Contents

Chapter 1

Philosopherers

(1.5)
$\color{red}{OrderTrichotomy}(<, S) := \forall_{x,y\in S}(x < y \veebar x = y \veebar y < x)$
$\color{red}{OrderTransitivity}(<, S) := \forall_{x,y,z\in S}\big((x < y \wedge y < z) \implies x < z\big)$
$\color{red}{Order}(<, S) := OrderTrichotomy(<, S) \wedge OrderTransitivity(<, S)$

(1.7)
$\color{red}{BoundedAbove}(E, S, <) := Order(<, S) \wedge E \subset S \wedge \exists_{\beta\in S}\forall_{x\in E}(x \leq \beta)$
$\color{red}{BoundedBelow}(E, S, <) := Order(<, S) \wedge E \subset S \wedge \exists_{\beta\in S}\forall_{x\in E}(\beta \leq x)$
$\color{red}{UpperBound}(\beta, E, S, <) := Order(<, S) \wedge E \subset S \wedge \beta \in S \wedge \forall_{x\in E}(x \leq \beta)$
$\color{red}{LowerBound}(\beta, E, S, <) := Order(<, S) \wedge E \subset S \wedge \beta \in S \wedge \forall_{x\in E}(\beta \leq x)$

(1.8)
$\color{red}{LUB}(\alpha, E, S, <) := UpperBound(\alpha, E, S, <) \wedge \forall_{\gamma}\big(\gamma < \alpha \implies \neg UpperBound(\gamma, E, S, <)\big)$
$\color{red}{GLB}(\alpha, E, S, <) := LowerBound(\alpha, E, S, <) \wedge \forall_{\beta}\big(\alpha < \beta \implies \neg LowerBound(\beta, E, S, <)\big)$

(1.10)
$\color{red}{LUBProperty}(S, <) := \forall_{E}\Big(\big(\emptyset \neq E \subset S \wedge BoundedAbove(E, S, <)\big) \implies \exists_{\alpha\in S}\big(LUB(\alpha, E, S, <)\big)\Big)$
$\color{red}{GLBProperty}(S, <) := \forall_{E}\Big(\big(\emptyset \neq E \subset S \wedge BoundedBelow(E, S, <)\big) \implies \exists_{\alpha\in S}\big(GLB(\alpha, E, S, <)\big)\Big)$

(1.11)
$\boxed{\text{LUBPropertyImpliesGLBProperty}}$ $LUBProperty(S, <) \implies GLBProperty(S, <)$

(1) $LUBProperty(S, <) \implies \ldots$ wts: 2

(1.1) $\big(\emptyset \neq B \subset S \wedge BoundedBelow(B, S, <)\big) \implies \ldots$ wts: 1.2

(1.1.1) $Order(<, S) \wedge \exists_{\delta'\in S}\big(LowerBound(\delta', B, S, <)\big)$ from: BoundedBelow, 1.1

(1.1.2) $|B| = 1 \implies \ldots$ wts: 1.1.3

(1.1.2.1) $\exists_{u'}(u' \in B) \blacksquare u := choice(\{u' | u' \in B\}) \blacksquare B = \{u\}$ from: 1.1.2

(1.1.2.2) $GLB(u, B, S, <) \blacksquare \exists_{\epsilon_0\in S}\big(GLB(\epsilon_0, B, S, <)\big)$

(1.1.3) $|B| = 1 \implies \exists_{\epsilon_0\in S}\big(GLB(\epsilon_0, B, S, <)\big)$

(1.1.4) $|B| \neq 1 \implies \ldots$ wts: 1.1.5

(1.1.4.1) $\forall_{E}\Big(\big(\emptyset \neq E \subset S \wedge BoundedAbove(E, S, <)\big) \implies \exists_{\alpha\in S}\big(LUB(\alpha, E, S, <)\big)\Big)$ from: LUBProperty, 1

(1.1.4.2) $L := \{s \in S | LowerBound(s, B, S, <)\}$

(1.1.4.3) $|B| > 1 \wedge OrderTrichotomy(<, S) \blacksquare \exists_{b_1{}'\in B}\exists_{b_0{}'\in B}(b_0{}' < b_1{}')$ from: Order, 1.1.1 wts: 1.1.4.7

(1.1.4.4) $b_1 := choice\Big(\{b_1{}' \in B | \exists_{b_0{}'\in B}(b_0{}' < b_1{}')\}\Big) \blacksquare \neg LowerBound(b_1, B, S, <)$ from: 1.1.4.2

