Evaluating Bayesian Networks from Multi-Agent Simulations

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Abstract

There are probabilistic methods for reasoning with evidence in court cases. Bayesian Networks have been suggested as one of these methods, but we do not know how suitable they would be in the domain of law, which is not traditionally a statistical field. This work investigates Bayesian Networks for criminal cases by creating multi-agent simulations of crimes, collecting frequency data on these simulations, using the K2 algorithm to generate Bayesian Networks from this data, and then evaluating the generated Bayesian Networks on structural, performance and human factor criteria. This is done for 3 simple simulations: one non-spatial simulation of a murder, one spatial simulation of a home-robbery with evidence, and one spatial simulation of a street robbery in a non-trivial environment.

We find that the Bayesian Networks are generally accurate and perform well even if we do not know the exact probabilities as generated by the simulation. However, it is implausible that human modellers could replicate the cpt's of the networks. Apart from that, private knowledge, conditioning on implicit factors and the effects of non-ideal operationalisation are factors that need to be solved before Bayesian Networks can be responsibly applied in court.

Contents

Chapter 1

Introduction

When we find evidence for a hypothesis that we have held in the back of our minds, our belief in the hypothesis increases. This is the basic idea behind reasoning with evidence. In a constellation of hypotheses and pieces of evidence, we want to construct a network that will lead us to believe as many true hypotheses as possible, given the evidence that we have.

However, evidence itself is elusive, and its connection to hypotheses is as well - how can we be sure that some evidence supports some hypothesis? Even if we know that it does, how can we express how strong the piece of evidence is? Some evidence is very weak, and only after a tedious process of ruling out other factors and careful investigation and collection of other pieces of evidence, we can come to a conclusion about a hypothesis. On the other hand, some evidence is so strong that it leaves no room for doubt.

We all have intuitions about evidence strength - but can we make these intuitions precise? Additionally, can we manage the complex realities of weak evidence for many different hypotheses?

We have a way of expressing how we should use evidence to update our beliefs in hypotheses. We can do this with the use of probabilities. When we reason probabilistically with evidence, we want to use the gold standard - Bayes Law. Let's say that we find some piece of evidence e, we want to know how our finding e affects our belief in some hypothesis h. We can express how much we believe h given e, using Bayes Law:

$$P(h|e) = \frac{P(e|h) \cdot P(h)}{P(e)}$$

This is the simplest form of Bayesian reasoning, one piece of evidence, and one hypothesis. But real life is not so simple, and we are constantly reasoning with many pieces of

evidence. Small hypotheses can serve as stepping stones to greater hypotheses. One way of handling the complexity of interacting pieces of evidence and hypotheses is by using Bayesian Networks.

A Bayesian Network is a directed, acyclic graph that represents the joint probability distribution over a set of events (Pearl, 1988). BNs have nodes and arcs. Formally, the nodes are random variables that represent events. In case of our Bayesian Networks, these random variables represent hypotheses and pieces of evidence. An arc represent a conditional relationships between two nodes. Conditional probability is expressed in the conditional probability table (cpt) of every node. A node that is not conditionally dependent on any other node (has no incoming arcs) is independent.

An example of a Bayesian Network can be seen in Figure 1.1. It expresses a simple reasoning step: how should our belief in the hypothesis that our train will be late in Groningen, change once we learn about the train station renovations in Zwolle?

In the figure, the node 'train_on_time_in_groningen' is a hypothesis. Nodes with one or more incoming arcs are conditionally dependent on their parent node(s). In our example network, the node 'station_renovation_zwolle' is conditionally dependent on its parent. The links between the nodes are links of conditional probability, and do not correspond to real-world causality, although they are sometimes interpreted as being causal (Dawid, 2008).
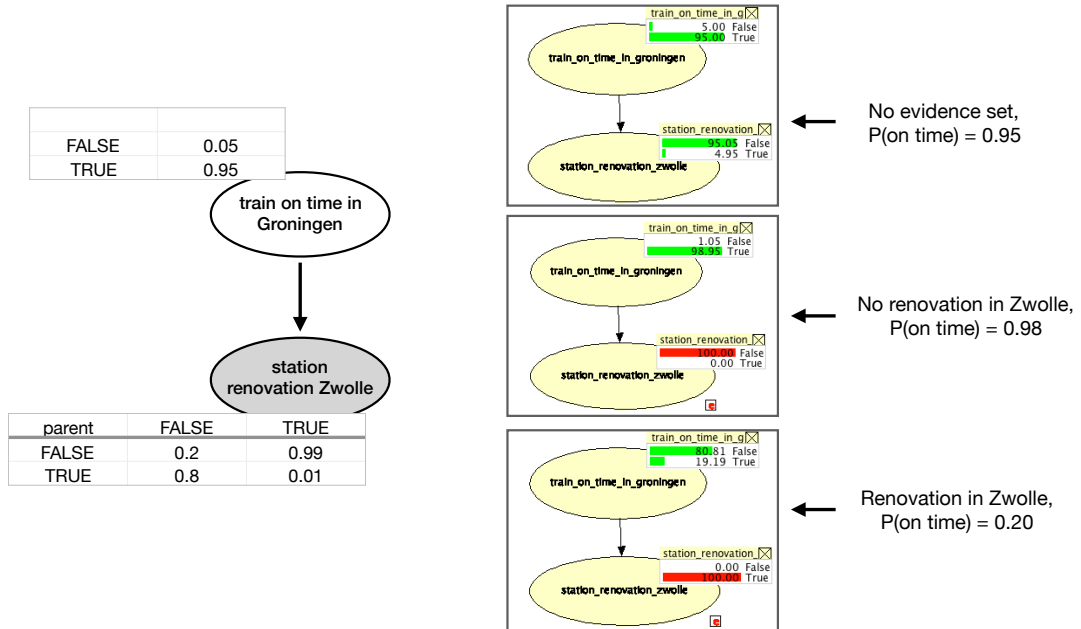

Figure 1.1: An example of a Bayesian Network, with and without evidence set.

We can reason with Bayesian Networks by setting evidence on one or more nodes that we can observe - our evidence. This means that we say whether some observable node is true or false, and use that information to update the network. We can now find a new posterior probability for a node that we cannot observe directly (yet). In Figure 1.1, this means that when we learn that there are station renovations going on at Zwolle, the our belief that our train will be late in Groningen increases from a probability of 0.01, to a probability of 0.8.

Using Bayesian Networks, we can make explicit how we think that we should reason about hypotheses given pieces of evidence. In our toy example, this is not very consequential, it is a very simple network. However, Bayesian reasoning and Bayesian Networks specifically, can be applied to many domains. One of which is AI and law, specifically, Bayesian Networks might be tools that could be useful in court cases, by organising evidence and hypotheses. A Bayesian Network might make explicit how reasoners should update their beliefs based on the evidence, which might increase transparency and fairness. In the ideal case, a Bayesian Network might stop a judge from making a reasoning error!

However, creating Bayesian Networks is very complex, especially in domains where probabilistic data is sparse. We also do not have a good idea if our methods for creating Bayesian Networks in criminal situations work, since it is difficult to test these networks empirically - you cannot ask a murderer to murder again, for the statistics. This is why we need a grounding to investigate whether our methods for building Bayesian Networks actually work.

This is why this project proposes to ground Bayesian Networks for crimes in multi-agent simulations of crimes. We can model crime cases using simulations to a level of realism we desire. We can see what probabilities emerge out of these simulations by running them multiple times, and see if we can capture this probabilistic information into Bayesian Networks. We can collect exactly the frequency information that we need. This means that we can focus on assessing the Bayesian Networks, since we know what frequency information they should replicate.

Our Research Questions:

- **Can we create Bayesian Networks that correctly reflect multi-agent simulations of crimes?**

- **If we can, under what assumptions do these networks function?**

Chapter 2 is an overview of the state of the art, Chapter 3 proposes a pipeline for creating simulations and automatic Bayesian Networks, and evaluation criteria that state when a Bayesian Network correctly reflects multi-agent simulations of crimes. Chapter 4, 5 and 6 are applications of this pipeline in three separate simulations: first a simple, non-spatial simulation based on (Vlek et al., 2015), a simple spatial simulation of a home-robbery, and

finally a spatial simulation of a street-robbery. For each of these simulations, some simple experiments are done to investigate several problematic aspects of Bayesian Networks. These simulations are then each evaluated based on the evaluation criteria set out in Chapter 3. Finally, we draw conclusions in Chapter 7.

You can interact with the simulation in Chapter 6 on `https://shielded-journey-34533.herokuapp.com`.

Code for this project can be found at `https://github.com/aludi/simulationTest`.

