ABSTAL1 QUIZ 3 November 20, 2014

Answer the following as indicated and in the given order. **SHOW YOUR SOLUTIONS.**

1. Answer the following questions. (20 Points)
   (a) If $H \leq S_n$, find $|H|$.
   (b) $|A_{10}| =$ ______.
   (c) How many reflections are there in the dihedral group D_n?
   (d) Given D_8, find $a^4 b * a^6$.
   (e) When is a left coset gH equal to H?
   (f) Find the intersection of the right cosets Hg_1 and Hg_2.
   (g) Every group of prime order is ______.
   (h) When is a homomorphism ϕ one-to-one?
   (i) When is a homomorphism ϕ onto?
   (j) Given a homomorphism ϕ, $|Ker\ \phi| \cdot |\phi(G)| =$ ______.

2. Determine if the following permutations are odd or even. (5 Points)
   (a) $\alpha = (4\ 1\ 7)(9\ 6\ 5\ 8\ 2)$
   (b) $\beta = (2\ 1\ 8\ 7)(6\ 4\ 2\ 5)$.

3. Enumerate the elements of D_3 and write its Cayley table. (10 Points)

4. Given the Cayley table below for a group $(G, *)$, find the distinct left and right cosets of $< y >$. (8 Points)

$*$	e	r	s	x	y	z
e	e	r	s	x	y	z
r	r	s	e	y	z	x
s	s	e	r	z	x	y
x	x	z	y	e	s	r
y	y	x	z	r	e	s
z	z	y	x	s	r	e

5. From number 4, find a mapping ϕ from $(< r >, *) \to (\mathbb{Z}_2, +)$ such that ϕ is a homomorphism. Verify that the mapping you chose is a homomorphism. (7 Points)

6. In each of the following, determine if the given function ϕ is a group homomorphism. (10 Points)
   (a) Let $\phi : (\mathbb{R}, +) \to (\mathbb{R}, +)$ where $\phi(x) = x^2$.
   (b) Let $\phi : (P_2, +) \to (P_2, +)$ where $\phi(f(x)) = f'(x)$. (P_2 is the set of all polynomials with real coefficients of degree less than or equal to 2).
   (c) Let $\phi : (\mathbb{Z}_{12}, +) \to (\mathbb{Z}_3, +)$ where $\phi(g) = g \pmod 3$.

7. For the given mapping $\phi : (\mathbb{Z}_{12}, +) \to (\mathbb{Z}_{12}, +)$ where $\phi(g) = 3g$, do the following: (15 Points)
   (a) Verify that ϕ is a homomorphism.
   (b) Determine if ϕ is one-to-one using the definition of one-to-one.
   (c) Determine if ϕ is onto using the definition of onto.
   (d) Verify your answer in (b) by finding $Ker\ \phi$.
   (e) Verify your answer in (c) by finding $\phi(G)$.
   (f) Is ϕ an isomorphism? Justify your answer.

BONUS. Find a mapping ϕ from D_3 to $(G, *)$ in number 4 such that ϕ is a homomorphism. (5 Points)