M showers, $m = 1...M$, are generated in a given energy bin, and the response evaluated for N_G grid cells tiling the detection plane. For shower m, $N_{S,m}$ cells are selected as candidates based on loose criteria (essentially, number of photons in field of view). Based on the full simulation, a fraction ϵ_m of these candidates passes final selection criteria (e.g. trigger, or trigger plus reconstruction plus selection cuts plus angular cuts, depending on the quantity under study); ϵ_m is derived using a detailed simulation of the instrument. The effective area for the given energy bin is then

$$A_{eff} = A_G \frac{\sum\limits_m \epsilon_m N_{S,m}}{\sum\limits_m N_G} = A_G \frac{\sum\limits_m \epsilon_m N_{S,m}}{M N_G}$$

where A_G is the total grid area tiled by N_G grid cells. Here, ϵ_m could be evaluated as

$$\epsilon_m = \frac{N_{A,m}}{N_{S,m}}$$

where $N_{A,m}$ is the number of candidate cells that pass final selection criteria, for shower m. This results in the familiar expression

$$A_{eff} = A_G \frac{\sum\limits_m N_{A,m}}{M N_G}$$

The difficulty with this definition of ϵ_m is the evaluation of its statistical error, since a given shower is used $N_{A,m}$ times and intensities in different grid cells will be strongly correlated. Conservatively assuming 100% correlation of candidate cells in a given shower, the relative error of A_{eff} is estimated as

$$\frac{\Delta A_{eff}}{A_{eff}} = \frac{\sqrt{\sum\limits_m N_{A,m}^2}}{\sum\limits_m N_{A,m}}$$

Rather than carrying out the time-consuming full simulation for $N_{S,m}$ cells and later effectively counting those (regarding their statistical weight) as one event, one can estimate ϵ_m by randomly selecting one of $N_{S,m}$ cells and test if that cell is accepted ($q_m = 1$) or rejected ($q_m = 0$), resulting in

$$A_{eff} = A_G \frac{\sum\limits_m q_m N_{S,m}}{M N_G} = A_G \frac{\sum\limits_{acc.} N_{S,m}}{M N_G}$$

where in the second case the sum runs over accepted events only. (We note that q_m and $N_{S,m}$ may be correlated; showers with high $N_{S,m}$ could be more likely to have $q = 1$.) The statistical error of A_{eff} is then given by

$$\frac{\Delta A_{eff}}{A_{eff}} = \frac{\sqrt{\sum\limits_{acc.} N_{S,m}^2}}{\sum\limits_{acc.} N_{S,m}}$$

Obviously, results are only reliable in case $A_{eff} \ll A_G$ and $\epsilon_m \ll 1$; efficiency of the simulation, on the other hand, dictates that the loose criteria are chosen such that ϵ_m is not too small.

CORRECTED: For average values over many experiments with M showers each, one has

$$\langle \sum_m N_{A,m} \rangle \equiv \langle \sum_m \epsilon_m N_{S,m} \rangle = \langle \sum_m q_m N_{S,m} \rangle \equiv \langle \sum_{acc.} N_{S,m} \rangle$$

Assuming that ϵ_m is relatively constant, $\epsilon_m \approx \epsilon \ll 1$

$$\langle \sum_m N_{A,m}^2 \rangle \equiv \langle \sum_m \epsilon_m^2 N_{S,m}^2 \rangle \approx \epsilon^2 \langle \sum_m N_{S,m}^2 \rangle$$

and

$$\frac{1}{M} \sum_m q_m \approx \epsilon$$

which implies that only approx. ϵ of the q_m are different from zero. Neglecting furthermore a possible correlation between q_m and $N_{S,m}$, one finds

$$\langle \sum_{acc.} N_{S,m}^2 \rangle \equiv \langle \sum_m q_m N_{S,m}^2 \rangle \approx \epsilon \langle \sum_m N_{S,m}^2 \rangle \approx \frac{1}{\epsilon} \langle \sum_m N_{A,m}^2 \rangle$$

and hence

$$\langle \frac{\sqrt{\sum_m N_{A,m}^2}}{\sum_m N_{A,m}} \rangle \approx \epsilon^{1/2} \ \langle \frac{\sqrt{\sum_{acc.} N_{S,m}^2}}{\sum_{acc.} N_{S,m}} \rangle$$

It is therefore not evident that the second method is neccessarily more efficient; for a given number M of showers, the statistical error of the second method is $1/\epsilon^{1/2}$ larger, hence requiring $1/\epsilon$ more showers, but saving a factor $\langle N_S \rangle$ in instrument simulations per shower.