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Project Report

Methods algorithms

1.Bisection Method

1. Start
2. Read x1, x2, e
3. Compute: f1 = f(x1) and f2 = f(x2)
4. If (f1*f2) > 0, then display initial guesses are wrong and go to (11).

Otherwise continue.

5. x = (x1 + x2)/2
6. If ([(x1 – x2)/x] < e), then display x and go to (11).
7. Else, f = f(x)
8. If ((f*f1) > 0), then x1 = x and f1 = f.
9. Else, x2 = x and f2 = f
10. Go to (5)
11. Stop

2.False Position Method

1. Start
2. Read values of x0, x1 and e
3. Computer function values f(x0) and f(x1)
4. Check whether the product of f(x0) and f(x1) is negative or not.

If it is positive take another initial guesses.

If it is negative then go to step 5.

5. Determine:

x = [x0*f(x1) – x1*f(x0)] / (f(x1) – f(x0))

6. Check whether the product of f(x1) and f(x) is negative or not.

If it is negative, then assign x0 = x;

If it is positive, assign x1 = x;

7. Check whether the value of f(x) is greater than 0.00001 or not.

If yes, go to step 5.

If no, go to step 8.

8. Display the root as x.

9. Stop

3.Fixed Point Method

1. Start
2. Define function as f(x)
3. Define convergent form g(x)
4. Input:

a. Initial guess x0

b. Tolerable Error e

c. Maximum Iteration N

5. Initialize iteration counter : step=1
6. Do

x1=g(x0)

step=step+1

If step>N

Go to (8)

End If

X0=x1

7. If ([(x1 – x0)/x1] < e)

8. Stop

4.Newton Raphson method

1. Start

2. Read x, e, n

a. x is the initial guess

b. e is the absolute error

c. n is for operating loop

3. Do for i =1 to n in step of 2

4. f = f(x)

5. f1 = f'(x)

6. If ([f1] < d), then display too small slope and go to 11.

7. x1 = x – f/f1

8. If ([(x1 – x)/x1] < e), the display the root as x1 and go to 11.

9. x = x1 and end loop

10. Display method does not converge due to oscillation.

11. Stop

5.Secant method

1. Start
2. Get values of x0, x1 and e
3. Compute f(x0) and f(x1)
4. Compute x2 = [x0*f(x1) – x1*f(x0)] / [f(x1) – f(x0)]
5. Test for accuracy of x2

If [(x2 – x1)/x2] > e

then assign x0 = x1 and x1 = x2

go to step 4

Else,

go to step 6

6. Display the required root as x2.
7. Stop

- Data structure used in all methods is array which is useful in the storage of each iteration’s error , input bounders , and root.

Sample runs

Equation 1: x^4+3x-4

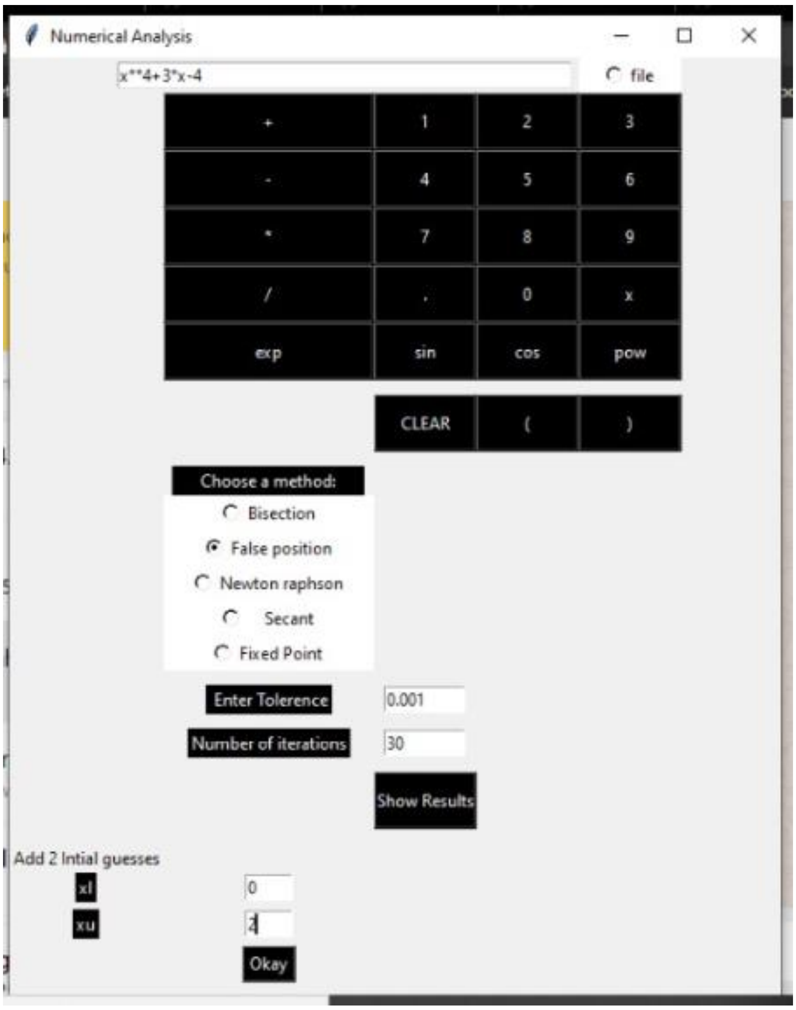
1) Bisection method

Numerical Analysis

Result of bisection method

Iteration	Xi	Xii	ERROR
1	0.0	1.0	1.0
2	1.0	1.5	0.3333333333333333
3	1.5	1.75	0.14285714285714285
4	1.75	1.875	0.06666666666666667
5	1.875	1.9375	0.03225806451612903
6	1.9375	1.96875	0.015873015873015872
7	1.96875	1.984375	0.007874015748031496
8	1.984375	1.9921875	0.00392156862745098
9	1.9921875	1.99609375	0.0019569471624266144
10	1.99609375	1.998046875	0.0009775171065493646
The Root	1.998046875		
The Number Of Iteration	10		
Elapsed_Time	0.0026009000000613014		

