Lecture Note: Week 7

MATH 461: Probability Theory, Spring 2021

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Lecture 18. The Uniform Random Variable (Sec 5.3)

Definition: Uniform random variable
A random variable X is a uniform random variable on (a,b) if its probability density function is

$$f(x) = \begin{cases} \frac{1}{b-a}, & a < x < b, \\ 0, & \text{otherwise.} \end{cases}$$

We denote by $X \sim U(a,b)$.

Example 1. Let X be a uniform random variable on $(0,10)$. Calculate $\mathbb{P}(X < 3)$, $\mathbb{P}(X > 6)$, and $\mathbb{P}(3 < X < 8)$.

Proposition 2. *If $X \sim U(a,b)$, then $\mathbb{E}[X] = \frac{a+b}{2}$, $\mathrm{Var}(X) = \frac{(a-b)^2}{12}$, and the distribution function is*

$$F(x) = \begin{cases} 0, & x < a, \\ \frac{1}{b-a}(x-a), & a \le x \le b, \\ 1, & x > b. \end{cases}$$

Example 3. A bus travels between the two cities A and B, which are 100 miles apart. If the bus has a breakdown, the distance from the breakdown to city A has a $U(0,100)$ distribution. There are bus service stations in city A, in B, and in the center of the route between A and B. It is suggested that it would be more efficient to have the three stations located 25, 50, and 75 miles, respectively, from A. Do you agree? Why?

Example 4. Find the density of $Y = \sqrt{U}$ where U is a uniform random variable on $[0,1]$.

Proposition 5. *Let X be a continuous random variable with cdf F_X. Let $g(x)$ be the inverse of F_X defined on $(0,1)$, that is, $g(x) = \inf\{t : F_X(t) \ge x\}$. If U is a uniform random variable on $[0,1]$, then $g(U)$ has the same distribution as X.*

References

[SR] Sheldon Ross, *A First Course in Probability*, 9th Edition, Pearson

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