6.838: Shape Analysis, 2021
Instructor: Justin Solomon
TAs: David Palmer and Paul Zhang

Probabilistic Optimal Transport based Diffeomorphic Registration

Peiqi Wang

MIT Department of Electrical Engineering and Computer Science

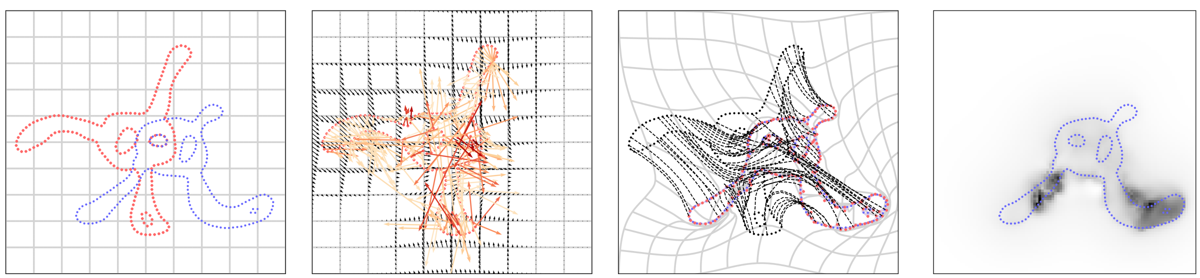

Figure 1: *The red bat is registered to blue amoeba, showing uncertainty of the transformation*

Abstract

1. Introduction

Diffeomorphic registration of shapes with unknown correspondence is an important step in medical data processing. The choice of similarity measure for correct matching distinguishes the different algorithms in this domain. Recent work explored the use of optimal transport distance as a global measure of similarity between discrete representation of shapes [FCVP17, FRTG19]. However, a point estimate of the transformation can generate errors left unnoticed by downstream processing pipelines. In addition, the solution is sensitive to hyperparameters of the model, requiring manual tuning for each new shape. Furthermore, the algorithm scales poorly due to the need to generate valid diffeomorphic transformations as well as to solve an optimal transport problem at each iteration.

We propose to extend optimal transport based diffeomorphic registration to probabilistic setting. Our method interprets the diffeomorphic transformation as a random variable, and estimates its parameters using variational inference. Naturally, the probabilistic formulation provides us with uncertainty estimates of both the transformation as well as the uncertainty of its effect on shapes. Hyperparameters such as the degree of smoothness of the transformation parameterizes the variational distribution, and thus can be optimized. In addition, we explored links to interdomain inducing variables as a way to reduce computation needed to generate valid diffeomorphic transformations [Fv09].

2. Related Work

3. Technical Approach

3.1. Diffeomorphic Registration

4. Diffeomorphic Registration

[BMTY05]

proposes lddmm for registering images. The goal of the paper is to register two images $I_0, I_1 : \Omega \to \mathbb{R}^d$ where $\Omega \subset \mathbb{R}^n$ (n=2 for 2d images) by computing a diffeomorphic coordinate transformation $\varphi : \Omega \to \Omega$ such that the pullback of I_0 by φ^{01}, i.e. $\varphi.I_0 = I \circ \varphi^{-1}$, is registered to the target image I_1. Previous work on non-rigid registration approximates φ as locally linear, e.g. $\varphi(x) = x + u(x)$ for some displacement vector field $u : \Omega \to \mathbb{R}^n$; However, this assumption breaks down when there is large displacement of objects within the two images. Instead of solving for the transformation directly, the paper proposes to solve for a time-varying velocity vector field $v_t : \Omega \to \mathbb{R}^n$ for $t \in [0,1]$ that dictates dynamics of time-varying transformations $\phi_t : \Omega \to \Omega$, $\frac{d}{dt}\phi_t = v_t(\phi_t) \quad \phi_0 = \mathrm{Id}$ and that the desired transformation φ is the endpoint of the above ODE problem, $\varphi = \phi_1 = \phi_0 + \int_0^1 v_t(\phi_t)\,dt$. It has been shown that if the velocity vectors are sufficiently smooth, then φ is a diffeomorphic map.

Let V be the space of velocity fields with norm $\|f\|_V = \|Lf\|_{L^2}$ for $L = (-\alpha\nabla^2 + \gamma)^{\beta} I$, we are interested to find velocity fields that are smooth and that the resulting pullback image is similar to the target image, captured by the follwing energy functional.

$$E(v) = \int_0^1 \|v_t\|_V^2 \, dt + \frac{1}{\sigma^2} \left\| I_0 \circ \phi_1^{-1} - I_1 \right\|_{L^2}^2 \tag{1}$$

where $v := \{v_t\}$ satisfies $\dot{\phi}_t = v_t(\phi_t)$. The velocity fields v_t can be optimized using gradient descent. Once an estimate of velocity field is estimated $\hat{v}$, the resulting transformation $\hat{\phi}$ can be computed via numerical integration of the ODE system. Of particular interest to this method is that the optimization can be interpreted as finding the (discretized) geodesic path on manifold of diffeomorphisms connecting I_0, I_1, and that the length of geodesic $\int_0^1 \|v_t\|_V \, dt$ is a metric distance between images connected via the diffeomorphism at the end point of the flow.

5. Results

Figures/tables illustrating the results of your work, as well as text interpreting these results. [PC20],

References

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