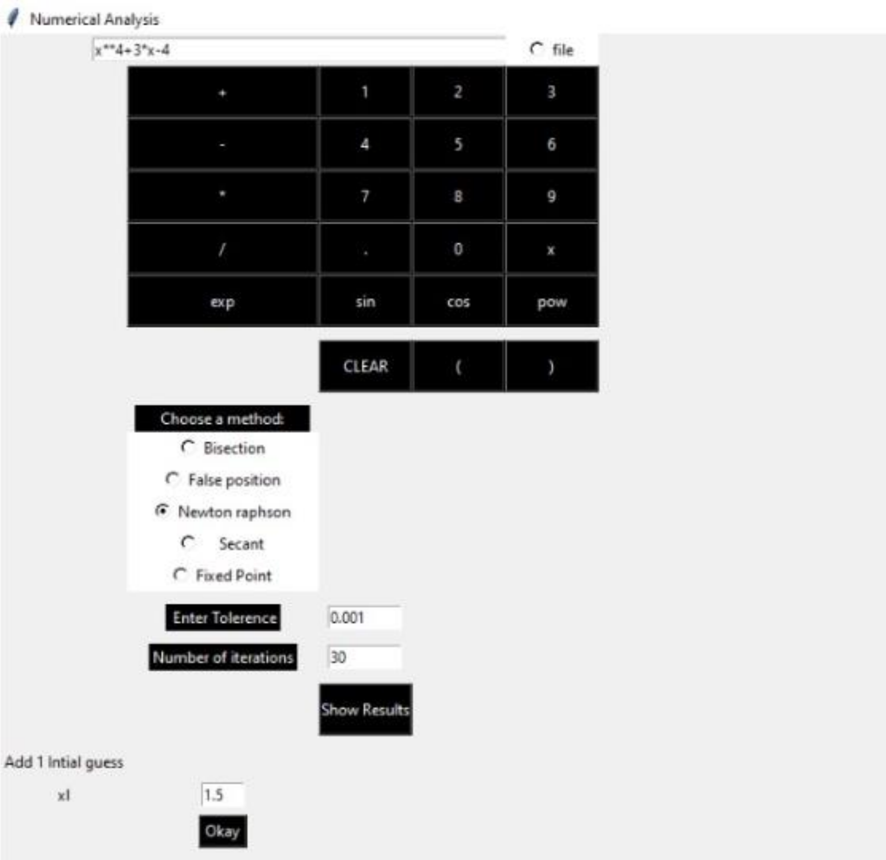
2)False position method



Result of false position method Go Back

Iteration	Xi	Xii	ERROR
1	0.0	0.36363636363636365	1.0
2	0.36363636363636365	0.5901253457305949	0.3837980926134097
3	0.5901253457305949	0.7379498006677122	0.20031776525091904
4	0.7379498006677122	0.8344085905662775	0.11560138640603257
5	0.8344085905662775	0.8964540687904178	0.06921211067495966
6	0.8964540687904178	0.9357609563354641	0.04200526563854096
7	0.9357609563354641	0.960363194174267	0.02561763923070398
8	0.960363194174267	0.9756307241657461	0.015648881911272648
9	0.9756307241657461	0.9850516605263794	0.009563900796430307
10	0.9850516605263794	0.9908437113777149	0.005845574619716726
11	0.9908437113777149	0.994396511751198	0.003572820631909136
12	0.994396511751198	0.9965726516682982	0.0021836239570263735
13	0.9965726516682982	0.9979043876858622	0.0013345326806832372
14	0.9979043876858622	0.9987189283605366	0.0008155854981255529
The Root	0.9987189283605366		
The Number Of Iteration	14		
Elapsed_Time	0.00512430000000208		

