Answer the following as indicated and in the given order. **SHOW YOUR SOLUTIONS.**

1. Answer the following questions. (20 Points)
   (a) If $a \in G$, where G is a group, $1 \le o(a) \le$ _____.
   (b) Is $G = \{1, -1, i, -i\}$ a subgroup of $\mathbb{C}^*$ under multiplication? Justify your answer.
   (c) If $a \in G$, where G is a group, $a^{-n} =$ _____.
   (d) Find $o(23)$ in $(\mathbb{Z}, +)$.
   (e) How many subgroups does $(\mathbb{Z}_5, +)$ have? Justify your answer.
   (f) If $H < G$ then _____.
   (g) The order of the symmetric group of order n is _____.
   (h) Find the intersection of all subgroups $C(g)$, g is an element of the group G.
   (i) If $Z(G) = G$ then _____.
   (j) Given $(\mathbb{Z}_6, +), 2^{453672} =$ _____.

2. Let

$$\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 6 & 8 & 1 & 2 & 3 & 5 & 7 \end{pmatrix}$$

$$\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 3 & 1 & 4 & 6 & 5 & 7 & 8 \end{pmatrix}$$

   Answer the following questions. (12 Points)
   (a) Write α in cycle notation.
   (b) Write β^{-1} in cycle notation.
   (c) Find $\beta\alpha$.
   (d) Write α as a product of transpositions.

3. Determine the order of the given elements of the given groups. Show your solution. (10 Points)
   (a) $G = (\mathbb{Z}_{10}, +), a = 3, 4$
   (b) $G = (\mathbb{Z}_7, \bullet), a = 2$
   (c) G is the group in the following number, $a = x^{-1}, yz$

4. Given the Cayley table below for a group $(G, *)$, (20 Points)

$*$	e	r	s	x	y	z
e	e	r	s	x	y	z
r	r	s	e	y	z	x
s	s	e	r	z	x	y
x	x	z	y	e	s	r
y	y	x	z	r	e	s
z	z	y	x	s	r	e

   (a) Is the group cyclic? Justify your answer.
   (b) Find $C(a)$ for all $a \in G$.
   (c) Find $Z(G)$.
   (d) Draw the subgroup lattice of the given group.

5. Use the two-step test to show that $H = \{12x + 21y \mid x,\ y \in \mathbb{Z}\}$ is a subgroup of $(\mathbb{Z}, +)$. (8 Points)

6. Show that every cyclic group is abelian. (5 Points)

BONUS. Show that all integers divisible by 2 is a subgroup of $(\mathbb{Z}, +)$. (5 Points)