Sum of Binomials

If $X \sim$ Bin(n, p) and $Y \sim$ Bin(n, p) are independent, then $X + Y \sim$ Bin$(n, 2p)$.

- (a) True
- X (b) False

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If $X \sim \text{Bin}(n, p)$ and $Y \sim \text{Bin}(m, p)$ and they are independent, then

$$X+Y \sim \text{Bin}(n+m, p).$$

Independent Random Variabls

Let X, Y be jointly distributed continuous random variables with joint density given by

$$f(x,y) = \begin{cases} 3(x - x^2y) & 0 < x < 1, 0 < y < 1, \\ 0, & \text{otherwise.} \end{cases}$$

Then X and Y are

- [x] (a) dependent.
- [ ] (b) independent.

$f_X(x) = 3x - \frac{3}{2}x^2y^2, \quad 0 < x < 1$

$f_Y(y) = \frac{3}{2} - y, \quad 0 < y < 1$

$\Rightarrow \quad f_X(x) \cdot f_Y(y) \neq f(x,y)$

Let X, Y be jointly distributed continuous random variables with joint density given by

$$f(x,y) = \begin{cases} 8xy & 0 < x < y < 1, \\ 0, & \text{otherwise.} \end{cases}$$

Then X and Y are

- [ ] (a) independent.
- [x] (b) dependent.

$f_X(x) = \int_x^1 8xy\, dy = 4x(1 - x^2)$

$0 < x < 1$

$f_Y(y) = \int_0^y 8xy\, dx = 4y^3, \quad 0 < y < 1$

$\Rightarrow \quad f_X(x) \cdot f_Y(y) \neq f(x,y).$

Let X, Y be jointly distributed continuous random variables with joint density given by

$$f(x,y) = \begin{cases} 4(x - xy) & 0 < x < 1, 0 < y < 1, \\ 0, & \text{otherwise.} \end{cases}$$

Then X and Y are

- [ ] (a) dependent.
- [x] (b) independent.

$f(x,y) = 4x \cdot \mathbb{1}_{(0,1)}(x) \cdot (1-y)\mathbb{1}_{(0,1)}(y)$

$= g(x) \cdot h(y)$

$\Rightarrow$ Independent.

Insurance Benefit

An insurance policy reimburses a loss up to a maximum benefit of 38. So if the loss is more than 38, the insurance will only pay 38. Suppose that the policyholder's loss, Y, follows a distribution with density function given by $f(y) = 81y^{-4}$, for $y > 3$ and 0 otherwise.

What is the expected value of the benefit paid under the insurance policy?

Answer = number (2 significant figures)

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$$\mathbb{E}[\min\{Y, 38\}]$$

$$= \int_3^\infty \min\{y, 38\} \cdot 81 y^{-4}\, dy$$

$$= \int_3^{38} 81 y^{-3}\, dy + \int_{38}^\infty 81 \cdot 38\, y^{-4}\, dy$$

$$= \frac{81}{2}\left(3^{-2} - 38^{-2}\right) + \frac{81 \cdot 38}{3} (38)^{-3}$$

$$= \frac{9}{2} - \frac{81}{2} \cdot \frac{1}{38^2} + \frac{81}{3} \cdot \frac{1}{38^2}$$

$$= \frac{9}{2} - \frac{81}{(38)^2} \cdot \frac{1}{6}$$

$$= 4.491$$

Sum of Uniform rvs

Let X and Y be independent continuous random variables with $X \sim$ Uniform$(0, 4)$ and $Y \sim$ Uniform$(0, 8)$.

Find the density of the random variable $Z = X + Y$ at the point $z = 9.3$.

Answer = number (2 significant figures)

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$$f_Z(a) = \int_{\mathbb{R}} f_X(x) \cdot f_Y(a-x)\, dx$$

$$= \frac{1}{32} \int_0^4 \mathbb{1}_{(0,8)}(a-x)\, dx$$

$$= \begin{cases} 0 & , a < 0 \text{ or } a \geq 12 \\ \frac{a}{32} & , 0 \leq a < 4 \\ \frac{1}{8} & , 4 \leq a < 8 \\ \frac{12-a}{32} & , 8 \leq a < 12 \end{cases}$$

$$\therefore f_Z(9.3) = \frac{12-9.3}{32} = 0.084375$$