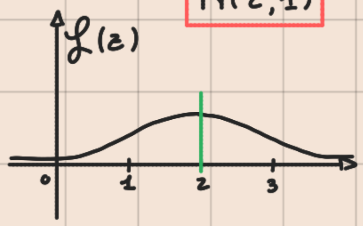
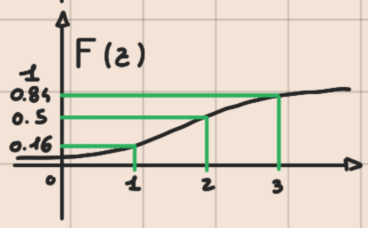
DISTRIBUZIONE NORMALE (o GAUSSIANA)

DISTRIBUZIONE NORMALE

$\boxed{N(2,1)}$





UNA V.A.A.C. X HA DISTRIBUZIONE NORMALE DI PARAMETRI:

$\mu \in \mathbb{R}$ E $\sigma^2 \in \mathbb{N}$ SE LA SUA DENSITÀ DI PROBABILITÀ f È DATA DA:

$$f(x) = \frac{1}{\sigma \cdot \sqrt{2\pi}} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

NOTAZIONE: $X \sim N(\mu, \sigma^2)$

NOTA: LA SUA IMPORTANZA È DOVUTA AL RUOLO CHE QUESTA DISTRIBUZIONE HA NEL TEOREMA DEL LIMITE CENTRALE.

DISTRIBUZIONE NORMALE - STANDARD

LA VARIABILE ALEATORIA $\boxed{Z \sim N(0,1)}$ È DETTA NORMALE STANDARD. CON f DEFINITA COME:

$$f(z) = \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{z^2}{2}}$$

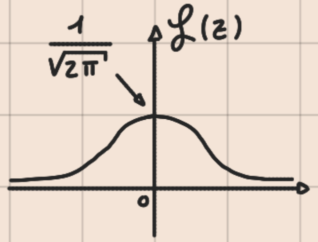
MENTRE LA FUNZIONE DI DISTRIBUZIONE È DEFINITA COME:

$$\Phi(z) = F_Z(z) = P(Z \leq z) = \frac{1}{\sqrt{2\pi}} \cdot \int_{-\infty}^{z} e^{-\frac{y^2}{2}} \cdot dy$$

INDIMOSTRABILE

DATO CHE NON POSSIAMO UTILIZZARLA DIRETTAMENTE, USIAMO INVECE LA TABELLA.

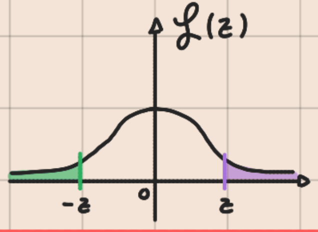
GRAFICO:



→ LA FUNZIONE f È PARI.

→ LE CODE CADONO ESPONENZIALMENTE, ANZI, ANCORA DI PIÙ.

→ OTTENUTA TRAMITE DERIVAZIONE EMPIRICA.



→ CASO z NEGATIVA:

→ LE DUE AREE SONO IDENTICHE!

$$\boxed{\Phi(-z) = 1 - \Phi(z)}$$

ESERCIZI D. NORMALE STANDARD

La funzione è pari

1) $P(Z > 0) = 1 - P(Z \leq 0) = 1 - F(0) = 1 - \phi(0) = 1 - 0.5 = \boxed{0.5}$.

2) $P(Z \geq 1.07) = 1 - P(Z < 1.07) = 1 - F(1.07) = 1 - \phi(1.07) = \boxed{1 - 0.85769}$.

Esempio z negativa: $\boxed{\phi(-z) = 1 - \phi(z)}$

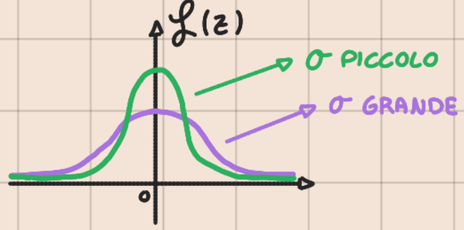
3) $P(Z < -1.45) = \phi(-1.45) = 1 - \phi(1.45) = \boxed{1 - 0.92647}$.

4) $P(Z > -0.06) = 1 - \phi(-0.06) = 1 - (1 - \phi(0.06)) = \phi(0.06) = \boxed{0.52392}$.

DISTRIBUZIONE NORMALE - GENERICA

• Significato del parametro: σ

Data $X \sim N(0, \sigma^2)$ teniamo μ a zero:



Abbiamo che:

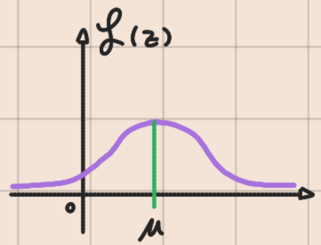
→ Se σ aumenta la campana si abbassa, quindi le code si alzano.

→ Viceversa se σ diminuisce la campana si alza, quindi le code si abbassano.

Formula con $\mu = 0$: $$f(x) = \frac{1}{\sigma \cdot \sqrt{2\pi}} \cdot e^{-\frac{x^2}{2\sigma^2}}$$

• Significato del parametro: μ

Data $X \sim N(\mu, 1)$ teniamo σ^2 ad uno:



Abbiamo che:

→ Il parametro μ indica la traslazione.