Chapter 2

State of the Art

We want to create grounded Bayesian Networks for reasoning with evidence. This can be done by creating multi-agent simulations, and building Bayesian Networks based on the events in these simulations. In this section, the relevant background is laid out on reasoning with evidence, Bayesian Networks, and multi-agent simulations.

2.1 Reasoning with evidence

We consider three main directions in the field of reasoning with evidence within law (Verheij et al., 2015). The first direction is via argumentation approaches, where hypotheses and evidence are represented as propositions that attack or support each other. The second direction is via scenario approaches, where more-or-less coherent hypotheses are combined into stories (Pennington and Hastie, 1993), (Wagenaar et al., 1993), which are supported with evidence. The third direction is via probabilistic approaches, where hypotheses and evidence are assigned probabilities, and the relation between hypothesis and evidence is represented with conditional probability. There are also be hybrids that combine or synthesise aspects of each approach, see, for example: (Bex, 2010) and (Timmer, 2016). This project does not consider the argumentative approach, and is mainly based on the work of Vlek (Vlek et al., 2015), (Vlek, 2016) and Fenton (Fenton et al., 2012), (Fenton et al., 2019). The networks in these papers describe whole criminal situations (scenarios), but use Bayesian Networks and hence have a probabilistic component.

2.2 Bayes and Bayesian Networks

We have already seen the power of Bayes's law in the introduction. It tells us how much we have to change our belief in a hypothesis once we find a piece of evidence. To do this,

we need to have probabilistic or frequentist information about the likelihood ratio and the prior.

Bayesian reasoning, without Bayesian Networks, have been used by Dahlman in 'event trees', specifying different branches of combinations and their probabilities (Dahlman, 2020).

However, just using Bayes law is very difficult, because sometimes we want to condition on multiple pieces of evidence. Doing all the calculations is tedious, hence we use the computational tools called Bayesian Networks.

Bayesian Networks can represent aspects of a criminal case or can attempt a scenario-like hybrid and represent the entire case - modelling actual crime cases (J Kadane, 1996), (Fenton et al., 2019), cases from fiction (Fenton et al., 2012) or fictionalized crime cases (van Leeuwen, 2019). In situations where the network represents aspects, the nets model DNA or blood-spatter evidence, for methods see (Ronald Meester, 2021).

Bayesian Networks can be built by hand, by experts or academics, in (proprietary) software like AgenaRisk or Hugin. They can also be built by hand in PyAgrum (Ducamp et al., 2020), a free Python software package, which does not have a GUI but has everything else. In this project, PyAgrum was used. Alternatively, Bayesian Networks can be automatically constructed from large datasets - PyAgrum also offers the opportunity for that. Automated Bayesian Network building is not plausible in the legal-evidence domain, because the data that we need is notoriously sparse.

2.2.1   Evaluating Bayesian Networks

Bayesian Networks are a promising tool, but there are a lot of open questions:

1. The use of Bayesian Networks is not straightforward, neither for the builder or for the interpreter. From de Koeijer et al. (2020): it is complex, time-consuming, hard to explain, and, the 'repeatability [...] leaves much to be desired. Node definitions and model structures are often directed by personal habits, resulting in different models for the same problem, depending on the expert'.

2. The granularity of the network - how do we know which nodes hypotheses and pieces of evidence to include? Ideally we would model as detailed as possible, but as we increase the number of nodes, we increase the complexity, which increases the probability of mistakes, and the time spend on the network.

3. The links: how do we know which events depend on each other?

4. The numbers - there's not just subjectivity in selecting the nodes, and drawing the links, but the probabilities that we have to fill in into the cpt themselves are the most obvious stumbling block. We can identify that there has to be some sort of

correlation between 'smoke' and 'fire', but how to express this in numbers? Fenton argues that we're just making something explicit that we would otherwise have left implicit. But what are the consequences of making something explicit, but doing it wrongly? How robust is the network against imprecise or wrong frequencies? If we use frequencies (or subjective probabilities based on frequencies), how do we decide what set of events is included when we start to count (this is the problem of the reference class, see (Allen and Pardo, 2007), (Colyvan et al., 2001)). Probabilities can be pure frequencies, or can be subjectively elicited from experts (Renooij, 2001), (Druzdzel and van der Gaag, 2000).

This project cannot address all of these problems, because it's too much. My main focus will be on problem 4. As we mentioned before, one of the problems that can plague Bayesian Network creation is the lack of well-defined and plentiful data. But what if we would have this data about criminal cases? Then we could test our methods for Bayesian Networks without worrying about the subjectivity and lack of data of the numbers in the cpt. If we can create a grounded 'data-generating' environments for criminal cases, we can test our Bayesian Networks against them.

2.3 Agent simulations

We can create such a data-generating environment for criminal cases by the use of multi-agent simulations. In a multi-agent simulation, agents observe and interact with their environment. The environment and the agent behaviour is fully controlled by its programming and all randomness can be accounted for. This means that we know exactly with which frequencies fully-specified events occur. Running the simulation multiple times generates a lot of data, which will serve as input to automatically build a Bayesian Network from data. This means we use a multi-agent simulations as a ground for our theory-testing. In this project, the simulations will be programmed in Python using the MESA framework (Kazil et al., 2020).

Multi-agent simulations have been used to investigate the criminal domain and to test out sociological theories. By creating spatially explicit simulations, the complex interactions of agents with their locations can be represented better than in traditional models - these agent-based models can model criminal hotspots (Bosse and Gerritsen, 2008), theories of behaviour (Gerritsen, 2015), and police strategies in urban crime (Haojie Zhu, 2021). Weaknesses identified with multi-agent models for criminological research are insufficiently complex models, unclarity on the effect of temporal resolution, and lack of a systemic approach (Haojie Zhu, 2021).

2.4 Conclusion

The two main ideas are Bayesian reasoning, specifically as implemented in Bayesian Networks, and computational simulations of agents. The simulation is supposed to be the grounding for the Bayesian Networks.

But why do we want to use Bayesian Networks in the first place? In the best possible case, a Bayesian Network would help us to make the correct reasoning steps, telling us exactly how to weigh each piece of evidence in the grand scale of the simulated crime. This is a normative approach - it's telling us how to reason, but it is not (and not meant to be) an empirical approach. Bayesian networks are not a reflection of 'how people actually reason' - If you open up our minds, you won't see Bayesian Networks in there. One of the problems for normative models in decision making (Colyvan, 2013), is that we can only test how well our normative models work, if we already know what a good outcome would be. In a sense, we're doing experimental model testing for normative Bayesian Networks in this project.

Chapter 3

Method

This chapter lays out the general method for creating agent-based simulations with automatic Bayesian Networks, as well as how to evaluate these Bayesian Networks.

We can consider an approach in four stages. First, we need a scenario, or set of alternative scenarios, that are to be modelled. These scenarios are written description of crime scenarios that contain the hypothetical events and evidence for these events, as postulated by the prosecutor and defence.

Second, based on this written description of the scenario(s) to be modelled, an agent-based simulation is created. In the simulation, all events of the scenario need to represented. The simulation can be as granular as desired. The simulation can be run multiple times, so that we can gain an understanding of possible underlying frequency information on the events in the scenario. The frequencies by which events occur in the simulation are collected.

Third, the collected frequencies are then used to automatically build a Bayesian Network on the simulation, using the K2 algorithm. Finally, the generated Bayesian Network is then evaluated based the criteria set out at the end of this chapter.

This four-step process will be explained in this chapter, and illustrated with a running example of a stabbing.

3.1   A Scenario

We start our process with one or more written scenarios. These scenarios can, in the first instance, be obtained from (abridged) court case descriptions. The scenarios should contain all and only those hypothesised events, and the evidence for such events, that are relevant to the case. Both the prosecution and the defence should be able to select relevant events and their evidence.

Example 1 *Here we will introduce our running example, loosely based on the first scenario in (Vlek et al., 2015): Mark and Jane live together. One day, a neighbour overhears that they've started to fight in the kitchen. Jane grabs a knife, and stabs Mark. Mark dies of the stab wounds, as confirmed by the forensic scientist at the scene. Jane's fingerprints are on the knife.*

3.2   An Agent-based Simulation

We need a way to transform the written text descriptions of the events in the scenario to observable events in the simulation. First we identify all relevant actors, objects and places in the scenario. Then, we divide all all events into three classes: either an event is 'evidence', a 'hypothesis', or an ultimate 'outcome'. The 'outcome' is what we are ultimately interested in, it represents the central fact of the case. Apart from it being the node of ultimate interest, it otherwise functions the same as a 'hypothesis' node.

