TA no. 8: Answers

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ENGR251 Thermodynamics I - Winter 2017

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Question no. 1

What is the general formula for the first law of thermodynamics?

Here is the most general formula for the first law of thermodynamics for open systems (or control volumes):

$$\frac{dE_{\text{system}}}{dt} = \dot{Q} - \dot{W} + \sum_{in} \dot{m}_{\text{in}} \left(h_{\text{in}} + \frac{v_{\text{in}}^2}{2} + gz_{\text{in}} \right) - \sum_{out} \dot{m}_{\text{out}} \left(h_{\text{out}} + \frac{v_{\text{out}}^2}{2} + gz_{\text{out}} \right) \tag{1}$$

All the other formulas come from this one. Notice that equation 1 is the rate form of the first law. We can multiply everything by dt to get the "regular" form. The rate form has one advantage; for open systems at steady state, nothing changes with time. As a result, the derivative of the total energy with respect to time (the left-hand side of equation 1) is equal to zero.

Before writing the formula at steady state, let us recall that $\dot{Q}$ and $\dot{W}$ are the **NET** heat and work transfers. By convention, heat transfer is positive if our system gains energy and negative if our system loses energy. Work transfer is the opposite. Accordingly, we can re-write them as follows:

$$\dot{Q} = \dot{Q}_{in} - \dot{Q}_{out} \tag{2}$$

$$\dot{W} = -\dot{W}_{in} + \dot{W}_{out} \tag{3}$$

At this point, all the terms having a subscript are positive. For example, let us assume that our system loses 20 kJ/s of energy by heat transfer. By convention, the left-hand side of equation 2 is negative such that $\dot{Q} = -20$ kJ/s. Now, let us look at the right hand side. We do not have any gains, but we have some losses. So, $\dot{Q}_{in}$ is equal to zero (no gains), but $\dot{Q}_{out} = 20$ kJ/s (some losses). Here, $\dot{Q}_{out}$ is positive because it has a subscript. Since there is a minus sign before $\dot{Q}_{out}$, the left-hand side of equation 2 is equal to the right-hand side – everything works.

Let us go back to the formula at steady state, which can be written as follows:

$$\sum_{in} \dot{m}_{\text{in}} \left(h_{\text{in}} + \frac{v_{\text{in}}^2}{2} + gz_{\text{in}} \right) + \dot{Q}_{in} + \dot{W}_{in} = \sum_{out} \dot{m}_{\text{out}} \left(h_{\text{out}} + \frac{v_{\text{out}}^2}{2} + gz_{\text{out}} \right) + \dot{Q}_{out} + \dot{W}_{out} \tag{4}$$

We only replaced equations 2 and 3 in equation 1. Remember that mass is conserved, so the change in mass of our open system with respect to time is equal to the net mass transfer to or from the system, which is expressed as:

$$\frac{dm_{\text{system}}}{dt} = \sum_{in} \dot{m}_{in} - \sum_{out} \dot{m}_{out} \tag{5}$$

At steady state, the left hand side is equal to zero, which leads to:

$$\sum_{in} \dot{m}_{in} = \sum_{out} \dot{m}_{out} \tag{6}$$

Imagine we have only one inlet and one outlet such that:

$$\dot{m}_{in} = \dot{m}_{out} = \dot{m} \tag{7}$$

We could therefore change the first law of thermodynamics one more time as follows:

$$\dot{m}\left(\Delta h + \Delta ke + \Delta pe\right) + \dot{Q} - \dot{W} = 0 \tag{8}$$

We can find different forms of the first law, but they all derive from equation 1. Speaking of which, it is interesting to notice that when there is no mass flow (closed system), equation 1 becomes:

$$\frac{dE_{\text{system}}}{dt} = \dot{Q} - \dot{W} \tag{9}$$

which is the first law for closed system (the one we used before). The change in the total energy of our system is given by:

$$\Delta E_{\text{system}} = \Delta U + \Delta KE + \Delta PE \tag{10}$$

As a result, for unsteady-flow processes (or transient processes), the left-hand side of equation 1 is no longer equal to zero, but is defined as:

$$\frac{dE_{\text{system}}}{dt} = \frac{d}{dt}\left[U + KE + PE\right] \tag{11}$$

This corresponds to the change in total energy of the nonflowing fluid within the defined control volume. Transient processes mean that we have some changes withing the control volume with time – for example, filling a tank with some fluid.

Question no. 2

What should we know about mass and volume flow rates?

