Structure from Motion (SfM)

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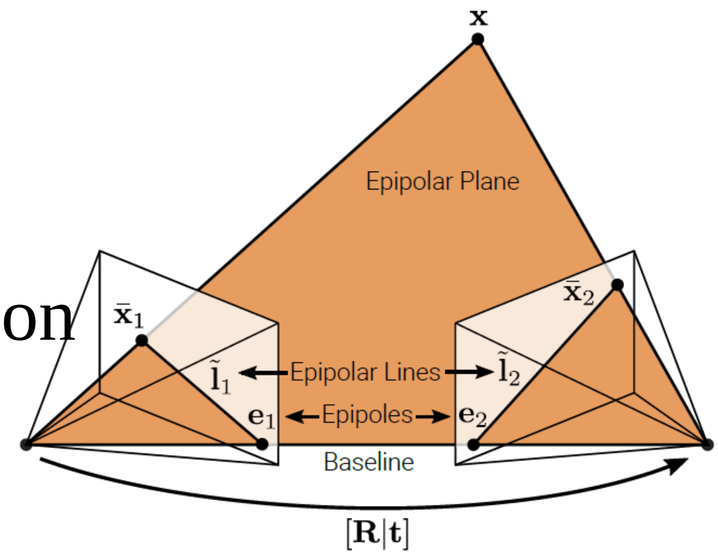
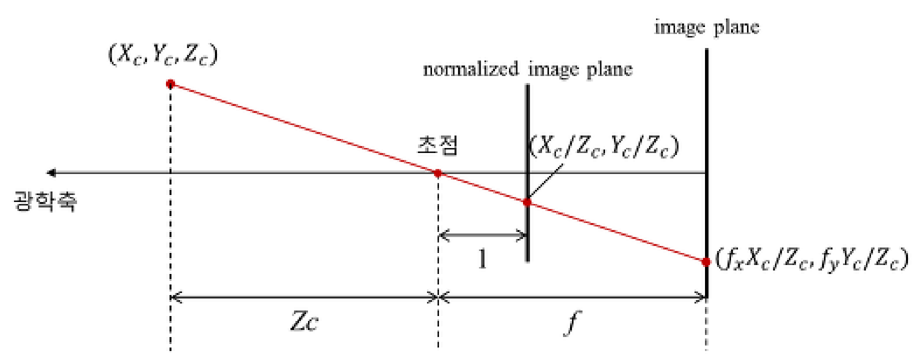


Camera Calibration

- 내부 파라미터 (Intrinsic parameter)
  - 최초 카메라 공정 과정에서 결정되는 파라미터
  - 초점거리(focal length) fx, fy : 렌즈 중심과 이미지 센서와의 거리, 픽셀단위로 표현
  - 주점(principal point) cx, cy : 카메라 렌즈의 중심
  - 비대칭 계수 skew_c : 이미지 센서의 y축 기울어진 정도

$$s\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & \mathrm{skew\_c}f_x & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \end{bmatrix}\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$= A[R \mid t]\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

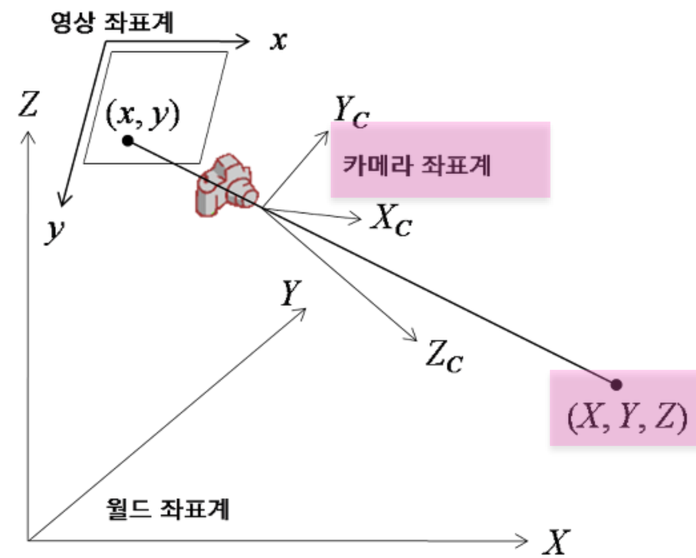


Camera Calibration

- 외부 파라미터 (Extrinsic Parameters)
  - 3차원 월드 좌표계와 카메라 좌표계의 상대적 위치를 정의
  - 영상을 획득한 당시의 카메라 위치관계를 설명
  - Rotation(회전) 3 X 3 행렬과 Translation (이동) 3 X 1 행렬로 구성
  - 카메라가 실제의 원점으로부터 얼마만큼 이동되었고, 얼마만큼 회전되었는지에 대한 정보

