ABSTAL1 QUIZ 4 November 27, 2014

Answer the following as indicated and in the given order. **SHOW YOUR SOLUTIONS.**

1. Consider the Cayley table below for a group $(G, *)$. (5 Points each)

$*$	e	r	s	x	y	z
e	e	r	s	x	y	z
r	r	s	e	y	z	x
s	s	e	r	z	x	y
x	x	z	y	e	s	r
y	y	x	z	r	e	s
z	z	y	x	s	r	e

   a. Find the centralizers of each element.
   b. Find all the subgroups generated by each element.
   c. Determine if $< r >$ is a normal subgroup by finding its distinct left and right cosets.
   d. Find all the conjugacy classes of G.
   e. Construct the Cayley table for the distinct right cosets of $H =< s >$ in G.

2. Let $G = (\mathbb{Z}_{20}, +), G' = (\mathbb{Z}_4, +)$ and define the homomorphism $\phi : G \to G'$ by

$$\phi(g) = \begin{cases} 0 & \text{if } 4 \text{ divides } g \\ 1 & \text{if } 4 \text{ divides } g+1 \\ 2 & \text{if } 4 \text{ divides } g+2 \\ 3 & \text{if } 4 \text{ divides } g+3 \end{cases}$$

   a. Find $K = Ker\ \phi$. (4 Points)
   b. Show that $K \lhd G$. (8 Points)
   c. Find $\phi(G)$. (4 Points)
   d. Construct the group table for G/K. (6 Points)
   e. Construct the group table for $\phi(G)$. (4 Points)
   f. If $\mu : G/K \to \phi(G)$ is given by $\mu(Kg) = \phi(g)$, find the image of each element of G/K under μ. (4 Points)