Communication Efficient Distributed Learning with Censored, Quantized, and Generalized Group ADMM

Chaouki ben Issaid
chaouki.benissaid@oulu.fi

Intelligent Connectivity and Networks/Systems (ICON) Group
Faculty of Information Technology and Electrical Engineering
University of Oulu

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Outline

Relevant Papers

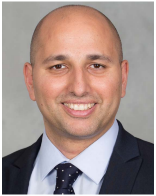
- Anis Elgabli, Jihong Park, Amrit S. Bedi, Mehdi Bennis and Vaneet Aggarwal, GADMM: Fast and Communication Efficient Framework for Distributed Machine Learning, Journal of Machine Learning Research, 2020.
- Anis Elgabli, Jihong Park, Amrit S Bedi, Chaouki Ben Issaid, Mehdi Bennis and Vaneet Aggarwal, Q-GADMM: Quantized Group ADMM for Communication Efficient Decentralized Machine Learning, IEEE Transactions on Communications, 2021.
- Chaouki Ben Issaid, Anis Elgabli, Jihong Park, Mehdi Bennis and Mérouane Debbah, Communication Efficient Distributed Learning with Censored, Quantized, and Generalized Group ADMM, arXiv:2009.06459, 2021.

Collaborators

Anis Elgabli
Postdoctoral Fellow
University of Oulu

Jihong Park
Lecturer
Deakin University

Mehdi Bennis
Associate Professor
University of Oulu



Mérouane Debbah
Director of Lagrange Mathematical and Computing Research Center
Huawei, Paris

Overview

Inroduction and Motivation

Centralized Learning

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$\rightarrow$ To design distributed and efficient learning algorithms to replace the centralized mode of operation.

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- Each node is usually assigned a local computation task
- Goal: to enable the nodes to converge towards the global minimizer of a central learning model with the help of a central node (parameter server/master).

Parameter Server Based Distributed Learning

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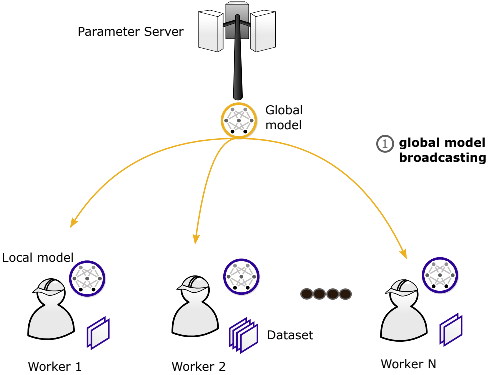
- $\boldsymbol{\Theta} \in \mathbb{R}^d$: global model parameter.
- $f_n : \mathbb{R}^d \to \mathbb{R}$: local cost function of worker n.

First Example: Mini-Batch SGD



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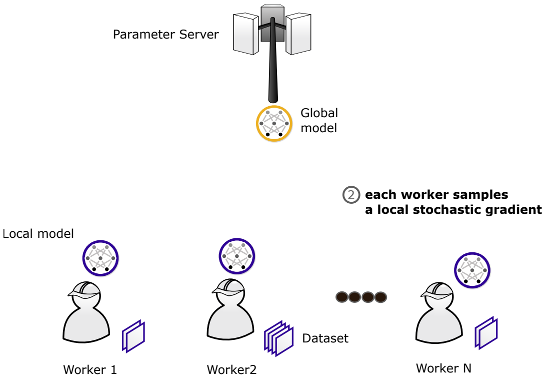

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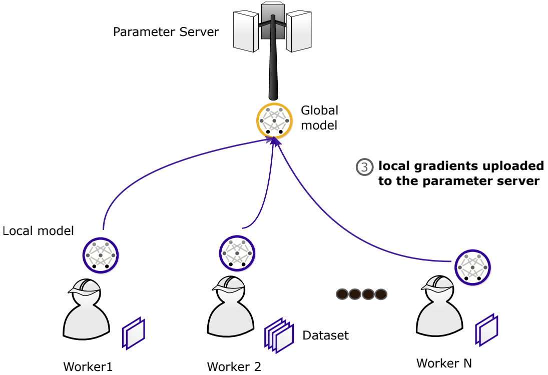