(1.1.4.5) $b_1 \notin L \blacksquare L \subset S$

(1.1.4.6) $\delta := choice\big(\{\delta' \in S | LowerBound(\delta', B, S, <)\}\big) \blacksquare \delta \in L \blacksquare \emptyset \neq L$ from: 1.1.1

(1.1.4.7) $\emptyset \neq L \subset S$ from: 1.1.4.5, 1.1.4.6

(1.1.4.8) $\forall_{y\in L}\big(LowerBound(y,B,S,<)\big)$ ▌ $\forall_{y\in L}\forall_{x\in B}(y\leq x)$ from: LowerBound, 1.1.4.2 wts: 1.1.4.10

(1.1.4.9) $\forall_{x\in B}\Big(x\in S\wedge\forall_{y\in L}(y\leq x)\Big)$ ▌ $\forall_{x\in B}\big(UpperBound(x,L,S,<)\big)$ from: UpperBound

(1.1.4.10) $\exists_{x\in S}\big(UpperBound(x,L,S,<)\big)$ ▌ $BoundedAbove(L,S,<)$

(1.1.4.11) $\emptyset\neq L\subset S\wedge BoundedAbove(L,S,<)$ from: 1.1.4.7, 1.1.4.10

(1.1.4.12) $\exists_{\alpha'\in S}\big(LUB(\alpha',L,S,<)\big)$ ▌ $\alpha:=choice\Big(\{\alpha'\in S|\big(LUB(\alpha',L,S,<)\big)\}\Big)$ from: 1.1.4.1 wts: 1.1.4.21

(1.1.4.13) $\forall_x\big(x\in B\implies UpperBound(x,L,S,<)\big)$ from: 1.1.4.9 wts: 1.1.4.17

(1.1.4.14) $\forall_x\big(\neg UpperBound(x,L,S,<)\implies x\notin B\big)$

(1.1.4.15) $\gamma<\alpha\implies\dots$ wts: 1.1.4.16

(1.1.4.15.1) $\neg UpperBound(\gamma,L,S,<)$ ▌ $\gamma\notin B$ from: LUB, 1.1.4.12, 1.1.4.14

(1.1.4.16) $\gamma<\alpha\implies\gamma\notin B$ ▌ $\gamma\in B\implies\gamma\geq\alpha$

(1.1.4.17) $\forall_{\gamma\in B}(\alpha\leq\gamma)$ ▌ $LowerBound(\alpha,B,S,<)$ from: LowerBound

(1.1.4.18) $\alpha<\beta\implies\dots$ wts: 1.1.4.19

(1.1.4.18.1) $\forall_{y\in L}(y\leq\alpha<\beta)$ ▌ $\forall_{y\in L}(y\neq\beta)$ from: LUB, 1.1.4.12, 1.1.4.18

(1.1.4.18.2) $\beta\notin L$ ▌ $\neg LowerBound(\beta,B,S,<)$ from: 1.1.4.2

(1.1.4.19) $\alpha<\beta\implies\neg LowerBound(\beta,B,S,<)$ ▌ $\forall_{\beta\in S}\big(\alpha<\beta\implies\neg LowerBound(\beta,B,S,<)\big)$

(1.1.4.20) $LowerBound(\alpha,B,S,<)\wedge\forall_{\beta\in S}\big(\alpha<\beta\implies\neg LowerBound(\beta,B,S,<)\big)$ from: 1.1.4.17, 1.1.4.19

(1.1.4.21) $GLB(\alpha,B,S,<)$ ▌ $\exists_{\epsilon_1\in S}\big(GLB(\epsilon_1,B,S,<)\big)$

(1.1.5) $|B|\neq 1\implies\exists_{\epsilon_1\in S}\big(GLB(\epsilon_1,B,S,<)\big)$

(1.1.6) $\Big(|B|=1\implies\exists_{\epsilon_0\in S}\big(GLB(\epsilon_0,B,S,<)\big)\Big)\wedge\Big(|B|\neq 1\implies\exists_{\epsilon_1\in S}\big(GLB(\epsilon_1,B,S,<)\big)\Big)$ from: 1.1.3, 1.1.5

(1.1.7) $(|B|=1\vee|B|\neq 1)\implies\exists_{\epsilon\in S}\big(GLB(\epsilon,B,S,<)\big)$ ▌ $\exists_{\epsilon\in S}\big(GLB(\epsilon,B,S,<)\big)$