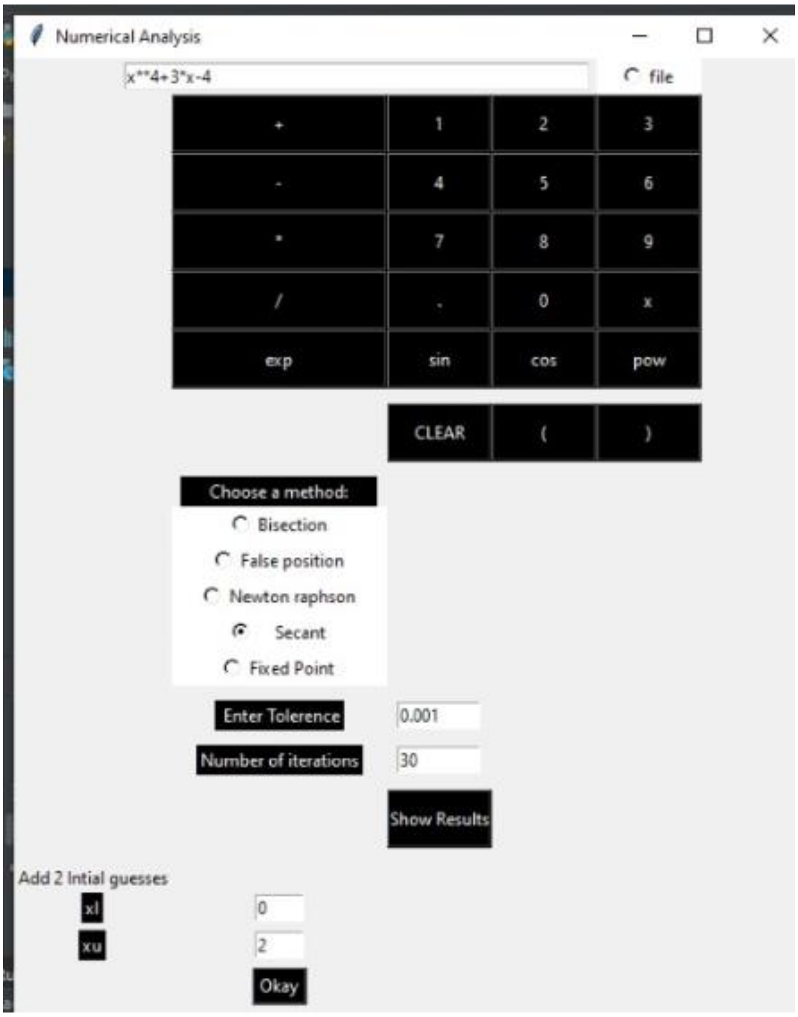
3)Newton raphson method



Result of NewtonRaphson method Go Back

Iteration	Xi	Xii	ERROR
1	0.0	1.3333333333333333	1.0
2	1.3333333333333333	1.0801186943620178	0.23443223443223438
3	1.0801186943620178	1.0053170636003428	0.07440600927809568
4	1.0053170636003428	1.000024182962787	0.005292752643115632
5	1.000024182962787	1.000000000501266	2.4182461508905476e-5
The Root	1.00000000050127		
The Number Of Iteration	5		
Elapsed_Time	0.1083438000000001		

4)Secant method



Numerical Analysis

Result of Secent method

Go Back

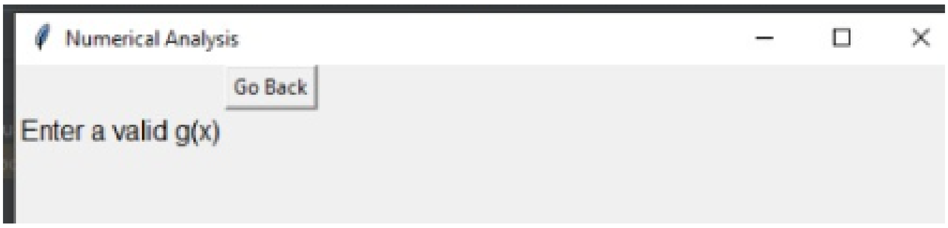
Iteration	Xi	Xii	ERROR
1	2.0	0.36363636363636365	4.499999999999999
2	0.36363636363636365	0.590125345730595	0.3837980926134098
3	0.590125345730595	1.199780388466869	0.5081388632425617
4	1.199780388466869	0.9301894926300669	0.2898236305320398
5	0.9301894926300669	0.9884484241185703	0.058939778815930266
6	0.9884484241185703	1.0007009941208889	0.012243987039387758
7	1.0007009941208889	0.9999930450879936	0.0007079539566527562
The Root	0.9999930450879936		
The Number Of Iteration	7		
Elapsed_Time	0.0030597999999990577		

5) Fixed point method

For f(x)= x^4+3x-4

If g(x)=(4-x^4)/3

Then, Output is

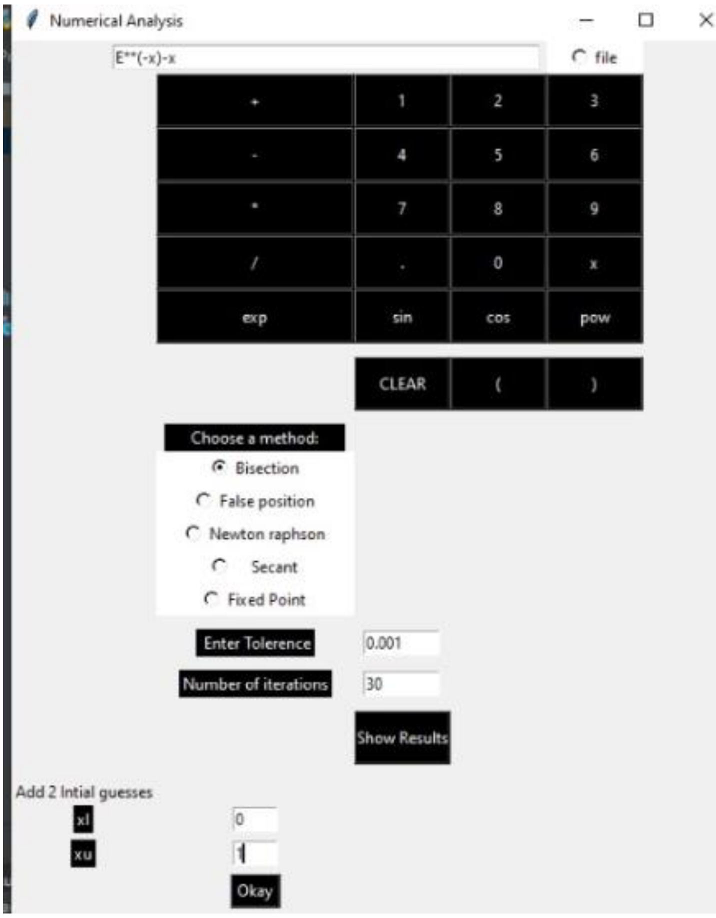


From the previous Example,

Newton raphson method terminates with the least iteration number (5)(bestmethod),However false position method is very slow with (14) iterations. g(x) in fixed point may diverge(ex, g(x)=(4-x^4)/3**) another input must be entered.**