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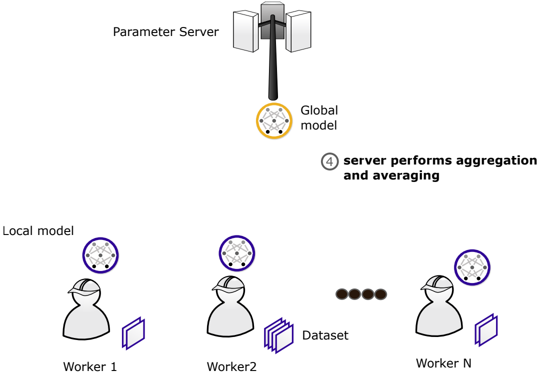

Second Example: Parameter Server ADMM

Constrained formulation of **(P1)**:

$$\min_{\boldsymbol{\Theta},\{\boldsymbol{\theta}_n\}_{n=1}^N} \sum_{n=1}^{N} f_n(\boldsymbol{\theta}_n) \tag{2}$$

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$$\boldsymbol{\theta}_n^{k+1} = \arg\min_{\boldsymbol{\theta}_n} \{ f_n(\boldsymbol{\theta}_n) + \langle \boldsymbol{\lambda}_n^k, \boldsymbol{\theta}_n - \boldsymbol{\Theta}^k \rangle + \frac{\rho}{2} \|\boldsymbol{\theta}_n - \boldsymbol{\Theta}^k\|^2 \}. \tag{3}$$

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2. Parameter server updates its model

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3. Worker n updates its dual variable

$$\boldsymbol{\lambda}_n^{k+1} = \boldsymbol{\lambda}_n^k + \rho\left(\boldsymbol{\theta}_n^{k+1} - \boldsymbol{\Theta}^{k+1}\right). \tag{5}$$

Towards Communication-Efficient Approaches

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This talk

Why Decentralized Learning?

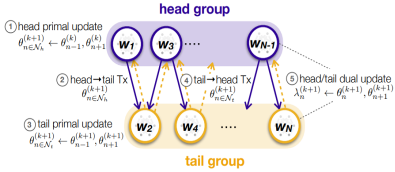
- Parameter server based architectures are not ideal for on-device intelligence settings for various reasons
  1. Transmission of information between the central node and the devices can be expensive especially when communication is conducted via multi-hop relays or when the devices are moving.
  2. Privacy and secrecy considerations where individual nodes may be reluctant to share information with a remote center.
  3. Efficiency depends on the slowest worker $\rightarrow$ as the number of workers increases, the uplink communication resources become the bottleneck.
- Alternative: extract information in a decentralized manner to avoid high latency and communication costs (number of active IoT devices is expected to be over 75 billion by 2025[1]).

[1] "Internet of things-number of connected devices worldwide 2015-2025", 2016.

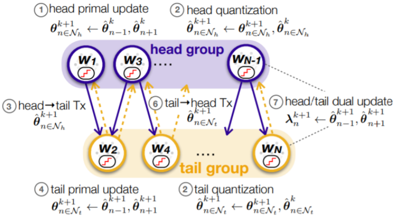
Censored and Quantized Generalized Group ADMM

Group ADMM

- A connected network wherein a set $\mathcal{V}$ of N workers aim to reach a consensus around a solution of the global optimization problem.



(a) GADMM [1]



(b) QGADMM [2]

[1] A. Elgabli, J. Park, AS Bedi, M. Bennis, and V. Aggarwal, "GADMM: Fast and Communication Efficient Framework for Distributed Machine Learning", JMLR, 21 (76), pp 1-39, 2020.

[2] A. Elgabli, J. Park, A. S. Bedi, C. Ben Issaid, M. Bennis and V. Aggarwal, "Q-GADMM: Quantized Group ADMM for Communication Efficient Decentralized Machine Learning," in IEEE TCOM, vol. 69, no. 1, pp. 164-181, Jan. 2021.

Problem Formulation

- The connections among workers are represented as an undirected communication graph $\mathcal{G}$ having the set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ of edges.

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- We assume that the communication graph $\mathcal{G}$ is **bipartite** and **connected**.
- In this case, workers are divided into two groups: a *head group* $\mathcal{H}$, and a *tail group* $\mathcal{T}$. Each head worker in $\mathcal{H}$ can only communicate with tail workers in $\mathcal{T}$, and vice versa.

