Homework 1

Math 461: Probability Theory, Spring 2021
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Due date: Feb 5, 2021

Each problem is worth 10 points and only five randomly chosen problems will be graded. Please indicate whom you worked with, it will not affect your grade in any way.

1. For years, area zip codes in the United States and Canada consisted of a sequence of five digits. The first digit was an integer between 2 and 9, the second digit was either 0 or 1, and the third to fifth digits were any integer from 1 to 9.
   (a) How many area codes were possible?
   (b) How many area codes starting with a 4 were possible?
   (c) How many area codes were possible with all distinct digits?
2. Five separate awards (best scholarship, best leadership qualities, and so on) are to be presented to selected students from a class of 25. How many different outcomes are possible if
   (a) a student can receive any number of awards?
   (b) a student can receive at most 1 award?
   (c) a student can receive at most 4 awards?
3. A person has 15 friends, of whom 6 will be invited to a party.
   (a) How many choices are there in total?
   (b) How many choices are there if 2 of the friends are feuding and will not attend together?
   (c) How many choices if 3 of the friends will only attend together?
4. How many different letter arrangements can be made from the letters
   (a) BLOCK (b) CLASS (c) ALLOWANCE (d) MISSISSIPPI
5. In how many ways can 2 chemistry, 3 mathematics books, and 4 biology book be arranged on a bookshelf if
   (a) the books can be arranged in any order?
   (b) the mathematics books must be together and the biology books must be together?
   (c) the biology books must be together, but the other books can be arranged in any order?
6. (a) How many ways are there to split a dozen people into 3 teams, where one team has 2 people, and the other two teams have 5 people each? The teams are unordered.
   (b) How many ways are there to split a dozen people into 3 teams, where each team has 4 people? The teams are unordered.
7. (a) How many paths are there from the point $(0, 0)$ to the point $(10, 10)$ in the plane such that each step either consists of going one unit up or one unit to the right?
   (b) How many paths are there from $(0, 0)$ to $(20, 20)$, where each step consists of going one unit up or one unit to the right, and the path has to go through $(10, 10)$?
8. Expand (a) $(2y + z^2)^5$ and (b) $(x + 2y + z)^3$.

9. A system is comprised of 5 components, each of which is either working or failed. Consider an experiment that consists of observing the status of each component, and let the outcome of the experiment be given by the vector $(x_1, x_2, x_3, x_4, x_5)$, where x_i is equal to 1 if component i is working and is equal to 0 if component i is failed.

   (a) How many outcomes are in the sample space of this experiment?

   (b) Suppose that the system will work if components 1 and 2 are both working, or if components 3 and 4 are both working, or if components 1, 3, and 5 are all working. Let W be the event that the system will work. Specify all the outcomes in W.

   (c) Let A be the event that components 4 and 5 are both failed. How many outcomes are contained in the event A?

   (d) Write out all the outcomes in the event AW.

10. A hospital administrator codes incoming patients suffering gunshot wounds according to whether they have insurance (coding 1 if they do and 0 if they do not) and according to their condition, which is rated as good (g), fair (f), or serious (s). Consider an experiment that consists of the coding of such a patient.

    (a) Give the sample space of this experiment.

    (b) Let A be the event that the patient is in serious condition. Specify the outcomes in A.

    (c) Let B be the event that the patient is uninsured. Specify the outcomes in B.

    (d) Give all the outcomes in the event $B^c \cup A$.