(1.2) $\big(\emptyset\neq B\subset S\wedge BoundedBelow(B,S,<)\big)\implies\exists_{\epsilon\in S}\big(GLB(\epsilon,B,S,<)\big)$

(1.3) $\forall_B\Big(\big(\emptyset\neq B\subset S\wedge BoundedBelow(B,S,<)\big)\implies\exists_{\epsilon\in S}\big(GLB(\epsilon,B,S,<)\big)\Big)$

(1.4) $GLBProperty(S,<)$

(2) $LUBProperty(S,<)\implies GLBProperty(S,<)$

(1.12)

$$Field(F,+,*):=\exists_{0,1\in F}\forall_{x,y,z\in F}\begin{pmatrix} x+y\in F \quad\wedge\quad x*y\in F \quad\wedge\\ x+y=y+x \quad\wedge\quad x*y=y*x \quad\wedge\\ (x+y)+z=x+(y+z) \quad\wedge\quad (x*y)*z=x*(y*z) \quad\wedge\\ \mathbb{1}\neq\mathbb{0} \quad\wedge\quad x*(y+z)=(x*y)+(x*z) \quad\wedge\\ \mathbb{0}+x=x \quad\wedge\quad \mathbb{1}*x=x \quad\wedge\\ \exists_{-x\in F}\big(x+(-x)=\mathbb{0}\big)\wedge\Big(x\neq\mathbb{0}\implies\exists_{1/x\in F}\big(x*(1/x)=\mathbb{1}\big)\Big)\end{pmatrix}$$

********************** $\big(Field(F,+,*)\wedge x,y,z\in F\big)\implies\dots$ **********************

(1.14)

AdditiveCancellation $(x+y=x+z)\implies y=z$

(1) $y=\mathbb{0}+y=\big(x+(-x)\big)+y=\big((-x)+x\big)+y=(-x)+(x+y)=\dots$ from: Field

(2) $(-x)+(x+z)=\big((-x)+x\big)+z=\big(x+(-x)\big)+z=\mathbb{0}+z=z$ from: Field

AdditiveIdentityUniqueness $(x+y=x)\implies y=\mathbb{0}$

(1) $x+y=x=\mathbb{0}+x=x+\mathbb{0}$ from: Field

(2) $y=0$ from: AdditiveCancellation

$\boxed{\text{AdditiveInverseUniqueness}}$ $(x + y = \mathbb{0}) \implies y = -x$

(1) $x + y = \mathbb{0} = x + (-x)$ from: Field

(2) $y = -x$ from: AdditiveCancellation

$\boxed{\text{DoubleNegative}}$ $x = -(-x)$

(1) $\mathbb{0} = x + (-x) = (-x) + x \;\blacksquare\; \mathbb{0} = (-x) + x$ from: Field

(2) $x = -(-x)$ from: AdditiveInverseUniqueness

(1.15)
$\boxed{\text{MultiplicativeCancellation}}$ $(x \neq 0 \land x * y = x * z) \implies y = z$ —

$\boxed{\text{MultiplicativeIdentityUniqueness}}$ $(x \neq 0 \land x * y = x) \implies y = \mathbb{1}$ —

$\boxed{\text{MultiplicativeInverseUniqueness}}$ $(x \neq 0 \land x * y = \mathbb{1}) \implies y = 1/x$ —

$\boxed{\text{DoubleReciprocal}}$ $(x \neq 0) \implies x = 1/(1/x)$ —

(1.16)
$\boxed{\text{Domination}}$ $\mathbb{0} * x = \mathbb{0}$

(1) $\mathbb{0} * x = (\mathbb{0} + \mathbb{0}) * x = \mathbb{0} * x + \mathbb{0} * x \;\blacksquare\; \mathbb{0} * x = \mathbb{0} * x + \mathbb{0} * x$ from: Field

(2) $\mathbb{0} * x = \mathbb{0}$ from: AdditiveIdentityUniqueness

$\boxed{\text{NonDomination}}$ $(x \neq \mathbb{0} \land y \neq \mathbb{0}) \implies x * y \neq \mathbb{0}$

(1) $(x \neq \mathbb{0} \land y \neq \mathbb{0}) \implies \ldots$

(1.1) $(x * y = 0) \implies \ldots$

(1.1.1) $\mathbb{1} = \mathbb{1} * \mathbb{1} = \big(x * (1/x)\big) * \big(y * (1/y)\big) = (x * y) * \big((1/x) * (1/y)\big) = \mathbb{0} * \big((1/x) * (1/y)\big) = \mathbb{0}$ from: Field, Domination, 1, 1.1

