Contents

Chapter 1

Philosopherers

(1.5)

$\mathit{OrderTrichotomy}(<, S) := \forall_{x,y \in S}(x < y \veebar x = y \veebar y < x)$
$\mathit{OrderTransitivity}(<, S) := \forall_{x,y,z \in S}\big((x < y \wedge y < z) \implies x < z\big)$
$\mathit{Order}(<, S) := \mathit{OrderTrichotomy}(<, S) \wedge \mathit{OrderTransitivity}(<, S)$

(1.7)

$\mathit{BoundedAbove}(E, S, <) := \mathit{Order}(<, S) \wedge E \subset S \wedge \exists_{\beta \in S}\forall_{x \in E}(x \leq \beta)$
$\mathit{BoundedBelow}(E, S, <) := \mathit{Order}(<, S) \wedge E \subset S \wedge \exists_{\beta \in S}\forall_{x \in E}(\beta \leq x)$
$\mathit{UpperBound}(\beta, E, S, <) := \mathit{Order}(<, S) \wedge E \subset S \wedge \beta \in S \wedge \forall_{x \in E}(x \leq \beta)$
$\mathit{LowerBound}(\beta, E, S, <) := \mathit{Order}(<, S) \wedge E \subset S \wedge \beta \in S \wedge \forall_{x \in E}(\beta \leq x)$

(1.8)

$\mathit{LUB}(\alpha, E, S, <) := \mathit{UpperBound}(\alpha, E, S, <) \wedge \forall_{\gamma}\big(\gamma < \alpha \implies \neg \mathit{UpperBound}(\gamma, E, S, <)\big)$
$\mathit{GLB}(\alpha, E, S, <) := \mathit{LowerBound}(\alpha, E, S, <) \wedge \forall_{\beta}\big(\alpha < \beta \implies \neg \mathit{LowerBound}(\beta, E, S, <)\big)$

(1.10)

$\mathit{LUBProperty}(S, <) := \forall_E\Big(\big(\emptyset \neq E \subset S \wedge \mathit{BoundedAbove}(E, S, <)\big) \implies \exists_{\alpha \in S}\big(\mathit{LUB}(\alpha, E, S, <)\big)\Big)$
$\mathit{GLBProperty}(S, <) := \forall_E\Big(\big(\emptyset \neq E \subset S \wedge \mathit{BoundedBelow}(E, S, <)\big) \implies \exists_{\alpha \in S}\big(\mathit{GLB}(\alpha, E, S, <)\big)\Big)$

(1.11)

$\boxed{\text{LUBPropertyImpliesGLBProperty}}$ $\quad \mathit{LUBProperty}(S, <) \implies \mathit{GLBProperty}(S, <)$

(1) $\mathit{LUBProperty}(S, <) \implies \dots$ — wts: 2

(1.1) $\big(\emptyset \neq B \subset S \wedge \mathit{BoundedBelow}(B, S, <)\big) \implies \dots$ — wts: 1.2

(1.1.1) $\mathit{Order}(<, S) \wedge \exists_{\delta' \in S}\big(\mathit{LowerBound}(\delta', B, S, <)\big)$ — from: BoundedBelow, 1.1

(1.1.2) $|B| = 1 \implies \dots$ — wts: 1.1.3

(1.1.2.1) $\exists_{u'}(u' \in B)$ ▮ $u := \mathit{choice}(\{u' | u' \in B\})$ ▮ $B = \{u\}$ — from: 1.1.2

(1.1.2.2) $\mathit{GLB}(u, B, S, <)$ ▮ $\exists_{\epsilon_0 \in S}\big(\mathit{GLB}(\epsilon_0, B, S, <)\big)$

(1.1.3) $|B| = 1 \implies \exists_{\epsilon_0 \in S}\big(\mathit{GLB}(\epsilon_0, B, S, <)\big)$

