Contents

Chapter 1

Philosopherers

(1.5)
$OrderTrichotomy(<,S) := \forall_{x,y\in S}(x < y \veebar x = y \veebar y < x)$
$OrderTransitivity(<,S) := \forall_{x,y,z\in S}\big((x < y \wedge y < z) \implies x < z\big)$
$Order(<,S) := OrderTrichotomy(<,S) \wedge OrderTransitivity(<,S)$

(1.7)
$BoundedAbove(E,S,<) := Order(<,S) \wedge E \subset S \wedge \exists_{\beta\in S}\forall_{x\in E}(x \leq \beta)$
$BoundedBelow(E,S,<) := Order(<,S) \wedge E \subset S \wedge \exists_{\beta\in S}\forall_{x\in E}(\beta \leq x)$
$UpperBound(\beta,E,S,<) := Order(<,S) \wedge E \subset S \wedge \beta \in S \wedge \forall_{x\in E}(x \leq \beta)$
$LowerBound(\beta,E,S,<) := Order(<,S) \wedge E \subset S \wedge \beta \in S \wedge \forall_{x\in E}(\beta \leq x)$

(1.8)
$LUB(\alpha,E,S,<) := UpperBound(\alpha,E,S,<) \wedge \forall_{\gamma}\big(\gamma < \alpha \implies \neg UpperBound(\gamma,E,S,<)\big)$
$GLB(\alpha,E,S,<) := LowerBound(\alpha,E,S,<) \wedge \forall_{\beta}\big(\alpha < \beta \implies \neg LowerBound(\beta,E,S,<)\big)$

(1.10)
$LUBProperty(S,<) := \forall_{E}\Big(\big(\emptyset \neq E \subset S \wedge BoundedAbove(E,S,<)\big) \implies \exists_{\alpha\in S}\big(LUB(\alpha,E,S,<)\big)\Big)$
$GLBProperty(S,<) := \forall_{E}\Big(\big(\emptyset \neq E \subset S \wedge BoundedBelow(E,S,<)\big) \implies \exists_{\alpha\in S}\big(GLB(\alpha,E,S,<)\big)\Big)$

(1.11)
$\boxed{\text{LUBPropertyImpliesGLBProperty}}$ $LUBProperty(S,<) \implies GLBProperty(S,<)$

(1) $LUBProperty(S,<) \implies \dots$ — wts: 2
(1.1) $\big(\emptyset \neq B \subset S \wedge BoundedBelow(B,S,<)\big) \implies \dots$ — wts: 1.2
(1.1.1) $Order(<,S) \wedge \exists_{\delta'\in S}\big(LowerBound(\delta',B,S,<)\big)$ — from: 1.1
(1.1.2) $|B| = 1 \implies \dots$ — wts: 1.1.3
(1.1.2.1) $\exists_{u'}(u' \in B)$ ▮ $u := choice(\{u' | u' \in B\})$ ▮ $B = \{u\}$ ▮ $GLB(u,B,S,<)$ — from: 1.1.2
(1.1.2.2) $\exists_{\epsilon_0\in S}\big(GLB(\epsilon_0,B,S,<)\big)$
(1.1.3) $|B| = 1 \implies \exists_{\epsilon_0\in S}\big(GLB(\epsilon_0,B,S,<)\big)$
(1.1.4) $|B| \neq 1 \implies \dots$ — wts: 1.1.5
(1.1.4.1) $\forall_{E}\Big(\big(\emptyset \neq E \subset S \wedge BoundedAbove(E,S,<)\big) \implies \exists_{\alpha\in S}\big(LUB(\alpha,E,S,<)\big)\Big)$ — from: 1
(1.1.4.2) $L := \{s \in S | LowerBound(s,B,S,<)\}$
(1.1.4.3) $|B| > 1 \wedge OrderTrichotomy(<,S)$ ▮ $\exists_{b_1'\in B}\exists_{b_0'\in B}(b_0' < b_1')$ — from: 1.1.1, wts: 1.1.4.7
(1.1.4.4) $b_1 := choice\Big(\{b_1' \in B | \exists_{b_0'\in B}(b_0' < b_1')\}\Big)$ ▮ $\neg LowerBound(b_1,B,S,<)$ — from: 1.1.4.2
(1.1.4.5) $b_1 \notin L$ ▮ $L \subset S$
(1.1.4.6) $\delta := choice\big(\{\delta' \in S | LowerBound(\delta',B,S,<)\}\big)$ ▮ $\delta \in L$ ▮ $\emptyset \neq L$ — from: 1.1
(1.1.4.7) $\emptyset \neq L \subset S$

