Contents

Chapter 1

Philosopherers

$\mathit{OrderTrichotomy}(<, S) := \forall_{x,y \in S}(x < y \veebar x = y \veebar y < x)$

$\mathit{OrderTransitivity}(<, S) := \forall_{x,y,z \in S}\big((x < y \wedge y < z) \implies x < z\big)$

$\mathit{Order}(<, S) := \mathit{OrderTrichotomy}(<, S) \wedge \mathit{OrderTransitivity}(<, S)$

$\mathit{BoundedAbove}(E, S, <) := \mathit{Order}(<, S) \wedge E \subset S \wedge \exists_{\beta \in S}\forall_{x \in E}(x \leq \beta)$

$\mathit{BoundedBelow}(E, S, <) := \mathit{Order}(<, S) \wedge E \subset S \wedge \exists_{\beta \in S}\forall_{x \in E}(\beta \leq x)$

$\mathit{UpperBound}(\beta, E, S, <) := \mathit{Order}(<, S) \wedge E \subset S \wedge \beta \in S \wedge \forall_{x \in E}(x \leq \beta)$

$\mathit{LowerBound}(\beta, E, S, <) := \mathit{Order}(<, S) \wedge E \subset S \wedge \beta \in S \wedge \forall_{x \in E}(\beta \leq x)$

$\mathit{LUB}(\alpha, E, S, <) := \mathit{UpperBound}(\alpha, E, S, <) \wedge \forall_{\gamma}\big(\gamma < \alpha \implies \neg\mathit{UpperBound}(\gamma, E, S, <)\big)$

$\mathit{GLB}(\alpha, E, S, <) := \mathit{LowerBound}(\alpha, E, S, <) \wedge \forall_{\beta}\big(\alpha < \beta \implies \neg\mathit{LowerBound}(\beta, E, S, <)\big)$

$\mathit{LUBProperty}(S, <) := \forall_E\Big(\big(\emptyset \neq E \subset S \wedge \mathit{BoundedAbove}(E, S, <)\big) \implies \exists_{\alpha \in S}\big(\mathit{LUB}(\alpha, E, S, <)\big)\Big)$

$\mathit{GLBProperty}(S, <) := \forall_E\Big(\big(\emptyset \neq E \subset S \wedge \mathit{BoundedBelow}(E, S, <)\big) \implies \exists_{\alpha \in S}\big(\mathit{GLB}(\alpha, E, S, <)\big)\Big)$

((LUBPropertyImpliesGLBProperty)) $\mathit{LUBProperty}(S, <) \implies \mathit{GLBProperty}(S, <)$

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(1))) $\mathit{LUBProperty}(S, <) \implies \ldots$ wts: 2

(1.1))) $\big(\emptyset \neq B \subset S \wedge \mathit{BoundedBelow}(B, S, <)\big) \implies \ldots$ wts: 1.2

(1.1.1))) $\mathit{Order}(<, S) \wedge \exists_{\delta' \in S}\big(\mathit{LowerBound}(\delta', B, S, <)\big)$ from: 1.1

(1.1.2))) $|B| = 1 \implies \ldots$ wts: 1.1.3

(1.1.2.1))) $\exists_{u'}(u' \in B)$ ■ $u := \mathit{choice}(\{u' | u' \in B\})$ ■ $B = \{u\}$ ■ $\mathit{GLB}(u, B, S, <)$ from: 1.1.2

(1.1.2.2))) $\exists_{\epsilon_0 \in S}\big(\mathit{GLB}(\epsilon_0, B, S, <)\big)$

(1.1.3))) $|B| = 1 \implies \exists_{\epsilon_0 \in S}\big(\mathit{GLB}(\epsilon_0, B, S, <)\big)$

(1.1.4))) $|B| \neq 1 \implies \ldots$ wts: 1.1.5

(1.1.4.1))) $\forall_E\Big(\big(\emptyset \neq E \subset S \wedge \mathit{BoundedAbove}(E, S, <)\big) \implies \exists_{\alpha \in S}\big(\mathit{LUB}(\alpha, E, S, <)\big)\Big)$ from: 1

(1.1.4.2))) $L := \{s \in S | \mathit{LowerBound}(s, B, S, <)\}$

(1.1.4.3))) $|B| > 1 \wedge \mathit{OrderTrichotomy}(<, S)$ ■ $\exists_{b_1{}' \in B}\exists_{b_0{}' \in B}(b_0{}' < b_1{}')$ from: 1.1.1 wts: 1.1.4.7

(1.1.4.4))) $b_1 := \mathit{choice}\Big(\{b_1{}' \in B | \exists_{b_0{}' \in B}(b_0{}' < b_1{}')\}\Big)$ ■ $\neg\mathit{LowerBound}(b_1, B, S, <)$ from: 1.1.4.2

(1.1.4.5))) $b_1 \notin L$ ■ $L \subset S$

(1.1.4.6))) $\delta := \mathit{choice}\big(\{\delta' \in S | \mathit{LowerBound}(\delta', B, S, <)\}\big)$ ■ $\delta \in L$ ■ $\emptyset \neq L$ from: 1.1

(1.1.4.7))) $\emptyset \neq L \subset S$

(1.1.4.8))) $\forall_{y \in L}\big(\mathit{LowerBound}(y, B, S, <)\big)$ ■ $\forall_{y \in L}\forall_{x \in B}(y \leq x)$ from: 1.1.4.2 wts: 1.1.4.10