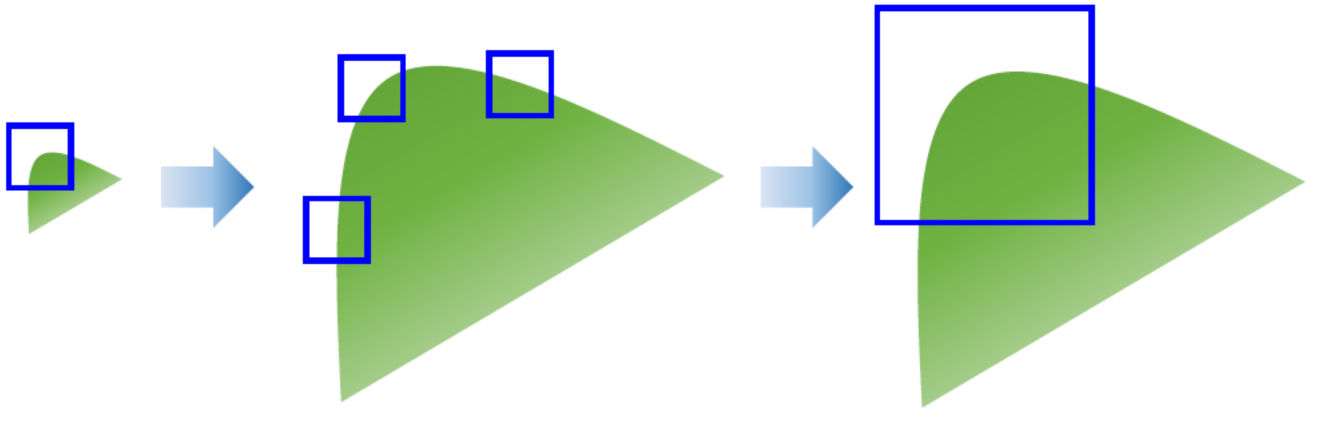
$$s\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & \text{skew\_c}f_x & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$= A[R \mid t] \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$



