Homework 3

Math 461: Probability Theory, Spring 2021
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Due date: Feb 19, 2021

Instruction

1. Each problem is worth 10 points and only five randomly chosen problems will be graded.
2. Convert a photocopy of your solutions to **one single pdf file** and upload it on Moodle.
3. Please indicate whom you worked with, it will not affect your grade in any way.

1. An urn contains 4 red and 8 black balls. Players A and B withdraw balls from the urn consecutively until a red ball is selected. Find the probability that A selects the red ball. (A draws the first ball, then B, and so on. There is no replacement of the balls drawn.)

2. Two fair dice are rolled. What is the conditional probability that none lands on 6 given that the dice land on different numbers?

3. Consider an urn containing 15 balls, of which 8 are red, 5 are green and 2 are blue. A sample of size 4 is to be drawn with replacement (without replacement). What is the conditional probability (**in each case**) that the first and third balls drawn will be red given that the sample drawn contains exactly 2 red balls?

4. A closet contains 12 pairs of shoes. If 7 shoes are randomly selected without replacement, find the probability that there will be (a) at least one complete pair? (b) exactly 2 complete pairs? (c) exactly 2 complete pairs given that there is at least one complete pair.

5. Consider 3 urns. Urn A contains 2 white and 4 red balls, urn B contains 8 white and 4 red balls, and urn C contains 1 white and 3 red balls. If 1 ball is selected from each urn, what is the probability that the ball chosen from urn A was white given that exactly 2 white balls were selected?

6. Urn I contains 2 white and 4 red balls, whereas urn II contains 1 white and 1 red ball. A ball is randomly chosen from urn I and put into urn II, and a ball is then randomly selected from urn II. What is
(a) the probability that the ball selected from urn II is white?
(b) the conditional probability that the transferred ball was white given that a white ball is selected from urn II?

7. Consider two boxes, one containing 1 black and 1 white marble, the other 2 black and 1 white marble. A box is selected at random, and a marble is drawn from it at random.
(a) What is the probability that the marble is black?
(b) What is the probability that the first box was the one selected given that the marble is white?

8. Die A has 4 red and 2 white faces, whereas die B has 2 red and 4 white faces. A fair coin is flipped once. If it lands on heads, the game continues with die A; if it lands on tails, then die B is to be used.
(a) Show that the probability of red at any throw is $1/2$.
(b) If the first two throws result in red, what is the probability of red at the third throw?
(c) If red turns up at the first two throws, what is the probability that it is die A that is being used?

9. Suppose that you continually collect coupons and that there are m different types. Suppose also that each time a new coupon is obtained, it is a type i coupon with probability $p_i, i = 1, 2, \ldots, m$. Suppose that you have just collected your n-th coupon. What is the probability that it is a new type?
Hint: Condition on the type of the n-th coupon.

10. Independent flips of a coin that lands on heads with probability p are made. What is the probability that the first four outcomes are
    (a) HHHH?
    (b) THHH?
    (c) What is the probability that the pattern THHH occurs before the pattern HHHH?
    Hint for part (c): How can the pattern HHHH occur first?