Then we select all relevant events from the written description and attempt to operationalise them in the simulation. This means that we create a way to measure if the events take place within the simulation. This is done by means of 'reporters', or random variables. These reporters report whether a given event has taken place within the simulation.

The process of operationalisation is very subjective and depends entirely on the modeller's assumptions and their time, knowledge or skill constraints. The behaviour of agents, as reflections of actual people, can be modelled in a way that reflect real sociological theories of behaviour (such as in Gerritsen (2015)), or with simple rules and a random number generator (such as here). The environment of the simulation can reflect a real-world geography with different affordances, or can be simplified. How a given event is operationalised is essential for the validity of the network: if it is unclear, or disputed, when an event is true in a simulation, the network should not be used.

Example 2 *We have modelled Jane and Mark as two agents in a simulation (Figure ??), with as relevant object a knife (blue dot). The only 'psychological' aspect that these agents have, is an anger level, which has 4 escalating states = {not angry, fighting, grabbing knife, stabbing}. Jane is quicker to increase to the next anger level.*
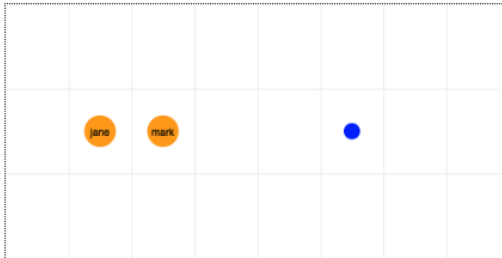
jane_and_mark_fight   *Both Mark and Jane have an anger level that is higher than 'not angry'*

jane_has_knife   *Jane has an anger level of 'grabbing knife' and moved to the location of the knife*
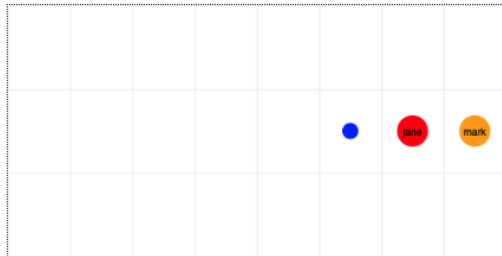
jane_stabs_mark_with_knife   *Jane has an anger level of 'stabbing', has the knife, and moved to Mark's position*
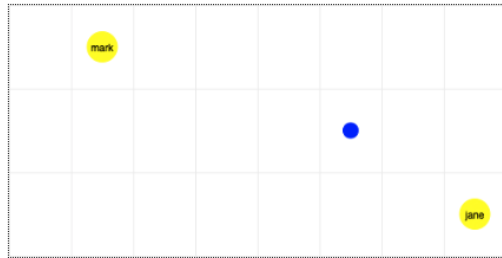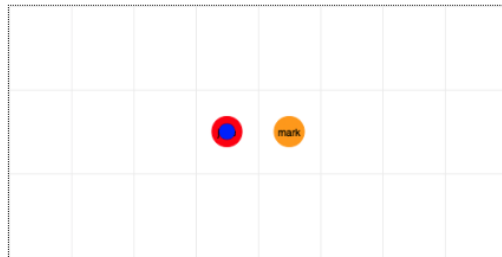
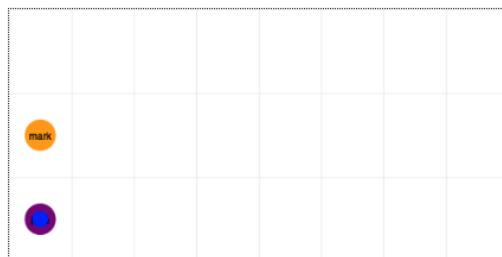(a) Initial stage of simulation: Mark and Jane are not fighting.

(b) Both Mark and Jane are angry, move towards each other to fight.

(c) Jane is so angry that she's grabbing the knife.

(d) Jane has the knife and moves with it.

(e) Jane's anger increases by a level, and she stabs Mark.

Figure 3.1: Simulation stages.

mark_dies   *Jane has stabbed Mark, and if a random number generator (uniform, 0, 100) produces a number > 60*

E_neighbour   *Either Mark and Jane have an anger level that is higher than 'not angry'*

E_prints   *True when Jane has knife*

E_stab_wounds   *True when Jane stabs Mark*

E_forensic   *True when Mark dies*

Then, the operationalisation itself: In the simulation, certain events can be brought about. In our example, this could be the events of 'jane_stabs_mark', or 'E_forensic'. We need a way to observe these states: this is where reporters come in. A reporter is a random variable that is reports the outcome of a relevant event in the simulation and is embedded in the code. If an event happens (or does not happen), the reporter reports that the event is true (or false). In essence, the reporter (R) is a random variable (RV) (for a further explanation of Random Variables, see Chapter 8). In short, a random variable maps an event (e) to a truth value:

$$R : e \rightarrow \{0, 1\}$$

Over which events we define reporters, depends on which events we deem relevant. We could, in theory, create a combinatorial explosive number of reporters for any possible event, like $jane\_in\_position\_(0, 0)$, $jane\_in\_position\_(1, 0)$, etc, but pragmatically, these very granular reporters will not help us in modelling the case. This means that we need to find a balance between over-specification and under-specification.

A reporter's value is 0 by default, but maps an event to 1 whenever it happens in the simulation. When we run the simulation in our example, we expect to see that, whenever Jane and Mark are fighting, R : 'jane_and_mark_fight' $\rightarrow 1$. At a later time-step, if Jane stabs Mark, we would see: R : 'jane_stabs_mark' $\rightarrow 1$.

No matter how many reporters we define, we can combine all reporters at the end of one run of the simulation into a global state G.

$$G = (e_0 \rightarrow \{0, 1\} \times e_1 \rightarrow \{0, 1\} \times ... \times e_n \rightarrow \{0, 1\})$$

or, for n reporters:

$$G = R_1 \times R_2 \times ... \times R_n$$

A global state that would represent that every one of the postulated events and evidence happened, except that Mark did not die of the stabbing, and that there was no forensic

scientist to determine that he died of the stab wounds, would be represented as (reporters in the same order as in the listing above):

$$1, 1, 1, 0, 1, 1, 1, 0$$

Then, we collect these global states over the number of runs that we do for each experiment, which results in the output O of this stage of the method, is a series of global states, one for each run:

$$O = (G_0, G_1, ...G_{runs})$$

3.3 Creating a Bayesian Network from a Simulation Automatically

The output of an experiment is the collection of runs O, where each run is the global state G of the simulation, as measured by the random variables R. The reporters in the simulation are random variables. These reporters become the nodes in the Bayesian Network.

The Bayesian Network is generated automatically, using the automated BN learner method as implemented in pyAgrum. There are several learners implemented in pyAgrum.[1] The algorithm used in this experiment to structure the Bayesian Network from the simulation data is the K2 algorithm (Cooper and Herskovits, 1992).

The K2 algorithm is a greedy search algorithm that attempts to maximise the posterior probability of the network by correctly connecting parent nodes to child nodes. It takes an ordering on the nodes as input. The first node in the ordering X_0 does not have any parents. For two nodes X_i and X_j, X_j cannot be the parent node of X_i if $j > i$: a node can only have a parent that is earlier in the ordering. The algorithm processes each node X_i by adding possible parent nodes to X_i (as constrained by the ordering), and maximising the score of the network. The algorithm stops if there are no parents to add, no parents improve the score, or the node has reached the maximal number of parents (Chen et al., 2008).

Due to this ordering constraint, the K2 algorithm is efficient. However, finding the ordering of the nodes as input for the network is not trivial. The ordering should be meaningful, to reflect causal or temporal information present in the domain. Through our simulation, we can find the temporal ordering of the nodes by collecting the temporal order in which hypothesis events occur. The evidence nodes are added at the end, to ensure that evidence nodes are never parents to hypothesis nodes.

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[1]An introduction to structure learning using PyAgrum: http://webia.lip6.fr/~phw/aGrUM/docs/last/notebooks/structuralLearning.ipynb.html

Example 3 *The temporal ordering of events in the simulation would be:*

{*'E_neighbour', 'jane_and_mark_fight', 'jane_has_knife', 'E_prints', 'jane_stabs_mark_with_knife', 'E_stab_wounds', 'mark_dies', 'E_forensic'* }

The ordering we want to give to K2 to save the temporal structure and the evidence-idiom Fenton et al. (2012):

{ *'jane_and_mark_fight', 'jane_has_knife', 'jane_stabs_mark_with_knife', 'mark_dies', 'E_neighbour', 'E_prints', 'E_stab_wounds', 'E_forensic'* }

The ordering is calculated as follows: at every run, we note down which hypothesis events happen in which order. We only note down the order, not the time-step at which an event occurs. We count how often each set of events occurs. We select the set of events that contains the most events, as this is likely the set that represents the entire 'causal' chain, or an entire scenario. If there are two sets of events of equal length, we pick the one that occurs most frequently. We add this set first to the ordering. We then look at the number of hypothesis nodes we have left, and add the largest set we can, until we have added all the hypothesis nodes to the ordering. Then we add the evidence nodes to the ordering.

