An Analysis of Fixed-Spread Liquidation Lending in DeFi *

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Abstract. We model and analyze fixed spread liquidation in DeFi lending as implemented by popular pooled lending protocols such as AAVE, JustLend, and Compound. Empirically, we observe that over 70% of liquidations occur in the absence of any downward price jumps. Then, considering a borrower who monitors their loan with an exponentially distributed horizon, we compute the liquidation cost incurred in closed form as a function of the monitoring frequency. We compare this cost against liquidation data obtained from AAVE protocol V2, and observe a match with our model assuming the borrowers monitor their loans $3-4$ times more often than they interact with the pool. Such borrowers must balance the financing cost against the likelihood of liquidation. We compute the optimal health factor in this situation assuming a financing rate for the collateral. Empirically, we observe that borrowers are more conservative compared to our model predictions indicating a very low financing and opportunity cost.

Keywords: DeFi · Lending · Liquidations.

1 Introduction

Lending is one of core functions of finance. In traditional as well as decentralized finance, the lenders are protected from counter-party risk through over-collateralization: the practice of using an asset valued more than the lent amount as a guarantee against the loan being repaid. In this paper, we model and study collateralized lending in decentralized finance (DeFi) as it is practiced by the most popular DeFi lending platforms today. In particular, three lending platforms (AAVE, JustLend, and Compound Finance) contribute \$10.76 billion in total value locked (TVL) of the \$14.73 billion TVL in DeFi lending platforms as of June 8, 2023. All three platforms use a protocol similar to the one we study here.

DeFi lending is most commonly implemented through lending pools (for example, the AAVE V2 lending pool). Users deposit their cryptocurrency tokens in the pool to earn interest. These deposits are lent by the platform to other users. To borrow, a user needs to have sufficient collateral available in their account to support their debt. A user can use multiple assets as collateral against their loan, which in turn, can also consist of multiple cryptocurrencies. A user continues to

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earn interest on their collateral assets even after borrowing against them. The interest rates charged by the platform, and those earned by the depositors depend on the utilization of the pool, and can change with the supply and demand of the particular cryptocurrencies. In the present work, we shall assume constant interest rates. We shall also assume that the collateral and the debt each consist of a single asset. We formalize our model and assumptions in Section 2.

Any collateralized lending protocol needs a mechanism to sell the collateral and repay the loan if the collateral value drops. In DeFi, this is handled by incentivizing third-party liquidators. These liquidators repay a fraction of the loan, and in return receive collateral in a larger amount. Because the liquidators receive payment from the borrower's collateral, a rational borrower always has an incentive to prevent liquidations. In particular, a rational borrower who continuously monitors their debt and collateral value would repay part of their loan if the collateral value drops to a certain threshold, possibly by using a flashloan to free up and sell some of the collateral, instead of letting the loan be liquidated and allowing liquidators to receive some of the collateral. Thus, the loan would never get liquidated if the price process is continuous, and the borrower monitors the loan continuously. However, in practice, loans do get liquidated, suggesting either that the price process has jumps or that the borrowers do not monitor their loans. In Section 4.3, we establish that a vast majority of liquidations occur when there was no downward jump in the price process in the preceding hour. Therefore, in this paper, we shall consider a "passive" borrower — one who is unable to monitor the state of their loan continuously — who takes out the loan but does not track the loan state until a later time.

Such a borrower can either face a high risk of liquidation, or deposit a large amount of collateral and weather the resulting opportunity cost. We consider the problem of balancing these trade-offs in Section 3. In Section 4, we use data to validate our model by comparing observed liquidation costs to model prediction, and the borrowers' health factors with our model optima.

Our goal is to use tools from quantitative finance, namely the risk neutral pricing framework of the Black-Scholes model, in order to better guide usage and design of these lending protocols. As such, throughout the paper, we assume a continuous price process.[1] In this setting, we make the following contributions:

1. We compute the cost of liquidation for a passive borrower and compute optimal quantity of the collateral that balances the cost of capital against the liquidation cost. Our results provide guidance to borrowers regarding the trade-offs between different levels of collateralization based on the volatility in the asset and the borrower's monitoring frequency and cost of capital.
2. Empirically, we observe that a vast majority of liquidations occur in time periods in which the null hypothesis of price continuity cannot be rejected.
3. We compute the average liquidation costs borne by the borrowers for different health factors and compare with model predictions, and observe a match

[1] In practice, the smart contracts rely on price oracles that update the prices at discrete time instances. We instead assume a continuously updating oracle in addition to a continuous price process.

assuming the loans are monitored three to four times more often than the frequency at which borrowers interact with the pool.

4. We compare the health factors maintained by the borrowers with the model optima and observe that the borrowers start with health factors that are, more often, higher than the model optima.