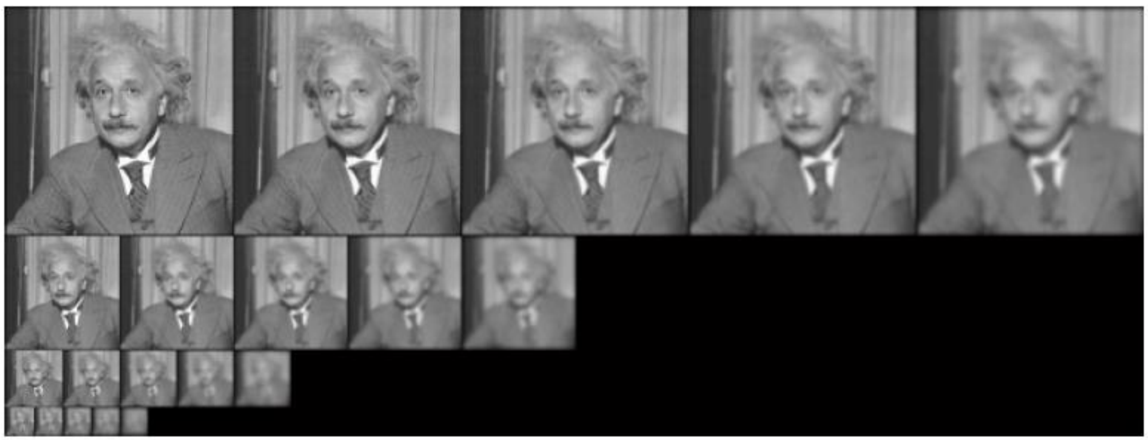
Feature Detection and Description

- Feature Detection

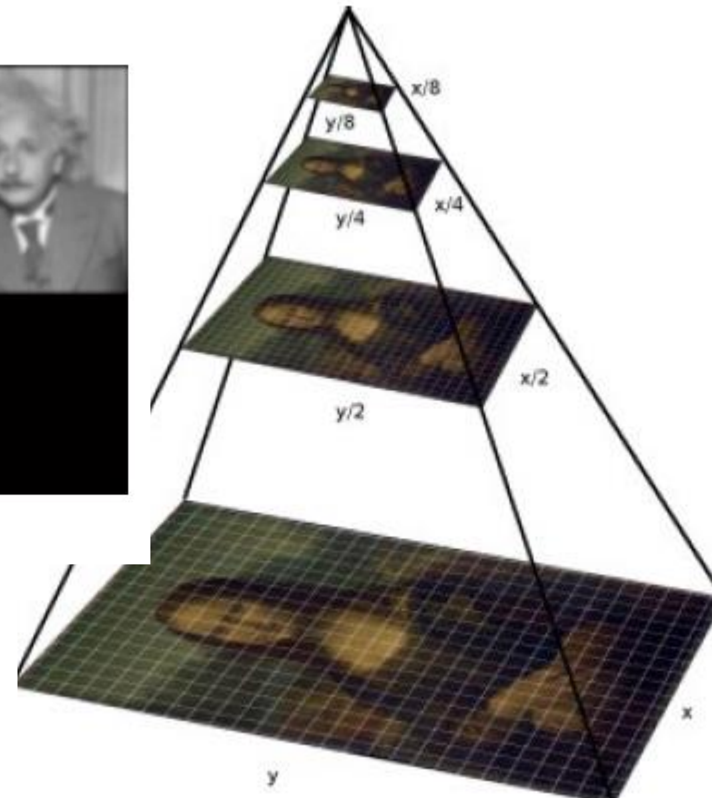


Feature Detection and Description

- Feature Detection
  - Difference of Gaussian
  - Image pyramid

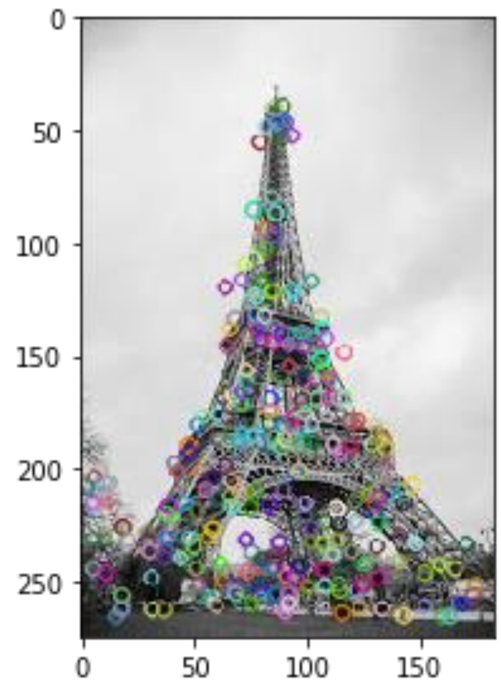


Scale-Space



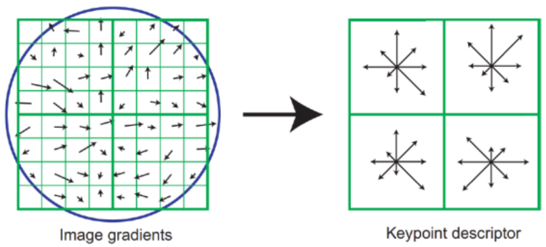
Feature Detection and Description

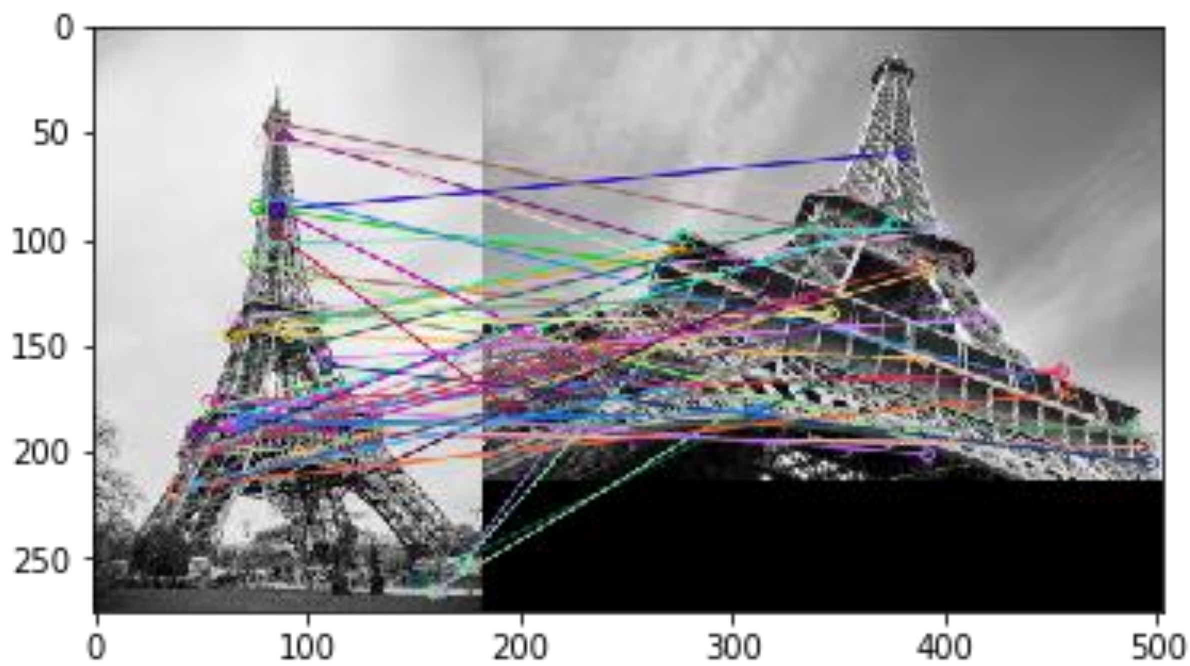
- Feature Detection
  - SIFT, SURF, BRISK, ORB, FAST
    - Despite > 20 years old, SIFT is still used today