For example, let's say we have 4 hypothesis nodes, $X_1, ...X_4$. In Run 1 we find X_2, X_3, X_4, in Run 2 we find X_1, in Run 3 X_2, X_3, X_4 again, but Run 4 is X_1, X_2, X_3. We select the longest sets first, and since X_2, X_3, X_4 occurs more often than the other one, we add it to the ordering first. The ordering is now X_2, X_3, X_4. We have 4 hypothesis nodes, so we need to add a set of length 1, which is X_1, which results in an ordering of X_2, X_3, X_4, X_1 that we add evidence to, and apply to the K2 algorithm.

Example 4 *We give the collection of global states O from our example to K2, as well as the temporal ordering as laid out in the previous example. This results in the a Bayesian Network that is shown in Figure 3.2.*

After the network is generated, we need to manually change a thing about it. The algorithm does not know how to handle impossible states, which are cases where combinations of valuations for parent nodes are impossible. For example, in Figure 3.2, the value of 'E_forensic'. 'E_forensic' has two parents: 'mark_dies', as well as 'E_neighbour'. In the simulation, it can never be the case that Mark dies, but the neighbour does not hear and report on fighting, or the combination of events $'mark\_dies' \to 1 \land 'E\_neighbour' \to 1$. However, the event for the thief stealing the object, can have both 'motive' and 'knowing about object' as parent nodes. The algorithm does not know how to deal with these situations and the resulting network will crash. This is why the probability for 'stealing' given the impossible combination of the parent state values, will be manually set to 0.
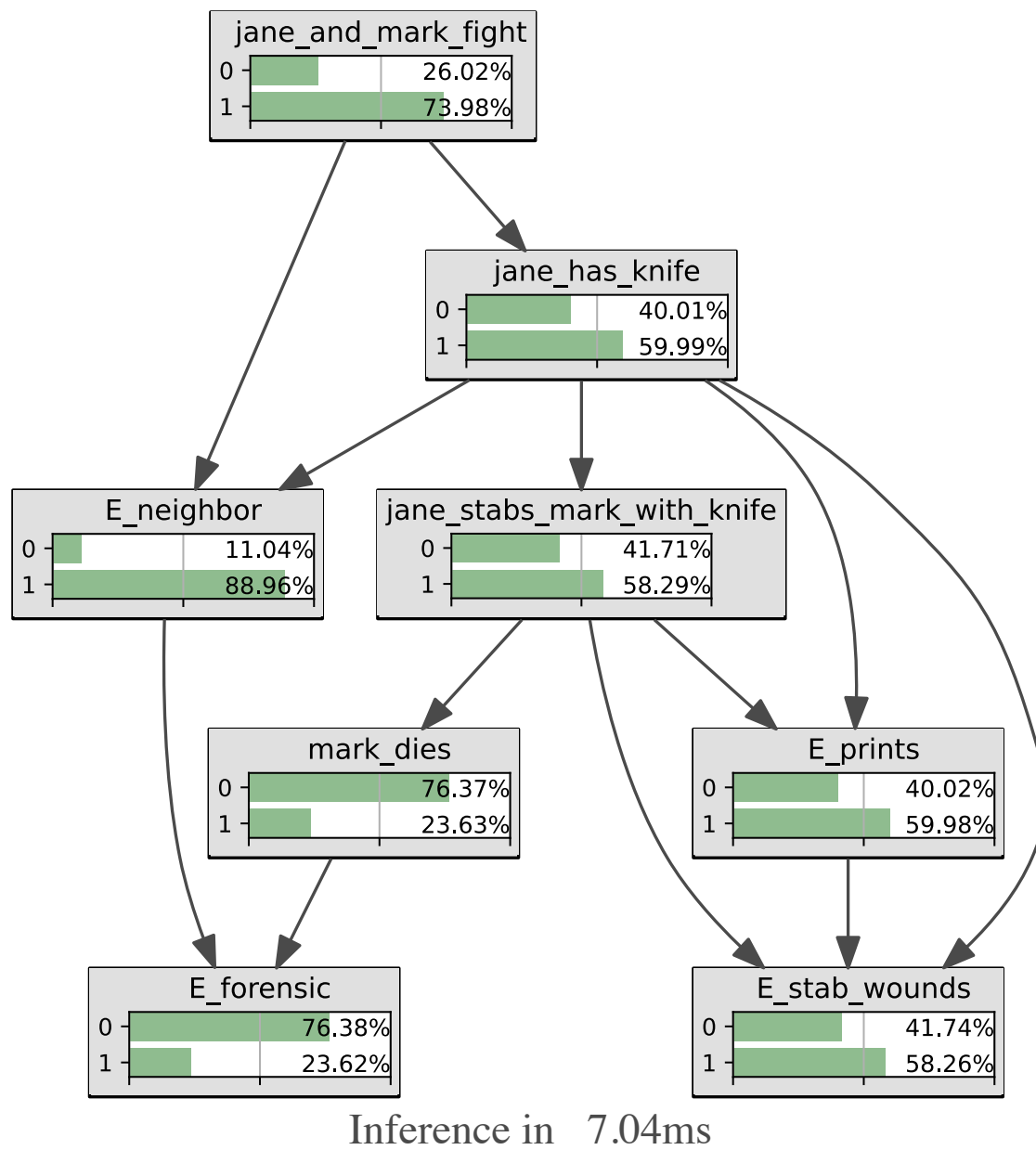
Figure 3.2: Example network.

3.4   Evaluating the Bayesian Network

Now we know that we can build Bayesian Networks automatically using the K2 algorithm based on data we generated in our simulation, we need to evaluate different aspects of the Bayesian Network: numerical, structural and performance criteria.

3.4.1   Numbers

1. **The conditional frequencies in the BN correspond to the conditional frequencies in the simulation**

   We take the collection of global states as the output of the simulation. For every event in the global state, we calculate the base-frequency at which they occur, so if event A occurs 40 times and $\neg A$ occurs 60 times in 100 runs, we find that the frequency of $A = 0.4$. Then, we compare these frequencies with the prior probabilities of the events in the BN.

   Given below (Table 3.1) are the correspondence between the frequencies in the simulation and in the BN. There are differences between the probabilities in the BN and frequencies in the simulation, but these are smaller than 0.01.

Conclusion	Frequency P(event)	BN P(event)
jane_and_mark_fight	0.74	0.7398
jane_has_knife	0.6	0.5999
jane_stabs_mark_with_knife	0.583	0.5829
mark_dies	0.236	0.2363
E_neighbour	0.89	0.8896
E_prints	0.6	0.5998
E_stab_wounds	0.583	0.5826
E_forensic	0.236	0.2352

   Table 3.1: Correspondences between simulation frequency and BN

2. **The values in the conditional probability tables are elicitable from human modellers**

   In real life, we do not have access to the true frequencies of events in real life. This means that we would have to discover them in some other ways. The CPT in question here, means including the parent node as determined by the K2 algorithm. Relevant questions to ask about the elicitability of CPTs of nodes in the BN are:

   (a) Could we discover the frequencies in the CPT using forensic-science methods?

(b) Could we discover the frequencies in the CPT using population statistics? [2]

If we have to answer both of these questions with 'no', then the CPTs are not elicitable at all. However, answering Question 1 or 2 with a 'yes' is not straightforward - we have to deal with the problem of the reference class.

To show what is meant by that problem, let's apply these questions to our example. We have the node 'jane_has_knife', given parent node 'jane_and_mark_fight', and we want to assign it some probability. However, we cannot find a 'population statistic' about 'jane_has_knife'. Are we going to the CBS and ask about all knife owners named Jane? This is not a meaningful population statistic. Hence, we need to generalise Jane to some class of person that we can find population statistics for. If we generalise 'Jane' to the reference class 'a random adult woman in the Netherlands', we could probably find population statistics about knife possession. However, if domestic violence is a pattern in Mark and Jane's relationship (as evidenced by the parent node 'jane_and_mark_fight'), we cannot just consider Jane to be 'a random adult woman in the Netherlands', but instead consider her 'an adult woman in a violent relationship in the Netherlands'. If we find more risk factors and learn more about Jane (high aggression, previous offences, etc.), the subset that we have to find a population statistic over gets smaller and smaller, until we have reached a subjective probability estimation. This is the case for the first question as well.

