Contents

Chapter 1

Philosopherers

(1.5)
$OrderTrichotomy(<,S) := \forall_{x,y\in S}(x<y \veebar x=y \veebar y<x)$
$OrderTransitivity(<,S) := \forall_{x,y,z\in S}\big((x<y \wedge y<z) \implies x<z\big)$
$Order(<,S) := OrderTrichotomy(<,S) \wedge OrderTransitivity(<,S)$

(1.7)
$BoundedAbove(E,S,<) := Order(<,S) \wedge E\subset S \wedge \exists_{\beta\in S}\forall_{x\in E}(x\le\beta)$
$BoundedBelow(E,S,<) := Order(<,S) \wedge E\subset S \wedge \exists_{\beta\in S}\forall_{x\in E}(\beta\le x)$
$UpperBound(\beta,E,S,<) := Order(<,S) \wedge E\subset S \wedge \beta\in S \wedge \forall_{x\in E}(x\le\beta)$
$LowerBound(\beta,E,S,<) := Order(<,S) \wedge E\subset S \wedge \beta\in S \wedge \forall_{x\in E}(\beta\le x)$

(1.8)
$LUB(\alpha,E,S,<) := UpperBound(\alpha,E,S,<) \wedge \forall_{\gamma}\big(\gamma<\alpha \implies \neg UpperBound(\gamma,E,S,<)\big)$
$GLB(\alpha,E,S,<) := LowerBound(\alpha,E,S,<) \wedge \forall_{\beta}\big(\alpha<\beta \implies \neg LowerBound(\beta,E,S,<)\big)$

(1.10)
$LUBProperty(S,<) := \forall_{E}\Big(\big(\emptyset\ne E\subset S \wedge BoundedAbove(E,S,<)\big) \implies \exists_{\alpha\in S}\big(LUB(\alpha,E,S,<)\big)\Big)$
$GLBProperty(S,<) := \forall_{E}\Big(\big(\emptyset\ne E\subset S \wedge BoundedBelow(E,S,<)\big) \implies \exists_{\alpha\in S}\big(GLB(\alpha,E,S,<)\big)\Big)$

(1.11)
$\boxed{\text{LUBPropertyImpliesGLBProperty}}$ $LUBProperty(S,<) \implies GLBProperty(S,<)$

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(1) $LUBProperty(S,<) \implies \dots$ wts: 2

(1.1) $\big(\emptyset\ne B\subset S \wedge BoundedBelow(B,S,<)\big) \implies \dots$ wts: 1.2

(1.1.1) $Order(<,S) \wedge \exists_{\delta'\in S}\big(LowerBound(\delta',B,S,<)\big)$ from: BoundedBelow, 1.1

(1.1.2) $|B|=1 \implies \dots$ wts: 1.1.3

(1.1.2.1) $\exists_{u'}(u'\in B)$ ▮ $u := choice(\{u' | u'\in B\})$ ▮ $B=\{u\}$ from: 1.1.2

(1.1.2.2) $GLB(u,B,S,<)$ ▮ $\exists_{\epsilon_0\in S}\big(GLB(\epsilon_0,B,S,<)\big)$

(1.1.3) $|B|=1 \implies \exists_{\epsilon_0\in S}\big(GLB(\epsilon_0,B,S,<)\big)$

(1.1.4) $|B|\ne 1 \implies \dots$ wts: 1.1.5

(1.1.4.1) $\forall_{E}\Big(\big(\emptyset\ne E\subset S \wedge BoundedAbove(E,S,<)\big) \implies \exists_{\alpha\in S}\big(LUB(\alpha,E,S,<)\big)\Big)$ from: LUBProperty, 1

(1.1.4.2) $L := \{s\in S | LowerBound(s,B,S,<)\}$

(1.1.4.3) $|B|>1 \wedge OrderTrichotomy(<,S)$ ▮ $\exists_{b_1'\in B}\exists_{b_0'\in B}(b_0' < b_1')$ from: Order, 1.1.1 wts: 1.1.4.7

