Contents

Chapter 1

Philosopherers

(1.5)

$\color{red}{OrderTrichotomy}(<, S) := \forall_{x,y\in S}(x < y \veebar x = y \veebar y < x)$
$\color{red}{OrderTransitivity}(<, S) := \forall_{x,y,z\in S}\big((x < y \wedge y < z) \implies x < z\big)$
$\color{red}{Order}(<, S) := OrderTrichotomy(<, S) \wedge OrderTransitivity(<, S)$

(1.7)

$\color{red}{BoundedAbove}(E, S, <) := Order(<, S) \wedge E \subset S \wedge \exists_{\beta\in S}\forall_{x\in E}(x \le \beta)$
$\color{red}{BoundedBelow}(E, S, <) := Order(<, S) \wedge E \subset S \wedge \exists_{\beta\in S}\forall_{x\in E}(\beta \le x)$
$\color{red}{UpperBound}(\beta, E, S, <) := Order(<, S) \wedge E \subset S \wedge \beta \in S \wedge \forall_{x\in E}(x \le \beta)$
$\color{red}{LowerBound}(\beta, E, S, <) := Order(<, S) \wedge E \subset S \wedge \beta \in S \wedge \forall_{x\in E}(\beta \le x)$

(1.8)

$\color{red}{LUB}(\alpha, E, S, <) := UpperBound(\alpha, E, S, <) \wedge \forall_{\gamma}\big(\gamma < \alpha \implies \neg UpperBound(\gamma, E, S, <)\big)$
$\color{red}{GLB}(\alpha, E, S, <) := LowerBound(\alpha, E, S, <) \wedge \forall_{\beta}\big(\alpha < \beta \implies \neg LowerBound(\beta, E, S, <)\big)$

(1.10)

$\color{red}{LUBProperty}(S, <) := \forall_{E}\Big(\big(\emptyset \ne E \subset S \wedge BoundedAbove(E, S, <)\big) \implies \exists_{\alpha\in S}\big(LUB(\alpha, E, S, <)\big)\Big)$
$\color{red}{GLBProperty}(S, <) := \forall_{E}\Big(\big(\emptyset \ne E \subset S \wedge BoundedBelow(E, S, <)\big) \implies \exists_{\alpha\in S}\big(GLB(\alpha, E, S, <)\big)\Big)$

(1.11)

$\boxed{\text{LUBPropertyImpliesGLBProperty}}$ $LUBProperty(S, <) \implies GLBProperty(S, <)$

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(1) $LUBProperty(S, <) \implies \ldots$ — wts: 2

(1.1) $\big(\emptyset \ne B \subset S \wedge BoundedBelow(B, S, <)\big) \implies \ldots$ — wts: 1.2

(1.1.1) $Order(<, S) \wedge \exists_{\delta'\in S}\big(LowerBound(\delta', B, S, <)\big)$ — from: BoundedBelow, 1.1

(1.1.2) $|B| = 1 \implies \ldots$ — wts: 1.1.3

(1.1.2.1) $\exists_{u'}(u' \in B) \;\blacksquare\; u := choice(\{u' | u' \in B\}) \;\blacksquare\; B = \{u\}$ — from: 1.1.2

(1.1.2.2) $GLB(u, B, S, <) \;\blacksquare\; \exists_{\epsilon_0\in S}\big(GLB(\epsilon_0, B, S, <)\big)$

(1.1.3) $|B| = 1 \implies \exists_{\epsilon_0\in S}\big(GLB(\epsilon_0, B, S, <)\big)$

(1.1.4) $|B| \ne 1 \implies \ldots$ — wts: 1.1.5

(1.1.4.1) $\forall_{E}\Big(\big(\emptyset \ne E \subset S \wedge BoundedAbove(E, S, <)\big) \implies \exists_{\alpha\in S}\big(LUB(\alpha, E, S, <)\big)\Big)$ — from: LUBProperty, 1

(1.1.4.2) $L := \{s \in S | LowerBound(s, B, S, <)\}$

(1.1.4.3) $|B| > 1 \wedge OrderTrichotomy(<, S) \;\blacksquare\; \exists_{{b_1}'\in B}\exists_{{b_0}'\in B}({b_0}' < {b_1}')$ — from: Order, 1.1.1; wts: 1.1.4.7

(1.1.4.4) $b_1 := choice\Big(\{{b_1}' \in B | \exists_{{b_0}'\in B}({b_0}' < {b_1}')\}\Big) \;\blacksquare\; \neg LowerBound(b_1, B, S, <)$ — from: 1.1.4.2