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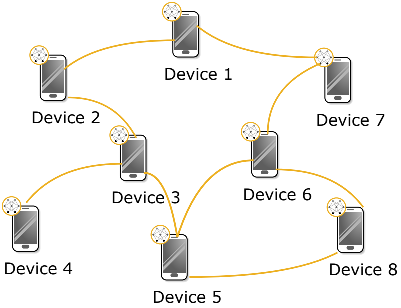
- In this case, workers are divided into two groups: a *head group* $\mathcal{H}$, and a *tail group* $\mathcal{T}$. Each head worker in $\mathcal{H}$ can only communicate with tail workers in $\mathcal{T}$, and vice versa.
- **(P1)** is equivalent to the following problem

$$\textbf{(P2)} \quad \min_{\{\boldsymbol{\theta}_n\}_{n=1}^N} \sum_{n=1}^N f_n(\boldsymbol{\theta}_n) \tag{6}$$

$$\text{s.t. } \boldsymbol{\theta}_n = \boldsymbol{\theta}_m, \forall (n, m) \in \mathcal{E},$$

Examples

- In wireless sensor networks, neighboring nodes can be devices that are within the range of radio broadcasting.
- In smart phone networks, the neighbors can be devices that are within the same local area network.



Schematic Illustration

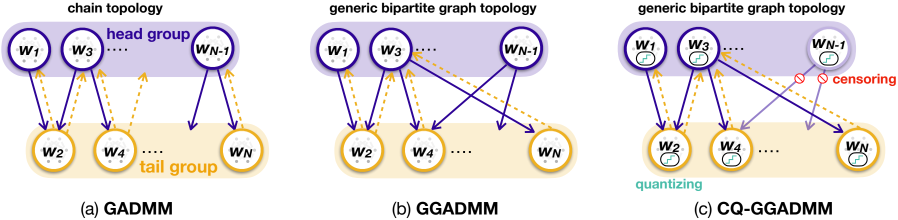


Figure 1: (a) *group ADMM (GADMM)*, the baseline algorithm under a chain topology, compared to our proposed (b) *generalized GADMM (GGADMM)* under a generic bipartite graph topology, and (c) *censored-and-quantized GGADMM (CQ-GGADMM)* that additionally applies link censoring for negligible updates after quantization.

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2. To reduce **the number of communication links per round** by exploiting a censoring approach that allows to exchange model parameters only when the updated quantized model is sufficiently changed from the previous quantized model.

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- **A4.** The local cost functions f_n have L_n-Lipschitz continuous gradient ($L_n > 0$).

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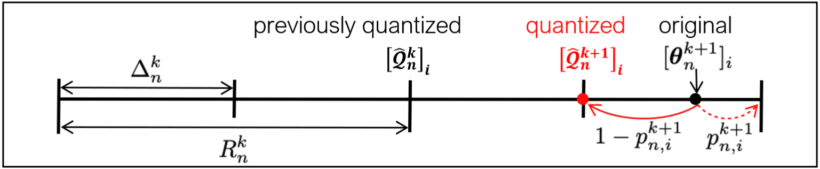
Communication-censoring Strategy

$$\hat{\theta}_n^{k+1} = \begin{cases} \hat{\mathcal{Q}}_n^{k+1}, & \text{if } \|\hat{\theta}_n^k - \hat{\mathcal{Q}}_n^{k+1}\| \geq \tau_0 \xi^{k+1} \\ \hat{\theta}_n^k, & \text{otherwise} \end{cases} \tag{7}$$

Heterogeneous Stochastic Quantization



- At iteration k, each worker n quantizes the difference between the current model and the previous quantized model vector: $\boldsymbol{\theta}_n^k - \hat{\mathcal{Q}}_n^{k-1} = Q_n(\boldsymbol{\theta}_n^k, \hat{\mathcal{Q}}_n^{k-1})$.

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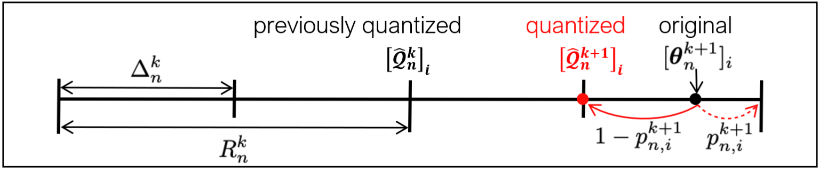

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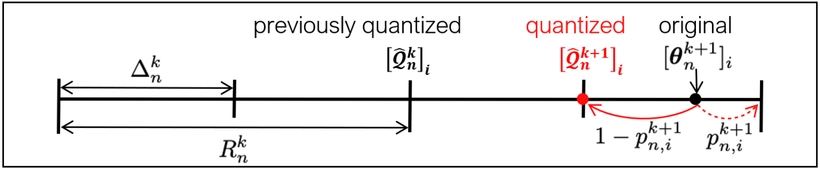

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- $\Delta_n^k = 2R_n^k/(2^{b_n^k} - 1)$: quantization step size where $2R_n^k$ is the quantization range.

