

Mathematical Logic Notes

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Chapter 1

Trigonometric Identities

1.1 Fundamental Identities

- $x^2 + y^2 = r^2$
 - $\tan(x) = \frac{\sin(x)}{\cos(x)}$
 - $\sin(-x) = -\sin(x)$
 - $\cos(-x) = \cos(x)$
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1.2 Pythagorean identities

- $\sin^2(x) + \cos^2(x) = 1$
 - $1 = \sec^2(x) - \tan^2(x)$
 - $1 = \csc^2(x) - \cot^2(x)$
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1.3 Sum and difference formulas

- $\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$
 - $\sin(a-b) = \sin(a)\cos(b) - \cos(a)\sin(b)$
 - $\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$
 - $\cos(a-b) = \cos(a)\cos(b) + \sin(a)\sin(b)$
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1.4 Double-angle and half-angle formulas

- $\sin(2a) = 2\sin(a)\cos(a)$
 - $\cos(2a) = \cos^2(a) - \sin^2(a)$
 - $\sin(a/2) = \sqrt{(1/2)(1 - \cos(a))}$
 - $\cos(a/2) = \sqrt{(1/2)(1 + \cos(a))}$
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1.5 Tangent angle formulas

- $\tan(a+b) = \frac{\tan(a)+\tan(b)}{1-\tan(a)\tan(b)}$
 - $\tan(a-b) = \frac{\tan(a)-\tan(b)}{1+\tan(a)\tan(b)}$
 - $\tan(2a) = \frac{2\tan(a)}{1-\tan^2(a)}$
 - $\tan(a/2) = \frac{1-\cos(a)}{\sin(a)}$
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1.6 Products as sums formulas

$$\begin{aligned} - \sin(a)\cos(b) &= (1/2)(\sin(a+b) + \sin(a-b)) \\ - \cos(a)\sin(b) &= (1/2)(\sin(a+b) - \sin(a-b)) \\ - \cos(a)\cos(b) &= (1/2)(\cos(a+b) + \cos(a-b)) \\ - \sin(a)\sin(b) &= (-1/2)(\cos(a+b) - \cos(a-b)) \end{aligned}$$

1.7 Sums as products formulas

$$\begin{aligned} - \sin(a) + \sin(b) &= 2\sin((1/2)(a+b))\cos((1/2)(a-b)) \\ - \sin(a) - \sin(b) &= 2\sin((1/2)(a-b))\cos((1/2)(a+b)) \\ - \cos(a) + \cos(b) &= 2\cos((1/2)(a+b))\sin((1/2)(a-b)) \\ - \cos(a) - \cos(b) &= -2\cos((1/2)(a+b))\sin((1/2)(a-b)) \end{aligned}$$