Equation 2: e^-x -x

Bisection method

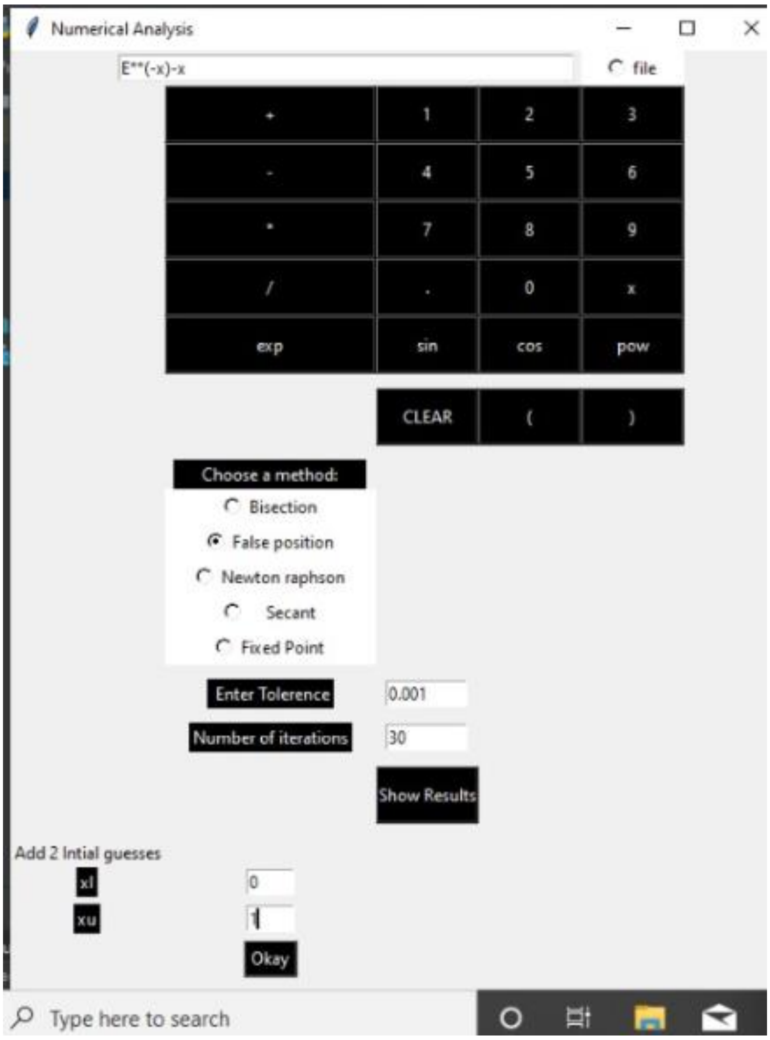


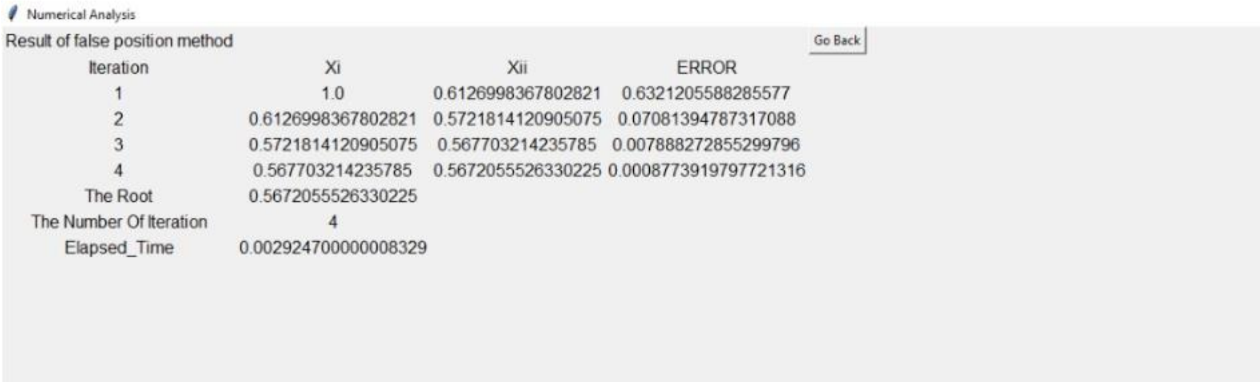
Result of bisection method

Go Back

Iteration	Xi	Xii	ERROR
1	0.0	0.5	1.0
2	1.0	0.75	0.3333333333333333
3	0.75	0.625	0.2
4	0.5	0.5625	0.1111111111111111
5	0.625	0.59375	0.05263157894736842
6	0.59375	0.578125	0.02702702702702703
7	0.578125	0.5703125	0.0136986301369863
8	0.5625	0.56640625	0.006896551724137931
9	0.5703125	0.568359375	0.003436426116838488
10	0.568359375	0.5673828125	0.0017211703958691911
11	0.56640625	0.56689453125	0.0008613264427217916
The Root	0.56689453125		
The Number Of Iteration	11		
Elapsed_Time	0.003158700000000181		

1)False position method





Numerical Analysis

Result of false position method

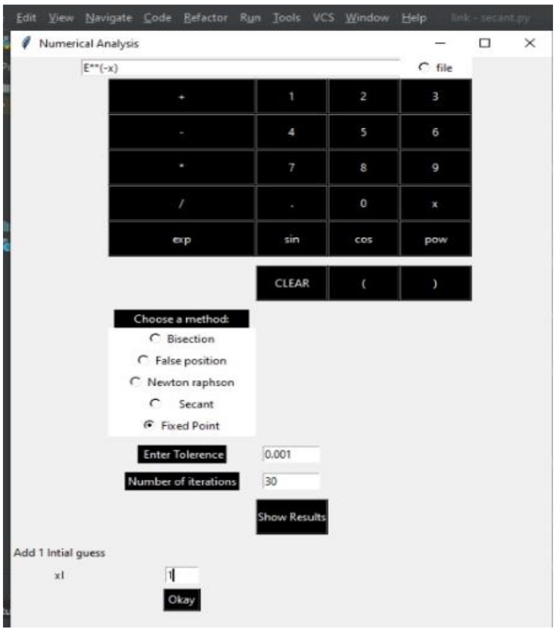
Go Back

Iteration	Xi	Xii	ERROR
1	1.0	0.6126998367802821	0.6321205588285577
2	0.6126998367802821	0.5721814120905075	0.07081394787317088
3	0.5721814120905075	0.567703214235785	0.007888272855299796
4	0.567703214235785	0.5672055526330225	0.0008773919797721316
The Root	0.5672055526330225		
The Number Of Iteration	4		
Elapsed_Time	0.002924700000008329		

3)Fixed point method

For f(x)=e^-x -x

If g(x)=e^(-x)