Feature Detection and Description

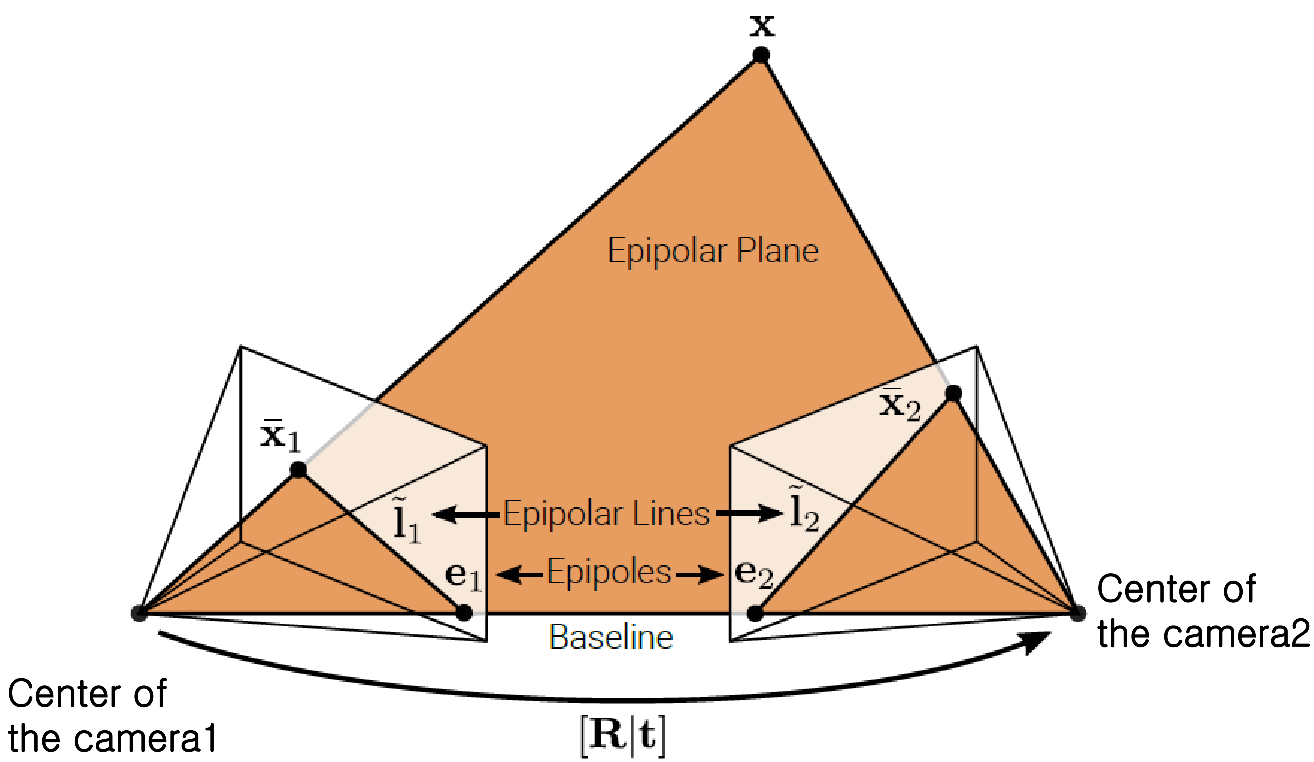
- Feature Description





Epipolar Geometry

- Epipolar Geometry



Epipolar Geometry

Essential Matrix

calibrated

- First Camera $P = K[I|0]$
- Second Camera $P' = K'[R|t]$

Essential Matrix

이 행렬은 정규화된 이미지 평면에서의 매칭쌍들 사이의 기하학적 관계를 설명한다.
즉, 카메라 내부[intrinsic]의 Matrix인 K를 제거한 좌표계에서의 변환관계를 나타낸다.

$$\vec{P}_{img}^{(1)} \equiv [I\ 0]\begin{bmatrix}P_W\\1\end{bmatrix} = P_W \longrightarrow \lambda_1\vec{P}_{img}^{(1)} = P_W$$

World coordinates.

$$\vec{P}_{img}^{(2)} \equiv [R\ t]\begin{bmatrix}P_W\\1\end{bmatrix} = RP_W + t \longrightarrow \lambda_2\vec{P}_{img}^{(2)} = RP_W + t$$

Eigenvalue

값을 대입해 한 식으로 표현하면

$$\lambda_2\vec{P}_{img}^{(2)} = \lambda_1 R\vec{P}_{img}^{(1)} + t$$

양변에 cross product t를 취한다.

$$\rightarrow \lambda_2 t\times\vec{P}_{img}^{(2)} = \lambda_1 t\times R\vec{P}_{img}^{(1)} + \underbrace{t\times t}_{=0}$$

$$\rightarrow \lambda_2 t\times\vec{P}_{img}^{(2)} = \lambda_1 t\times R\vec{P}_{img}^{(1)}$$

Cross product of two same vector is equal to 0 : $r\times r = |r^2|\sin\theta$

양변에 p image 2를 곱해준다.

$$\rightarrow \underbrace{\lambda_2\vec{P}_{img}^{(2)} t\times\vec{P}_{img}^{(2)}}_{=0} = \lambda_1\vec{P}_{img}^{(2)} t\times R\vec{P}_{img}^{(1)}$$

$$\rightarrow 0 = \lambda_1\vec{P}_{img}^{(2)} t\times R\vec{P}_{img}^{(1)}$$

$\lambda(u\cdot v\times u)$ 관계에서
$\lambda(v\cdot u\times u)$로 표현할 수 있으므로
$\lambda(v\cdot 0) = 0$으로 소거된다.

$$\rightarrow \vec{P}_{img}^{(2)}[t]_\times R\vec{P}_{img}^{(1)} = 0$$

$$\rightarrow \vec{P}_{img}^{(2)} E\vec{P}_{img}^{(1)} = 0$$

→ Essential Matrix [3x3 matrix]

Epipolar Geometry

- Fundamental Matrix
  - uncalibrated

카메라 내부[intrinsic]의 Matrix인 K까지 포함한 이미지 평면에서의 p, p' 의 매칭쌍들 사이의 기하학적 관계를 설명한다.