(1.1.4.5) $b_1 \notin L \;\blacksquare\; L \subset S$

(1.1.4.6) $\delta := choice\big(\{\delta' \in S | LowerBound(\delta', B, S, <)\}\big) \;\blacksquare\; \delta \in L \;\blacksquare\; \emptyset \ne L$ — from: 1.1.1

(1.1.4.7) $\emptyset \ne L \subset S$ — from: 1.1.4.5, 1.1.4.6

(1.1.4.8) $\forall_{y\in L}\big(LowerBound(y,B,S,<)\big) \;\blacksquare\; \forall_{y\in L}\forall_{x\in B}(y \le x)$ — from: LowerBound, 1.1.4.2; wts: 1.1.4.10

(1.1.4.9) $\forall_{x\in B}\Big(x\in S \wedge \forall_{y\in L}(y\le x)\Big) \;\blacksquare\; \forall_{x\in B}\big(UpperBound(x,L,S,<)\big)$ — from: UpperBound

(1.1.4.10) $\exists_{x\in S}\big(UpperBound(x,L,S,<)\big) \;\blacksquare\; BoundedAbove(L,S,<)$

(1.1.4.11) $\emptyset \ne L \subset S \wedge BoundedAbove(L,S,<)$ — from: 1.1.4.7, 1.1.4.10

(1.1.4.12) $\exists_{\alpha'\in S}\big(LUB(\alpha',L,S,<)\big) \;\blacksquare\; \alpha := choice\Big(\{\alpha'\in S | \big(LUB(\alpha',L,S,<)\big)\}\Big)$ — from: 1.1.4.1; wts: 1.1.4.21

(1.1.4.13) $\forall_x\big(x\in B \implies UpperBound(x,L,S,<)\big)$ — from: 1.1.4.9; wts: 1.1.4.17

(1.1.4.14) $\forall_x\big(\neg UpperBound(x,L,S,<) \implies x\notin B\big)$

(1.1.4.15) $\gamma<\alpha \implies \dots$ — wts: 1.1.4.16

(1.1.4.15.1) $\neg UpperBound(\gamma,L,S,<) \;\blacksquare\; \gamma\notin B$ — from: LUB, 1.1.4.12, 1.1.4.14

(1.1.4.16) $\gamma<\alpha \implies \gamma\notin B \;\blacksquare\; \gamma\in B \implies \gamma\ge\alpha$

(1.1.4.17) $\forall_{\gamma\in B}(\alpha\le\gamma) \;\blacksquare\; LowerBound(\alpha,B,S,<)$ — from: LowerBound

(1.1.4.18) $\alpha<\beta \implies \dots$ — wts: 1.1.4.19

(1.1.4.18.1) $\forall_{y\in L}(y\le\alpha<\beta) \;\blacksquare\; \forall_{y\in L}(y\ne\beta)$ — from: LUB, 1.1.4.12, 1.1.4.18

(1.1.4.18.2) $\beta\notin L \;\blacksquare\; \neg LowerBound(\beta,B,S,<)$ — from: 1.1.4.2

(1.1.4.19) $\alpha<\beta \implies \neg LowerBound(\beta,B,S,<) \;\blacksquare\; \forall_{\beta\in S}\big(\alpha<\beta \implies \neg LowerBound(\beta,B,S,<)\big)$

(1.1.4.20) $LowerBound(\alpha,B,S,<) \wedge \forall_{\beta\in S}\big(\alpha<\beta \implies \neg LowerBound(\beta,B,S,<)\big)$ — from: 1.1.4.17, 1.1.4.19

(1.1.4.21) $GLB(\alpha,B,S,<) \;\blacksquare\; \exists_{\epsilon_1\in S}\big(GLB(\epsilon_1,B,S,<)\big)$

(1.1.5) $|B|\ne 1 \implies \exists_{\epsilon_1\in S}\big(GLB(\epsilon_1,B,S,<)\big)$

(1.1.6) $\Big(|B|=1 \implies \exists_{\epsilon_0\in S}\big(GLB(\epsilon_0,B,S,<)\big)\Big) \wedge \Big(|B|\ne 1 \implies \exists_{\epsilon_1\in S}\big(GLB(\epsilon_1,B,S,<)\big)\Big)$ — from: 1.1.3, 1.1.5

