Quantum state tomography

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1 Team

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2 Background

Quantum tomography (QT) is applied on a source of systems, to determine what the quantum state is of the output of that source. Unlike a measurement on a single system, which determines the system's current state after the measurement (in general, the act of making a measurement alters the quantum state), quantum tomography works to determine the state prior to the measurements.

It is used in quantum computing to examine the result of quantum algorithm (i.e. state of a qubit) or in quantum conmmunications.

The computational part of QT requires to solve system of linear equations. What makes NLA algorithms useful for this problem.

3 Problem formulation

Quantum state may be represented as a density matrix in a Hilbert spase $\rho \in \mathcal{H}^{d\times d}$. And has following properties:

- $\rho = \rho^*$
- $\rho \geq 0$
- $Tr[\rho] = 1$

where * denotes hermitian conjugation.

Measurements in quantum mechanics are probabilistic and can be described using a positive-operators valued measure (POVM) formaism. Operator E_j

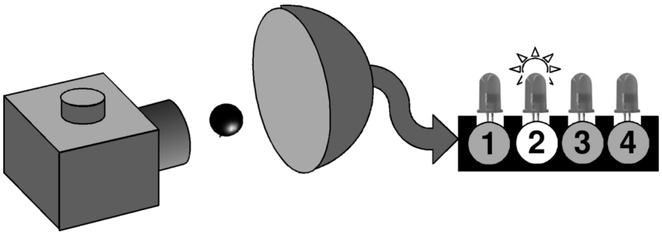

Figure 1: Intuitive approach to POVM formalism.

from a POVM set ($\sum_j E_j = I$) corresponds to "j" measurement outcome. The probability of getting j outcome after single measurement is

$$p_j = Tr[E_j \rho]$$

. Informally speaking, measurement process can be described as follows. A measurement device has N led indicators. While perfoming measurement on an input state ρ a certain j led lit up. It coresponds to acting of E_j operator on a system.

A state ρ of a qubit can be parameterized in the following way:

$$\rho = \frac{I + \sum_k (a_k \cdot \sigma_k)}{2} \qquad k = 1, 2, 3,$$

where $a_k \in \mathbb{R}$ and σ_k are Puali matrices with properties:

- $\sigma_i \sigma_j = \delta_{ij} + i\epsilon^{ijk}\sigma_k$
- $Tr[\sigma_i] = 0$

Here δ_{ij} denotes Kroneker symbol and ϵ^{ijk} is Levi-Civita symbol.

One can use vector notation:

$$\rho = \begin{pmatrix} I & \sigma_1 & \sigma_2 & \sigma_3 \end{pmatrix} \begin{pmatrix} 1 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix} = \mathbf{\Sigma}^T \mathbf{a}.$$

Measurement operator can also be parameterized in the same notation:

$$E_j = s_0 I + \sum_k s_{k,j} \sigma_k = \mathbf{\Sigma}^T \mathbf{s_j},$$

And $s_{k,j} \in \mathbb{R}$.

We can rewrite probability of j-th outcome in i-th measurement as follow

$$p_j^{(i)} = (\mathbf{s_j^{(i)}})^{\mathbf{T}} \mathbf{a}$$

And for the set all measurements, i.e. the experiment statistics:

$$\mathbf{p} = \mathbf{S}\mathbf{a}$$

After solving that system one can recovery a quantum state $\hat{\rho}$. But there is a problem which may happens after solution of this particular problem: state ρ is not physical (i.e. $Tr[\rho] > 1$ or $\rho < 0$). And hence, it is necessery to find the nearest state $\tilde{\rho}$ which is physical by these conditions.

4 Data

Data of physical experiment will be obtained via numerical modeling of finite dimensional quantum system's measurement.

5 Related works

For quantum QT linear inverse method usually have been used [1]. But it does not guarantee the physical results. To handle this problem Maximum Likelihood method was proposed, but it requires a lot of iterations to find estimated state [3, 4, 2].

6 Scope

During this project our team consider the following tasks:

- Simulate an experiment of a quantum state measurement;
- Apply different approaches to the problem of QT, s.t. PseudoInverse, Iterative Methods, Semidefinite Programming;
- Compare mentioned methods in terms of complexity and accuracy;

7 Evaluation

As evaluation criteria for the project we considering the following:

- Overview of considered approaches;
- Numerical benchmarking of algorithms.

References

[1] Martin Ringbauer *Exploring Quantum Foundations with Single Photons*

[2] Alessandro Bisio, Giulio Chiribella, Giacomo Mauro, D'Ariano, Stefano Facchini, Paolo Perinotti *Optimal quantum tomography*
`https://arxiv.org/abs/1702.08751`

[3] Takanori Sugiyama, Peter S. Turner, Mio Murao *Precision-guaranteed quantum tomography*
`https://arxiv.org/abs/1306.4191`

[4] Jaroslav Rehacek, Berthold-Georg Englert, Dagomir Kaszlikowski *Minimal qubit tomography*
`https://arxiv.org/abs/quant-ph/0405084`