(1.1.4.8) $\forall_{y\in L}\big(LowerBound(y,B,S,<)\big)$ ▌ $\forall_{y\in L}\forall_{x\in B}(y\leq x)$ from: 1.1.4.2 wts: 1.1.4.10

(1.1.4.9) $\forall_{x\in B}\Big(x\in S\wedge\forall_{y\in L}(y\leq x)\Big)$ ▌ $\forall_{x\in B}\big(UpperBound(x,L,S,<)\big)$

(1.1.4.10) $\exists_{x\in S}\big(UpperBound(x,L,S,<)\big)$ ▌ $BoundedAbove(L,S,<)$

(1.1.4.11) $\emptyset\neq L\subset S\wedge BoundedAbove(L,S,<)$

(1.1.4.12) $\exists_{\alpha'\in S}\big(LUB(\alpha',L,S,<)\big)$ ▌ $\alpha:=choice\Big(\{\alpha'\in S|\big(LUB(\alpha',L,S,<)\big)\}\Big)$ from: 1.1.4.1 wts: 1.1.4.21

(1.1.4.13) $\forall_{x}\big(x\in B\implies UpperBound(x,L,S,<)\big)$ from: 1.1.4.9 wts: 1.1.4.17

(1.1.4.14) $\forall_{x}\big(\neg UpperBound(x,L,S,<)\implies x\notin B\big)$

(1.1.4.15) $\gamma<\alpha\implies\ldots$ wts: 1.1.4.16

(1.1.4.15.1) $\neg UpperBound(\gamma,L,S,<)$ ▌ $\gamma\notin B$ from: 1.1.4.12, 1.1.4.14

(1.1.4.16) $\gamma<\alpha\implies\gamma\notin B$ ▌ $\gamma\in B\implies\gamma\geq\alpha$

(1.1.4.17) $\forall_{\gamma\in B}(\alpha\leq\gamma)$ ▌ $LowerBound(\alpha,B,S,<)$

(1.1.4.18) $\alpha<\beta\implies\ldots$ wts: 1.1.4.19

(1.1.4.18.1) $\forall_{y\in L}(y\leq\alpha<\beta)$ ▌ $\forall_{y\in L}(y\neq\beta)$ from: 1.1.4.12

(1.1.4.18.2) $\beta\neq L$ ▌ $\neg LowerBound(\beta,B,S,<)$

(1.1.4.19) $\alpha<\beta\implies\neg LowerBound(\beta,B,S,<)$ ▌ $\forall_{\beta\in S}\big(\alpha<\beta\implies\neg LowerBound(\beta,B,S,<)\big)$

(1.1.4.20) $LowerBound(\alpha,B,S,<)\wedge\forall_{\beta\in S}\big(\alpha<\beta\implies\neg LowerBound(\beta,B,S,<)\big)$ from: 1.1.4.17, 1.1.4.19

(1.1.4.21) $GLB(\alpha,B,S,<)$ ▌ $\exists_{\epsilon_1\in S}\big(GLB(\epsilon_1,B,S,<)\big)$

(1.1.5) $|B|\neq 1\implies\exists_{\epsilon_1\in S}\big(GLB(\epsilon_1,B,S,<)\big)$

(1.1.6) $\Big(|B|=1\implies\exists_{\epsilon_0\in S}\big(GLB(\epsilon_0,B,S,<)\big)\Big)\wedge\Big(|B|\neq 1\implies\exists_{\epsilon_1\in S}\big(GLB(\epsilon_1,B,S,<)\big)\Big)$ from: 1.1.3, 1.1.5

(1.1.7) $(|B|=1\vee|B|\neq 1)\implies\exists_{\epsilon\in S}\big(GLB(\epsilon,B,S,<)\big)$ ▌ $\exists_{\epsilon\in S}\big(GLB(\epsilon,B,S,<)\big)$

(1.2) $\big(\emptyset\neq B\subset S\wedge BoundedBelow(B,S,<)\big)\implies\exists_{\epsilon\in S}\big(GLB(\epsilon,B,S,<)\big)$

