Contents

Chapter 1

Philosopherers

$OrderTrichotomy(<, S) := \forall_{x,y \in S}(x < y \veebar x = y \veebar y < x)$
$OrderTransitivity(<, S) := \forall_{x,y,z \in S}\big((x < y \wedge y < z) \implies x < z\big)$
$Order(<, S) := OrderTrichotomy(<, S) \wedge OrderTransitivity(<, S)$
$BoundedAbove(E, S, <) := Order(<, S) \wedge E \subset S \wedge \exists_{\beta \in S}\forall_{x \in E}(x \leq \beta)$
$BoundedBelow(E, S, <) := Order(<, S) \wedge E \subset S \wedge \exists_{\beta \in S}\forall_{x \in E}(\beta \leq x)$
$UpperBound(\beta, E, S, <) := Order(<, S) \wedge E \subset S \wedge \beta \in S \wedge \forall_{x \in E}(x \leq \beta)$
$LowerBound(\beta, E, S, <) := Order(<, S) \wedge E \subset S \wedge \beta \in S \wedge \forall_{x \in E}(\beta \leq x)$
$LUB(\alpha, E, S, <) := UpperBound(\alpha, E, S, <) \wedge \forall_{\gamma}\big(\gamma < \alpha \implies \neg UpperBound(\gamma, E, S, <)\big)$
$GLB(\alpha, E, S, <) := LowerBound(\alpha, E, S, <) \wedge \forall_{\beta}\big(\alpha < \beta \implies \neg LowerBound(\beta, E, S, <)\big)$
$LUBProperty(S, <) := \forall_E\Big(\big(\emptyset \neq E \subset S \wedge BoundedAbove(E, S, <)\big) \implies \exists_{\alpha \in S}\big(LUB(\alpha, E, S, <)\big)\Big)$
$GLBProperty(S, <) := \forall_E\Big(\big(\emptyset \neq E \subset S \wedge BoundedBelow(E, S, <)\big) \implies \exists_{\alpha \in S}\big(GLB(\alpha, E, S, <)\big)\Big)$
((LUBPropertyImpliesGLBProperty)) $LUBProperty(S, <) \implies GLBProperty(S, <)$

(1)	$LUBProperty(S, <) \implies \ldots$	wts: 2
(1.1)	$\big(\emptyset \neq B \subset S \wedge BoundedBelow(B, S, <)\big) \implies \ldots$	wts: 1.2
(1.1.1)	$Order(<, S) \wedge \exists_{\delta' \in S}\big(LowerBound(\delta', B, S, <)\big)$	from: 1.1
(1.1.2)	$\lvert B \rvert = 1 \implies \ldots$	wts: 1.1.3
(1.1.2.1)	$\exists_{u'}(u' \in B) \;\blacksquare\; u := choice(\{u' \mid u' \in B\}) \;\blacksquare\; B = \{u\} \;\blacksquare\; GLB(u, B, S, <)$	from: 1.1.2
(1.1.2.2)	$\exists_{\epsilon_0 \in S}\big(GLB(\epsilon_0, B, S, <)\big)$	
(1.1.3)	$\lvert B \rvert = 1 \implies \exists_{\epsilon_0 \in S}\big(GLB(\epsilon_0, B, S, <)\big)$	
(1.1.4)	$\lvert B \rvert \neq 1 \implies \ldots$	wts: 1.1.5
(1.1.4.1)	$\forall_E\Big(\big(\emptyset \neq E \subset S \wedge BoundedAbove(E, S, <)\big) \implies \exists_{\alpha \in S}\big(LUB(\alpha, E, S, <)\big)\Big)$	from: 1
(1.1.4.2)	$L := \{s \in S \mid LowerBound(s, B, S, <)\}$	
(1.1.4.3)	$\lvert B \rvert > 1 \wedge OrderTrichotomy(<, S) \;\blacksquare\; \exists_{b_1' \in B}\exists_{b_0' \in B}(b_0' < b_1')$	from: 1.1.1 wts: 1.1.4.7
(1.1.4.4)	$b_1 := choice\Big(\{b_1' \in B \mid \exists_{b_0' \in B}(b_0' < b_1')\}\Big) \;\blacksquare\; \neg LowerBound(b_1, B, S, <)$	from: 1.1.4.2
(1.1.4.5)	$b_1 \notin L \;\blacksquare\; L \subset S$	
(1.1.4.6)	$\delta := choice\big(\{\delta' \in S \mid LowerBound(\delta', B, S, <)\}\big) \;\blacksquare\; \delta \in L \;\blacksquare\; \emptyset \neq L$	from: 1.1
(1.1.4.7)	$\emptyset \neq L \subset S$	
(1.1.4.8)	$\forall_{y \in L}\big(LowerBound(y, B, S, <)\big) \;\blacksquare\; \forall_{y \in L}\forall_{x \in B}(y \leq x)$	from: 1.1.4.2 wts: 1.1.4.10
(1.1.4.9)	$\forall_{x \in B}\Big(x \in S \wedge \forall_{y \in L}(y \leq x)\Big) \;\blacksquare\; \forall_{x \in B}\big(UpperBound(x, L, S, <)\big)$	
(1.1.4.10)	$\exists_{x \in S}\big(UpperBound(x, L, S, <)\big) \;\blacksquare\; BoundedAbove(L, S, <)$	
(1.1.4.11)	$\emptyset \neq L \subset S \wedge BoundedAbove(L, S, <)$	

