_{t-m} \end{bmatrix} + \Gamma u_t + \varphi_t,$$

where $\xi \sim N(0, \mathrm{Q})$, $\varphi \sim N(0, R)$ with $R = 0$,

$$\Gamma = [\alpha, \beta_1 w_0, \dots, \beta_1 w_m, \dots, \beta_k w_0, \dots, \beta_k w_m]_{1\times(k\widetilde{m}+1)},$$

$$u_t' = [1, f_{1t}, \dots, f_{1,t-m}, \dots, f_{k,t} \dots, f_{k,t-m}]_{1\times(k\widetilde{m}+1)},$$

and $\widetilde{m} = m + 1$.