(1.1.7) $(|B|=1 \vee |B|\ne 1) \implies \exists_{\epsilon\in S}\big(GLB(\epsilon,B,S,<)\big) \;\blacksquare\; \exists_{\epsilon\in S}\big(GLB(\epsilon,B,S,<)\big)$

(1.2) $\big(\emptyset\ne B\subset S \wedge BoundedBelow(B,S,<)\big) \implies \exists_{\epsilon\in S}\big(GLB(\epsilon,B,S,<)\big)$

(1.3) $\forall_B\Big(\big(\emptyset\ne B\subset S \wedge BoundedBelow(B,S,<)\big) \implies \exists_{\epsilon\in S}\big(GLB(\epsilon,B,S,<)\big)\Big)$

(1.4) $GLBProperty(S,<)$

(2) $LUBProperty(S,<) \implies GLBProperty(S,<)$

(1.12)

$$Field(F,+,*) := \exists_{\mathbb{0},\mathbb{1}\in F}\forall_{x,y,z\in F}\left(\begin{array}{l} x+y\in F \quad\wedge\quad x*y\in F \quad\wedge \\ x+y=y+x \quad\wedge\quad x*y=y*x \quad\wedge \\ (x+y)+z=x+(y+z) \quad\wedge\quad (x*y)*z=x*(y*z) \quad\wedge \\ \mathbb{1}\ne\mathbb{0} \quad\wedge\quad x*(y+z)=(x*y)+(x*z) \quad\wedge \\ \mathbb{0}+x=x \quad\wedge\quad \mathbb{1}*x=x \quad\wedge \\ \exists_{-x\in F}\big(x+(-x)=\mathbb{0}\big) \wedge \Big(x\ne\mathbb{0} \implies \exists_{1/x\in F}\big(x*(1/x)=\mathbb{1}\big)\Big) \end{array}\right)$$

************************ $\big(Field(F,+,*) \wedge x,y,z\in F\big) \implies \dots$ ************************

(1.14.a)

AdditiveCancellation $(x+y=x+z) \implies y=z$

(1) $y=\mathbb{0}+y=\big(x+(-x)\big)+y=\big((-x)+x\big)+y=(-x)+(x+y)=\dots$ — from: Field

(2) $(-x)+(x+z)=\big((-x)+x\big)+z=\big(x+(-x)\big)+z=\mathbb{0}+z=z$ — from: Field

(1.14.b)

AdditiveIdentityUniqueness $(x+y=x) \implies y=\mathbb{0}$

(1) $x+y=x=\mathbb{0}+x=x+\mathbb{0}$ — from: Field

(2) $y=0$ — from: AdditiveCancellation

(1.14.c)
AdditiveInverseUniqueness $(x + y = \mathbb{0}) \implies y = -x$

(1) $x + y = \mathbb{0} = x + (-x)$ — from: Field

(2) $y = -x$ — from: AdditiveCancellation

(1.14.d)
DoubleNegative $x = -(-x)$

(1) $\mathbb{0} = x + (-x) = (-x) + x$ ▮ $\mathbb{0} = (-x) + x$ — from: Field

(2) $x = -(-x)$ — from: AdditiveInverseUniqueness

(1.15.a)
MultiplicativeCancellation $(x \neq 0 \land x * y = x * z) \implies y = z$ —

(1.15.b)
MultiplicativeIdentityUniqueness $(x \neq 0 \land x * y = x) \implies y = \mathbb{1}$ —

(1.15.c)
MultiplicativeInverseUniqueness $(x \neq 0 \land x * y = \mathbb{1}) \implies y = 1/x$ —

(1.15.d)
DoubleReciprocal $(x \neq 0) \implies x = 1/(1/x)$ —

(1.16.a)
Domination $\mathbb{0} * x = \mathbb{0}$

(1) $\mathbb{0} * x = (\mathbb{0} + \mathbb{0}) * x = \mathbb{0} * x + \mathbb{0} * x$ ▮ $\mathbb{0} * x = \mathbb{0} * x + \mathbb{0} * x$ — from: Field

(2) $\mathbb{0} * x = \mathbb{0}$ — from: AdditiveIdentityUniqueness

(1.16.b)
NonDomination $(x \neq \mathbb{0} \land y \neq \mathbb{0}) \implies x * y \neq \mathbb{0}$

(1) $(x \neq \mathbb{0} \land y \neq \mathbb{0}) \implies \ldots$

(1.1) $(x * y = 0) \implies \ldots$

(1.1.1) $\mathbb{1} = \mathbb{1} * \mathbb{1} = \big(x * (1/x)\big) * \big(y * (1/y)\big) = (x * y) * \big((1/x) * (1/y)\big) = \mathbb{0} * \big((1/x) * (1/y)\big) = \mathbb{0}$ — from: Field, Domination, 1, 1.1

