TA no. 7: Answers

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ENGR251 Thermodynamics I - Winter 2017

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Question no. 1

How can we use the specific heat capacity at constant volume to find the change in internal energy of an ideal gas during an expansion or a compression (where its volume changes)?

Let us first recall the definition of the specific heat capacity of an object, that is, the amount of heat per unit mass required to raise its temperature, per degree temperature increase, such that:

$$c \triangleq \frac{\delta q}{dT} \tag{1}$$

This is nothing more than a slope – a proportionality constant (which is different from one material to another) between the change in temperature and the change in heat transfer. It should be noted that the specific heat capacity is assumed to be a constant if the temperature change is small enough (an infinitesimal change dT). For large temperature variations, the specific heat capacity is no longer constant.
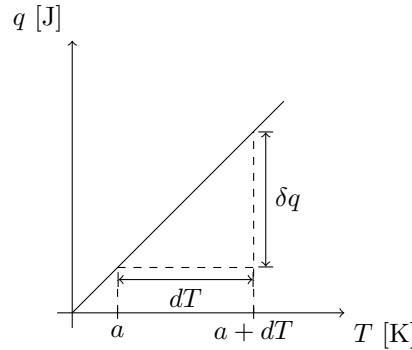

Figure 1: A small change in temperature requires a small addition of heat

Using the first law of thermodynamics for closed systems (assuming that the macroscopic kinetic energy and potential energy associated with external forces can be neglected), we have:

$$du = \delta q - \delta w \tag{2}$$

We can plug the first law in equation 1 such that:

$$c = \frac{du + \delta w}{dT} = \frac{du + pd\nu}{dT} = \frac{du}{dT} + \frac{pd\nu}{dT} \tag{3}$$

Now, we are interested in the coefficient c_v, which corresponds to the amount of heat required when the volume is maintained constant.

$$c_v = \left(\frac{du}{dT} + \frac{pd\nu}{dT} \right)_v = \left(\frac{du}{dT} \right)_v + \left(\frac{pd\nu}{dT} \right)_v \tag{4}$$

When the volume is kept constant, the volume does not change, so $d\nu = 0$. For this reason, we can remove the second term from the equation such that:

$$c_v = \left(\frac{du}{dT}\right)_v \tag{5}$$

We have a general definition for the specific heat capacity (often called specific heat only) at constant volume. Physically, when the volume is kept constant, the object cannot expand, so the temperature goes up more rapidly than when the object can expand – since when the object expands, it loses energy to its surroundings in order to make some room. Let us go back to the initial definition, equation 1, and rewrite it as follows:

$$\delta q = c\,dT \tag{6}$$

So, at constant volume, the temperature of our substance should increase more rapidly for the same amount of energy given by heat transfer. In other words, for the same δq, we have a lager dT, so c_v must be smaller than the general c – so that δq remains the same.

So far, no problem, but how can we use c_v when the system undergoes expansion or compression processes? Technically, if the volume of the system changes, its internal energy is affected. Remember that for simple compressible systems, the state postulate tells us that the state of our system is completely specified by two independent intensive properties. As a result, we can define the internal energy of our system as follows:

$$du = \left(\frac{\partial u}{\partial T}\right)_v dT + \left(\frac{\partial u}{\partial \nu}\right)_T d\nu \tag{7}$$

The (specific) internal energy is a function of both temperature and (specific) volume $u \equiv u(T, \nu)$ – where the partial derivatives are the derivatives of u with respect to one of the variables, keeping the other one constant.

Anyway, this is just some technicality. Let us go back to our problem. We have an ideal gas. The ideal gas model assumes that the internal energy is a function of the temperature only, so $u \equiv u(T)$. Based on this assumption, equation 7 becomes:

$$du = \left(\frac{\partial u}{\partial T}\right)_v dT \tag{8}$$

If we look at the equation 5, we see that we can rewrite our equation as follows:

$$du = \left(\frac{\partial u}{\partial T}\right)_v dT = c_v dT \tag{9}$$

You might argue that equation 5 does not involve a partial derivative, but since u depends on one variable only (the temperature) for an ideal gas, the partial derivative is actually a regular derivative. To sum up, no matter the process (expansion or compression), for an ideal gas, the change in internal energy is well approximated by the specific heat capacity at constant volume times the change in temperature – provided that this change is small enough.

Question no. 2

Why does the thermal efficiency η_{th} does not take into account Q_L (during a Carnot cycle)?