(1.1.2) $\mathbb{1} = \mathbb{0} \land \mathbb{1} \neq \mathbb{0} \;\blacksquare\; \bot$ from: Field

(1.2) $(x * y = 0) \implies \bot \;\blacksquare\; x * y \neq 0$

(2) $(x \neq \mathbb{0} \land y \neq \mathbb{0}) \implies x * y \neq 0$

$\boxed{\text{NegationCommutativity}}$ $(-x) * y = -(x * y) = x * (-y)$

(1) $x * y + (-x) * y = (x + -x) * y = \mathbb{0} * y = \mathbb{0} \;\blacksquare\; x * y + (-x) * y = \mathbb{0}$ from: Field, Domination wts: 2

(2) $(-x) * y = -(x * y)$ from: AdditiveInverseUniqueness

(3) $x * y + x * (-y) = x * (y + -y) = x * \mathbb{0} = \mathbb{0} \;\blacksquare\; x * y + x * (-y) = \mathbb{0}$ from: Field, Domination wts: 4

(4) $x * (-y) = -(x * y)$ from: AdditiveInverseUniqueness

(5) $(-x) * y = -(x * y) = x * (-y)$ from: 2, 4

$\boxed{\text{NegativeMultiplication}}$ $(-x) * (-y) = x * y$

(1) $(-x) * (-y) = -\big(x * (-y)\big) = -\big(-(x * y)\big) = x * y$ from: NegationCommutativity, DoubleNegative

(1.17)

$$OrderedField(F, +, *, <) := \begin{pmatrix} Field(F, +, *) \quad \land \quad Order(<, F) \quad \land \\ \forall_{x,y,z \in F}(y < z \implies x + y < x + z) \quad \land \\ \forall_{x,y \in F}\big((x > \mathbb{0} \land y > \mathbb{0}) \implies x * y > \mathbb{0}\big) \end{pmatrix}$$

************************ $\big(OrderedField(F, +, *, <) \land x, y, z \in F\big) \implies \ldots$ ************************

(1.18)
$\boxed{\text{NegationOnOrder}}$ $x > \mathbb{0} \iff -x < \mathbb{0}$

(1) $x > \mathbb{0} \implies \dots$

(1.1) $\mathbb{0} = (-x) + x > (-x) + \mathbb{0} = -x$ ▌ $\mathbb{0} > -x$ ▌ $-x < \mathbb{0}$ — from: OrderedField

(2) $x > \mathbb{0} \implies -x < \mathbb{0}$

(3) $-x < \mathbb{0} \implies \dots$

(3.1) $\mathbb{0} = x + (-x) < x + \mathbb{0} = x$ ▌ $\mathbb{0} < x$ ▌ $x > \mathbb{0}$ — from: OrderedField

(4) $-x < \mathbb{0} \implies x > \mathbb{0}$

(5) $x > \mathbb{0} \implies -x < \mathbb{0} \wedge -x < \mathbb{0} \implies x > \mathbb{0}$ ▌ $x > \mathbb{0} \iff -x < \mathbb{0}$ — from: 2, 4

PositiveFactorPreservesOrder $(x > \mathbb{0} \wedge y < z) \implies x * y < x * z$

(1) $(x > \mathbb{0} \wedge y < z) \implies \dots$

(1.1) $(-y) + z > (-y) + y = \mathbb{0}$ ▌ $z + (-y) = \mathbb{0}$ — from: OrderedField

(1.2) $x * (z + (-y)) > \mathbb{0}$ ▌ $x * z + x * (-y) > \mathbb{0}$ — from: OrderedField

(1.3) $x * z = \mathbb{0} + x * z = \big(x * y + -(x * y)\big) + x * z = \big(x * y + x * (-y)\big) + x * z = \dots$ — from: Field, NegationCommutativity

(1.4) $x * y + \big(x * z + x * (-y)\big) > x * y + \mathbb{0} = x * y$ — from: Field, 1.2

(1.5) $x * z > x * y$ — from: 1.3, 1.4

(2) $(x > \mathbb{0} \wedge y < z) \implies x * z > x * y$

NegativeFactorFlipsOrder $(x < \mathbb{0} \wedge y < z) \implies x * y > x * z$

(1) $(x < \mathbb{0} \wedge y < z) \implies \dots$

(1.1) $-x > \mathbb{0}$ — from: NegationOnOrder

(1.2) $(-x) * y < (-x) * z$ ▌ $\mathbb{0} = x * y + (-x) * y < x * y + (-x) * z$ ▌ $\mathbb{0} < x * y + (-x) * z$ — from: PositiveFactorPreservesOrder