Here are the two formulas about mass and volume flow rates:

$$\dot{V} = A \times v \tag{12}$$

$$\dot{m} = \rho \times \dot{V} = \frac{\dot{V}}{\nu} = \rho \times A \times v = \frac{A \times v}{\nu} \tag{13}$$

When you do not want to remember more formulas, you need to come up with a way to derive them. Let us focus on a small volume element, which is nothing more than a small volume. You know that a volume is equal to an area times a distance. So, a small volume element dV is equal to:

$$dV = A \times dl \tag{14}$$

A small distance element dl is equal to some velocity times a small time interval:

$$dl = v \times dt \tag{15}$$

Always check the units! Intuitively, you know that the velocity is [m/s], so multiplying some velocity by some time [s] gives you a distance [m]. If we plug equation 15 into equation 14, we have:

$$dV = A \times (v \times dt) = A \times v \times dt \tag{16}$$

Remember that the volume flow rate (or the rate of change in volume) is the derivative of the volume with respect to time:

$$\dot{V} = \frac{dV}{dt} = \frac{A \times v \times dt}{dt} = A \times v \tag{17}$$

We can double check the units! Here: $[\mathrm{m}^2] \cdot [\mathrm{m/s}] = [\mathrm{m}^3/\mathrm{s}]$. The area times a velocity is equal to a volume flow rate.

Now, let us focus on some mass. We have already seen that the mass of something uniform is equal to $\delta m = \rho \times dV$. If we plug equation 16 into this one, you have:

$$\delta m = \rho \times (A \times v \times dt) = \rho \times A \times v \times dt \tag{18}$$

Same reasoning as before, if we want the mass flow rate (the rate of change in mass):

$$\dot{m} = \frac{\delta m}{dt} = \frac{\rho \times A \times v \times dt}{dt} = \rho \times A \times v \tag{19}$$

We can replace ρ in the previous equation such that:

$$\dot{m} = \frac{A \times v}{\nu} \tag{20}$$

Question no. 3

What equations should we know?

I am not sure exactly, but I believe everything we saw so far could be summarized with the following equations:

$$\frac{dE_{\text{system}}}{dt} = \dot{Q} - \dot{W} + \sum_{in} \dot{m}_{\text{in}} \left(h_{\text{in}} + \frac{v_{\text{in}}^2}{2} + g z_{\text{in}} \right) - \sum_{out} \dot{m}_{\text{out}} \left(h_{\text{out}} + \frac{v_{\text{out}}^2}{2} + g z_{\text{out}} \right) \tag{21}$$

$$\Delta E = \Delta U + \Delta KE + \Delta PE \tag{22}$$

$$H = U + pV \tag{23}$$

$$\delta W_b = pdV \tag{24}$$

$$c = \frac{\delta q}{dT} \tag{25}$$

$$\dot{V} = A \times v \tag{26}$$

$$m = \rho V = \frac{V}{\nu} \tag{27}$$

All other formulas can be derived from the ones I listed. For example, the first law for closed systems (not in the rate form). No mass can enter our system, so:

$$dE_{\text{system}} = dU + dKE + dPE = Q - W \tag{28}$$

No need to remember that when you know the first two equations.

How to express the change in enthalpy?

$$dh = du + dp\nu + pd\nu \tag{29}$$

$$c = \frac{\delta q}{dT} = \frac{du + dw}{dT} = \frac{du + pd\nu}{dT} \tag{30}$$

For constant-pressure processes,

$$dh = du + pd\nu = c_p dT \tag{31}$$

No need to remember that when we know the first five equations.

Same idea for constant-volume processes,

$$c_\nu = \left(\frac{du + pd\nu}{dT} \right)_\nu = \frac{du}{dT} \tag{32}$$

$$du = c_\nu dT \tag{33}$$

Maybe we also need:

$$du = \left(\frac{\partial u}{\partial T}\right)_\nu dT + \left(\frac{\partial u}{\partial \nu}\right)_T d\nu \tag{34}$$

$$dh = \left(\frac{\partial u}{\partial T}\right)_p dT + \left(\frac{\partial u}{\partial p}\right)_T dp \tag{35}$$

These formulas are useful for ideal gases, where $u = u(T)$ and $h = h(T)$, so:

$$du = \left(\frac{\partial u}{\partial T}\right)_\nu dT = c_\nu dT \tag{36}$$

$$dh = \left(\frac{\partial h}{\partial T}\right)_p dT = c_p dT \tag{37}$$

No matter whether or not we have a constant-volume process for the change in internal energy or a constant-pressure process for the change in enthalpy.

For liquids and solids, that is incompressible substances ($d\nu = 0$),

$$du = c_\nu dT \tag{38}$$

$$dh = du + \nu dp = c_\nu dT + \nu dp \tag{39}$$

Most of the time, the change in pressure is small compared to the other term (especially in solids), so $dh \approx du$. For constant-pressure processes, $dh = du$.