(1.1.4.4) $b_1 := choice\Big(\{b_1'\in B | \exists_{b_0'\in B}(b_0' < b_1')\}\Big)$ ▮ $\neg LowerBound(b_1,B,S,<)$ from: 1.1.4.2

(1.1.4.5) $b_1\notin L$ ▮ $L\subset S$

(1.1.4.6) $\delta := choice\big(\{\delta'\in S | LowerBound(\delta',B,S,<)\}\big)$ ▮ $\delta\in L$ ▮ $\emptyset\ne L$ from: 1.1.1

(1.1.4.7) $\emptyset\ne L\subset S$ from: 1.1.4.5, 1.1.4.6

(1.1.4.8) $\forall_{y\in L}\big(\mathit{LowerBound}(y,B,S,<)\big)$ ▮ $\forall_{y\in L}\forall_{x\in B}(y\leq x)$ — from: LowerBound, 1.1.4.2; wts: 1.1.4.10

(1.1.4.9) $\forall_{x\in B}\Big(x\in S\wedge\forall_{y\in L}(y\leq x)\Big)$ ▮ $\forall_{x\in B}\big(\mathit{UpperBound}(x,L,S,<)\big)$ — from: UpperBound

(1.1.4.10) $\exists_{x\in S}\big(\mathit{UpperBound}(x,L,S,<)\big)$ ▮ $\mathit{BoundedAbove}(L,S,<)$

(1.1.4.11) $\emptyset\neq L\subset S\wedge\mathit{BoundedAbove}(L,S,<)$ — from: 1.1.4.7, 1.1.4.10

(1.1.4.12) $\exists_{\alpha'\in S}\big(\mathit{LUB}(\alpha',L,S,<)\big)$ ▮ $\alpha:=\mathit{choice}\Big(\{\alpha'\in S|\big(\mathit{LUB}(\alpha',L,S,<)\big)\}\Big)$ — from: 1.1.4.1; wts: 1.1.4.21

(1.1.4.13) $\forall_x\big(x\in B\implies\mathit{UpperBound}(x,L,S,<)\big)$ — from: 1.1.4.9; wts: 1.1.4.17

(1.1.4.14) $\forall_x\big(\neg\mathit{UpperBound}(x,L,S,<)\implies x\notin B\big)$

(1.1.4.15) $\gamma<\alpha\implies\ldots$ — wts: 1.1.4.16

(1.1.4.15.1) $\neg\mathit{UpperBound}(\gamma,L,S,<)$ ▮ $\gamma\notin B$ — from: LUB, 1.1.4.12, 1.1.4.14

(1.1.4.16) $\gamma<\alpha\implies\gamma\notin B$ ▮ $\gamma\in B\implies\gamma\geq\alpha$

(1.1.4.17) $\forall_{\gamma\in B}(\alpha\leq\gamma)$ ▮ $\mathit{LowerBound}(\alpha,B,S,<)$ — from: LowerBound

(1.1.4.18) $\alpha<\beta\implies\ldots$ — wts: 1.1.4.19

(1.1.4.18.1) $\forall_{y\in L}(y\leq\alpha<\beta)$ ▮ $\forall_{y\in L}(y\neq\beta)$ — from: LUB, 1.1.4.12, 1.1.4.18

(1.1.4.18.2) $\beta\notin L$ ▮ $\neg\mathit{LowerBound}(\beta,B,S,<)$ — from: 1.1.4.2

(1.1.4.19) $\alpha<\beta\implies\neg\mathit{LowerBound}(\beta,B,S,<)$ ▮ $\forall_{\beta\in S}\big(\alpha<\beta\implies\neg\mathit{LowerBound}(\beta,B,S,<)\big)$