(1.1.4) $|B| \neq 1 \implies \dots$ — wts: 1.1.5

(1.1.4.1) $\forall_E\Big(\big(\emptyset \neq E \subset S \wedge \mathit{BoundedAbove}(E, S, <)\big) \implies \exists_{\alpha \in S}\big(\mathit{LUB}(\alpha, E, S, <)\big)\Big)$ — from: LUBProperty, 1

(1.1.4.2) $L := \{s \in S | \mathit{LowerBound}(s, B, S, <)\}$

(1.1.4.3) $|B| > 1 \wedge \mathit{OrderTrichotomy}(<, S)$ ▮ $\exists_{b_1' \in B}\exists_{b_0' \in B}(b_0' < b_1')$ — from: Order, 1.1.1; wts: 1.1.4.7

(1.1.4.4) $b_1 := \mathit{choice}\Big(\{b_1' \in B | \exists_{b_0' \in B}(b_0' < b_1')\}\Big)$ ▮ $\neg \mathit{LowerBound}(b_1, B, S, <)$ — from: 1.1.4.2

(1.1.4.5) $b_1 \notin L$ ▮ $L \subset S$

(1.1.4.6) $\delta := \mathit{choice}\big(\{\delta' \in S | \mathit{LowerBound}(\delta', B, S, <)\}\big)$ ▮ $\delta \in L$ ▮ $\emptyset \neq L$ — from: 1.1.1

(1.1.4.7) $\emptyset \neq L \subset S$ — from: 1.1.4.5, 1.1.4.6

(1.1.4.8) $\forall_{y \in L}\big(LowerBound(y, B, S, <)\big) \blacksquare \forall_{y \in L}\forall_{x \in B}(y \leq x)$ from: LowerBound, 1.1.4.2 wts: 1.1.4.10

(1.1.4.9) $\forall_{x \in B}\Big(x \in S \land \forall_{y \in L}(y \leq x)\Big) \blacksquare \forall_{x \in B}\big(UpperBound(x, L, S, <)\big)$ from: UpperBound

(1.1.4.10) $\exists_{x \in S}\big(UpperBound(x, L, S, <)\big) \blacksquare BoundedAbove(L, S, <)$

(1.1.4.11) $\emptyset \neq L \subset S \land BoundedAbove(L, S, <)$ from: 1.1.4.7, 1.1.4.10

(1.1.4.12) $\exists_{\alpha' \in S}\big(LUB(\alpha', L, S, <)\big) \blacksquare \alpha := choice\Big(\{\alpha' \in S | \big(LUB(\alpha', L, S, <)\big)\}\Big)$ from: 1.1.4.1 wts: 1.1.4.21

(1.1.4.13) $\forall_x\big(x \in B \implies UpperBound(x, L, S, <)\big)$ from: 1.1.4.9 wts: 1.1.4.17

(1.1.4.14) $\forall_x\big(\neg UpperBound(x, L, S, <) \implies x \notin B\big)$

(1.1.4.15) $\gamma < \alpha \implies \ldots$ wts: 1.1.4.16

(1.1.4.15.1) $\neg UpperBound(\gamma, L, S, <) \blacksquare \gamma \notin B$ from: LUB, 1.1.4.12, 1.1.4.14

(1.1.4.16) $\gamma < \alpha \implies \gamma \notin B \blacksquare \gamma \in B \implies \gamma \geq \alpha$

(1.1.4.17) $\forall_{\gamma \in B}(\alpha \leq \gamma) \blacksquare LowerBound(\alpha, B, S, <)$ from: LowerBound

(1.1.4.18) $\alpha < \beta \implies \ldots$ wts: 1.1.4.19

(1.1.4.18.1) $\forall_{y \in L}(y \leq \alpha < \beta) \blacksquare \forall_{y \in L}(y \neq \beta)$ from: LUB, 1.1.4.12, 1.1.4.18

(1.1.4.18.2) $\beta \notin L \blacksquare \neg LowerBound(\beta, B, S, <)$ from: 1.1.4.2

(1.1.4.19) $\alpha < \beta \implies \neg LowerBound(\beta, B, S, <) \blacksquare \forall_{\beta \in S}\big(\alpha < \beta \implies \neg LowerBound(\beta, B, S, <)\big)$