→ Con $\mu = 0$ la mediana = asse ordinate.

Formula con $\sigma^2 = 1$: $$f(x) = \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{(x-\mu)^2}{2}}$$

• Relazione tra le F(x) della Normale standard e generica:

Sia $\boxed{X \sim N(\mu, \sigma^2)}$ allora la funzione di distribuzione è data in termini della ϕ:

• $\boxed{F(x) = P(X \leq x) = \phi\left(\frac{x-\mu}{\sigma}\right)}$ → Così si può utilizzare la tabella della gaussiana standard per calcolare F.

Formule quantile percentile

• $p = P(X \leq q_p) = F_X(q_p)$ ⇒ $\boxed{\phi(q_p) = p}$ Gaussiana Standard

⇒ $\boxed{\phi\left(\frac{q_p - \mu}{\sigma}\right) = p}$ Gaussiana Generalizzata

• $\boxed{x = q_p = \phi^{-1}(p)}$

• $\boxed{\phi(-q_p) = 1 - \phi(q_p)}$ • $p < 0.5$ con $0 \leq p \leq 1$ ⇒ $\boxed{q_{0.1} < 0}$ ⇒ $\boxed{\phi(-q_{0.1})}$

ESERCIZI D. NORMALE GENERICA

2) Sia $X \sim N(4, 25)$: $\boxed{\mu = 4}$, $\sigma^2 = 25 \Rightarrow \boxed{\sigma = 5}$ con $\sigma > 0$.

- $P(X \leq 5) = \phi\left(\frac{5-4}{5}\right) = \phi\left(\frac{1}{5}\right) = \phi(0.2) = \boxed{0.57926}$.
- $P(X \geq 2) = 1 - \phi\left(\frac{2-4}{5}\right) = 1 - \phi\left(-\frac{2}{5}\right) = 1 - \phi(-0.4) = \phi(0.4) = \boxed{0.65542}$.

3) Determinare il 10^{mo} percentile $(q_{0.1}$ o $P_{10})$:

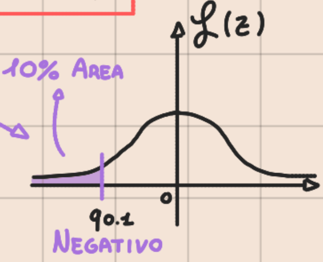
- Gaussiana Standard $Z \sim N(0,1)$

1) • $0.1 = p = P(X \leq q_p) = F_X(q_p) \overset{\text{Standard}}{=} \phi(q_p) \Rightarrow \boxed{\phi(q_p) = p}$.

• $p < 0.5$ con $0 \leq p \leq 1 \Rightarrow \boxed{q_{0.1} < 0}$.

• $\boxed{\phi(-x) = 1 - \phi(x)}$, $\boxed{q_p = x} \Rightarrow \boxed{\phi(-q_p) = 1 - \phi(q_p)}$.

• $\boxed{x = q_p = \phi^{-1}(p)}$.



2) $\phi(-q_{0.1}) = 1 - \phi(q_{0.1}) = 1 - 0.1 = 0.9 \approx 0.89973$

3) $\phi(-q_{0.1}) = 0.9$

$-q_{0.1} = \phi^{-1}(0.9) = 1.28$

$q_{0.1} = \boxed{-1.28}$

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56750	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76731	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84850	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92786	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95819	.95907	.95994	.96080	.9616	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
2.0	.97725	.97778	.97831	.97882	.97933	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
2.8	.99745	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861

Alcune formule esplicative

$\boxed{Q(p) = x \quad F(x) = p}$

$\boxed{F(Q(p)) = p}$

$\boxed{Q(p) = q_p}$

$\boxed{x = Q(p) = F^{-1}(p)}$

$\boxed{F(x) = \phi(x)}$

$\boxed{p = F(x) = Q^{-1}(x)}$

$\boxed{\phi(-x) = 1 - \phi(x)}$

$\boxed{\phi(-q_p) = 1 - \phi(q_p)}$

- Gaussiana Generalizzata $Z \sim N(21, 3) \Rightarrow \mu = 21$, $\sigma = \sqrt{3}$

1) • $0.1 = p = P(X \leq q_p) = F_X(q_p) \overset{\text{Gener.}}{=} \phi\left(\frac{q_p - \mu}{\sigma}\right) \Rightarrow \boxed{\phi\left(\frac{q_p - \mu}{\sigma}\right) = p}$.

• $p < 0.5$ con $0 \leq p \leq 1 \Rightarrow \boxed{q_{0.1} < 0}$.

2) $\phi\left(-\frac{q_p - \mu}{\sigma}\right) = 1 - \phi\left(\frac{q_p - \mu}{\sigma}\right) = 1 - 0.1 = 0.9$.

3) • $\phi\left(-\frac{q_p - \mu}{\sigma}\right) = 0.9$

• $-\frac{q_p - \mu}{\sigma} = \phi^{-1}(0.9)$

• $q_p = -\phi^{-1}(0.9) \cdot \sigma + \mu$

$= -1.28 \cdot \sqrt{3} + 21 = \boxed{18.783}$.