Answer the following questions completely.

1. Let W be a subset of $\mathbb{R}_3$ defined by $W = \left\{ \begin{bmatrix} a-b+2c \\ 2a-2c+3d \\ a+b+d \end{bmatrix} |a,b,c,d \in \mathbb{R} \right\}$.

   (a) Find a set S that spans W.

   (b) Find the subset of S that forms a basis for W. What is the dimension of W?

2. Recall that the set $M_{3,3}$ is the set of all 3×3 matrices. Let W be the set of all diagonal 3×3 matrices.

   (a) Show that W is a subspace of $M_{3,3}$.

   (b) Find a basis for W. *(5 pts)*

3. Let $V = M_{3,3}$. A matrix is *circulant*, if the $(i+1)st$ row can be obtained by shifting the ith row entries on place to the right with a wrap around. Examples of 3×3 circulant matrices are:

$$\begin{bmatrix} 2 & -1 & 3 \\ 3 & 2 & -1 \\ -1 & 3 & 2 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

   Let W be the set of all circulant 3×3 matrices.

   (a) Show that W is a subspace of $M_{3,3}$.

   (b) Find a basis for W.

   (c) What is the dimension of W?

4. Find a basis for $\mathbb{R}^3$ that includes $(1,2,3)$.

5. Find a basis for $\mathbb{R}^3$ that is a subset of

$$S = \{(1,0,2), (-3,-4,4), (1,1,-1), (2,1,3)\}.$$

6. Given $A = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 1 & -4 & -5 \end{bmatrix}$.

   (a) Find the rank of A.

   (b) Is A singular or nonsingular? Support your answer.

   (c) Give a basis for the row space of A.

   (d) Find the solution set of the homogenous system $AX = O$. What is the dimension of this solution set?

7. Suppose $S = \{\alpha_1, \alpha_2, \alpha_3\}$ is a linearly independent set of vectors in a vector space V. Prove that $T = \{\beta_1, \beta_2, \beta_3\}$ is also linearly independent set if $\beta_1 = \alpha_1 + \alpha_2$, $\beta_2 = \alpha_1 + \alpha_3$ and $\beta_3 = \alpha_2 + \alpha_3$.

8. Let A be an $n \times n$ matrix and λ be a scalar. Show that the set W consisting of all vectors $\alpha \in \mathbb{R}^n$ such that $A\alpha = \lambda\alpha$ is a subspace of $\mathbb{R}_n$.