(1.1.4.20) $\mathit{LowerBound}(\alpha,B,S,<)\wedge\forall_{\beta\in S}\big(\alpha<\beta\implies\neg\mathit{LowerBound}(\beta,B,S,<)\big)$ — from: 1.1.4.17, 1.1.4.19

(1.1.4.21) $\mathit{GLB}(\alpha,B,S,<)$ ▮ $\exists_{\epsilon_1\in S}\big(\mathit{GLB}(\epsilon_1,B,S,<)\big)$

(1.1.5) $|B|\neq 1\implies\exists_{\epsilon_1\in S}\big(\mathit{GLB}(\epsilon_1,B,S,<)\big)$

(1.1.6) $\Big(|B|=1\implies\exists_{\epsilon_0\in S}\big(\mathit{GLB}(\epsilon_0,B,S,<)\big)\Big)\wedge\Big(|B|\neq 1\implies\exists_{\epsilon_1\in S}\big(\mathit{GLB}(\epsilon_1,B,S,<)\big)\Big)$ — from: 1.1.3, 1.1.5

(1.1.7) $(|B|=1\vee|B|\neq 1)\implies\exists_{\epsilon\in S}\big(\mathit{GLB}(\epsilon,B,S,<)\big)$ ▮ $\exists_{\epsilon\in S}\big(\mathit{GLB}(\epsilon,B,S,<)\big)$

(1.2) $\big(\emptyset\neq B\subset S\wedge\mathit{BoundedBelow}(B,S,<)\big)\implies\exists_{\epsilon\in S}\big(\mathit{GLB}(\epsilon,B,S,<)\big)$

(1.3) $\forall_B\Big(\big(\emptyset\neq B\subset S\wedge\mathit{BoundedBelow}(B,S,<)\big)\implies\exists_{\epsilon\in S}\big(\mathit{GLB}(\epsilon,B,S,<)\big)\Big)$

(1.4) $\mathit{GLBProperty}(S,<)$

(2) $\mathit{LUBProperty}(S,<)\implies\mathit{GLBProperty}(S,<)$

(1.12)

$$\mathit{Field}(F,+,*):=\exists_{\mathbb{0},\mathbb{1}\in F}\forall_{x,y,z\in F}\left(\begin{array}{l}x+y\in F\quad\wedge\quad x*y\in F\quad\wedge\\ x+y=y+x\quad\wedge\quad x*y=y*x\quad\wedge\\ (x+y)+z=x+(y+z)\quad\wedge\quad(x*y)*z=x*(y*z)\quad\wedge\\ \mathbb{1}\neq\mathbb{0}\quad\wedge\quad x*(y+z)=(x*y)+(x*z)\quad\wedge\\ \mathbb{0}+x=x\quad\wedge\quad\mathbb{1}*x=x\quad\wedge\\ \exists_{-x\in F}\big(x+(-x)=\mathbb{0}\big)\wedge\Big(x\neq\mathbb{0}\implies\exists_{1/x\in F}\big(x*(1/x)=\mathbb{1}\big)\Big)\end{array}\right)$$

*********************** $\big(\mathit{Field}(F,+,*)\wedge x,y,z\in F\big)\implies\ldots$ ***********************

(1.14)

AdditiveCancellation $(x+y=x+z)\implies y=z$

(1) $y=\mathbb{0}+y=\big(x+(-x)\big)+y=\big((-x)+x\big)+y=(-x)+(x+y)=\ldots$ — from: Field

(2) $(-x)+(x+z)=\big((-x)+x\big)+z=\big(x+(-x)\big)+z=\mathbb{0}+z=z$ — from: Field

AdditiveIdentityUniqueness $(x+y=x)\implies y=\mathbb{0}$

(1) $x+y=x=\mathbb{0}+x=x+\mathbb{0}$ — from: Field

(2) $y=0$ — from: AdditiveCancellation

$\boxed{\text{AdditiveInverseUniqueness}}$ $(x + y = \mathbb{0}) \implies y = -x$