(1.3) $\forall_{B}\Big(\big(\emptyset\neq B\subset S\wedge BoundedBelow(B,S,<)\big)\implies\exists_{\epsilon\in S}\big(GLB(\epsilon,B,S,<)\big)\Big)$

(1.4) $GLBProperty(S,<)$

(2) $LUBProperty(S,<)\implies GLBProperty(S,<)$

(1.12)

$$Field(F,+,*):=\exists_{\mathbb{0},\mathbb{1}\in F}\forall_{x,y,z\in F}\begin{pmatrix} x+y\in F\quad\wedge\quad x*y\in F\quad\wedge\\ x+y=y+x\quad\wedge\quad x*y=y*x\quad\wedge\\ (x+y)+z=x+(y+z)\quad\wedge\quad(x*y)*z=x*(y*z)\quad\wedge\\ \mathbb{1}\neq\mathbb{0}\quad\wedge\quad x*(y+z)=(x*y)+(x*z)\quad\wedge\\ \mathbb{0}+x=x\quad\wedge\quad\mathbb{1}*x=x\quad\wedge\\ \exists_{-x\in F}\big(x+(-x)=\mathbb{0}\big)\wedge\Big(x\neq\mathbb{0}\implies\exists_{1/x\in F}\big(x*(1/x)=\mathbb{1}\big)\Big)\end{pmatrix}$$

*********************** $\big(Field(F,+,*)\wedge x,y,z\in F\big)\implies\ldots$ ***********************

(1.14.a)

$\boxed{\text{AdditiveCancellation}}$ $(x+y=x+z)\implies y=z$

(1) $y=\mathbb{0}+y=\big(x+(-x)\big)+y=\big((-x)+x\big)+y=(-x)+(x+y)=\ldots$

(2) $(-x)+(x+z)=\big((-x)+x\big)+z=\big(x+(-x)\big)+z=\mathbb{0}+z=z$

(1.14.b)

$\boxed{\text{AdditiveIdentityUniqueness}}$ $(x+y=x)\implies y=\mathbb{0}$

(1) $x+y=x=\mathbb{0}+x=x+\mathbb{0}$

(2) $y=0$ from: AdditiveCancellation

(1.14.c)

AdditiveInverseUniqueness $(x+y=\mathbb{0}) \implies y=-x$

(1) $x+y=\mathbb{0}=x+(-x)$	
(2) $y=-x$	from: AdditiveCancellation

(1.14.d)

DoubleNegative $x=-(-x)$

(1) $\mathbb{0}=x+(-x)=(-x)+x \;\blacksquare\; \mathbb{0}=(-x)+x$	
(2) $x=-(-x)$	from: AdditiveInverseUniqueness

(1.15.a)

MultiplicativeCancellation $(x \neq 0 \wedge x*y=x*z) \implies y=z$ —

(1.15.b)

MultiplicativeIdentityUniqueness $(x \neq 0 \wedge x*y=x) \implies y=\mathbb{1}$ —

(1.15.c)

MultiplicativeInverseUniqueness $(x \neq 0 \wedge x*y=\mathbb{1}) \implies y=1/x$ —

(1.15.d)

DoubleReciprocal $(x \neq 0) \implies x=1/(1/x)$ —

(1.16.a)

Domination $\mathbb{0}*x=\mathbb{0}$

(1) $\mathbb{0}*x=(\mathbb{0}+\mathbb{0})*x=\mathbb{0}*x+\mathbb{0}*x \;\blacksquare\; \mathbb{0}*x=\mathbb{0}*x+\mathbb{0}*x$	
(2) $\mathbb{0}*x=\mathbb{0}$	from: AdditiveIdentityUniqueness

(1.16.b)

NonDomination $(x \neq \mathbb{0} \wedge y \neq \mathbb{0}) \implies x*y \neq \mathbb{0}$

(1) $(x \neq \mathbb{0} \wedge y \neq \mathbb{0}) \implies \ldots$	
(1.1) $(x*y=0) \implies \ldots$	
(1.1.1) $\mathbb{1}=\mathbb{1}*\mathbb{1}=\big(x*(1/x)\big)*\big(y*(1/y)\big)=(x*y)*\big((1/x)*(1/y)\big)=\mathbb{0}*\big((1/x)*(1/y)\big)=\mathbb{0}$	from: 1, 1.1, Domination
(1.1.2) $\mathbb{1}=\mathbb{0} \wedge \mathbb{1} \neq \mathbb{0}$ ※	
(1.2) $(x*y=0) \implies \perp \;\blacksquare\; x*y \neq 0$	
(2) $(x \neq \mathbb{0} \wedge y \neq \mathbb{0}) \implies x*y \neq 0$	

