

College Committee

A club consists of 7 seniors, 8 juniors, and 10 sophomores. An organizing committee of size 5 is chosen randomly (with all subsets of size 5 equally likely).

Find the probability that there are exactly 4 seniors in the committee.

Answer= number (3 significant figures)

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Solution

$$\begin{aligned} P(\text{exactly 4 seniors}) &= \frac{\binom{7}{4} \cdot \binom{8+10}{1}}{\binom{7+8+10}{5}} \\ &= 0.0119 \end{aligned}$$

Random Injective Maps

Consider the two sets $A = \{1, 2, \dots, 6\}$ and $B = \{1, 2, \dots, 9\}$. A function f from A to B is constructed by assigning a random element from B to $f(x)$ for all $x \in A$.

What is the probability that the function f is injective, that is $f(x) \neq f(y)$ for all $x \neq y$ in A .

Answer= number (3 significant figures)



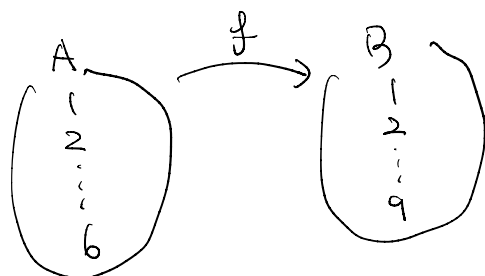
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Solution

Total outcomes = 9^6



For each element in A , there are $|B|=9$ options.

If f is injective, then $1 \in A$ has 9 options, $2 \in A$ has 8 options, ...

$$\begin{aligned} \text{Thus, } P_{\text{prob}} &= \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4}{9^6} \\ &= 0.114. \end{aligned}$$

Random Box

Consider two boxes, one containing 5 black and 6 white marble, the other 3 black and 3 white marble. A box is selected at random (each choice is equally likely), and a marble is drawn from it at random (each choice is equally likely).

What is the probability that the **first box was the one selected** given that the **marble is white**?

Answer =



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Solution

B: choose black

W: choose white

F: 1st box chosen

S: 2nd box chosen.

$$P(F|W) = \frac{P(W|F)P(F)}{P(W|F)P(F) + P(W|S)P(S)}$$

Since $P(F) = P(S) = \frac{1}{2}$, $P(W|F) = \frac{6}{5+6}$, $P(W|S) = \frac{3}{3+3}$,

$$P(F|W) = \frac{\frac{6}{11} \times \frac{1}{2}}{\frac{6}{11} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2}} = 0.522$$

Random Voter

A total of 16 percent of the voters in a certain city classify themselves as Independents, whereas 63 percent classify themselves as Liberals and 21 percent say that they are Conservatives.

In a recent local election, 66 percent of the Independents, 55 percent of the Liberals, and 36 percent of the Conservatives voted. A voter is chosen at random.

Given that this person voted in the local election, what is the probability that he or she is an Independent?

Answer =



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Solution.

V : Voted.

I : Independents

L : Liberals

C : Conservatives

$$P(I) = 0.16$$

$$P(L) = 0.63$$

$$P(C) = 0.21$$

$$P(V|I) = 0.66$$

$$P(V|L) = 0.55$$

$$P(V|C) = 0.36$$

$$P(I|V) =$$

$$\frac{P(V|I) P(I)}{P(V|I) P(I) + P(V|L) P(L) + P(V|C) P(C)}$$

$$= 0.200$$