(1) $x + y = \mathbb{0} = x + (-x)$	from: Field
(2) $y = -x$	from: AdditiveCancellation

$\boxed{\text{DoubleNegative}}$ $x = -(-x)$

(1) $\mathbb{0} = x + (-x) = (-x) + x$ ▮ $\mathbb{0} = (-x) + x$	from: Field
(2) $x = -(-x)$	from: AdditiveInverseUniqueness

(1.15)

$\boxed{\text{MultiplicativeCancellation}}$ $(x \neq 0 \wedge x * y = x * z) \implies y = z$ —

$\boxed{\text{MultiplicativeIdentityUniqueness}}$ $(x \neq 0 \wedge x * y = x) \implies y = \mathbb{1}$ —

$\boxed{\text{MultiplicativeInverseUniqueness}}$ $(x \neq 0 \wedge x * y = \mathbb{1}) \implies y = 1/x$ —

$\boxed{\text{DoubleReciprocal}}$ $(x \neq 0) \implies x = 1/(1/x)$ —

(1.16)

$\boxed{\text{Domination}}$ $\mathbb{0} * x = \mathbb{0}$

(1) $\mathbb{0} * x = (\mathbb{0} + \mathbb{0}) * x = \mathbb{0} * x + \mathbb{0} * x$ ▮ $\mathbb{0} * x = \mathbb{0} * x + \mathbb{0} * x$	from: Field
(2) $\mathbb{0} * x = \mathbb{0}$	from: AdditiveIdentityUniqueness

$\boxed{\text{NonDomination}}$ $(x \neq \mathbb{0} \wedge y \neq \mathbb{0}) \implies x * y \neq \mathbb{0}$

(1) $(x \neq \mathbb{0} \wedge y \neq \mathbb{0}) \implies \ldots$	
(1.1) $(x * y = 0) \implies \ldots$	
(1.1.1) $\mathbb{1} = \mathbb{1} * \mathbb{1} = \big(x * (1/x)\big) * \big(y * (1/y)\big) = (x * y) * \big((1/x) * (1/y)\big) = \mathbb{0} * \big((1/x) * (1/y)\big) = \mathbb{0}$	from: Field, Domination, 1, 1.1
(1.1.2) $\mathbb{1} = \mathbb{0} \wedge \mathbb{1} \neq \mathbb{0}$ ▮ $\perp$	from: Field
(1.2) $(x * y = 0) \implies \perp$ ▮ $x * y \neq 0$	
(2) $(x \neq \mathbb{0} \wedge y \neq \mathbb{0}) \implies x * y \neq 0$	

$\boxed{\text{NegationCommutativity}}$ $(-x) * y = -(x * y) = x * (-y)$

(1) $x * y + (-x) * y = (x + -x) * y = \mathbb{0} * y = \mathbb{0}$ ▮ $x * y + (-x) * y = \mathbb{0}$	from: Field, Domination wts: 2
(2) $(-x) * y = -(x * y)$	from: AdditiveInverseUniqueness
(3) $x * y + x * (-y) = x * (y + -y) = x * \mathbb{0} = \mathbb{0}$ ▮ $x * y + x * (-y) = \mathbb{0}$	from: Field, Domination wts: 4
(4) $x * (-y) = -(x * y)$	from: AdditiveInverseUniqueness
(5) $(-x) * y = -(x * y) = x * (-y)$	from: 2, 4

$\boxed{\text{NegativeMultiplication}}$ $(-x) * (-y) = x * y$

(1) $(-x) * (-y) = -\big(x * (-y)\big) = -\big(-(x * y)\big) = x * y$	from: NegationCommutativity, DoubleNegative

