Course Selection

To fulfill the requirements for a certain degree, a student can choose to take any 5 out of a list of 21 courses, with the constraint that at least 1 of the 5 courses must be a Mathemetics course.

Suppose that 8 of the 21 courses are Mathemetics courses.

How many choices are there for which 5 courses to take?

Answer = number (3 significant figures)

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Solution

The number of choosing 5 out of 21 courses

$$= \binom{21}{5}$$

The number of choosing 5 from 21 without Math

$$= \binom{21-8}{5} = \binom{13}{5}$$

So, the answer is

$$\binom{21}{5} - \binom{13}{5} = \frac{21\times 20\times 19\times 18\times 17}{5\times 4\times 3\times 2\times 1} - \frac{13\times 12\times 11\times 10\times 9}{5\times 4\times 3\times 2\times 1}$$

$$= 19062.$$

Couple Seat

In how many ways can 12 people be seated in a row if there are 6 married couples and each couple must sit together? (Write answer in integer.)

Answer = number (3 significant figures)

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Arrangement of 6 couples = $6!$

Arrangements of each couples = $(2!)^6$

$\therefore$ ANS = $6! \times (2!)^6$.

Friends and Party (Version 1)

A person has 12 friends, of whom 5 will be invited to a party.

How many choices are there if 2 of the friends are feuding and will not attend together?

Write only integer answer.

Answer= number (3 significant figures)

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Total number $= \binom{12}{5}$

Number of invitation that 2 friends attend together $= \binom{12-2}{5-2} = \binom{10}{2}$

$\therefore$ ANSWER $= \binom{12}{5} - \binom{10}{2}$

$= 672$

Friends and Party (Version 2)

A person has 12 friends, of whom 5 will be invited to a party.

How many choices are there if 2 of the friends ~~are feuding and will not attend~~ will attend only together?

Write only integer answer.

Answer= number (3 significant figures)

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It will be either

(i) two friends attend together

$$= \binom{12-2}{5-2} = \binom{10}{3}$$

or

(ii) two friends don't attend

$$= \binom{12-2}{5} = \binom{10}{5}$$

$$\therefore \text{ANSWER} = \binom{10}{3} + \binom{10}{5} = 132$$

Block Arrangements

A child has 16 blocks, of which 8 are black, 4 are red, 2 is white, and 2 is blue. (Blocks in each color are indistinguishable.)

If the child puts the blocks in a line, how many arrangements are possible?

Write only integer answer.

Answer= number (4 significant figures)

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This is the second interpretation of multinomial coefficient:

$n = 16, \quad n_1 = 8, \quad n_2 = 4, \quad n_3 = n_4 = 2$

$\Rightarrow \binom{16}{8, 4, 2, 2}$ is the answer.

$= 5\,405\,400.$