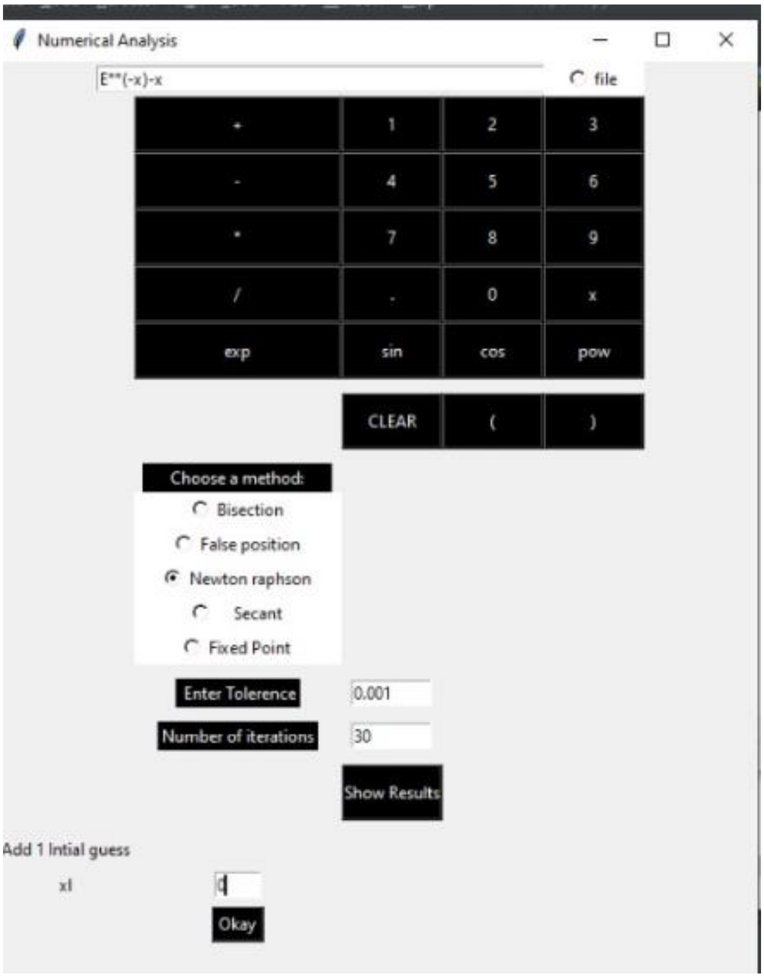
Then,Output is

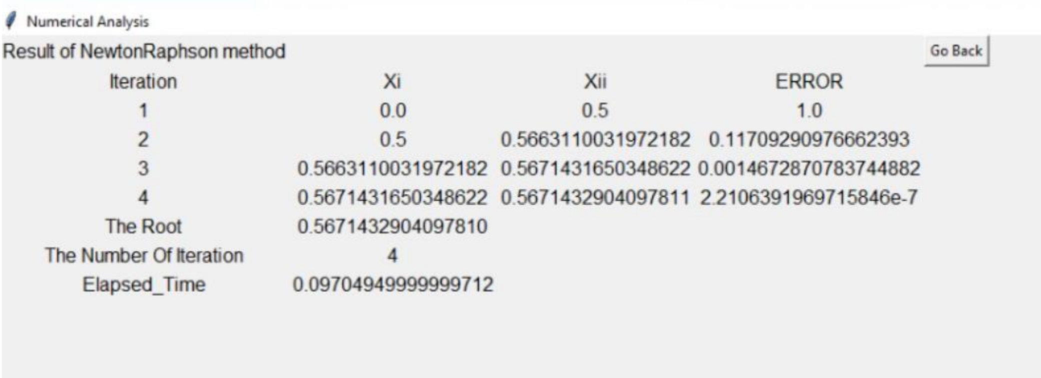
Numerical Analysis

Result of Fixed Point method | Go Back

Iteration	Xi	Xii	ERROR
1	1.0	0.36787944117144233	1.718281828459045
2	0.36787944117144233	0.6922006275553464	0.46853639461338437
3	0.6922006275553464	0.5004735005636368	0.3830914659333314
4	0.5004735005636368	0.6062435350855974	0.17446789681151248
5	0.6062435350855974	0.545395785975027	0.11156622525381316
6	0.545395785975027	0.5796123355033789	0.059033508144086734
7	0.5796123355033789	0.5601154613610891	0.03480866979624528
8	0.5601154613610891	0.571143115080177	0.019308039312598228
9	0.571143115080177	0.5648793473910495	0.011088682420515694
10	0.5648793473910495	0.5684287250290607	0.006244191918328175
11	0.5684287250290607	0.5664147331468833	0.003555684137995690 8
12	0.5664147331468833	0.5675566373282835	0.0020119651613549187
13	0.5675566373282835	0.5669089119214953	0.0011425564022143186
14	0.5669089119214953	0.5672762321755697	0.0006475156779716674
The Root	0.5672762321755697		
The Number Of Iteration	14		
Elapsed_Time	0.031433700000000 9		

