Probability Density?

Let $f(x)$ and $g(x)$ be two probability density functions. Then $0.2f(x) + 0.8g(x)$ is also a probability density function.

- (a) False
- (b) True

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Case 1   If $\underbrace{af(x) + (1-a)g(x)}_{= h(x)}$ is given

(for example, $a = 0.2$ and $1-a = 0.8$)

then $h(x) \geq 0$ for all x and

$$\int h(x)\,dx = \int \left(af(x) + (1-a)g(x)\right) dx$$

$$= a\int f(x)\,dx + (1-a)\int g(x)\,dx$$

$$= a + (1-a)$$

$$= 1$$

$\therefore$ h is a pdf.

Case 2   If $\underbrace{af(x) + bg(x)}_{= h(x)}$ is given

where a or b is negative.

Then $h(x) < 0$ for some x.

For instance $a = \frac{3}{2}$, $b = -\frac{1}{2}$.

$$f(x) = \begin{cases} 1, & x \in (0,1) \\ 0, & \text{o.w} \end{cases}, \quad g(x) = \begin{cases} 1, & x \in (1,2) \\ 0, & \text{o.w} \end{cases}$$

Then $h(x) < 0$ for $x \in (1,2)$.

Random Circle

A circle with a random radius $R \sim U(1,2)$ is generated. Let X be its area.

Find $\mathbb{E}[X]$.

Answer = number (3 significant figures)

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$$X = \pi R^2$$

$$\mathbb{E}[X] = \pi\, \mathbb{E}[R^2] = \pi \cdot \frac{1}{3}(1^2 + 1\cdot 2 + 2^2)$$

$$= \pi \cdot \frac{7}{3}$$

(Note: If $U \sim \text{unif}(a,b)$ then

$$\mathbb{E}[U^2] = \frac{1}{3}(a^2 + ab + b^2)$$).

Density Function

Let $f(x)$ be a probability density function given by

$$f(x) = \begin{cases} cx^3(4-x), & 0 < x < 4, \\ 0, & \text{otherwise.} \end{cases}$$

Find the value of c?

Answer = number (3 significant figures)

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Since f is pdf

$$1 = \int f(x)\,dx = \int_0^4 c\, x^3(4-x)\,dx$$

$$= c\left[4\int_0^4 x^3\,dx - \int_0^4 x^4\,d\,dx\right]$$

$$= c\left[4^4 - \frac{1}{5}4^5\right]$$

$$= c \cdot \frac{4^4}{5} \qquad \therefore\ c = \frac{5}{4^4}$$

Uniform Probability

Let U be a uniform random variable on $(9, 32)$. Find $\mathbb{P}(U \in (14, 19) \cup (24, 28))$.

Answer = number (3 significant figures)

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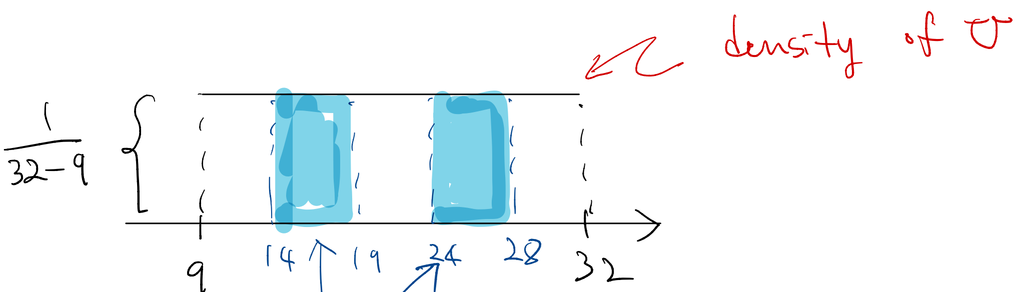

$$\text{The area} = P(U \in (14,19) \cup (24,28))$$

$$= \frac{1}{23} \cdot \left((19-14) + (28-24) \right)$$

$$= \frac{9}{23}.$$