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- Define the function $q_n(\cdot)$ as

$$[q_n(\boldsymbol{\theta}_n^k)]_i = \begin{cases} \lceil [c_n(\boldsymbol{\theta}_n^k)]_i \rceil & \text{with probability } p_{n,i}^k, \\ \lfloor [c_n(\boldsymbol{\theta}_n^k)]_i \rfloor & \text{with probability } 1 - p_{n,i}^k, \end{cases} \tag{9}$$

where $[c_n(\boldsymbol{\theta}_n^k)]_i = \frac{1}{\Delta_n^k} \left([\boldsymbol{\theta}_n^k]_i - [\hat{\boldsymbol{\mathcal{Q}}}_n^{k-1}]_i + R_n^k \right)$.

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- At the neighbours of worker n: $\hat{\boldsymbol{\mathcal{Q}}}_n^k = \hat{\boldsymbol{\mathcal{Q}}}_n^{k-1} + \Delta_n^k q_n(\boldsymbol{\theta}_n^k) - R_n^k \mathbf{1}$.

CQ-GGADMM Steps

1. Primal variables for head workers are found using

$$\boldsymbol{\theta}_n^{k+1} = \arg\min_{\boldsymbol{\theta}_n} \; f_n(\boldsymbol{\theta}_n) + \langle \boldsymbol{\theta}_n, \boldsymbol{\alpha}_n^k - \rho \sum_{m \in \mathcal{N}_n} \hat{\boldsymbol{\theta}}_m^k \rangle + \frac{\rho}{2} d_n \|\boldsymbol{\theta}_n\|^2. \quad (10)$$

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2. Primal variables update for tail workers is done as follow

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3. Dual variable of each worker is updated locally

$$\boldsymbol{\alpha}_n^{k+1} = \boldsymbol{\alpha}_n^k + \rho \sum_{m\in\mathcal{N}_n} (\hat{\boldsymbol{\theta}}_n^{k+1} - \hat{\boldsymbol{\theta}}_m^{k+1}), \; \forall n \in \mathcal{V}. \tag{12}$$

where $\boldsymbol{\alpha}_n = \sum_{m\in\mathcal{N}_n} \boldsymbol{\lambda}_{n,m}$

Main Theorem

Convergence of CQ-GGADMM

Suppose that assumptions **A1-A4** hold and the dual variable $\boldsymbol{\alpha}$ is initialized such that $\boldsymbol{\alpha}^0$ lies in the column space of the oriented incidence matrix $\boldsymbol{M}_{-}$.
For sufficiently small ρ, the sequence of iterates of CQ-GGADMM converges linearly with a rate $(1+\delta_2)/2$ where $\delta_2 = \max\{(1+\kappa)^{-1}, \psi^2\}$ and $\psi = \max\{\xi, \omega\}$.

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Convergence of CQ-GGADMM

Suppose that assumptions **A1-A4** hold and the dual variable $\boldsymbol{\alpha}$ is initialized such that $\boldsymbol{\alpha}^0$ lies in the column space of the oriented incidence matrix $\boldsymbol{M}_-$.
For sufficiently small ρ, the sequence of iterates of CQ-GGADMM converges linearly with a rate $(1+\delta_2)/2$ where $\delta_2 = \max\{(1+\kappa)^{-1}, \psi^2\}$ and $\psi = \max\{\xi, \omega\}$.

The constant κ depends on:

- the network topology through the maximum and minimum non-zero singular values of $\boldsymbol{M}_-$,
- the properties of the local objective functions ($\mu = \min_{1\le n\le N} \mu_n$ and $L = \max_{1\le n\le N} L_n$),
- the penalty parameter ρ.

Numerical Results

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- **Communication energy:**
  - Consumed energy defined as $E = P\tau$.
  - Transmission power: $P = \tau D^2 N_0 B_n (2^{R/B_n} - 1)$.
  - Upload/download transmission time: $\tau = 1ms$.
  - Transmission rate: $R = (32d/1ms) bits/sec$.
  - Power spectral density: $N_0 = 10^{-6} W/Hz$.