(1.1.4.12) $\exists_{\alpha'\in S}\big(LUB(\alpha', L, S, <)\big) \blacksquare \alpha := choice\Big(\{\alpha' \in S | \big(LUB(\alpha', L, S, <)\big)\}\Big)$ from: 1.1.4.1 wts: 1.1.4.21

(1.1.4.13) $\forall_x\big(x \in B \implies UpperBound(x, L, S, <)\big)$ from: 1.1.4.9 wts: 1.1.4.17

(1.1.4.14) $\forall_x\big(\neg UpperBound(x, L, S, <) \implies x \notin B\big)$

(1.1.4.15) $\gamma < \alpha \implies \dots$ wts: 1.1.4.16

(1.1.4.15.1) $\neg UpperBound(\gamma, L, S, <) \blacksquare \gamma \notin B$ from: 1.1.4.12, 1.1.4.14

(1.1.4.16) $\gamma < \alpha \implies \gamma \notin B \blacksquare \gamma \in B \implies \gamma \geq \alpha$

(1.1.4.17) $\forall_{\gamma\in B}(\alpha \leq \gamma) \blacksquare LowerBound(\alpha, B, S, <)$

(1.1.4.18) $\alpha < \beta \implies \dots$ wts: 1.1.4.19

(1.1.4.18.1) $\forall_{y\in L}(y \leq \alpha < \beta) \blacksquare \forall_{y\in L}(y \neq \beta)$ from: 1.1.4.12

(1.1.4.18.2) $\beta \neq L \blacksquare \neg LowerBound(\beta, B, S, <)$

(1.1.4.19) $\alpha < \beta \implies \neg LowerBound(\beta, B, S, <) \blacksquare \forall_{\beta\in S}\big(\alpha < \beta \implies \neg LowerBound(\beta, B, S, <)\big)$

(1.1.4.20) $LowerBound(\alpha, B, S, <) \land \forall_{\beta\in S}\big(\alpha < \beta \implies \neg LowerBound(\beta, B, S, <)\big)$ from: 1.1.4.17, 1.1.4.19

(1.1.4.21) $GLB(\alpha, B, S, <) \blacksquare \exists_{\epsilon_1\in S}\big(GLB(\epsilon_1, B, S, <)\big)$

(1.1.5) $|B| \neq 1 \implies \exists_{\epsilon_1\in S}\big(GLB(\epsilon_1, B, S, <)\big)$

(1.1.6) $\Big(|B| = 1 \implies \exists_{\epsilon_0\in S}\big(GLB(\epsilon_0, B, S, <)\big)\Big) \land \Big(|B| \neq 1 \implies \exists_{\epsilon_1\in S}\big(GLB(\epsilon_1, B, S, <)\big)\Big)$ from: 1.1.3, 1.1.5

(1.1.7) $(|B| = 1 \lor |B| \neq 1) \implies \exists_{\epsilon\in S}\big(GLB(\epsilon, B, S, <)\big) \blacksquare \exists_{\epsilon\in S}\big(GLB(\epsilon, B, S, <)\big)$

(1.2) $\big(\emptyset \neq B \subset S \land BoundedBelow(B, S, <)\big) \implies \exists_{\epsilon\in S}\big(GLB(\epsilon, B, S, <)\big)$

(1.3) $\forall_B\Big(\big(\emptyset \neq B \subset S \land BoundedBelow(B, S, <)\big) \implies \exists_{\epsilon\in S}\big(GLB(\epsilon, B, S, <)\big)\Big)$

(1.4) $GLBProperty(S, <)$

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(2) $LUBProperty(S, <) \implies GLBProperty(S, <)$

$Field(F, +, *) := \exists_{\mathbb{0},\mathbb{1}\in F}\forall_{x,y,z\in F}($
$x + y \in F \land x * y \in F \land$
$x + y = y + x \land x * y = y * x \land$
$(x + y) + z = x + (y + z) \land (x * y) * z = x * (y * z) \land$
$\mathbb{0} + x = x \land (\mathbb{1} \neq 0 \land \mathbb{1} * x = x) \land$
$\exists_{-x\in F})\big(x + (-x) = \mathbb{0}\big) \land ()\land$

Chapter 2

First Chapter

(1) First

(1.1) Second

(1.2) Third

(2) Fourth

This will be an empty chapter and I will put some text here

$$\sum_{i=0}^{\infty} a_i x^i \tag{2.1}$$

The equation 2.1 shows a sum that is divergent. This formula will later be used in the page **??**.
For further references see Something Linky or go to the next url: http://www.sharelatex.com or open the next file File.txt
It's also possible to link directly any word or any sentence in your document. supwithitSup With It Theorem
If you read this text, you will get no information. Really? Is there no information?

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