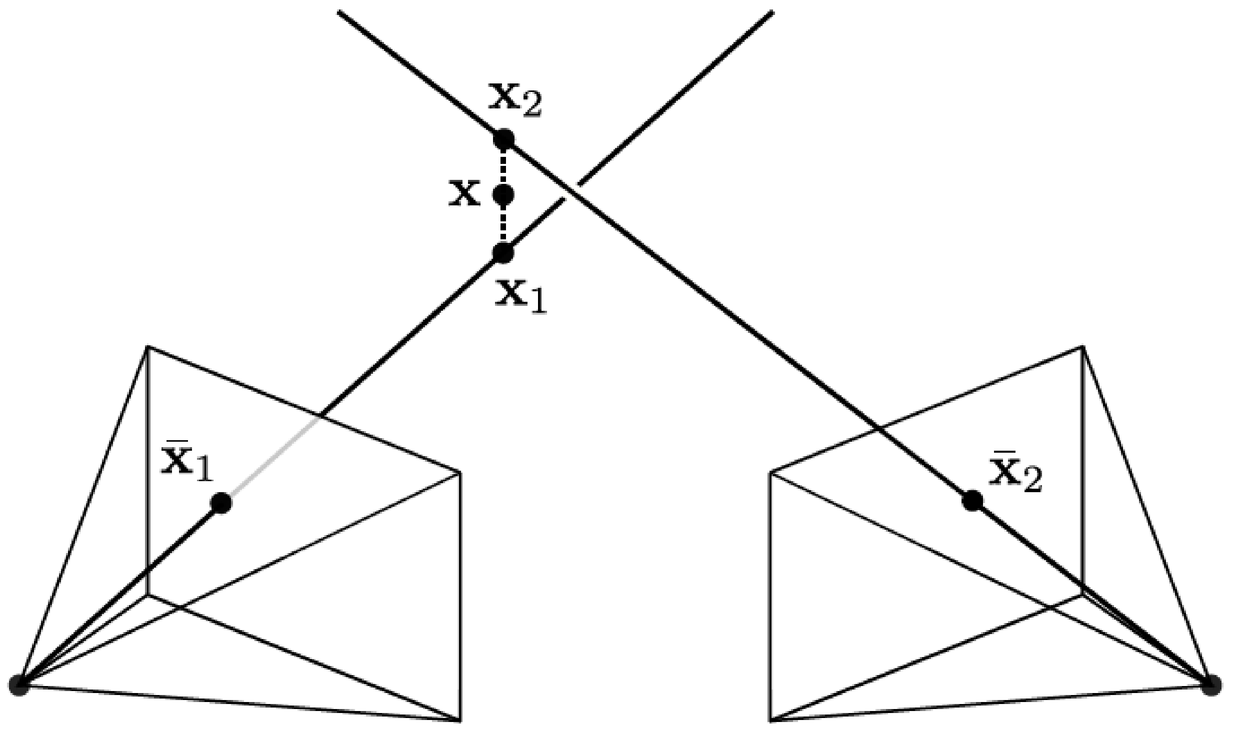
$$\vec{P}_{img}^{(2)T} K_2^{-T} [t]_{\times} R K_1^{-1} \vec{P}_{img}^{(1)} = 0$$

$$\rightarrow \vec{P}_{img}^{(2)T} F \vec{P}_{img}^{(1)}$$

이처럼 앞에서 구한 Essemtial matrix 에 k1의 역행렬, k2의 전치행렬을 구한다음 그것의 역행렬을 같이 곱해주면 Fundamental Matrix가 나오게된다.