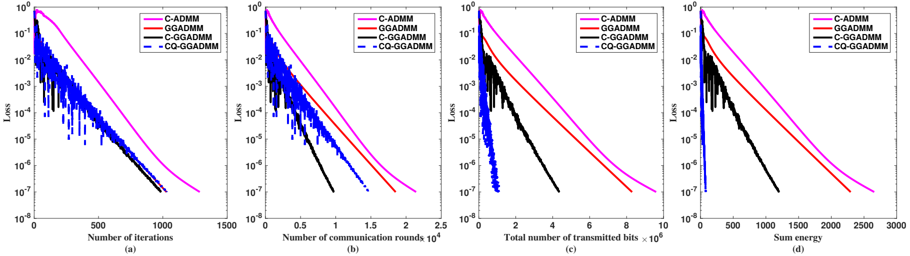


Figure 2: *Linear regression* results on real dataset showing: (a) loss w.r.t. # iterations; (b) loss w.r.t. # communication rounds; (c) loss w.r.t. # transmitted bits; (d) energy efficiency (loss w.r.t. total energy), the number of workers is 18.

Logistic Regression

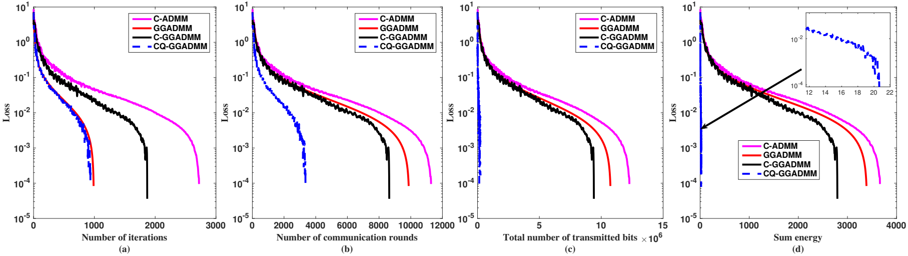
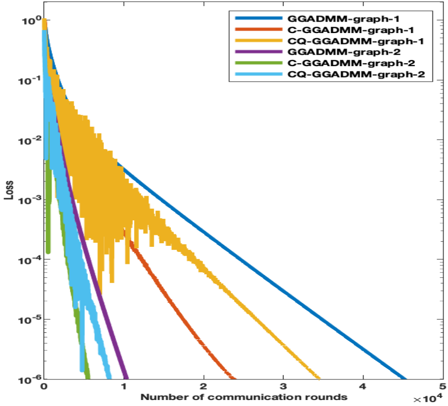


Figure 3: *Logistic regression* results on real dataset showing: (a) loss w.r.t. # iterations; (b) loss w.r.t. # communication rounds; (c) loss w.r.t. # transmitted bits; (d) energy efficiency (loss w.r.t. total energy), the number of workers is 18.

Impact of the Network Graph Density



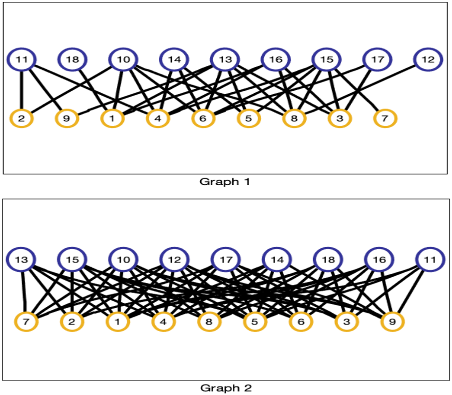


Figure 4: *Effect of the graph density on the performance of the algorithms*: loss w.r.t. # communication rounds (left), Graph 1: Sparse graph (right top); Graph 2: dense graph (right bottom). The number of workers is 18, and the task is linear regression on real dataset.

Conclusion and Future Work

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  - Time-varying topology.
  - Communication and computation efficiency: running few local SGD instead of solving the exact local problem.

Thank you!

- Chaouki Ben Issaid, Anis Elgabli, Jihong Park, Mehdi Bennis and Mérouane Debbah, Communication Efficient Distributed Learning with Censored, Quantized, and Generalized Group ADMM, arXiv:2009.06459, 2021.