(1.1.4.20) $LowerBound(\alpha, B, S, <) \land \forall_{\beta \in S}\big(\alpha < \beta \implies \neg LowerBound(\beta, B, S, <)\big)$ from: 1.1.4.17, 1.1.4.19

(1.1.4.21) $GLB(\alpha, B, S, <) \blacksquare \exists_{\epsilon_1 \in S}\big(GLB(\epsilon_1, B, S, <)\big)$

(1.1.5) $|B| \neq 1 \implies \exists_{\epsilon_1 \in S}\big(GLB(\epsilon_1, B, S, <)\big)$

(1.1.6) $\Big(|B| = 1 \implies \exists_{\epsilon_0 \in S}\big(GLB(\epsilon_0, B, S, <)\big)\Big) \land \Big(|B| \neq 1 \implies \exists_{\epsilon_1 \in S}\big(GLB(\epsilon_1, B, S, <)\big)\Big)$ from: 1.1.3, 1.1.5

(1.1.7) $(|B| = 1 \lor |B| \neq 1) \implies \exists_{\epsilon \in S}\big(GLB(\epsilon, B, S, <)\big) \blacksquare \exists_{\epsilon \in S}\big(GLB(\epsilon, B, S, <)\big)$

(1.2) $\big(\emptyset \neq B \subset S \land BoundedBelow(B, S, <)\big) \implies \exists_{\epsilon \in S}\big(GLB(\epsilon, B, S, <)\big)$

(1.3) $\forall_B\Big(\big(\emptyset \neq B \subset S \land BoundedBelow(B, S, <)\big) \implies \exists_{\epsilon \in S}\big(GLB(\epsilon, B, S, <)\big)\Big)$

(1.4) $GLBProperty(S, <)$

(2) $LUBProperty(S, <) \implies GLBProperty(S, <)$

(1.12)

$$Field(F, +, *) := \exists_{\mathbb{0}, \mathbb{1} \in F}\forall_{x,y,z \in F}\begin{pmatrix} x + y \in F \quad \land \quad x * y \in F \quad \land \\ x + y = y + x \quad \land \quad x * y = y * x \quad \land \\ (x + y) + z = x + (y + z) \quad \land \quad (x * y) * z = x * (y * z) \quad \land \\ \mathbb{1} \neq \mathbb{0} \quad \land \quad x * (y + z) = (x * y) + (x * z) \quad \land \\ \mathbb{0} + x = x \quad \land \quad \mathbb{1} * x = x \quad \land \\ \exists_{-x \in F}\big(x + (-x) = \mathbb{0}\big) \land \Big(x \neq \mathbb{0} \implies \exists_{1/x \in F}\big(x * (1/x) = \mathbb{1}\big)\Big) \end{pmatrix}$$

$$************************ \big(Field(F, +, *) \land x, y, z \in F\big) \implies \ldots ************************$$

(1.14)

$\boxed{\text{AdditiveCancellation}}$ $(x + y = x + z) \implies y = z$

(1) $y = \mathbb{0} + y = \big(x + (-x)\big) + y = \big((-x) + x\big) + y = (-x) + (x + y) = \ldots$ from: Field

(2) $(-x) + (x + z) = \big((-x) + x\big) + z = \big(x + (-x)\big) + z = \mathbb{0} + z = z$ from: Field

$\boxed{\text{AdditiveIdentityUniqueness}}$ $(x + y = x) \implies y = \mathbb{0}$

(1) $x + y = x = \mathbb{0} + x = x + \mathbb{0}$ from: Field

(2) $y = 0$ from: AdditiveCancellation

$\boxed{\text{AdditiveInverseUniqueness}}$ $(x + y = \mathbb{0}) \implies y = -x$