(1.3) $\mathbb{0} < (-x) * (-y + z)$ ▌ $\mathbb{0} > x * (-y + z)$ ▌ $\mathbb{0} > -(x * y) + x * z$ — from: NegationOnOrder

(1.4) $x * y > x * z$

(2) $(x < \mathbb{0} \wedge y < z) \implies x * y > x * z$

SquareIsPositive $(x \neq \mathbb{0}) \implies x * x > \mathbb{0}$

(1) $(x > \mathbb{0}) \implies x * x > 0$ — from: OrderedField

(2) $(x < \mathbb{0}) \implies \dots$

(2.1) $-x > \mathbb{0}$ ▌ $x * x = (-x) * (-x) > \mathbb{0}$ ▌ $x * x > \mathbb{0}$ — from: NegationOnOrder, OrderedField, NegativeMultiplication

(3) $(x < \mathbb{0}) \implies x * x > \mathbb{0}$

(4) $x \neq \mathbb{0} \implies (x > \mathbb{0} \vee x < \mathbb{0}) \implies x * x > \mathbb{0}$ ▌ $x \neq \mathbb{0} \implies x * x > \mathbb{0}$ — from: OrderTrichotomy, 1, 3

OneIsPositive $\mathbb{1} > \mathbb{0}$

(1) $\mathbb{1} \neq \mathbb{0}$ ▌ $\mathbb{1} = \mathbb{1} * \mathbb{1} > \mathbb{0}$ — from: Field, SquareIsPositive

ReciprocationOnOrder $(0 < x < y) \implies 0 < 1/y < 1/x$

(1) $(0 < x < y) \implies \dots$

(1.1) $x * (1/x) = \mathbb{1} > \mathbb{0}$ ▌ $x * (1/x) > 0$ — from: Field, OneIsPositive

(1.2) $1/x < \mathbb{0} \implies x * (1/x) < 0 \wedge x * (1/x) > 0 \implies \perp$ ▌ $1/x > \mathbb{0}$ — from: NegativeFactorFlipsOrder, 1

(1.3) $y * (1/y) = \mathbb{1} > \mathbb{0}$ ▌ $y * (1/y) > 0$ — from: Field, OneIsPositive

(1.4) $1/y < \mathbb{0} \implies y * (1/y) < 0 \wedge y * (1/y) > 0 \implies \perp$ ▌ $1/y > \mathbb{0}$ — from: NegativeFactorFlipsOrder, 1

(1.5) $(1/x) * (1/y) > \mathbb{0}$ — from: OrderedField

(1.6) $\mathbb{0} < 1/y = \big((1/x) * (1/y)\big) * x < \big((1/x) * (1/y)\big) * y = 1/x$ — from: OrderedField, 1, 1.4, 1.5

(1.19)
FieldQ $OrderedField(\mathbb{Q}, +, *, <)$ —

$Subfield(K, F, +, *) := Field(F, +, *) \wedge K \subset F \wedge Field(K, +, *)$
$OrderedSubfield(K, F, +, *, <) := OrderedField(F, +, *, <) \wedge K \subset F \wedge OrderedField(K, +, *, <)$
$CutI(\alpha) := \emptyset \neq \alpha \subset \mathbb{Q}$
$CutII(\alpha) := \forall_{p \in \alpha} \forall_{q \in \mathbb{Q}} (q < p \implies q \in \alpha)$
$CutIII(\alpha) := \forall_{p \in \alpha} \exists_{r \in \alpha} (p < r)$
$SetR := \mathbb{R} := \{\alpha \in \mathbb{Q} | CutI(\alpha) \wedge CutII(\alpha) \wedge CutIII(\alpha)\}$

CutCorollaryI $(\alpha \in \mathbb{R} \wedge p \in \alpha \wedge q \in \mathbb{Q} \wedge q \notin \alpha) \implies p < q$