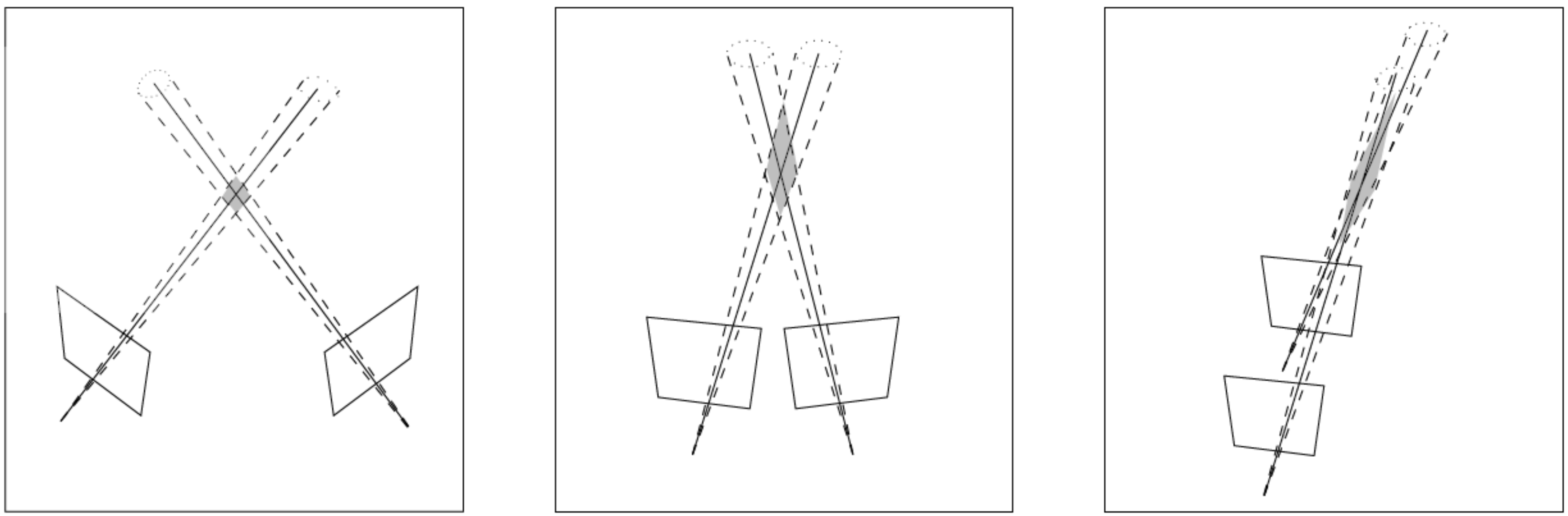
Triangulation

- .