(1.1.4.9))) $\forall_{x \in B}\Big(x \in S \wedge \forall_{y \in L}(y \leq x)\Big)$ ■ $\forall_{x \in B}\big(\mathit{UpperBound}(x, L, S, <)\big)$

(1.1.4.10))) $\exists_{x \in S}\big(\mathit{UpperBound}(x, L, S, <)\big)$ ■ $\mathit{BoundedAbove}(L, S, <)$

(1.1.4.11))) $\emptyset \neq L \subset S \wedge \mathit{BoundedAbove}(L, S, <)$

(1.1.4.12))) $\exists_{\alpha' \in S}\big(LUB(\alpha', L, S, <)\big)$ ▮ $\alpha := choice\Big(\{\alpha' \in S | \big(LUB(\alpha', L, S, <)\big)\}\Big)$ from: 1.1.4.1 wts: 1.1.4.21

(1.1.4.13))) $\forall_x\big(x \in B \implies UpperBound(x, L, S, <)\big)$ from: 1.1.4.9 wts: 1.1.4.17

(1.1.4.14))) $\forall_x\big(\neg UpperBound(x, L, S, <) \implies x \notin B\big)$

(1.1.4.15))) $\gamma < \alpha \implies \ldots$ wts: 1.1.4.16

(1.1.4.15.1))) $\neg UpperBound(\gamma, L, S, <)$ ▮ $\gamma \notin B$ from: 1.1.4.12, 1.1.4.14

(1.1.4.16))) $\gamma < \alpha \implies \gamma \notin B$ ▮ $\gamma \in B \implies \gamma \geq \alpha$

(1.1.4.17))) $\forall_{\gamma \in B}(\alpha \leq \gamma)$ ▮ $LowerBound(\alpha, B, S, <)$

(1.1.4.18))) $\alpha < \beta \implies \ldots$ wts: 1.1.4.19

(1.1.4.18.1))) $\forall_{y \in L}(y \leq \alpha < \beta)$ ▮ $\forall_{y \in L}(y \neq \beta)$ from: 1.1.4.12

(1.1.4.18.2))) $\beta \neq L$ ▮ $\neg LowerBound(\beta, B, S, <)$

(1.1.4.19))) $\alpha < \beta \implies \neg LowerBound(\beta, B, S, <)$ ▮ $\forall_{\beta \in S}\big(\alpha < \beta \implies \neg LowerBound(\beta, B, S, <)\big)$

(1.1.4.20))) $LowerBound(\alpha, B, S, <) \wedge \forall_{\beta \in S}\big(\alpha < \beta \implies \neg LowerBound(\beta, B, S, <)\big)$ from: 1.1.4.17, 1.1.4.19

(1.1.4.21))) $GLB(\alpha, B, S, <)$ ▮ $\exists_{\epsilon_1 \in S}\big(GLB(\epsilon_1, B, S, <)\big)$

(1.1.5))) $|B| \neq 1 \implies \exists_{\epsilon_1 \in S}\big(GLB(\epsilon_1, B, S, <)\big)$

(1.1.6))) $\Big(|B| = 1 \implies \exists_{\epsilon_0 \in S}\big(GLB(\epsilon_0, B, S, <)\big)\Big) \wedge \Big(|B| \neq 1 \implies \exists_{\epsilon_1 \in S}\big(GLB(\epsilon_1, B, S, <)\big)\Big)$ from: 1.1.3, 1.1.5

(1.1.7))) $(|B| = 1 \vee |B| \neq 1) \implies \exists_{\epsilon \in S}\big(GLB(\epsilon, B, S, <)\big)$ ▮ $\exists_{\epsilon \in S}\big(GLB(\epsilon, B, S, <)\big)$

(1.2))) $\big(\emptyset \neq B \subset S \wedge BoundedBelow(B, S, <)\big) \implies \exists_{\epsilon \in S}\big(GLB(\epsilon, B, S, <)\big)$

(1.3))) $\forall_B\Big(\big(\emptyset \neq B \subset S \wedge BoundedBelow(B, S, <)\big) \implies \exists_{\epsilon \in S}\big(GLB(\epsilon, B, S, <)\big)\Big)$

(1.4))) $GLBProperty(S, <)$

(2))) $LUBProperty(S, <) \implies GLBProperty(S, <)$

$Field(F, +, *) := \exists_{0, \mathbb{1} \in F}\forall_{x,y,z \in F}($
$x + y \in F \wedge x * y \in F \wedge$
$x + y = y + x \wedge x * y = y * x \wedge$
$(x + y) + z = x + (y + z) \wedge (x * y) * z = x * (y * z) \wedge$
$\mathbb{0} + x = x \wedge (\mathbb{1} \neq 0 \wedge \mathbb{1} * x = x) \wedge$
$\exists_{-x \in F}\big(x + (-x) = \mathbb{0}\big) \wedge () \wedge$
$)$

$$F = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

Chapter 2

First Chapter

This will be an empty chapter and I will put some text here

$$\sum_{i=0}^{\infty} a_i x^i \tag{2.1}$$

The equation 2.1 shows a sum that is divergent. This formula will later be used in the page **??**.
For further references see Something Linky or go to the next url: http://www.sharelatex.com or open the next file File.txt
It's also possible to link directly any word or any sentence in your document. supwithitSup With It Theorem
If you read this text, you will get no information. Really? Is there no information?

For instance this sentence. supwithit