(1) $x + y = \mathbb{0} = x + (-x)$ — from: Field

(2) $y = -x$ — from: AdditiveCancellation

$\boxed{\text{DoubleNegative}}$ $x = -(-x)$

(1) $\mathbb{0} = x + (-x) = (-x) + x$ ▮ $\mathbb{0} = (-x) + x$ — from: Field

(2) $x = -(-x)$ — from: AdditiveInverseUniqueness

(1.15)

$\boxed{\text{MultiplicativeCancellation}}$ $(x \neq 0 \land x * y = x * z) \implies y = z$ —

$\boxed{\text{MultiplicativeIdentityUniqueness}}$ $(x \neq 0 \land x * y = x) \implies y = \mathbb{1}$ —

$\boxed{\text{MultiplicativeInverseUniqueness}}$ $(x \neq 0 \land x * y = \mathbb{1}) \implies y = 1/x$ —

$\boxed{\text{DoubleReciprocal}}$ $(x \neq 0) \implies x = 1/(1/x)$ —

(1.16)

$\boxed{\text{Domination}}$ $\mathbb{0} * x = \mathbb{0}$

(1) $\mathbb{0} * x = (\mathbb{0} + \mathbb{0}) * x = \mathbb{0} * x + \mathbb{0} * x$ ▮ $\mathbb{0} * x = \mathbb{0} * x + \mathbb{0} * x$ — from: Field

(2) $\mathbb{0} * x = \mathbb{0}$ — from: AdditiveIdentityUniqueness

$\boxed{\text{NonDomination}}$ $(x \neq \mathbb{0} \land y \neq \mathbb{0}) \implies x * y \neq \mathbb{0}$

(1) $(x \neq \mathbb{0} \land y \neq \mathbb{0}) \implies \ldots$

(1.1) $(x * y = 0) \implies \ldots$

(1.1.1) $\mathbb{1} = \mathbb{1} * \mathbb{1} = \big(x * (1/x)\big) * \big(y * (1/y)\big) = (x * y) * \big((1/x) * (1/y)\big) = \mathbb{0} * \big((1/x) * (1/y)\big) = \mathbb{0}$ — from: Field, Domination, 1, 1.1

(1.1.2) $\mathbb{1} = \mathbb{0} \land \mathbb{1} \neq \mathbb{0}$ ▮ $\perp$ — from: Field

(1.2) $(x * y = 0) \implies \perp$ ▮ $x * y \neq 0$

(2) $(x \neq \mathbb{0} \land y \neq \mathbb{0}) \implies x * y \neq 0$

$\boxed{\text{NegationCommutativity}}$ $(-x) * y = -(x * y) = x * (-y)$

(1) $x * y + (-x) * y = (x + -x) * y = \mathbb{0} * y = \mathbb{0}$ ▮ $x * y + (-x) * y = \mathbb{0}$ — from: Field, Domination; wts: 2

(2) $(-x) * y = -(x * y)$ — from: AdditiveInverseUniqueness

(3) $x * y + x * (-y) = x * (y + -y) = x * \mathbb{0} = \mathbb{0}$ ▮ $x * y + x * (-y) = \mathbb{0}$ — from: Field, Domination; wts: 4

(4) $x * (-y) = -(x * y)$ — from: AdditiveInverseUniqueness

(5) $(-x) * y = -(x * y) = x * (-y)$ — from: 2, 4

$\boxed{\text{NegativeMultiplication}}$ $(-x) * (-y) = x * y$

(1) $(-x) * (-y) = -\big(x * (-y)\big) = -\big(-(x * y)\big) = x * y$ — from: NegationCommutativity, DoubleNegative

(1.17)

$$OrderedField(F, +, *, <) := \begin{pmatrix} Field(F, +, *) \quad \land \quad Order(<, F) \quad \land \\ \forall_{x,y,z \in F}(y < z \implies x + y < x + z) \quad \land \\ \forall_{x,y \in F}\big((x > \mathbb{0} \land y > \mathbb{0}) \implies x * y > \mathbb{0}\big) \end{pmatrix}$$

$$*********************** \ \big(OrderedField(F, +, *, <) \land x, y, z \in F\big) \implies \ldots \ ***********************$$