This problem is unavoidable and it is unclear how we can resolve it. This is a question for future research. However, in this idealised situation, let us always take the most generous interpretation of a node - meaning that we interpret a node as if there might be a reference class. As example, let us divide the nodes into the relevant categories (Table 3.2).

Node	Forensic science	Population Statistics
jane_and_mark_fight		x
jane_has_knife		x
jane_stabs_mark_with_knife		x
mark_dies	x	
E_neighbour		x
E_prints	x	
E_stab_wounds	x	
E_forensic	x	

Table 3.2: Theoretical elicitability

An important thing to note is that these are **not** assessments about the actual prob-

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[2]Guessing.

ability values in the CPTs in the generated BN, but instead only generous theoretical assessments about the deemed possibility of elicitation. The actual probability values in the generated BN will not line up with the forensic or statistical probabilities that we could or would find in real life, since the simulation is very simplified.

3. **The BN is robust against imprecision in the conditional probability tables**

We can reduce the precision of the values inside the CPTs to investigate the robustness of the BN (Oniśko and Druzdzel, 2013). We generate one Bayesian Network, with the data we collected from the simulation. Then, we create many new networks, with this network as a basis. We do not change the number of nodes, nor the structure of the Bayesian network. We only change the values in each node's CPT. In this case, we round the values in the CPT, such that the BN becomes increasingly less precise. The intuition behind this is, that when expert users are going to elicit the probabilities, we do not know how robust the network is against smaller and larger imprecisions in the elicited probabilities. By simulating such imprecisions, we can compare the predictive power of the more imprecise networks to the ground truth of the 'real' network.

We round every value in the cpts c to each of

$$i, i \in \{\text{no disturbance}, 0.01, 0.05, 0.1, 0.125, 0.2, 0.25, 0.33, 0.5\}$$

according to

$$floor(\frac{c}{i} + 0.5) \cdot i.$$

We also add some imprecision into these networks - so that we never have the values of 0 and 1 in our cpts, as this blocks the propagation of evidence throughout the network. Instead, where we would round to 0 or 1, we round to $1 = \epsilon$ and ϵ, where $\epsilon = 0.01$.

3.4.2   Structural

1. **The BN represents all events of the scenario**

We have to define relevance in a meaningful way. The scenario can be split into many different (implicit and explicit) events. The simulation should attempt to model the level of granularity as given in the scenario description, with more detail given in areas where there is conflict between reasoners.

In our example, we modelled every fact as described by the written scenario description. All nodes in the network reflect an event that happened in the scenario. However, the rules that underlie the agent's behaviour (increasing anger, goal-directed

movement) are not represented explicitly in the scenario. Using more or less complex behaviour would result in different BNs. Testing different models of criminal behaviour lies outside the scope for this project.

2. **The BN has temporal ordering of the hypothesis nodes**

   We want the BN to follow the temporal structure of the simulation and original scenario. The temporal ordering given to the K2 algorithm should ensure this - however, sometimes there are events that can happen in arbitrary order, or contradict each other with no clear way of determining which events happens first (as they happen exclusively).

   One example of contradictory events, which are not represented in the example network: the simulated 'jane_stabs_mark_with_knife', compared to the non-simulated 'mark_stabbed_himself'. Apart from the contradictory events or arbitrary temporal events, the hypothesis nodes in the example network are ordered chronologically, in the same order as in the simulation.

3. **The BN follows the evidence-idiom**

   We want to reason from evidence to hypotheses, or, we can observe evidence in real life (or in the simulation), and from there, infer the posterior probability of its parent-hypothesis node. We do not want to do this the other way around, setting a value in a hypothesis node, and looking at the resulting posterior probability of the parent-evidence. This is why evidence must always be the child node of a hypothesis. This construction is also called the evidence idiom (Fenton et al., 2012). Evidence can be the parent of other evidence, as evidence can be dependent on each other (**?**).

   For example, in the network that was generated for our example, we see that the network fulfils the criterium: There is not a hypothesis node that has an evidence node as a parent.

4. **The BN represents multiple alternative scenarios**

   If we want to model both the scenario as postulated by the defence, and by the prosecution, and let the collective evidence speak, then we need to be able to represent multiple conflicting stories within one BN. Representing different scenarios is not just setting some hypothesis node to false, but instead, it is creating a separate hypothesis node, that contradicts the first node.

   In our example, this would look like, instead of 'jane_stabs_mark_with_knife', the defence would say that 'mark_stabbed_himself'. These alternative scenarios are not represented in the scenario, simulation or network in this chapter.

3.4.3   Predictive

1. **The BN only predicts high probability of the output node given relevant evidence truth values**

   We want to use the BN to give a probability estimation of the ultimate output node. We want the probability of the output node to be high if the given evidence supports it, and the probability of this node to be low, when the evidence does not support it. We can test this: given a certain set of evidence, vary the assigned truth values of these evidence nodes, and switch the evidence nodes on in chronological order. We can plot the results, showing a progression of the posterior given a certain set of true or false evidence. Examples of these progression plots are shown in Figure 3.3.

   From these plots, we can see that we can only find a convincing posterior probability for 'mark_dies' when we have the forensic report, the rest of the evidence increases the posterior probability of the node, but never higher than 0.4. This is the expected behaviour - given the operationalisation fo the 'E_forensic' evidence, we expect it (and only it) to weight heavily on the posterior for 'mark_dies'. The other evidence is weaker.

2. **We can know when the BN predicts the wrong outcome**

   We investigate all possible permutations of evidence values. There are also impossible permutations of evidence, these are states of evidence that are not allowed by constraints of the simulation, such as the forensic report stating that Mark is dead, but Mark not having stab wounds. These impossible states are not tested. Over the possible evidence states, we show for which evidence states, the network predicts the wrong value for the output node. This means, that in the simulation, given some evidence, the output would happen, but in the BN, given that same set of evidence, the output does not happen. The states for which this occurs are states where the BN is failing. In our example, this is never the case: for all possible evidence states, the network predicts the correct response in the output node.
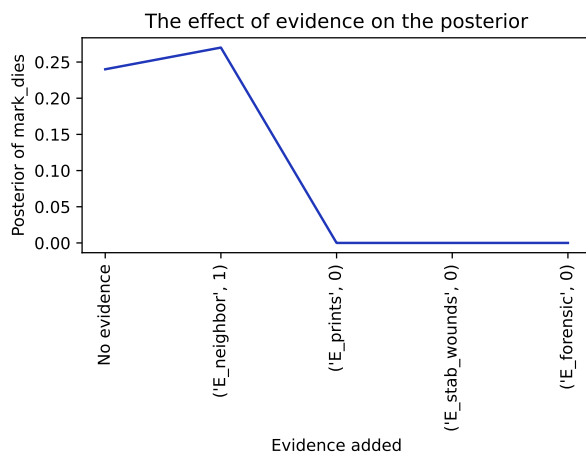
3.5   Conclusion

In this chapter, we have explained the method for creating and evaluating Bayesian Networks from a simulation based on a written scenario.
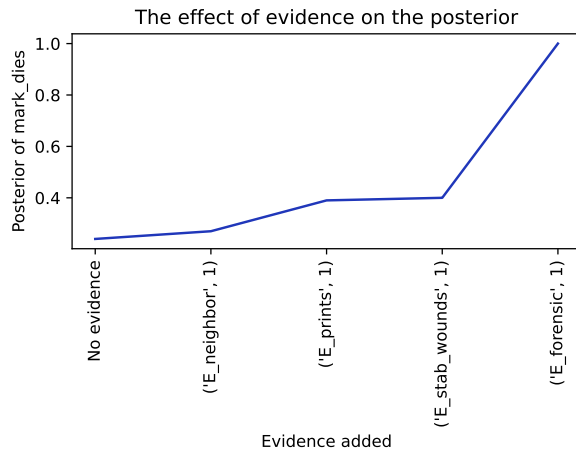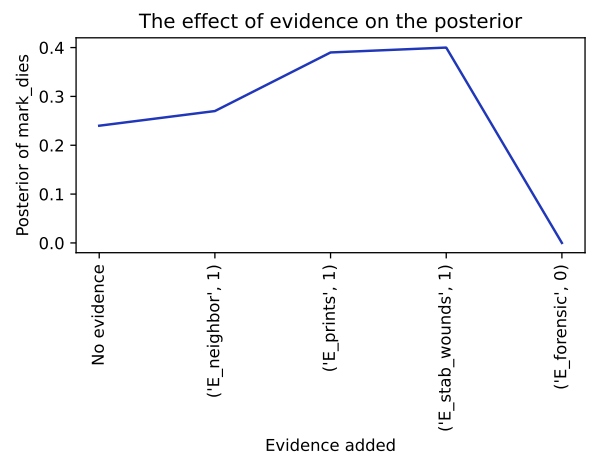
(a) All evidence is true.
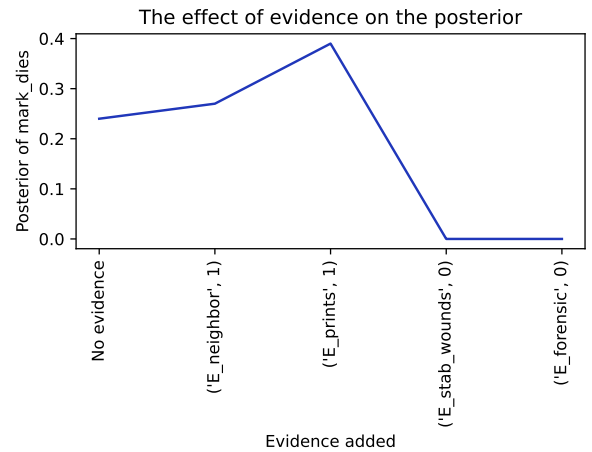
(b) Fight, and Jane's prints on the knife.