(1) $(\alpha \in \mathbb{R} \wedge p \in \alpha \wedge q \in \mathbb{Q} \wedge q \notin \alpha) \implies \ldots$
(1.1) $\forall_{p' \in \alpha} \forall_{q' \in \mathbb{Q}} (q' < p' \implies q' \in \alpha)$ from: CutII, 1
(1.2) $q < p \implies q \in \alpha$ ▌ $q \notin \alpha \implies q \geq p$ from: 1
(1.3) $(q \notin \alpha) \implies \ldots$
(1.3.1) $q \geq p$ from: 1.2
(1.3.2) $(q = p) \implies (p \in \alpha \wedge p \notin \alpha) \implies \perp$ ▌ $q \neq p$ from: 1, 1.3
(1.3.3) $q \geq p \wedge q \neq p$ ▌ $p < q$
(1.4) $q \notin \alpha \implies p < q$ ▌ $p < q$ from: 1
(2) $(\alpha \in \mathbb{R} \wedge p \in \alpha \wedge q \in \mathbb{Q} \wedge q \notin \alpha) \implies p < q$

CutCorollaryII $(\alpha \in \mathbb{R} \wedge r, s \in \mathbb{Q} \wedge r < s \wedge r \notin \alpha) \implies s \notin \alpha$

(1) $(\alpha \in \mathbb{R} \wedge r, s \in \mathbb{Q} \wedge r < s \wedge r \notin \alpha) \implies \ldots$
(1.1) $\forall_{s' \in \alpha} \forall_{r' \in \mathbb{Q}} (r' < s' \implies r' \in \alpha)$ from: CutII, 1
(1.2) $s \in \alpha \implies \big(r \in \mathbb{Q} \implies (r < s \implies r \in \alpha)\big)$ ▌ $s \in \alpha \implies r \in \alpha$ from: 1, 1.1
(1.3) $r \notin \alpha \implies s \notin \alpha$ ▌ $s \notin \alpha$ from: 1, 1.2
(2) $(\alpha \in \mathbb{R} \wedge r, s \in \mathbb{Q} \wedge r < s \wedge r \notin \alpha) \implies s \notin \alpha$

$<R(\alpha, \beta) := \alpha, \beta \in \mathbb{R} \wedge \alpha \subset \beta$

OrderTrichotomyR $OrderTrichotomy(\mathbb{R}, <R)$

(1) $(\alpha, \beta \in \mathbb{R}) \implies \ldots$
(1.1) $\neg(\alpha < R\beta \vee \alpha = \beta) \implies \ldots$
(1.1.1) $\alpha \not\subset \beta \wedge \alpha \neq \beta$ from: <R, 1.1
(1.2) $\neg(\alpha < R\beta \vee \alpha = \beta) \implies \beta < R\alpha$
(2) $(\alpha, \beta \in \mathbb{R}) \implies (\alpha < R\beta \veebar \alpha = \beta \veebar \alpha < R\beta)$
(3) $\forall_{\alpha, \beta \in \mathbb{R}} (\alpha < R\beta \veebar \alpha = \beta \veebar \alpha < R\beta)$
(4) $OrderTrichotomy(\mathbb{R}, <R)$

OrderTransitivityR $OrderTransitivity(\mathbb{R}, <R)$

(1) $(\alpha, \beta, \gamma \in \mathbb{R}) \implies \ldots$
(1.1) 123123
(2) $(\alpha, \beta, \gamma \in \mathbb{R}) \implies \big((\alpha < R\beta \wedge \beta < R\gamma) \implies \alpha < R\gamma\big)$
(3) $\forall_{\alpha, \beta, \gamma \in \mathbb{R}} \big((\alpha < R\beta \wedge \beta < R\gamma) \implies \alpha < R\gamma\big)$
(4) $OrderTransitivity(\mathbb{R}, <R)$

OrderR $Order(<R, \mathbb{R})$ —

ExistenceOfR $\exists_{\mathbb{R}} \big(LUBProperty(\mathbb{R}, <) \wedge OrderedSubfield(\mathbb{Q}, \mathbb{R}, +, *, <)\big)$

(1) 123123

TODO: - name all properties - hyperlink all definitions ???

Chapter 2

First Chapter

(1) First
- (1.1) Second
- (1.2) Third

(2) Fourth

This will be an empty chapter and I will put some text here

$$\sum_{i=0}^{\infty} a_i x^i \tag{2.1}$$

The equation 2.1 shows a sum that is divergent. This formula will later be used in the page **??**.
For further references see Something Linky or go to the next url: http://www.sharelatex.com or open the next file File.txt
It's also possible to link directly any word or any sentence in your document. supwithitSup With It Theorem
If you read this text, you will get no information. Really? Is there no information?

For instance this sentence. supwithit