

# Lecture Note: Week 7

MATH 461: Probability Theory, Spring 2021  
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## Lecture 18. The Uniform Random Variable (Sec 5.3)

**Definition: Uniform random variable**

A random variable  $X$  is a uniform random variable on  $(a, b)$  if its probability density function is

$$f(x) = \begin{cases} \frac{1}{b-a}, & a < x < b, \\ 0, & \text{otherwise.} \end{cases}$$

We denote by  $X \sim U(a, b)$ .

**Example 1.** If  $X$  is uniformly distributed over  $(0, 10)$ , calculate the probability that

- (i)  $X < 3$ ,
- (ii)  $X > 6$ ,
- (iii)  $3 < X < 8$ ,

**Proposition 2.** If  $X \sim U(a, b)$ , then  $\mathbb{E}[X] = \frac{a+b}{2}$ ,  $\text{Var}(X) = \frac{(b-a)^2}{12}$ , and the distribution function is

$$F(x) = \begin{cases} 0, & x < a, \\ \frac{1}{b-a}(x-a), & a \leq x \leq b, \\ 1, & x > b. \end{cases}$$

**Example 3.** A bus travels between the two cities  $A$  and  $B$ , which are 100 miles apart. If the bus has a breakdown, the distance from the breakdown to city  $A$  has a  $U(0, 100)$  distribution. There are bus service stations in city  $A$ , in  $B$ , and in the center of the route between  $A$  and  $B$ . It is suggested that it would be more efficient to have the three stations located 25, 50, and 75 miles, respectively, from  $A$ . Do you agree? Why?

## References

[SR] Sheldon Ross, *A First Course in Probability*, 9th Edition, Pearson

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