(1.17)

$$OrderedField(F, +, *, <) := \begin{pmatrix} Field(F, +, *) \quad \wedge \quad Order(<, F) \quad \wedge \\ \forall_{x,y,z \in F}(y < z \implies x + y < x + z) \quad \wedge \\ \forall_{x,y \in F}\big((x > \mathbb{0} \wedge y > \mathbb{0}) \implies x * y > \mathbb{0}\big) \end{pmatrix}$$

$$*********************** \ \big(OrderedField(F, +, *, <) \wedge x, y, z \in F\big) \implies \ldots \ ***********************$$

(1.18)

$\boxed{\text{NegationOnOrder}}$ $x > \mathbb{0} \iff -x < \mathbb{0}$

(1) $x > \mathbb{0} \implies \ldots$

(1.1) $\mathbb{0} = (-x) + x > (-x) + \mathbb{0} = -x \;\blacksquare\; \mathbb{0} > -x \;\blacksquare\; -x < \mathbb{0}$ — from: OrderedField

(2) $x > \mathbb{0} \implies -x < \mathbb{0}$

(3) $-x < \mathbb{0} \implies \ldots$

(3.1) $\mathbb{0} = x + (-x) < x + \mathbb{0} = x \;\blacksquare\; \mathbb{0} < x \;\blacksquare\; x > \mathbb{0}$ — from: OrderedField

(4) $-x < \mathbb{0} \implies x > \mathbb{0}$

(5) $x > \mathbb{0} \implies -x < \mathbb{0} \wedge -x < \mathbb{0} \implies x > \mathbb{0} \;\blacksquare\; x > \mathbb{0} \iff -x < \mathbb{0}$ — from: 2, 4

PositiveFactorPreservesOrder $(x > \mathbb{0} \wedge y < z) \implies x * y < x * z$

(1) $(x > \mathbb{0} \wedge y < z) \implies \ldots$

(1.1) $(-y) + z > (-y) + y = \mathbb{0} \;\blacksquare\; z + (-y) = \mathbb{0}$ — from: OrderedField

(1.2) $x * (z + (-y)) > \mathbb{0} \;\blacksquare\; x * z + x * (-y) > \mathbb{0}$ — from: OrderedField

(1.3) $x * z = \mathbb{0} + x * z = \big(x * y + -(x * y)\big) + x * z = \big(x * y + x * (-y)\big) + x * z = \ldots$ — from: Field, NegationCommutativity

(1.4) $x * y + \big(x * z + x * (-y)\big) > x * y + \mathbb{0} = x * y$ — from: Field, 1.2

(1.5) $x * z > x * y$ — from: 1.3, 1.4

(2) $(x > \mathbb{0} \wedge y < z) \implies x * z > x * y$

NegativeFactorFlipsOrder $(x < \mathbb{0} \wedge y < z) \implies x * y > x * z$

(1) $(x < \mathbb{0} \wedge y < z) \implies \ldots$

(1.1) $-x > \mathbb{0}$ — from: NegationOnOrder

(1.2) $(-x) * y < (-x) * z \;\blacksquare\; \mathbb{0} = x * y + (-x) * y < x * y + (-x) * z \;\blacksquare\; \mathbb{0} < x * y + (-x) * z$ — from: PositiveFactorPreservesOrder

(1.3) $\mathbb{0} < (-x) * (-y + z) \;\blacksquare\; \mathbb{0} > x * (-y + z) \;\blacksquare\; \mathbb{0} > -(x * y) + x * z$ — from: NegationOnOrder

(1.4) $x * y > x * z$

(2) $(x < \mathbb{0} \wedge y < z) \implies x * y > x * z$

SquareIsPositive $(x \neq \mathbb{0}) \implies x * x > \mathbb{0}$

(1) $(x > \mathbb{0}) \implies x * x > 0$ — from: OrderedField

(2) $(x < \mathbb{0}) \implies \ldots$

(2.1) $-x > \mathbb{0} \;\blacksquare\; x * x = (-x) * (-x) > \mathbb{0} \;\blacksquare\; x * x > \mathbb{0}$ — from: NegationOnOrder, OrderedField, NegativeMultiplication