4)Newton raphson





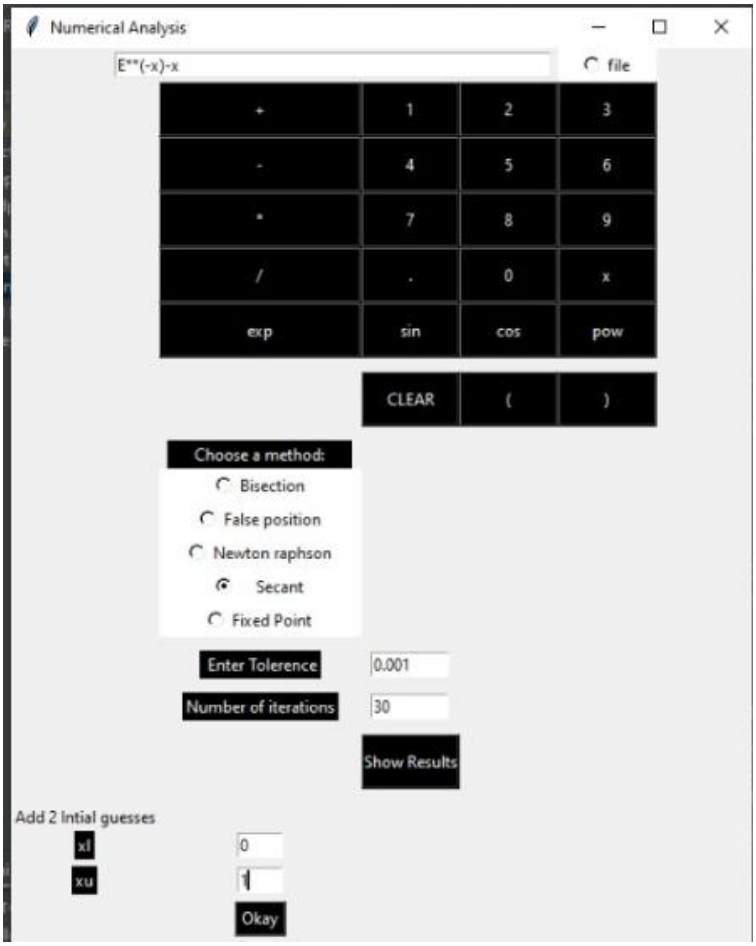
Numerical Analysis

Result of NewtonRaphson method

Go Back

Iteration	Xi	Xii	ERROR
1	0.0	0.5	1.0
2	0.5	0.5663110031972182	0.11709290976662393
3	0.5663110031972182	0.5671431650348622	0.0014672870783744882
4	0.5671431650348622	0.5671432904097811	2.2106391969715846e-7
The Root	0.5671432904097810		
The Number Of Iteration	4		
Elapsed_Time	0.09704949999999712		

5)Secant method



Numerical Analysis

Result of Secent method

Go Back

Iteration	Xi	Xii	ERROR
1	1.0	0.6126998367802821	0.6321205588285577
2	0.6126998367802821	0.5638383891610742	0.08665860388099504
3	0.5638383891610742	0.5671703584197446	0.005874723897690938
4	0.5671703584197446	0.5671433066049633	4.7698376171079925e-5
The Root	0.5671433066049633		
The Number Of Iteration	4		
Elapsed_Time	0.0019622000000012463		

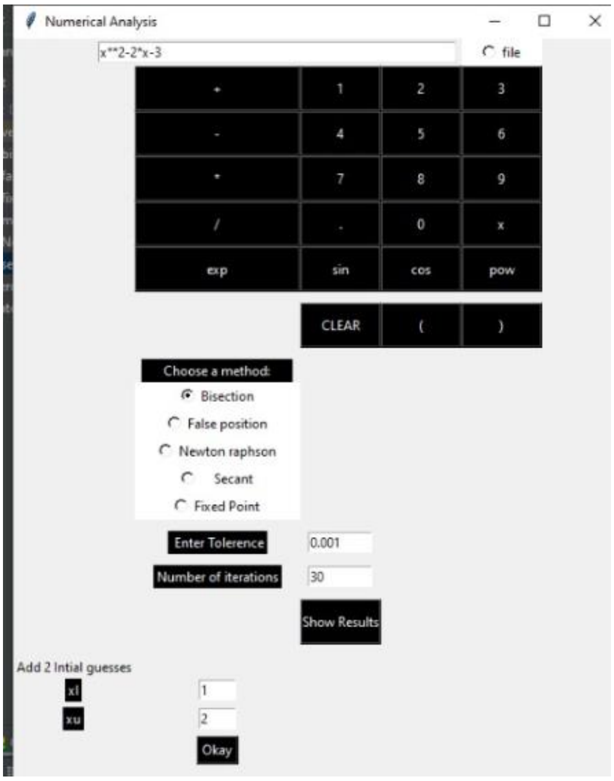
From Previous Example,

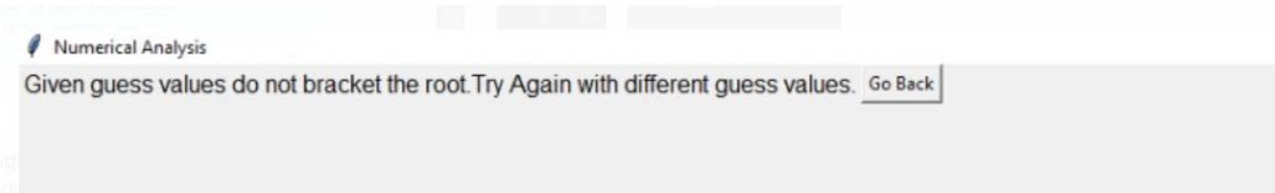
Bisection method has the most number of iterations (11).Secant ,newton raphson , and false position have the same number of iteration(4) but Newton Raphson has highest rate of error decrement(best method).g(x) may converge after huge number of iterations (14).

Equation 3: x^2-2x-3

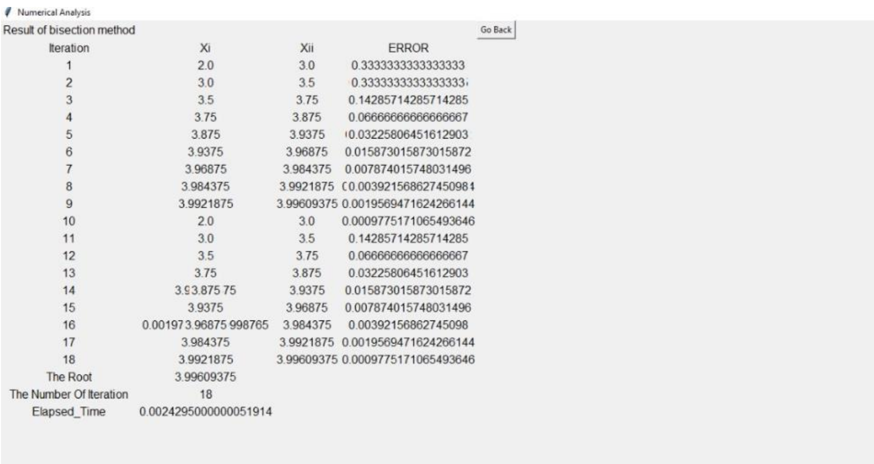
1)Bisection method

Input boundaries [1,2]





Input boundries [2,4]

