

# Homework 4

Math 461: Probability Theory, Spring 2021

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Due date: Feb 26, 2021

## Instruction

1. Each problem is worth 10 points and only five randomly chosen problems will be graded.
2. Convert a photocopy of your solutions to **one single pdf file** and upload it on Moodle.
3. Please indicate whom you worked with, it will not affect your grade in any way.

1. A simplified model for the movement of the price of a stock supposes that on each day the stock's price either moves up 1 unit with probability  $p$  or moves down 1 unit with probability  $1 - p$ . The changes on different days are assumed to be independent.
  - (a) What is the probability that after 2 days the stock will be at its original price?
  - (b) What is the probability that after 3 days the stock's price will have increased by 1 unit?
  - (c) Given that after 3 days the stock's price has increased by 1 unit, what is the probability that it went up on the first day?
2.  $A$  and  $B$  play a series of games. Each game is independently won by  $A$  with probability  $p$  and by  $B$  with probability  $1 - p$ . They stop when the total number of wins of one of the players is two greater than that of the other player. The player with the greater number of total wins is declared the winner of the series.
  - (a) Find the probability that a total of 4 games are played.
  - (b) Find the probability that  $A$  is the winner of the series.
3. An urn contains 12 balls, of which 4 are white. Three player— $A$ ,  $B$ , and  $C$ —successively draw from the urn,  $A$  first, then  $B$ , then  $C$ , then  $A$ , and so on. The winner is the first one to draw a white ball. Find the probability of winning for each player if
  - (a) each ball is replaced after it is drawn;
  - (b) the balls that are withdrawn are not replaced.
4. Find an example that three events  $A, B, C$  are pairwise independent (that is,  $A$  and  $B$  are independent,  $B$  and  $C$  are independent, and  $C$  and  $A$  are independent) but not independent.
5. Two balls are chosen randomly from an urn containing 8 white and 4 black. Suppose that we win \$2 for each black ball selected and we lose \$1 for each white ball selected. Let  $X$  denote our winnings. What are the possible values of  $X$ , and what are the probabilities associated with each value?
6. Five men and five women are ranked according to their scores on an examination. Assume that no two scores are alike and all  $10!$  possible rankings are equally likely. Let  $X$  be rank of the best scoring woman (ranks are w.r.t. decreasing values of score, so that rank 1 has the best score, rank 2 has the second best score and so on). Find the pmf of  $X$ .
7. A salesman has scheduled two appointments to sell encyclopedias. His first appointment will lead to a sale with probability .4, and his second will lead independently to a sale with probability .6. Any sale made is equally likely to be either for the deluxe model, which costs \$1000, or the standard model, which costs \$500. Determine the possible values and probabilities of  $X$ , the total dollar value of all sales.

8. Five distinct numbers are randomly distributed to five players numbered 1 through 5. Whenever two players compare their numbers, the one with the higher one is declared the winner. Initially, players 1 and 2 compare their numbers; the winner then compares her number with that of player 3, and so on. Let  $X$  denote the number of times player 1 is a winner. Find  $\mathbb{P}(X = i), i = 0, 1, 2, 3, 4$ .
9. Four buses carrying 148 students from the same school arrive at a football stadium. The buses carry, respectively, 40, 33, 25, and 50 students. One of the students is randomly selected. Let  $X$  denote the number of students that were on the bus carrying the randomly selected student. One of the 4 bus drivers is also randomly selected. Let  $Y$  denote the number of students on her bus.
- (a) Which of  $\mathbb{E}(X)$  or  $\mathbb{E}(Y)$  do you think is larger? Why?
  - (b) Find the pmf of  $X$  and  $Y$ .
  - (c) Compute  $\mathbb{E}(X)$  and  $\mathbb{E}(Y)$ .
10. On a multiple-choice exam with 3 possible answers (that is, one answer among 3 choices) for each of the 5 questions, what is the probability that a student will get 4 or more correct answers just by guessing?