(1.16.c)

NegationCommutativity $(-x)*y=-(x*y)=x*(-y)$

(1) $x*y+(-x)*y=(x+-x)*y=\mathbb{0}*y=\mathbb{0} \;\blacksquare\; x*y+(-x)*y=\mathbb{0}$	from: Domination wts: 2
(2) $(-x)*y=-(x*y)$	from: AdditiveInverseUniqueness
(3) $x*y+x*(-y)=x*(y+-y)=x*\mathbb{0}=\mathbb{0} \;\blacksquare\; x*y+x*(-y)=\mathbb{0}$	from: Domination wts: 4
(4) $x*(-y)=-(x*y)$	from: AdditiveInverseUniqueness
(5) $(-x)*y=-(x*y)=x*(-y)$	from: 2, 4

(1.16.d)

NegativeMultiplication $(-x)*(-y)=x*y$

(1) $(-x)*(-y)=-\big(x*(-y)\big)=-\big(-(x*y)\big)=x*y$	from: NegationCommutativity, DoubleNegative

(1.17)

$$\color{red}{OrderedField}(F,+,*,<) := \begin{pmatrix} \color{cyan}{Field}(F,+,*) \quad \wedge \quad \color{cyan}{Order}(<,F) \quad \wedge \\ \forall_{x,y,z\in F}(y<z \implies x+y<x+z) \quad \wedge \\ \forall_{x,y\in F}\big((x>\mathbb{0} \wedge y>\mathbb{0}) \implies x*y>\mathbb{0}\big) \end{pmatrix}$$

$$************************ \big(\color{cyan}{OrderedField}(F,+,*,<) \wedge x,y,z \in F\big) \implies \ldots ************************$$

(1.18.a)

NegationFlipsOrder $x > \mathbb{0} \iff -x < \mathbb{0}$

(1) $x > \mathbb{0} \implies \ldots$

(1.1) $\mathbb{0} = (-x)+x > (-x)+\mathbb{0} = -x$ ▌ $\mathbb{0} > -x$ ▌ $-x < \mathbb{0}$

(2) $x > \mathbb{0} \implies -x < \mathbb{0}$

(3) $-x < \mathbb{0} \implies \ldots$

(3.1) $\mathbb{0} = x+(-x) < x+\mathbb{0} = x$ ▌ $\mathbb{0} < x$ ▌ $x > \mathbb{0}$

(4) $-x < \mathbb{0} \implies x > \mathbb{0}$

(5) $x > \mathbb{0} \implies -x < \mathbb{0} \wedge -x < \mathbb{0} \implies x > \mathbb{0}$ ▌ $x > \mathbb{0} \iff -x < \mathbb{0}$ from: 2, 4

(1.18.a)

PositiveFactorPreservesOrder $(x > \mathbb{0} \wedge y < z) \implies x*y < x*z$

(1) $(x > \mathbb{0} \wedge y < z) \implies \ldots$

(1.1) $(-y)+z > (-y)+y = \mathbb{0}$ ▌ $z+(-y) = \mathbb{0}$

(1.2) $x*(z+(-y)) > \mathbb{0}$ ▌ $x*z+x*(-y) > \mathbb{0}$

(1.3) $x*z = \mathbb{0}+x*z = \big(x*y+-(x*y)\big)+x*z = \big(x*y+x*(-y)\big)+x*z = \ldots$ from: NegationCommutativity

(1.4) $x*y+\big(x*z+x*(-y)\big) > x*y+\mathbb{0} = x*y$ from: 1.2

(1.5) $x*z > x*y$ from: 1.3, 1.4

(2) $(x > \mathbb{0} \wedge y < z) \implies x*z > x*y$

Chapter 2

First Chapter

(1) First

  (1.1) Second

  (1.2) Third

(2) Fourth

This will be an empty chapter and I will put some text here

$$\sum_{i=0}^{\infty} a_i x^i \tag{2.1}$$

The equation 2.1 shows a sum that is divergent. This formula will later be used in the page **??**.
For further references see Something Linky or go to the next url: http://www.sharelatex.com or open the next file File.txt
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