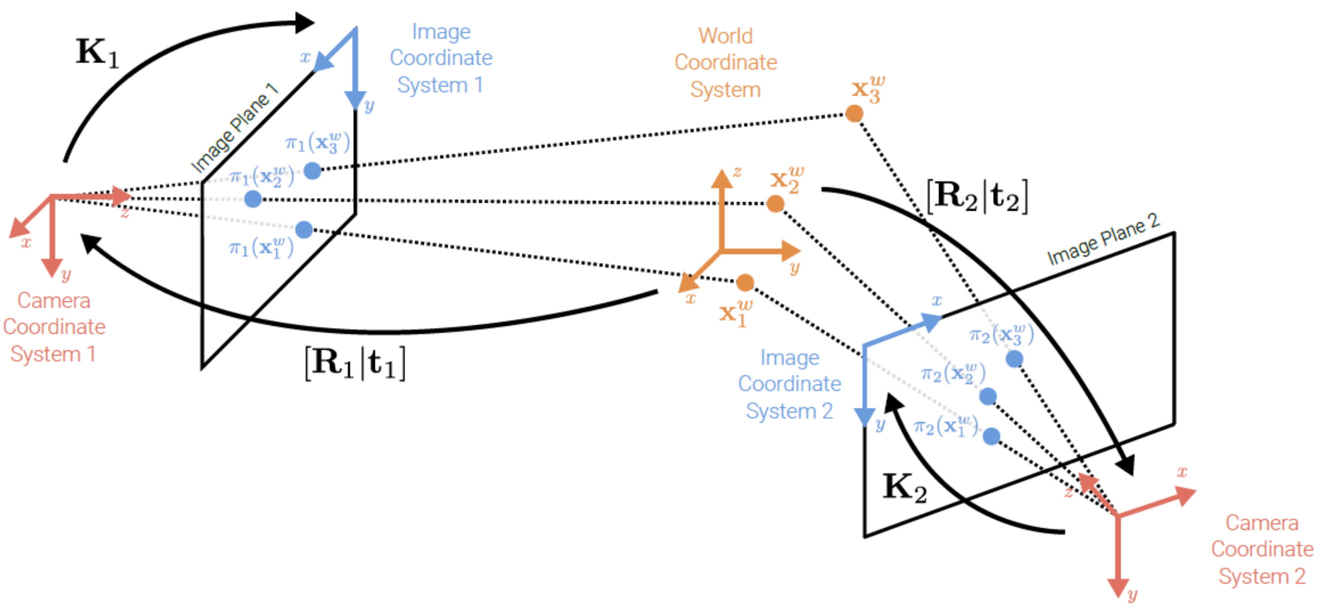
Triangulation

- Triangulation Uncertainty



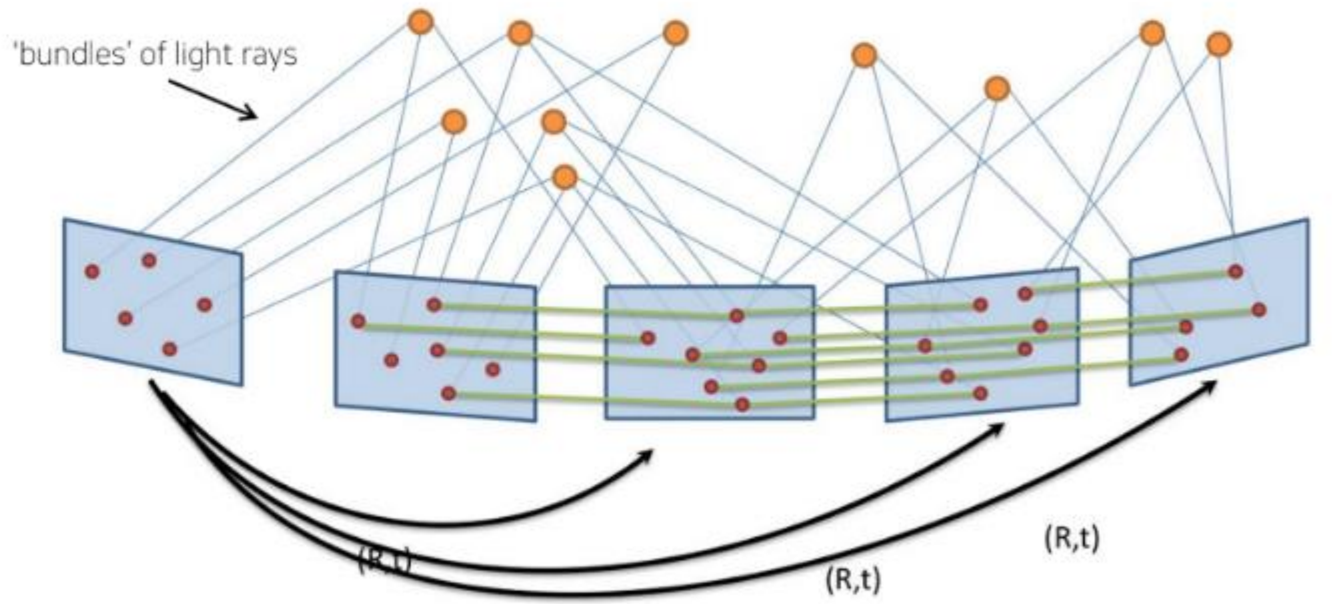
Bundle Adjustment

- Adjust Bundle of ray



Bundle Adjustment

- Adjust Bundle of ray



Bundle Adjustment

reprojection error

$$\arg\min_{P} E(P) = \sum \| proj(M) - m \|^2$$

M: 3D point

m: M과 대응되는 Image에서의 좌표

proj(M): 3D point를 2D Image Coordinate로 projection

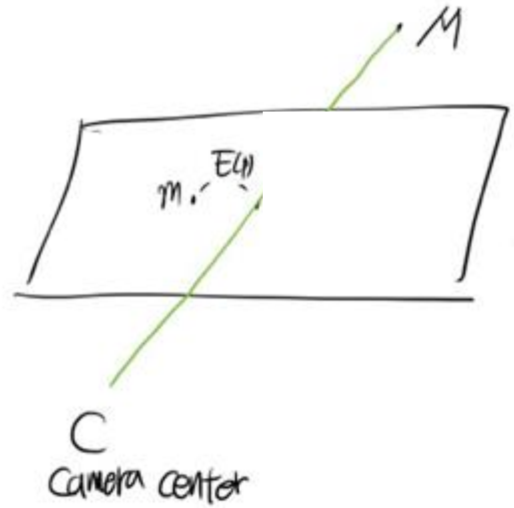
여기에 사용되는 파라미터는

K: Calibration matrix (Intrinsic matrix) 알고있음

