Math 461: Midterm 1 Review

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Midterm 1 Information

Date: Wednesday March 10, Available 8am – 12pm (noon), 60 min exam

Location: Prairielearn

Test format:

(1) The exam will close 60 minutes after starting or at noon, whichever is earlier.

(2) There are three parts: True/False (3 questions, worth 15 pts), Multiple Choice (3 questions, worth 24 pts), Free response (3 questions, worth 36 pts)

(3) You have to submit answers in Prairielearn; & scan and upload the answer pdf file.

Review - Chapter 1.

1.2 The **Generalized basic principle of Counting** is

1.3 Number of permutations (i.e. orderings) of n distinct objects is

Number of permutations of n objects in which n_1 are alike, n_2 are alike, ... and n_r are alike is

1.4 Number of different ways to select k objects out of n distinct objects is
Equivalently, number of subsets of k elements of a set with n elements is
Simplify:

$$\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{n} =$$

The **Binomial theorem:**

1.5 The number of ways to split n distinct objects in r groups, with $n_1, n_2, \ldots, n_r$ objects in each group is

It coincides with the number of orderings of n objects in which n_1 are alike, n_2 are alike,..., n_r are alike.
The **Multinomial theorem** states that

1.6 Number of distinct, strictly positive integer solutions of $x_1 + x_2 + \cdots + x_r = n$ is

Number of distinct, non-negative integer solutions of $x_1 + x_2 + \cdots + x_r = n$ is

Review - Chapter 2.

2.2 Define **Sample Space**, **Event**.

The operations with events (sets) are

The basic properties of the operations are

2.3 Define **Probability** including its three axioms

2.4 The basic properties of probabilities are (Complement of an event, monotonicity)

Inclusion-Exclusion principle (probability of union of two or more events)

Qn. Suppose A, B and C are events with $\mathbb{P}(A) = .43, \mathbb{P}(B) = .40, \mathbb{P}(C) = 0.32$, $\mathbb{P}(A \cap B) = 0.29, \mathbb{P}(A \cap C) = 0.22, \mathbb{P}(B \cap C) = 0.20$ and $\mathbb{P}(A \cap B \cap C) = 0.15$. Find $\mathbb{P}(A' \cap B' \cap C')$.

How to compute probability of events in an sample space with equally likely outcomes

Review - Chapter 3.

3.2 Define **Conditional Probability**

If $\mathbb{P}(A) = 0.4, \mathbb{P}(B) = 0.5, \mathbb{P}(B \mid A) = 0.75$, find $\mathbb{P}(A \mid B)$ and $\mathbb{P}(A \mid B')$.

Multiplication Rule

3.3 **LoTP - Law of Total Probability**

Bayes Rule

3.4 Define **Independent Events** (for two, three and n number of events)

Review - Chapter 4.

4.1 Define a **Random Variable**

4.2 Define a **Discrete Random Variable**

Define **PMF**

Properties of PMF

4.3 Define **Expectation** of X

4.4 Compute **Expectation** of $g(X)$

X is a random variable taking values in $S = \{0, 1, 2, 3, \ldots\}$ with pmf $\mathbb{P}(X = 0) = p$, $\mathbb{P}(X = k) = \frac{1}{2^k k!}, k = 1, 2, 3, \ldots$. Find the value of p that would make this a valid probability model. Find $\mathbb{E}(X), \mathbb{E}(2^X)$.

4.5 Define **Variance**

How to compute $\mathrm{Var}(X) = \mathbb{E}(X^2) - (\mathbb{E}X)^2$

4.6 Define **Independent Trials**

Define a **Bernoulli Random Variable**

Define a **Binomial Random Variable**

Define a **Poisson Random Variable**

Write the interpretation of a Binomial random variable:

Find the mean and variance of these random variables

Under what assumptions Binomial Distribution can be assumed?

Is it appropriate to use Binomial model

(1) A fair coin is tossed 3 times. $X =$ Number of H's.

(2) A box contains 40 parts, 10 of which are defective. A person takes 7 parts out of the box with replacement. $X =$ number of defective parts selected.

(3) Seven members of the same family are tested for a particular food allergy. $X =$ number of family members who are allergic to this particular food.

Approximation to Poisson

When can one approximate a binomial to a Poisson?

Explain Poisson Paradigm.