Using the first law of thermodynamics for closed systems (assuming that the macroscopic kinetic energy and potential energy associated with external forces can be neglected), we have:

$$du = \delta q - \delta w \tag{10}$$

A cycle means that we start from one state and return to this initial state at the end. Since the internal energy is a state function (which does not depend on the path), we have:

$$du = u_2 - u_1 = 0 \tag{11}$$

The final state and initial state being the same state, the change in internal energy is zero. As a result, we have:

$$\delta q = \delta w \tag{12}$$

Remember that we defined the thermal efficiency η_{th} as the desired output divided by the required input:

$$\eta_{\text{th}} = \frac{\text{desired}}{\text{required}} = \frac{W}{Q_H} \tag{13}$$

We have a heat engine. So we want to take energy from a heat source and produce some work – for example, a steam locomotive. Let us rewrite the thermal efficiency using the fact that the heat transfer is equal to the work:

$$\eta_{\text{th}} = \frac{W}{Q_H} = \frac{Q}{Q_H} = \frac{Q_H - Q_L}{Q_H} \tag{14}$$

The amount of energy lost by heat transfer Q_L is taken into account in the work. This can be shown in figure 2. Since the heat leaving the system is negative, you have a minus sign (but Q_L is a positive quantity). Otherwise, you would have $Q_H + Q_L$ but Q_L would be a negative quantity.
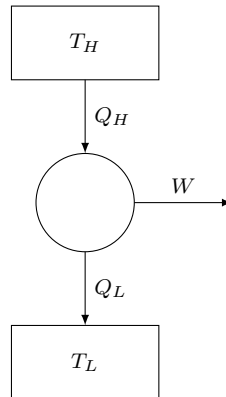

Figure 2: Engine converting heat into work

The thermal efficiency tells you how much work you can get from the heat source. If you do not have Q_L, you convert everything (Q_H) into work – so you have an efficiency of 100%. Unfortunately, you have some losses, so you do not produce as much work as you would like. Energy being conserved, if you lose some heat, you produce less work. The thermal efficiency is therefore the ratio of the work output to the heat input.

Question no. 3

Is there any methodology we can apply to solve thermodynamics problems?

So, first of all, you should define your system. What is your system? Is it open, closed, isolated? Then, you need to sketch what is happening and understand the process. What is going on physically? It might seem silly, but it does help. As a result, you have a sketch with your system in its initial state and in its final state. I know you do not have much time during an exam, so it is difficult, but you should pause at this point to really understand what is going on. How does my system go from its initial state to its final state? It also helps to write what you know about each state (pressure, temperature, and so on).
Then, what are we asked? What are we looking for (mass, energy, pressure, velocity)? At this point, you need to write principles equations – basically, what you know about the unknown. When you are asked for example to find the amount of heat transfer, you know that it is probably related to the first law of thermodynamics. If you are asked about the final pressure and you have a gas, it is

likely that you have to use the ideal gas law (or maybe the change in pressure with depth in a fluid). Either you know exactly where to go to solve your problem or you write everything you know about the unknown. They are asking for the volume, the volume could be:

$$pV = nRT \tag{15}$$

$$m = \rho V = \frac{V}{\nu} \tag{16}$$

$$V = A \times L = A \times v \times \Delta t \tag{17}$$

Now, you need to check what you have. You do not have a gas, forget about the ideal gas law (it is useless, you cannot use it for a liquid). I have the velocity, so I should go with the third equation. You have to match what you know with what is given. Sometimes, you need to make a couple of assumptions. Your gas is an ideal gas. You have a steady flow process. Anything that is not too far-fetched and will help you solve your problem. Do not assume that you have a constant-volume process if you have a piston-cylinder device, just to get rid of the boundary work.

You will either have everything you need to get your unknown or you need to dig a little deeper – maybe you have to look at the tables, maybe you need to use another equation to find the missing property. If you are stuck, you should go back to your system and its process. Maybe you missed something. The boundary work can be divided into two parts – when the pressure does not change, then when it changes because the volume is the same. What is happening?

Once you have everything, you just need to substitute the knowns into your equation (where you have isolated your unknown). Make sure that units are consistent. Double-check that your results make sense. Your system is being compressed, so it receives energy by work. Your work should be negative. The final temperature should be higher than the initial one if there is no heat loss. Use your common sense. The volume is fixed, you have a rigid tank, the system is closed, the mass cannot change, so the specific volume remains constant. What will happen to the process? I add energy by heat transfer, the molecules start moving faster, but the system cannot expand, so the pressure should increase. Try to relate your problem to real-life scenarios. If you have some water in a pan that you are trying to bring to a boil. It will turn into vapor at some point. If you put a lid on compressing a lot your liquid water, there is no chance that it will turn into vapor immediately. You need to supply a lot of energy which will increase the temperature of the water.

I do understand that it is not always easy, especially when you do not have much time. I hope it helps anyway.