(1.18)

$\boxed{\text{NegationOnOrder}}$ $x > \mathbb{0} \iff -x < \mathbb{0}$

(1) $x > \mathbb{0} \implies \dots$

(1.1) $\mathbb{0} = (-x) + x > (-x) + \mathbb{0} = -x$ ▮ $\mathbb{0} > -x$ ▮ $-x < \mathbb{0}$ — from: OrderedField

(2) $x > \mathbb{0} \implies -x < \mathbb{0}$

(3) $-x < \mathbb{0} \implies \dots$

(3.1) $\mathbb{0} = x + (-x) < x + \mathbb{0} = x$ ▮ $\mathbb{0} < x$ ▮ $x > \mathbb{0}$ — from: OrderedField

(4) $-x < \mathbb{0} \implies x > \mathbb{0}$

(5) $x > \mathbb{0} \implies -x < \mathbb{0} \wedge -x < \mathbb{0} \implies x > \mathbb{0}$ ▮ $x > \mathbb{0} \iff -x < \mathbb{0}$ — from: 2, 4

PositiveFactorPreservesOrder $(x > \mathbb{0} \wedge y < z) \implies x * y < x * z$

(1) $(x > \mathbb{0} \wedge y < z) \implies \dots$

(1.1) $(-y) + z > (-y) + y = \mathbb{0}$ ▮ $z + (-y) = \mathbb{0}$ — from: OrderedField

(1.2) $x * (z + (-y)) > \mathbb{0}$ ▮ $x * z + x * (-y) > \mathbb{0}$ — from: OrderedField

(1.3) $x * z = \mathbb{0} + x * z = \big(x * y + -(x * y)\big) + x * z = \big(x * y + x * (-y)\big) + x * z = \dots$ — from: Field, NegationCommutativity

(1.4) $x * y + \big(x * z + x * (-y)\big) > x * y + \mathbb{0} = x * y$ — from: Field, 1.2

(1.5) $x * z > x * y$ — from: 1.3, 1.4

(2) $(x > \mathbb{0} \wedge y < z) \implies x * z > x * y$

NegativeFactorFlipsOrder $(x < \mathbb{0} \wedge y < z) \implies x * y > x * z$

(1) $(x < \mathbb{0} \wedge y < z) \implies \dots$

(1.1) $-x > \mathbb{0}$ — from: NegationOnOrder

(1.2) $(-x) * y < (-x) * z$ ▮ $\mathbb{0} = x * y + (-x) * y < x * y + (-x) * z$ ▮ $\mathbb{0} < x * y + (-x) * z$ — from: PositiveFactorPreservesOrder

(1.3) $\mathbb{0} < (-x) * (-y + z)$ ▮ $\mathbb{0} > x * (-y + z)$ ▮ $\mathbb{0} > -(x * y) + x * z$ — from: NegationOnOrder

(1.4) $x * y > x * z$

(2) $(x < \mathbb{0} \wedge y < z) \implies x * y > x * z$

SquareIsPositive $(x \neq \mathbb{0}) \implies x * x > \mathbb{0}$

(1) $(x > \mathbb{0}) \implies x * x > 0$ — from: OrderedField

(2) $(x < \mathbb{0}) \implies \dots$

(2.1) $-x > \mathbb{0}$ ▮ $x * x = (-x) * (-x) > \mathbb{0}$ ▮ $x * x > \mathbb{0}$ — from: NegationOnOrder, OrderedField, NegativeMultiplication

(3) $(x < \mathbb{0}) \implies x * x > \mathbb{0}$

(4) $x \neq \mathbb{0} \implies (x > \mathbb{0} \vee x < \mathbb{0}) \implies x * x > \mathbb{0}$ ▮ $x \neq \mathbb{0} \implies x * x > \mathbb{0}$ — from: OrderTrichotomy, 1, 3

OneIsPositive $\mathbb{1} > \mathbb{0}$

(1) $\mathbb{1} \neq \mathbb{0}$ ▮ $\mathbb{1} = \mathbb{1} * \mathbb{1} > \mathbb{0}$ — from: Field, SquareIsPositive