(1.1.2) $\mathbb{1} = \mathbb{0} \land \mathbb{1} \neq \mathbb{0}$ ▮ $\perp$ — from: Field

(1.2) $(x * y = 0) \implies \perp$ ▮ $x * y \neq 0$

(2) $(x \neq \mathbb{0} \land y \neq \mathbb{0}) \implies x * y \neq 0$

(1.16.c)
NegationCommutativity $(-x) * y = -(x * y) = x * (-y)$

(1) $x * y + (-x) * y = (x + -x) * y = \mathbb{0} * y = \mathbb{0}$ ▮ $x * y + (-x) * y = \mathbb{0}$ — from: Field, Domination; wts: 2

(2) $(-x) * y = -(x * y)$ — from: AdditiveInverseUniqueness

(3) $x * y + x * (-y) = x * (y + -y) = x * \mathbb{0} = \mathbb{0}$ ▮ $x * y + x * (-y) = \mathbb{0}$ — from: Field, Domination; wts: 4

(4) $x * (-y) = -(x * y)$ — from: AdditiveInverseUniqueness

(5) $(-x) * y = -(x * y) = x * (-y)$ — from: 2, 4

(1.16.d)
NegativeMultiplication $(-x) * (-y) = x * y$

(1) $(-x) * (-y) = -\big(x * (-y)\big) = -\big(-(x * y)\big) = x * y$ — from: NegationCommutativity, DoubleNegative

(1.17)

$$OrderedField(F,+,*,<) := \begin{pmatrix} Field(F,+,*) \quad \wedge \quad Order(<,F) \quad \wedge \\ \forall_{x,y,z\in F}(y<z \implies x+y<x+z) \quad \wedge \\ \forall_{x,y\in F}\big((x>\mathbb{0}\wedge y>\mathbb{0}) \implies x*y>\mathbb{0}\big) \end{pmatrix}$$

************************ $\big(OrderedField(F,+,*,<)\wedge x,y,z\in F\big) \implies \ldots$ ************************

(1.18.a)

$\boxed{\text{NegationOnOrder}}$ $x>\mathbb{0} \iff -x<\mathbb{0}$

(1) $x>\mathbb{0} \implies \ldots$

(1.1) $\mathbb{0}=(-x)+x>(-x)+\mathbb{0}=-x \;\blacksquare\; \mathbb{0}>-x \;\blacksquare\; -x<\mathbb{0}$ — from: OrderedField

(2) $x>\mathbb{0} \implies -x<\mathbb{0}$

(3) $-x<\mathbb{0} \implies \ldots$

(3.1) $\mathbb{0}=x+(-x)<x+\mathbb{0}=x \;\blacksquare\; \mathbb{0}<x \;\blacksquare\; x>\mathbb{0}$ — from: OrderedField

(4) $-x<\mathbb{0} \implies x>\mathbb{0}$

(5) $x>\mathbb{0} \implies -x<\mathbb{0}\wedge -x<\mathbb{0} \implies x>\mathbb{0} \;\blacksquare\; x>\mathbb{0} \iff -x<\mathbb{0}$ — from: 2, 4

(1.18.b)

$\boxed{\text{PositiveFactorPreservesOrder}}$ $(x>\mathbb{0}\wedge y<z) \implies x*y<x*z$

(1) $(x>\mathbb{0}\wedge y<z) \implies \ldots$

(1.1) $(-y)+z>(-y)+y=\mathbb{0} \;\blacksquare\; z+(-y)=\mathbb{0}$ — from: OrderedField

(1.2) $x*(z+(-y))>\mathbb{0} \;\blacksquare\; x*z+x*(-y)>\mathbb{0}$ — from: OrderedField

(1.3) $x*z=\mathbb{0}+x*z=\big(x*y+-(x*y)\big)+x*z=\big(x*y+x*(-y)\big)+x*z=\ldots$ — from: Field, NegationCommutativity

(1.4) $x*y+\big(x*z+x*(-y)\big)>x*y+\mathbb{0}=x*y$ — from: Field, 1.2

(1.5) $x*z>x*y$ — from: 1.3, 1.4

(2) $(x>\mathbb{0}\wedge y<z) \implies x*z>x*y$

(1.18.c)