(3) $(x < \mathbb{0}) \implies x * x > \mathbb{0}$

(4) $x \neq \mathbb{0} \implies (x > \mathbb{0} \vee x < \mathbb{0}) \implies x * x > \mathbb{0} \;\blacksquare\; x \neq \mathbb{0} \implies x * x > \mathbb{0}$ — from: OrderTrichotomy, 1, 3

OneIsPositive $\mathbb{1} > \mathbb{0}$

(1) $\mathbb{1} \neq \mathbb{0} \;\blacksquare\; \mathbb{1} = \mathbb{1} * \mathbb{1} > \mathbb{0}$ — from: Field, SquareIsPositive

ReciprocationOnOrder $(0 < x < y) \implies 0 < 1/y < 1/x$

(1) $(0 < x < y) \implies \ldots$

(1.1) $x * (1/x) = \mathbb{1} > \mathbb{0} \;\blacksquare\; x * (1/x) > 0$ — from: Field, OneIsPositive

(1.2) $1/x < \mathbb{0} \implies x * (1/x) < 0 \wedge x * (1/x) > 0 \implies \perp \;\blacksquare\; 1/x > \mathbb{0}$ — from: NegativeFactorFlipsOrder, 1

(1.3) $y * (1/y) = \mathbb{1} > \mathbb{0} \;\blacksquare\; y * (1/y) > 0$ — from: Field, OneIsPositive

(1.4) $1/y < \mathbb{0} \implies y * (1/y) < 0 \wedge y * (1/y) > 0 \implies \perp \;\blacksquare\; 1/y > \mathbb{0}$ — from: NegativeFactorFlipsOrder, 1

(1.5) $(1/x) * (1/y) > \mathbb{0}$ — from: OrderedField

(1.6) $\mathbb{0} < 1/y = \big((1/x) * (1/y)\big) * x < \big((1/x) * (1/y)\big) * y = 1/x$ — from: OrderedField, 1, 1.4, 1.5

(1.19)
FieldQ $OrderedField(\mathbb{Q},+,*,<)$ —

$Subfield(K,F,+,*) := Field(F,+,*) \land K \subset F \land Field(K,+,*)$
$OrderedSubfield(K,F,+,*,<) := OrderedField(F,+,*,<) \land K \subset F \land OrderedField(K,+,*,<)$
$CutI(\alpha) := \emptyset \neq \alpha \subset \mathbb{Q}$
$CutII(\alpha) := \forall_{p\in\alpha}\forall_{q\in\mathbb{Q}}(q < p \implies q \in \alpha)$
$CutIII(\alpha) := \forall_{p\in\alpha}\exists_{r\in\alpha}(p < r)$
$SetR := \mathbb{R} := \{\alpha \in \mathbb{Q} | CutI(\alpha) \land CutII(\alpha) \land CutIII(\alpha)\}$

CutCorollaryI $(\alpha \in \mathbb{R} \land p \in \alpha \land q \in \mathbb{Q} \land q \notin \alpha) \implies p < q$

(1) $(\alpha \in \mathbb{R} \land p \in \alpha \land q \in \mathbb{Q} \land q \notin \alpha) \implies \ldots$
(1.1) $\forall_{p'\in\alpha}\forall_{q'\in\mathbb{Q}}(q' < p' \implies q' \in \alpha)$ from: CutII, 1
(1.2) $q < p \implies q \in \alpha$ ▮ $q \notin \alpha \implies q \geq p$ from: 1
(1.3) $(q \notin \alpha) \implies \ldots$
(1.3.1) $q \geq p$ from: 1.2
(1.3.2) $(q = p) \implies (p \in \alpha \land p \notin \alpha) \implies \perp$ ▮ $q \neq p$ from: 1, 1.3
(1.3.3) $q \geq p \land q \neq p$ ▮ $p < q$
(1.4) $q \notin \alpha \implies p < q$ ▮ $p < q$ from: 1
(2) $(\alpha \in \mathbb{R} \land p \in \alpha \land q \in \mathbb{Q} \land q \notin \alpha) \implies p < q$