ReciprocationOnOrder $(0 < x < y) \implies 0 < 1/y < 1/x$

(1) $(0 < x < y) \implies \dots$

(1.1) $x * (1/x) = \mathbb{1} > \mathbb{0}$ ▮ $x * (1/x) > 0$ — from: Field, OneIsPositive

(1.2) $1/x < \mathbb{0} \implies x * (1/x) < 0 \wedge x * (1/x) > 0 \implies \bot$ ▮ $1/x > \mathbb{0}$ — from: NegativeFactorFlipsOrder, 1

(1.3) $y * (1/y) = \mathbb{1} > \mathbb{0}$ ▮ $y * (1/y) > 0$ — from: Field, OneIsPositive

(1.4) $1/y < \mathbb{0} \implies y * (1/y) < 0 \wedge y * (1/y) > 0 \implies \bot$ ▮ $1/y > \mathbb{0}$ — from: NegativeFactorFlipsOrder, 1

(1.5) $(1/x) * (1/y) > \mathbb{0}$ — from: OrderedField

(1.6) $\mathbb{0} < 1/y = \big((1/x) * (1/y)\big) * x < \big((1/x) * (1/y)\big) * y = 1/x$ — from: OrderedField, 1, 1.4, 1.5

(1.19)
FieldQ $OrderedField(\mathbb{Q},+,*,<)$ —

$Subfield(K,F,+,*) := Field(F,+,*) \wedge K \subset F \wedge Field(K,+,*)$
$OrderedSubfield(K,F,+,*,<) := OrderedField(F,+,*,<) \wedge K \subset F \wedge OrderedField(K,+,*,<)$
$CutI(\alpha) := \emptyset \neq \alpha \subset \mathbb{Q}$
$CutII(\alpha) := \forall_{p\in\alpha}\forall_{q\in\mathbb{Q}}(q < p \implies q \in \alpha)$
$CutIII(\alpha) := \forall_{p\in\alpha}\exists_{r\in\alpha}(p < r)$
$SetR := \mathbb{R} := \{\alpha \in \mathbb{Q} | CutI(\alpha) \wedge CutII(\alpha) \wedge CutIII(\alpha)\}$

CutCorollaryI $(\alpha \in \mathbb{R} \wedge p \in \alpha \wedge q \in \mathbb{Q} \wedge q \notin \alpha) \implies p < q$

(1) $(\alpha \in \mathbb{R} \wedge p \in \alpha \wedge q \in \mathbb{Q} \wedge q \notin \alpha) \implies \ldots$
(1.1) $\forall_{p'\in\alpha}\forall_{q'\in\mathbb{Q}}(q' < p' \implies q' \in \alpha)$ from: CutII, 1
(1.2) $q < p \implies q \in \alpha$ ▌ $q \notin \alpha \implies q \geq p$ from: 1
(1.3) $(q \notin \alpha) \implies \ldots$
(1.3.1) $q \geq p$ from: 1.2
(1.3.2) $(q = p) \implies (p \in \alpha \wedge p \notin \alpha) \implies \perp$ ▌ $q \neq p$ from: 1, 1.3
(1.3.3) $q \geq p \wedge q \neq p$ ▌ $p < q$
(1.4) $q \notin \alpha \implies p < q$ ▌ $p < q$ from: 1
(2) $(\alpha \in \mathbb{R} \wedge p \in \alpha \wedge q \in \mathbb{Q} \wedge q \notin \alpha) \implies p < q$

CutCorollaryII $(\alpha \in \mathbb{R} \wedge r, s \in \mathbb{Q} \wedge r < s \wedge r \notin \alpha) \implies s \notin \alpha$