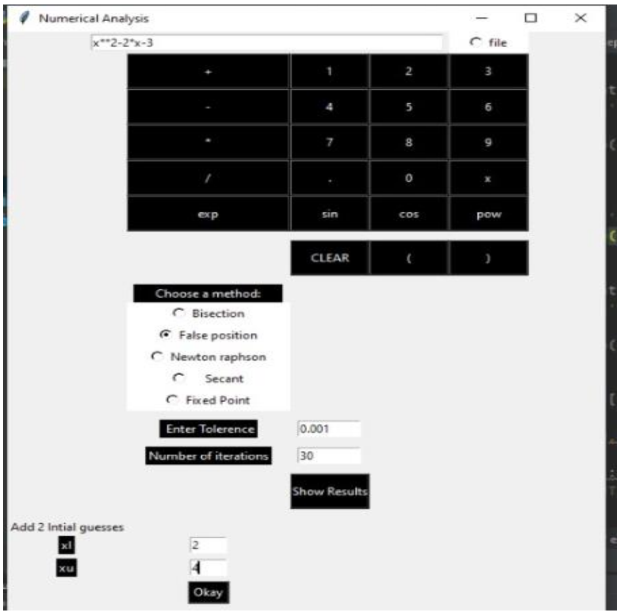

Numerical Analysis

Result of bisection method

Go Back

Iteration	Xi	Xii	ERROR
1	2.0	3.0	0.3333333333333333
2	3.0	3.5	0.333333333333333
3	3.5	3.75	0.14285714285714285
4	3.75	3.875	0.06666666666666667
5	3.875	3.9375	0.03225806451612903
6	3.9375	3.96875	0.015873015873015872
7	3.96875	3.984375	0.007874015748031496
8	3.984375	3.9921875	0.00392156862745098
9	3.9921875	3.99609375	0.0019569471624266144
10	2.0	3.0	0.0009775171065493646
11	3.0	3.5	0.14285714285714285
12	3.5	3.75	0.06666666666666667
13	3.75	3.875	0.03225806451612903
14	3.93.875 75	3.9375	0.015873015873015872
15	3.9375	3.96875	0.007874015748031496
16	0.00197 3.96875 998765	3.984375	0.00392156862745098
17	3.984375	3.9921875	0.0019569471624266144
18	3.9921875	3.99609375	0.0009775171065493646
The Root	3.99609375		
The Number Of Iteration	18		
Elapsed_Time	0.002429500000051914		

2)False position method

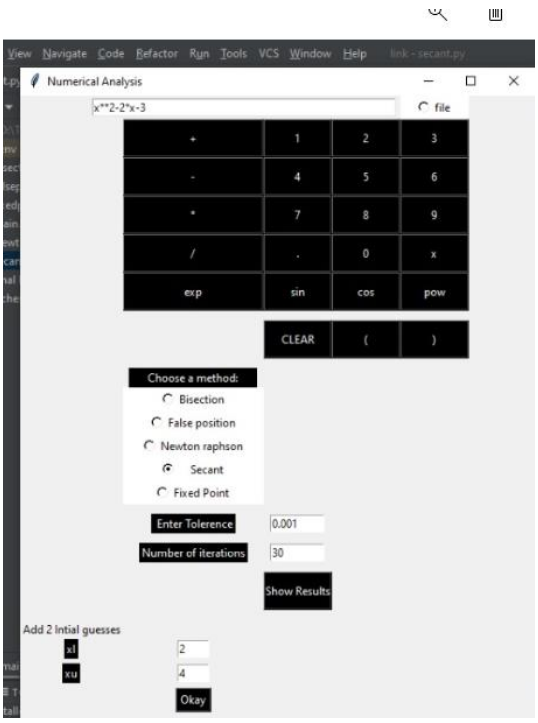


Numerical Analysis

Result of false position method

Iteration	Xi	Xii	ERROR
1	2.0	2.75	0.2727272727272727
2	2.75	2.9473684210526314	0.06696428571428566
3	2.9473684210526314	2.9893617021276597	0.0140475744521447
4	2.9893617021276597	2.997867803837953	0.0028373838564207054
5	2.997867803837953	2.99957337883959	0.000568605860309734
The Root	2.99957337883959		
The Number Of Iteration	5		
Elapsed_Time	0.002136300000000091		

3)Secant method



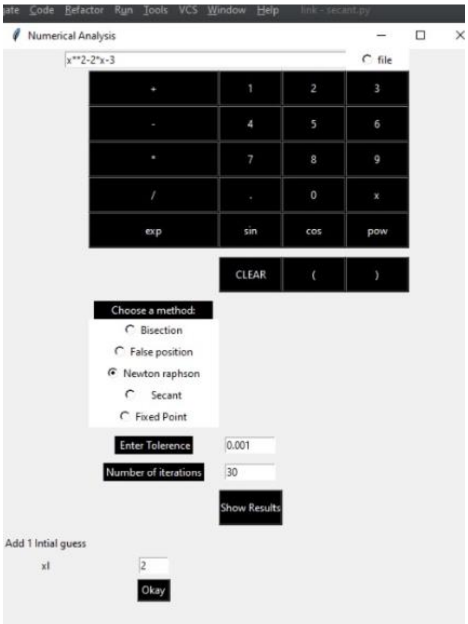
Numerical Analysis

Result of Secent method

Go Back

Iteration	Xi	Xii	ERROR
1	4.0	2.75	0.45454545454545453
2	2.75	2.947368421052632	0.06696428571428581
3	2.947368421052632	3.0035587188612096	0.018707907208780038
4	3.0035587188612096	2.9999525931544517	0.0012020608975577335
5	2.9999525931544517	2.9999999578600827	1.5788235432083898e-5
The Root	2.9999999578600827		
The Number Of Iteration	5		
Elapsed_Time	0.0025958999999957655		