(c) Only the neighbour hears them fight.

(d) Only the death report is false.

Figure 3.3: Effect of evidence on posterior.

Chapter 4

Simulation of Grote Markt

In the previous chapter, we have laid out the process from creating a Bayesian Network from a written scenario of a case, and laid out criteria on how to evaluate that network. In this chapter, the method is applied to a simulation of a robbery at the Grote Markt, the main square of Groningen.

By placing agents in a 'real' spatial environment, they are constrained by geography in some manner - they cannot see through buildings, and cannot move through them. These affordances result in more interesting behaviours for the agents, and are a first step towards using this approach to less abstracted societal issues.

4.1 Scenario

There are two agents walking around the area of Grote Markt. One of the agents is old and carries a valuable object. The other agent is young, and might potentially steal the object. If the potential thief sees the potential victim, it decides whether the potential victim is vulnerable enough to steal from, and the object is valuable enough. If both of these conditions are fulfilled, the agent becomes a potential thief, and now has a motive to steal from the old agent. The agent will attempt to sneak up on the victim, and steal the object from the old agent. At some point, the old agent realises that their valuable object is gone. This is the first scenario. In the second scenario, the old agent simply dropped the valuable object, and after a while notices that it is gone. As evidence, we have a psychological report that estimates whether the young agent is capable of the crime, we might have video footage of the thief stealing the object, and we have the fact that the potential thief shows up on camera.

Additionally, outside the described scenario, we also have access to the 'mental' state of the potential thief, so we know whether they actually consider the old agent vulnerable, and

whether they find the object itself valuable. We also know if the thief is actually intending to sneak up on the other agent.

4.2 Simulation

We extract the relevant agents from the scenario, we need two of them: a potential thief, and a potential victim. Every agent has an age, to determine whether they are vulnerable or not - old people are considered more vulnerable. Every agent also has an object of a certain value, the thief's object has a value of -1, and the other agent's object has a value of 1000, to make it a tempting target. An agent decides if it wants to steal something by making a very simple risk-calculation based on their risk threshold: if the object is more expensive than their risk threshold, they will attempt to steal it (contributing to 'motive'). Every agent also has a goal state, this is the location at the edge of the map. When they enter their goal state, they are essentially removed from the simulation, as they leave the relevant area. Every simulation was run for 100 timesteps, or until both agents are in their goal states. The simulation itself was ran 500 times. The behaviour of the agents is shown in Figure 4.1. Agents are placed randomly on the map.

The operationalisation of the random variables is described below. The output node is the node 'stealing_1_0':

seen_1_0   if the victim agent is in the line of sight of the thief agent.

know_valuable_1_0   if the value is higher than the risk threshold.

know_vulnerable_1_0   if the victim's age is older than the thief's threshold age.

motive_1_0   if the object is more valuable than the risk threshold, the victim's age is older than the thief's threshold age, and the thief is not already stealing from someone else.

sneak_1_0   if the thief has targeted the victim (has a motive) , but is not yet in the same position.

stealing_1_0   if the thief has targeted the victim, the object's value is greater than the risk threshold, and the thief's position is the same as the victim's position.

object_dropped_accidentally_0   at every epoch, there is a 1/500 probability that the victim agent will drop the object by accident.

E_vulnerable_1_0   the thief decides that the target is vulnerable (know_vulnerable_1_0 is true).

E_valuable_1_0   the thief decides that the object is valuable (know_valuable_1_0 is true).

E_sneak_1_0   the agent sneaks up on the target (sneak_1_0 is true).

E_camera_1   the thief is seen on any one of the cameras.

E_psych_report_1_0   if the thief has a motive, we draw an estimated risk threshold and an estimated age threshold from two normal distributions (mean = victim age, sd = 20), (mean = value good, sd = 100). If the victim is older than the thief's minimal age threshold, and the object more valuable, we estimate that the psych profile states that the victim fits the thief's profile, else not.

E_camera_1   the thief is seen on any one of the cameras.

E_object_gone_0   if the object is dropped accidentally, or if the object has been stolen.

E_camera_seen_stealing_1_0   if any camera sees the thief during the state in which it is stealing.

The environment for the agents was created by converting a map image into an agent-readable world. This was done by writing a method to convert screenshots of maps into an agent-readable environment. The maps were screenshotted from `http://maps.stamen.com/terrain/#18/53.21618/6.57225` and converted into greyscale. Then, they were transformed into a grid of a given size. The average greyscale value of each cell in the grid was taken and coded as either 'accessible' or 'non-accessible'. On the greyscale map, the color of the buildings was in the range of (189, 199) - cells within this range were coded as 'inaccessible', since agents cannot walk through buildings. All other cells were 'accessible'. This resulted in a map shared by all agents that constrained their movements. There are 5 cameras placed randomly on the 'accessible' cells on the map, each with a visual radius of 8. Additionally, we used this map to calculate the sight lines of both cameras and agents. An agent or a camera cannot see another agent if there is an 'inaccessible' grid cell on the sight line between the two.

4.3   Network

The network is shown in Figure 4.3.

4.4   Evaluation

4.4.1   Numbers

1. **The conditional frequencies in the BN correspond to the conditional frequencies in the simulation**

2. **The values in the conditional probability tables are elicitable from human modellers**

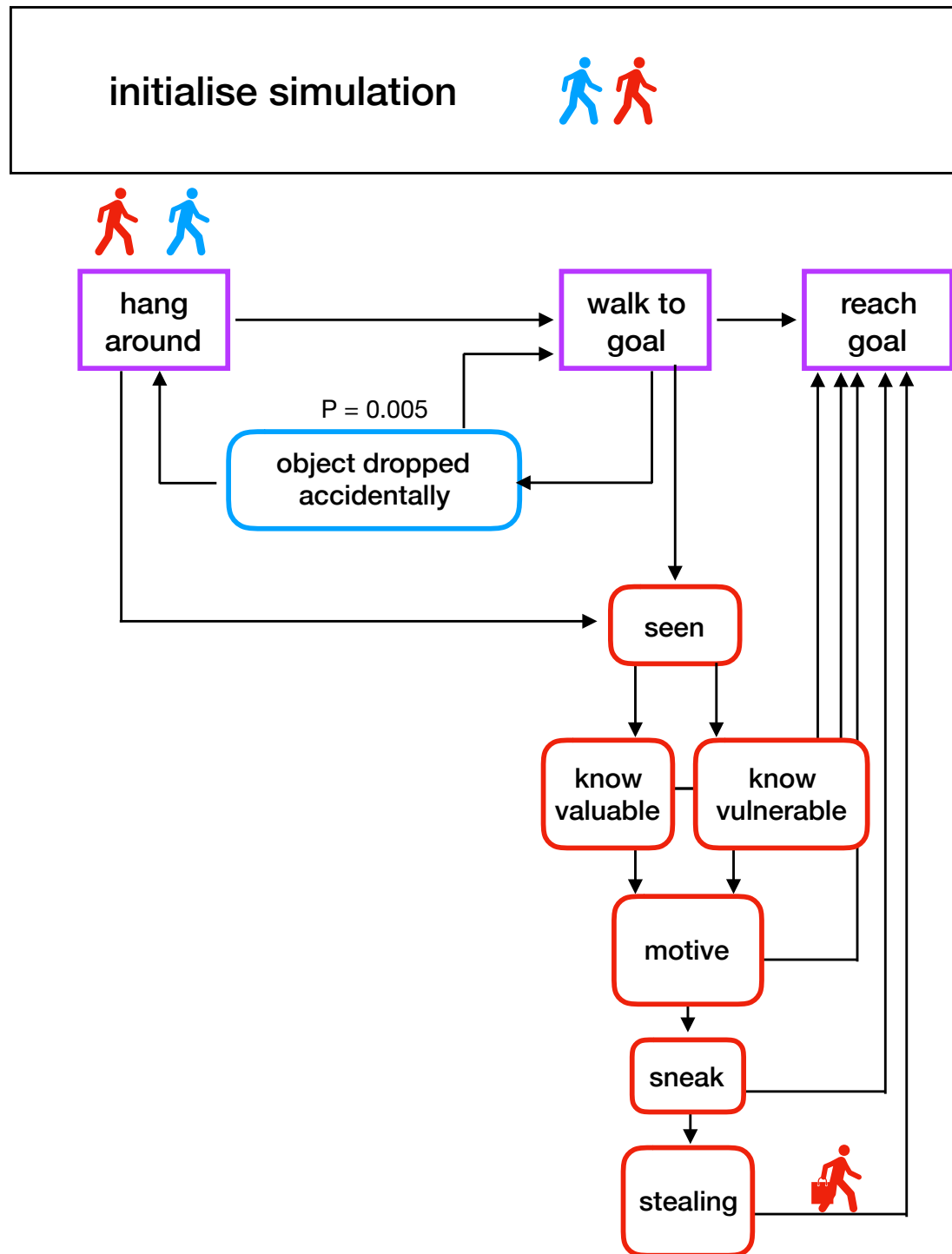3. **The BN is robust against imprecision in the conditional probability tables**