$\boxed{\text{NegativeFactorFlipsOrder}}$ $(x<\mathbb{0}\wedge y<z) \implies x*y>x*z$

(1) $(x<\mathbb{0}\wedge y<z) \implies \ldots$

(1.1) $-x>\mathbb{0}$ — from: NegationOnOrder

(1.2) $(-x)*y<(-x)*z \;\blacksquare\; \mathbb{0}=x*y+(-x)*y<x*y+(-x)*z \;\blacksquare\; \mathbb{0}<x*y+(-x)*z$ — from: PositiveFactorPreservesOrder

(1.3) $\mathbb{0}<(-x)*(-y+z) \;\blacksquare\; \mathbb{0}>x*(-y+z) \;\blacksquare\; \mathbb{0}>-(x*y)+x*z$ — from: NegationOnOrder

(1.4) $x*y>x*z$

(2) $(x<\mathbb{0}\wedge y<z) \implies x*y>x*z$

(1.18.d)

$\boxed{\text{SquareIsPositive}}$ $(x\neq\mathbb{0}) \implies x*x>\mathbb{0}$

(1) $(x>\mathbb{0}) \implies x*x>0$ — from: OrderedField

(2) $(x<\mathbb{0}) \implies \ldots$

(2.1) $-x>\mathbb{0} \;\blacksquare\; x*x=(-x)*(-x)>\mathbb{0} \;\blacksquare\; x*x>\mathbb{0}$ — from: NegationOnOrder, OrderedField, NegativeMultiplication

(3) $(x<\mathbb{0}) \implies x*x>\mathbb{0}$

(4) $x\neq\mathbb{0} \implies (x>\mathbb{0}\vee x<\mathbb{0}) \implies x*x>\mathbb{0} \;\blacksquare\; x\neq\mathbb{0} \implies x*x>\mathbb{0}$ — from: OrderTrichotomy, 1, 3

(1.18.d)

$\boxed{\text{OneIsPositive}}$ $\mathbb{1}>\mathbb{0}$

(1) $\mathbb{1}\neq\mathbb{0} \;\blacksquare\; \mathbb{1}=\mathbb{1}*\mathbb{1}>\mathbb{0}$ — from: Field, SquareIsPositive

(1.18.e)

$\boxed{\text{ReciprocationOnOrder}}$ $(0 < x < y) \implies 0 < 1/y < 1/x$

(1) $(0 < x < y) \implies \ldots$

(1.1) $x * (1/x) = \mathbb{1} > \mathbb{0}$ ▌ $x * (1/x) > 0$ from: Field, OneIsPositive

(1.2) $1/x < \mathbb{0} \implies x * (1/x) < 0 \wedge x * (1/x) > 0 \implies \perp$ ▌ $1/x > \mathbb{0}$ from: NegativeFactorFlipsOrder, 1

(1.3) $y * (1/y) = \mathbb{1} > \mathbb{0}$ ▌ $y * (1/y) > 0$ from: Field, OneIsPositive

(1.4) $1/y < \mathbb{0} \implies y * (1/y) < 0 \wedge y * (1/y) > 0 \implies \perp$ ▌ $1/y > \mathbb{0}$ from: NegativeFactorFlipsOrder, 1

(1.5) $(1/x) * (1/y) > \mathbb{0}$ from: OrderedField

(1.6) $\mathbb{0} < 1/y = \big((1/x) * (1/y)\big) * x < \big((1/x) * (1/y)\big) * y = 1/x$ from: OrderedField, 1, 1.4, 1.5

(1.19)

$Subfield(K, F, +, *) := Field(F, +, *) \wedge K \subset F \wedge Field(K, +, *)$

$OrderedSubfield(K, F, +, *, <) := OrderedField(F, +, *, <) \wedge K \subset F \wedge OrderedField(K, +, *, <)$

$\boxed{\text{ExistenceOfR}}$ $\exists_{\mathbb{R}} \big(LUBProperty(\mathbb{R}, <) \wedge OrderedSubfield(\mathbb{Q}, \mathbb{R}, +, *, <) \big)$ Plan: - - - - -

(1) OH BOI

Chapter 2

First Chapter

(1) First

(1.1) Second

(1.2) Third

(2) Fourth

This will be an empty chapter and I will put some text here

$$\sum_{i=0}^{\infty} a_i x^i \tag{2.1}$$

The equation 2.1 shows a sum that is divergent. This formula will later be used in the page **??**.
For further references see Something Linky or go to the next url: http://www.sharelatex.com or open the next file File.txt
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