4)Newton Raphson



Numerical Analysis

Result of NewtonRaphson method

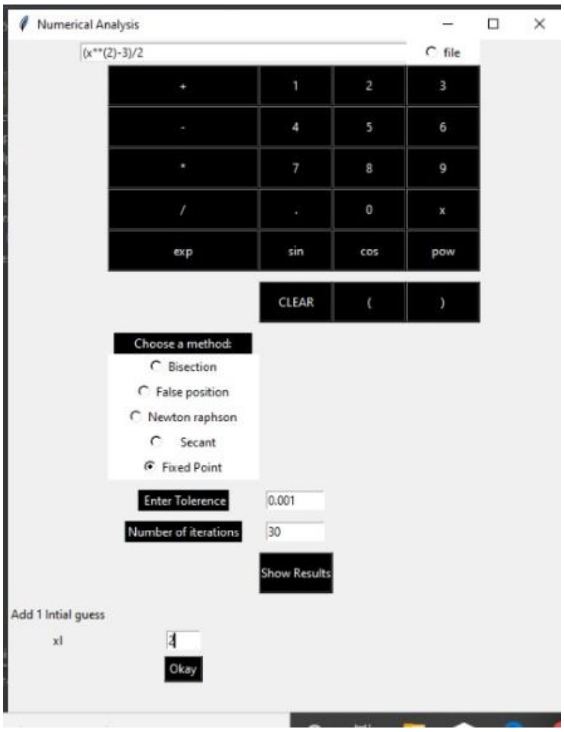
Go Back

Iteration	Xi	Xii	ERROR
1	2.0	3.5	0.42857142857142855
2	3.5	3.05	0.14754098360655746
3	3.05	3.0006097560975613	0.016460069091647867
4	3.0006097560975613	3.0000000929222947	0.00020322105212759834
The Root	3.00000009292229		
The Number Of Iteration	4		
Elapsed_Time	0.095487600000002		

5)Fixed point method

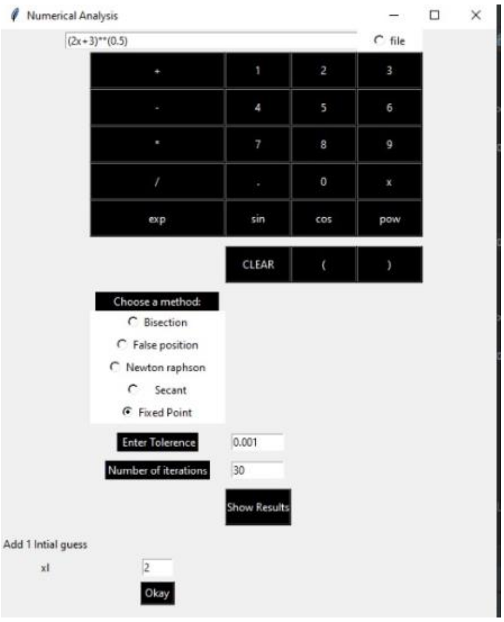
for f(x)=x^2-2*x-3

if g(x)=(x**(2)-3)/2



Then,Output is

if g(x)=(2x+3)^(0.5)



Then, output is

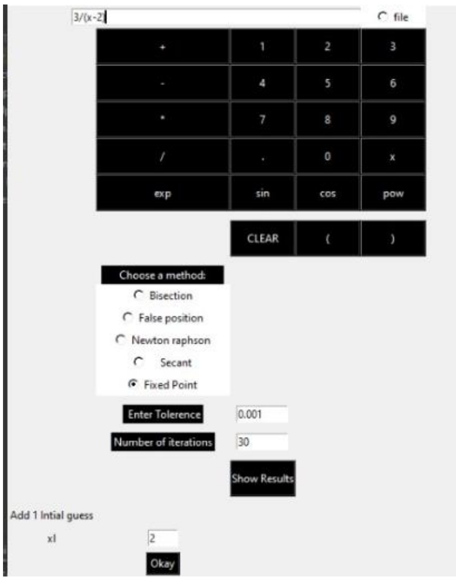
Numerical Analysis

Result of Fixed Point method

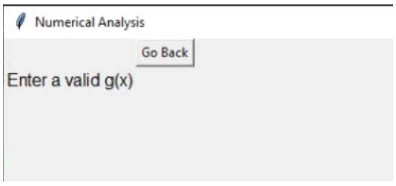
Go Back

Iteration	Xi	Xii	ERROR
1	2.0	2.6457513110645907	0.24407105398154558
2	2.6457513110645907	2.879496939072723	0.0811758556976987
3	2.879496939072723	2.959559743972986	0.0270522685217986
4	2.959559743972986	2.986489492354857	0.009017191739937086
5	2.986489492354857	2.9954931121118795	0.0030057220698046405
6	2.9954931121118795	2.998497327699953	0.001001907042010884
7	2.998497327699953	2.9994990674110746	0.0003339690023595638
The Root	2.9994990674110746		
The Number Of Iteration	7		
Elapsed_Time	0.08742839999999674		

If g(x)=3/(x-2)



Then ,output is



From the previous example,

In Bisection method if roots don't exist between interval then gauss value will not bracket the root (ex, [1,2] root is 3) however, if root is within the boundaries (ex, [2,4] root is 3) gauss value will bracket the root but will conclude large number of iterations so its not the best method to be used. Newton Raphson method has least number of iterations(4) therefore is the best method for this function. Fixed point method disadvantage that it may diverge because it depends on g(x) as g(x)=(x**(2)-3)/2 and g(x)=3/(x-2).

Pitfalls of methods

1)bisection method

1) Rate of Convergence is slow

2) need to define lower and upper initial guesses

3) Can't Detect Multiple Roots

4) Relies on Sign Changes.

2)false position

1)Always check if f(xi) equals to zero

2)as it is trial and error method in some cases it may take large time span to calculate correct root and thereby slowing down the process

3)It is used to calculate only a single unknown equation

3) newton raphson

1)convergence may be slow

2) if there is an inflection point then the function diverges

3) a local maximum or local minimum cause oscillation

4)Secant method

1)it may not converge

2)there is no guaranteed error bound for the computed iterates.

5)Fixed point method

1)Divergence is possible

2) If the equation has more than 1 root, and f(x) is continuous then this method may miss one or more roots

Gui sample user input