Figure 4.1: The behaviour of the agents. The nodes with rounded edges correspond to the nodes in the Bayesian Network.

(a) map of environment - 2 agents

(b) Camera locations are randomly initialized
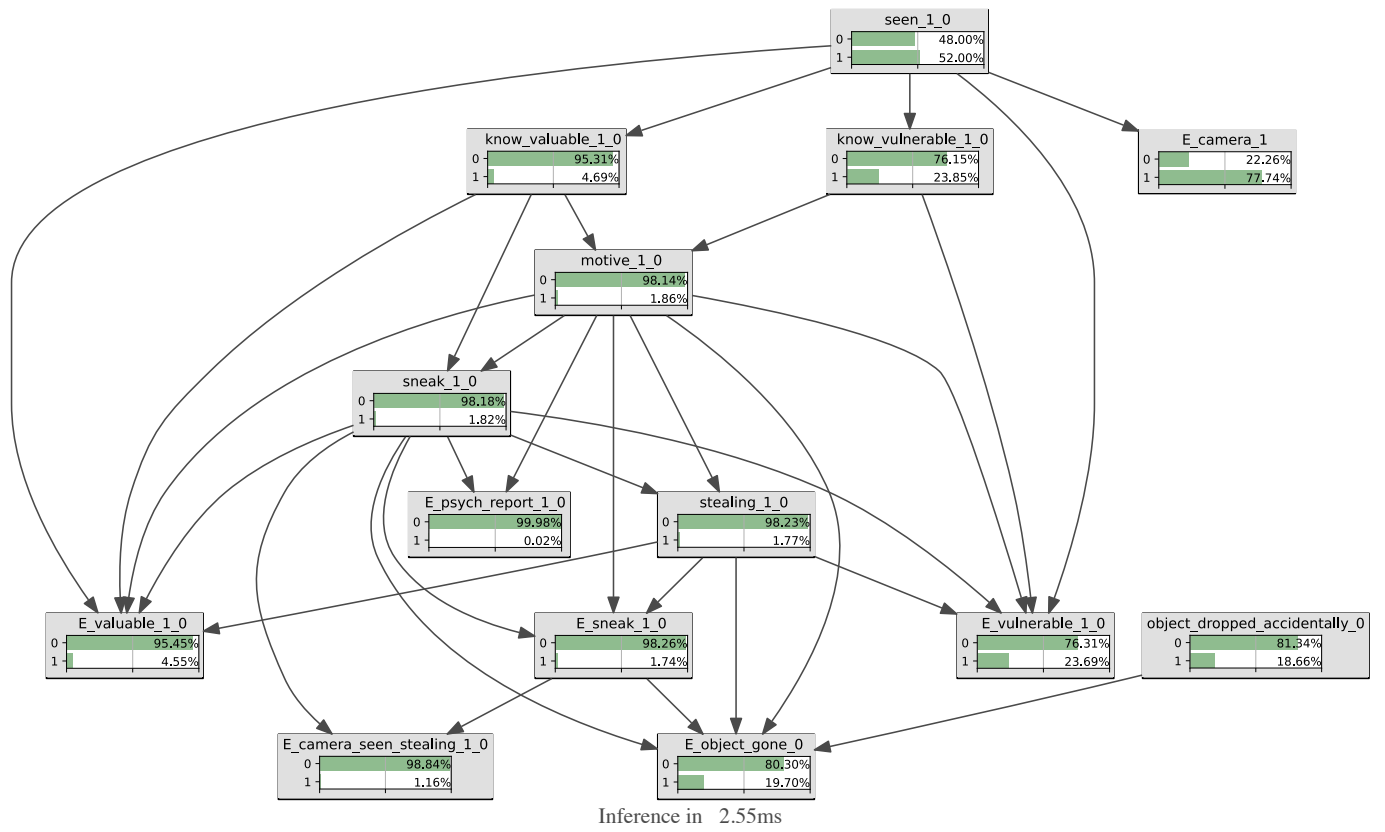
Figure 4.2: The Grote Markt environment

Figure 4.3: Network of Grote Markt simulation.

4.4.2 Structural

1. **The BN represents all events of the scenario**

2. **The BN has temporal ordering of the hypothesis nodes**

3. **The BN follows the evidence-idiom**

4. **The BN represents multiple alternative scenarios**

4.4.3 Predictive Skills

1. **The BN only predicts high probability of the output node given relevant evidence truth values**

   We see here that the posterir

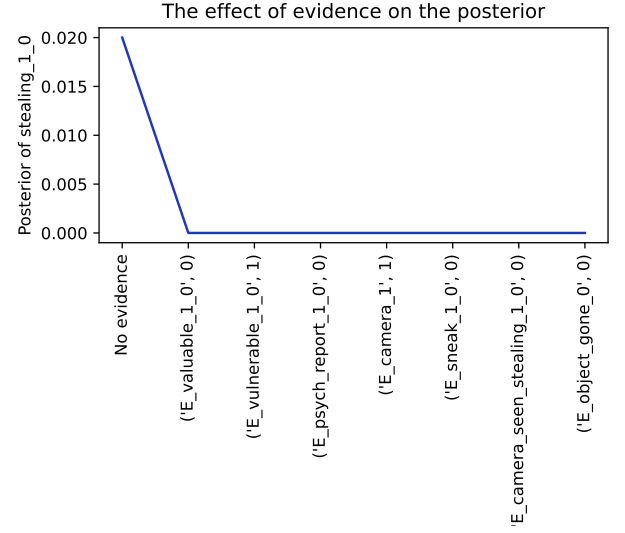2. **We can know when the BN predicts the wrong outcome**

4.5 Discussion

4.5.1 Problems

1. **Does the BN not depend on private knowledge?**

2. **Could we apply this BN to a different location?**
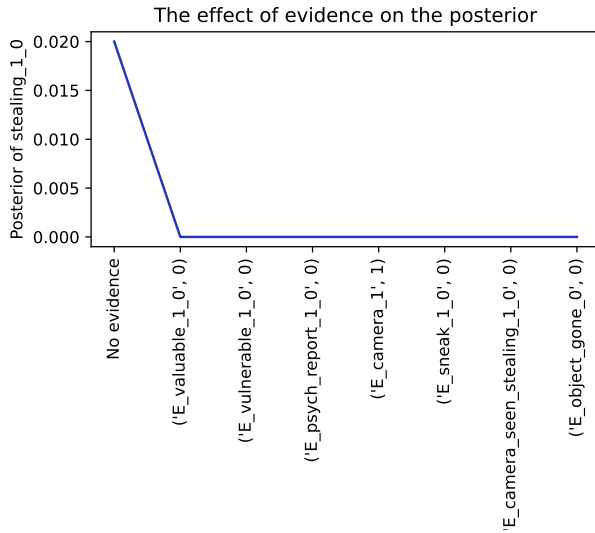
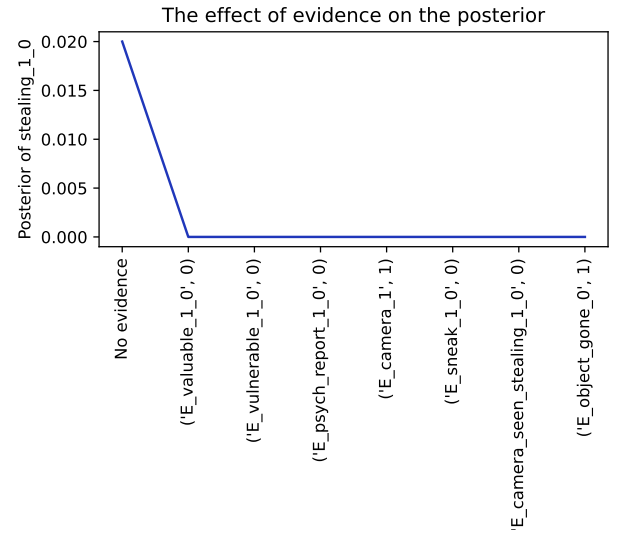3. **Identity and guilt?**