(1) $(\alpha \in \mathbb{R} \wedge r, s \in \mathbb{Q} \wedge r < s \wedge r \notin \alpha) \implies \ldots$
(1.1) $\forall_{s'\in\alpha}\forall_{r'\in\mathbb{Q}}(r' < s' \implies r' \in \alpha)$ from: CutII, 1
(1.2) $s \in \alpha \implies \big(r \in \mathbb{Q} \implies (r < s \implies r \in \alpha)\big)$ ▌ $s \in \alpha \implies r \in \alpha$ from: 1, 1.1
(1.3) $r \notin \alpha \implies s \notin \alpha$ ▌ $s \notin \alpha$ from: 1, 1.2
(2) $(\alpha \in \mathbb{R} \wedge r, s \in \mathbb{Q} \wedge r < s \wedge r \notin \alpha) \implies s \notin \alpha$

$<<(\alpha, \beta) := \alpha, \beta \in \mathbb{R} \wedge \alpha \subset \beta$

OrderTrichotomyR $OrderTrichotomy(\mathbb{R}, <<)$

(1) $(\alpha, \beta \in \mathbb{R}) \implies \ldots$
(1.1) $\neg(\alpha << \beta \vee \alpha = \beta) \implies \ldots$
(1.1.1) $\alpha \not\subset \beta \wedge \alpha \neq \beta$ from: «, 1.1
(1.1.2) $\exists_{p'}(p' \in \alpha \wedge p' \notin \beta)$ ▌ $p := \{p' | p' \in \alpha \wedge p' \notin \beta\}$
(1.1.3) $q \in \beta \implies \ldots$
(1.1.3.1) 123123
(1.1.4) $q \in \beta \implies q \in \alpha$
(1.1.5) $\forall_{q\in\beta}(q \in \alpha)$ ▌ $\beta \subseteq \alpha$
(1.1.6) $\beta \subset \alpha$ ▌ $\beta << \alpha$
(1.2) $\neg(\alpha << \beta \vee \alpha = \beta) \implies \beta << \alpha$
(1.3) $\ldots$
(1.4) $\neg(\alpha << \beta \vee \alpha = \beta) \vee (\alpha << \beta \vee \alpha = \beta)$ ▌ $(\beta << \alpha) \vee (\alpha << \beta \vee \alpha = \beta)$
(1.5) 123
(2) $(\alpha, \beta \in \mathbb{R}) \implies (\alpha << \beta \veebar \alpha = \beta \veebar \alpha << \beta)$
(3) $\forall_{\alpha,\beta\in\mathbb{R}}(\alpha << \beta \veebar \alpha = \beta \veebar \alpha << \beta)$
(4) $OrderTrichotomy(\mathbb{R}, <<)$

OrderTransitivityR $OrderTransitivity(\mathbb{R}, <<)$

(1) $(\alpha, \beta, \gamma \in \mathbb{R}) \implies \ldots$

(1.1) 123123

(2) $(\alpha, \beta, \gamma \in \mathbb{R}) \implies \big((\alpha << \beta \land \beta << \gamma) \implies \alpha << \gamma\big)$

(3) $\forall_{\alpha,\beta,\gamma\in\mathbb{R}}\big((\alpha << \beta \land \beta << \gamma) \implies \alpha << \gamma\big)$

(4) $OrderTransitivity(\mathbb{R}, <<)$

OrderR $Order(<<, \mathbb{R})$ —

ExistenceOfR $\exists_{\mathbb{R}}\big(LUBProperty(\mathbb{R}, <) \land OrderedSubfield(\mathbb{Q}, \mathbb{R}, +, *, <)\big)$

(1) 123123

TODO: - name all properties - hyperlink all definitions ???

Chapter 2

First Chapter

(1) First

  (1.1) Second

  (1.2) Third

(2) Fourth

This will be an empty chapter and I will put some text here

$$\sum_{i=0}^{\infty} a_i x^i \tag{2.1}$$

The equation 2.1 shows a sum that is divergent. This formula will later be used in the page **??**.
For further references see Something Linky or go to the next url: http://www.sharelatex.com or open the next file File.txt
It's also possible to link directly any word or any sentence in your document. supwithitSup With It Theorem
If you read this text, you will get no information. Really? Is there no information?

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