R: Rotation matrix

T: transpose vector

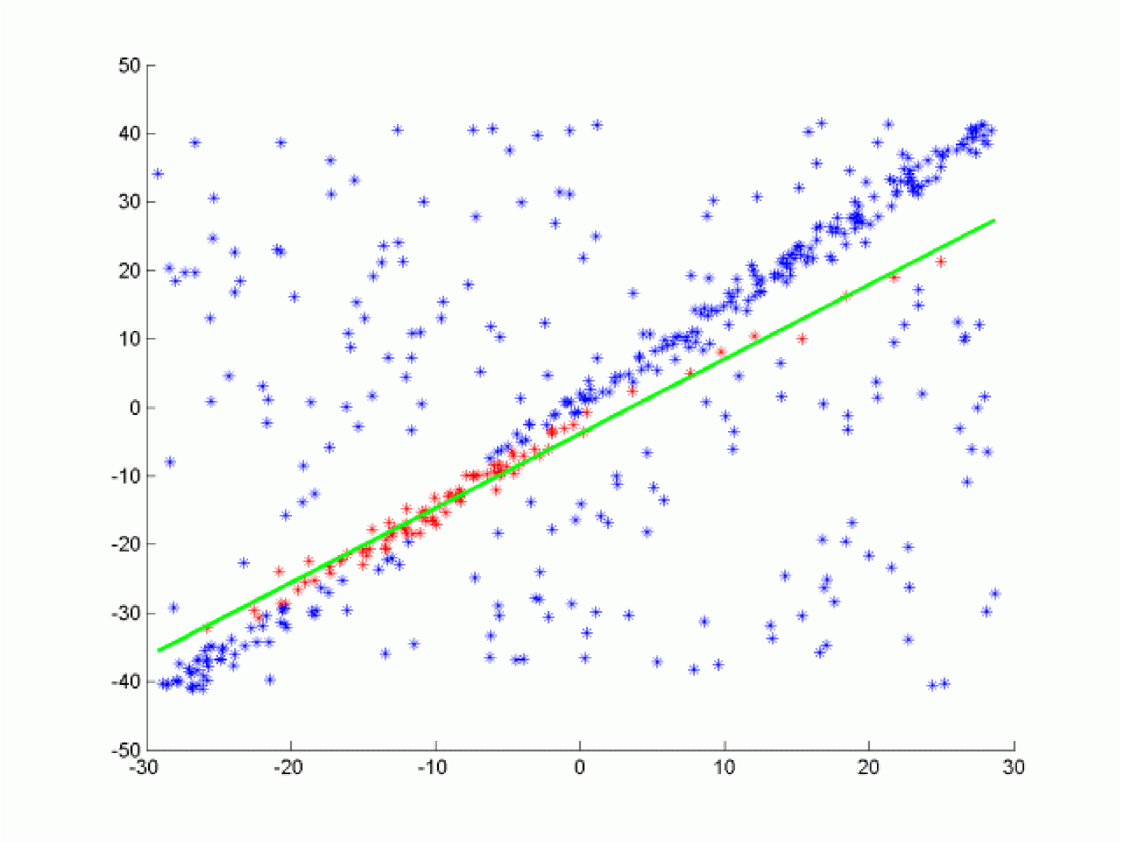
(R, T) ⇒ extrinsic matrix 구해야함



$$proj(M) = K[R|t]M$$

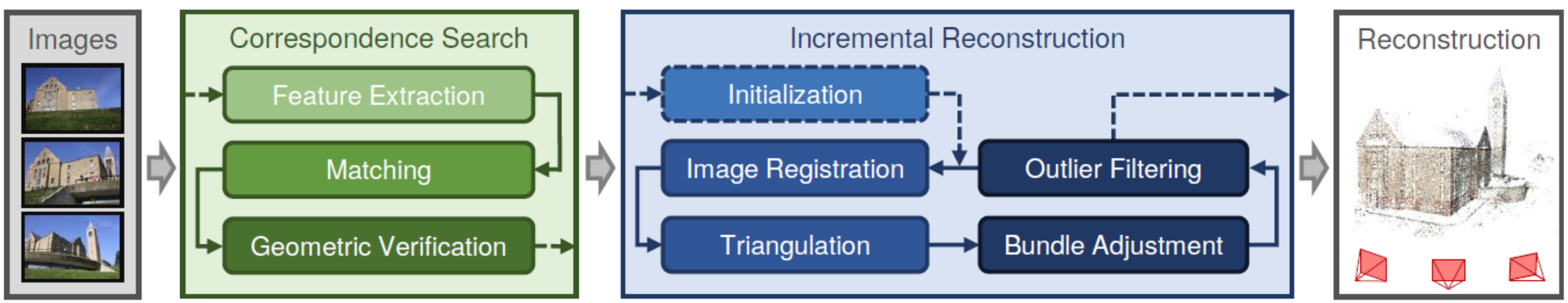
RANSAC

- Random Sample Consensus



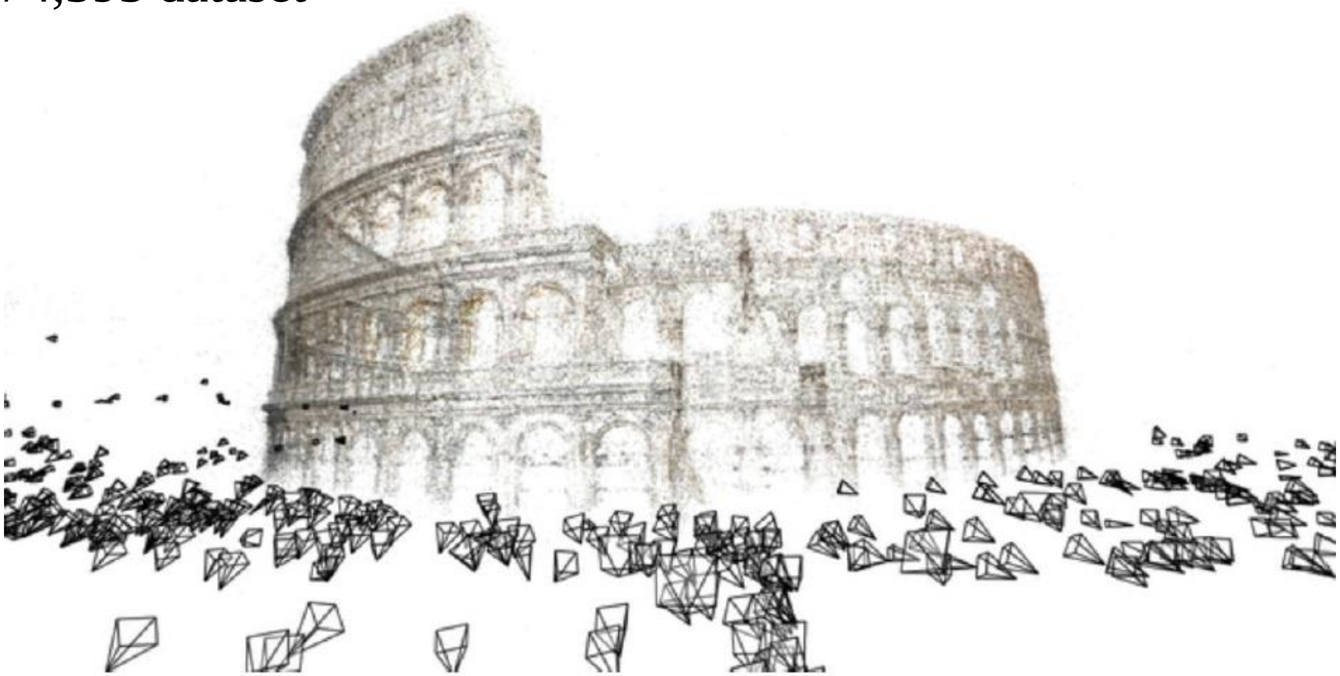
Summary

- Incremental Structure from Motion
  - COLMAP



Application

- Structure from Motion
  - COLMAP
    - 74,393 dataset



Application

- Structure from Motion
  - COLMAP

Application

- Structure from Motion
  - COLMAP

Question

- Factorization