4.5.2 Legal Interpretation
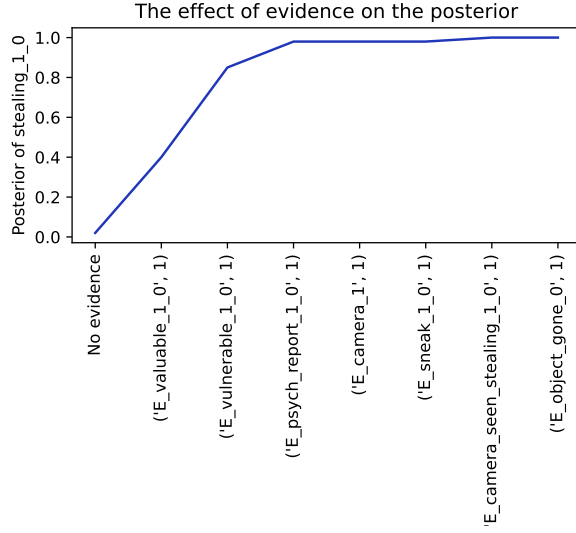
(a) All evidence is true.

(b) Agent only has assessed vulnerable.

(c) Agent was only seen on camera.

(d) Agent was seen on camera and object is gone.

Figure 4.4: Effect of evidence on posterior.

Chapter 5

Conclusion

Our research questions, and answers:

- **Can we create Bayesian Networks that correctly reflect multi-agent simulations of crimes?**

  We can create Bayesian Networks that correctly reflect multi-agent simulations of crimes. For different types of simulations, the Bayesian Networks that were generated using the pipeline had high accuracy, low RMSE, generally fulfilled the structural criteria we set out. The probabilities in these networks reflected the probabilities we found in the simulation.

  The networks were even robust under imprecision, which means that even though experts might disagree on elicited probability values, or there are imprecisions in the measuring tools, the performance of the network and the conclusion that it gets to in the end, might remain correct.

  We can also show how we should update our posterior probability of an event given a set of evidence and hence show that different pieces of evidence have different strengths. The effect of evidence on the posterior lines up with the evidence as reported in the simulation, as well as with modeller's intuition about the evidence. The visualisation of the posterior probability of the outcome node is a useful interpretative tool for reasoning. Different sets of evidence can also be compared with each other, so that we could compare the effect of different evidence and different scenarios on the posterior probability of the ultimate output node.

  However, the networks that we generated using the pipelines did have some features that are hard on modellers, some nodes had too many parents to easily fill a cpt. We might want to trade-off high network scores for user ease, and restrict the number of

parents that a node can have. Additionally, the probabilities that were used in the network would be difficult for a human modeller to elicit or discover. However, since we have our robustness to imprecision, this might not be a large practical problem. Additionally, the temporal ordering of the nodes is not correct when two alternative scenarios are represented within one network.

- **If we can, under what assumptions do these networks function?**

  The assumptions under which the networks are correct, are too strict for this approach to be applied one-on-one to the real world. There are three problems that were identified in this work. These are the meaning of the reporters, private knowledge, and implicit conditioning. These problems are all abundantly present in the real world and it is not clear how to we should fix them. This means that Bayesian Networks might remain a good tool for seeing how evidence affects hypotheses, but might not be ready for the real world.

Reporters and random variables

Fundamentally, the problem is that the nodes in Bayesian Networks are random variables. They are not pragmatic, dialogical, argumentation, or logical sentences, but mathematical objects. They are random variables, and a random variable implies an observation procedure that maps a world state to a truth value. This means, that a node implies that we know how to measure if it is true or not in the real world.

In our simulation, this is really not a problem. We have an observation procedure: if a certain state occurs, we have the reporter in the same place as when the state change occurs, and the reporter reports exactly and only that. Hence, Bayesian Networks might work for subsections of reality that have a clear observation procedure. For example, reasoning with DNA evidence or other valid forensic evidence. In these cases, we know exactly what it means for the node to be true, since we know the measurement associated with the random variable.

However, for many of the events that we encounter in our simulation, or in Bayesian Networks in the state of the art, we do not know how we are determining that the node is true or not. We do not know which operationalisation is used to determine the truth value, and if that operationalisation is correct or not. This information has to be included with the networks, otherwise we cannot interpret what a node actually means.

Private knowledge

This problem is very clear: in our simulation, we know when private information is true or false, even if we have no way of knowing this private knowledge in real life.

If we remove this knowledge from our network, we find that this effects the posterior probability of the outcome node to such an extent that we fall below a 'comfortable' threshold of guilt. This implies that we can only generate a network that correctly and legally reasons about evidence if everyone in the process would help. This is not feasible.

Conditioning on implicit parameters

We find that if we change the parameters of the simulations, such as using a different map, the probabilities for events change. It means that there is outside influence - things that are not nodes, can influence the cpt's of the network, as if there is an invisible 'environment' node that is the parent of all the nodes in the network. This means that if we see a Bayesian Network and we do not know exactly the context in which it was created, we cannot assume that it generalises to a different environment, even if on the face of it, the nodes would be the same. We cannot take parts of that network (with probabilities) and put it in a different one, because the Bayesian Network is implicitly defined over the environment for which it was originally created, and not any other.

5.1   Future Work

It is difficult to assess the human criteria for Bayesian Networks. We need formal methods to investigate whether a person could really 'think' of a dependence relationship between two nodes, or estimate the correct probabilities. The ad-hoc, subjective way that the human criteria were assessed in this thesis are not very satisfactory. New standards need to be invented that correspond to objective facts about people's ability to estimate probability and investigate conditional relations between nodes.

The simulations as tested here are still very simplified - they are only appropriate as a baseline for testing Bayesian Networks, and for nothing else. Testing Bayesian Networks on more complex simulations would likely result in a reduced accuracy due to inherent and irreducible uncertainty, and lay bare many more problems that do not emerge from these simple simulations. Hence, testing on more complex models is necessary to fully understand the limitations of Bayesian Networks.

The values of ϵ can be reduced from 0.01 to smaller probabilities, to discover how this would influence accuracy and precision of networks, especially in situations that are more complex than the simulations we used here. If there are inherent features to the simulation that makes the frequency of some event smaller than 0.01, it is important to know how that would interact with an ϵ of 0.01. Hence, this should be investigated further.

As the K2 algorithm used depends on a given node ordering, the node-ordering algorithm

used in this project resulted generally in successfully replicating the temporal structure of scenarios, however the merging of scenarios caused problems, and resulted in a missing temporal structure. This can be fixed relatively easily, by taking the average time-step at which an event occurs into account, instead of only the ordering relative to other events.

While we discussed the problem of operationalisation in the discussion sections of these chapters, this was not investigated directly. This makes it unclear how big the effect of inefficient, insufficient or invalid operationalisation on the structural, performance and human criteria of the network would be. This problem can be investigated in simulation in simple ways - for instance, by changing the accuracy of reporting, such that a reporter might sometimes report an event wrongly. Alternative operationalisations for the 'same' random variables or events should also be investigated. Investigating the effects of non-ideal operationalisation is essential for knowing if we can generalise our Bayesian Networks beyond the bounds of simulation.

Testing or generating Bayesian Network idioms from simulations. The dream is to get "plug and play" Bayesian Network idioms - preconnected structures, perhaps even with some probabilities attached that you can add evidence to and adapt and combine if necessary. Using simulations, we can test the granularity of these possible idioms, to simulate a crime at larger and smaller resolution (more or fewer events) to see how well the idioms can capture it.

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