CutCorollaryII $(\alpha \in \mathbb{R} \land r, s \in \mathbb{Q} \land r < s \land r \notin \alpha) \implies s \notin \alpha$

(1) $(\alpha \in \mathbb{R} \land r, s \in \mathbb{Q} \land r < s \land r \notin \alpha) \implies \ldots$
(1.1) $\forall_{s'\in\alpha}\forall_{r'\in\mathbb{Q}}(r' < s' \implies r' \in \alpha)$ from: CutII, 1
(1.2) $s \in \alpha \implies \big(r \in \mathbb{Q} \implies (r < s \implies r \in \alpha)\big)$ ▮ $s \in \alpha \implies r \in \alpha$ from: 1, 1.1
(1.3) $r \notin \alpha \implies s \notin \alpha$ ▮ $s \notin \alpha$ from: 1, 1.2
(2) $(\alpha \in \mathbb{R} \land r, s \in \mathbb{Q} \land r < s \land r \notin \alpha) \implies s \notin \alpha$

$<R(\alpha,\beta) := \alpha,\beta \in \mathbb{R} \land \alpha \subset \beta$

OrderTrichotomyR $OrderTrichotomy(\mathbb{R}, <R)$

(1) $(\alpha,\beta \in \mathbb{R}) \implies \ldots$
(1.1) $\neg(\alpha < R\beta \lor \alpha = \beta) \implies \ldots$
(1.1.1) $\alpha \not\subset \beta \land \alpha \neq \beta$ from: <R, 1.1
(1.2) $\neg(\alpha < R\beta \lor \alpha = \beta) \implies \beta < R\alpha$
(2) $(\alpha,\beta \in \mathbb{R}) \implies (\alpha < R\beta \veebar \alpha = \beta \veebar \alpha < R\beta)$
(3) $\forall_{\alpha,\beta\in\mathbb{R}}(\alpha < R\beta \veebar \alpha = \beta \veebar \alpha < R\beta)$
(4) $OrderTrichotomy(\mathbb{R}, <R)$

OrderTransitivityR $OrderTransitivity(\mathbb{R}, <R)$

(1) $(\alpha,\beta,\gamma \in \mathbb{R}) \implies \ldots$
(1.1) 123123
(2) $(\alpha,\beta,\gamma \in \mathbb{R}) \implies \big((\alpha < R\beta \land \beta < R\gamma) \implies \alpha < R\gamma\big)$
(3) $\forall_{\alpha,\beta,\gamma\in\mathbb{R}}\big((\alpha < R\beta \land \beta < R\gamma) \implies \alpha < R\gamma\big)$
(4) $OrderTransitivity(\mathbb{R}, <R)$

OrderR $Order(<R, \mathbb{R})$ —

ExistenceOfR $\exists_{\mathbb{R}}\big(LUBProperty(\mathbb{R},<) \land OrderedSubfield(\mathbb{Q},\mathbb{R},+,*,<)\big)$

(1) 123123

TODO: - name all properties - hyperlink all definitions ???

Chapter 2

First Chapter

- (1) First
  - (1.1) Second
  - (1.2) Third
- (2) Fourth

This will be an empty chapter and I will put some text here

$$\sum_{i=0}^{\infty} a_i x^i \tag{2.1}$$

The equation 2.1 shows a sum that is divergent. This formula will later be used in the page **??**.
For further references see Something Linky or go to the next url: http://www.sharelatex.com or open the next file File.txt
It's also possible to link directly any word or any sentence in your document. supwithitSup With It Theorem
If you read this text, you will get no information. Really? Is there no information?

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