Related Papers. With growing interest in DeFi protocols, there have been quite a few recent papers that study DeFi lending. A large number of these papers study the inherent fragility of these protocols. [3] systematize the knowledge about lending pools and formally model user interactions with such pools. Using this model, they are able to analyze the vulnerabilities in the lending platforms. [21] study liquidations in such lending protocols from an empirical standpoint. They consider potentials to abuse the existing system and hurt the borrower, and suggest an alternative liquidation mechanism. [18] observe an inherent systemic fragility in the DeFi lending markets when there is price impact to trading. This stems from liquidators selling the collateral and thereby, moving the prices further, causing cascading liquidations of other loans. [24] also study such liquidation spirals, and recommend changes to existing liquidation protocols to prevent their harmful effects. [20] propose a new financial instrument called a "reversible call option" to mitigate liquidations, and potentially strengthen the market.

Other related papers focus on the ways attackers can take advantage of this fragility, and suggest methods to address them. [26] systematize the knowledge about attacks on DeFi protocols in general. [8] present a novel governance attack strategy that would have allowed an attacker to steal 0.5bn USD worth of collateral, and mint an unlimited supply of DAI tokens. [10] argue for redundancy in program logic to minimize severity and frequency of DeFi attacks and provide a novel algorithm to implement it for smart contracts. [6] present a framework for analyzing risk in fixed-spread lending protocols and show that the liquidation incentive, which is necessary to keep loans solvent, also acts as an incentive for the liquidators to manipulate prices and cause liquidations of loans having a health factor close to one.

Closer to this work are papers that model and analyze the design and usage of DeFi lending platforms. [12] evaluate the economic security of the Compound protocol. [9] empirically examine the data on interest rates, lending pool utilization, and conduct a liquidity study of the markets for DAI, ETH and USDC across AAVE, Compound and dYdX. [2] formally analyze lending pools using a statistical analyzer and show how such an analysis can be used to find threshold and reward parameters that reduce the risk of unrecoverable loans. [22] investigate how the Compound protocol is used by its users, and identify systemic risk arising from such usage. [25] develop an evaluation model for DeFi lending protocols that can be used by participants to decide which protocol to use.

There are also papers that use lending protocols as a tool to design other applications, and analyze them. [13] study non-custodial stablecoins that arise from lending markets, and suggest design improvements for their long-term stability. [14] develop a stochastic model to study such non-custodial stablecoins.

[5] suggest allowing liquidity provider (LP) shares on constant function market makers to be borrowed to improve capital efficiency for LPs.

2 Model

A DeFi loan is a contract between a borrower and a lender. Under the contract, the borrower, at time $t = 0$, borrows V_0 units of currency $\mathcal{C}$ (numéraire) against N_0 units of asset (collateral) $\mathcal{B}$. Both the borrowed amount and the collateral accrue interest with time at constant rates γ_V and γ_C respectively. At time t, the loan amount V_t and the collateral amount N_t evolve so that $dV_t = V_t\gamma_V\,dt,\ dN_t = N_t\gamma_C\,dt$. Note here that both the loan and the collateral accrue interest. The lending platform is able to pay this interest to the depositors because it rehypothecates the collateral as a loan to other borrowers. The borrow rate and deposit rate (sometimes known as the "supply" rate) are determined by the supply and demand of the respective assets, and are important tools the platform employs to balance supply and demand of each asset. The risks a platform faces due to this practice is out of the scope of this paper.

Let the spot price of the collateral $\mathcal{B}$ be p_t. We assume p_t evolves as per the Black-Scholes model. The price process evolves stochastically and has continuous sample paths. Specifically, consider a filtered probability space $\left(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, \mathbb{Q}\right)$ satisfying the usual conditions. We assume the price of the collateral, p_t, to be an Itô process on this space given by a geometric Brownian motion, i.e., $dp_t = \mu p_t\,dt + \sigma p_t\,dB_t$, where B_t is the standard Brownian motion, μ is the expected rate of return and σ is the return volatility. We also assume that $\mathbb{Q}$ is a risk-neutral measure. In other words, $\mu = r$, the risk-free rate.

The total value of the collateral in the loan at time t is given by $N_t p_t$. While the loan is unpaid, the borrower does not have access to their collateral, and so as long as $V_t < N_t p_t$, the borrower has an incentive to pay back the loan and free up the collateral since the value of the collateral exceeds the value of the loan. Let T be the first time at the borrower observes the loan after its origination. We call T the monitoring horizon. The idea is that when the loan is originated, the borrower must decide the level of collaterallization, and is unable to adjust the collateral over the interval $(0, T)$. At time T, the borrower again observes the state of the market and the state of the loan, and can revisit this decision.

The lender is exposed to the market risk of the collateral. As is common with collateralized loans, this is addressed by over collateralization. This is implemented by defining a borrowing capacity, c_t, for the borrower given by $c_t := N_t p_t \ell$, where $\ell \in (0, 1)$ is known as the liquidation threshold. Similar to loans in traditional finance, the lender can conclude that the borrower has defaulted once $c_t < V_t$ and liquidate the collateral. Having a borrowing capacity less than the value of the collateral, thus, gives the lender some respite from the market risk associated with the collateral. The assets used in DeFi, however, are much more volatile than those in traditional finance and thus further safeguards are desirable.

These safeguards are implemented by paying a third party, known as a liquidator, to liquidate some fraction of the loan once $c_t < V_t$. The ratio $h_t :=$

c_t/V_t is called the *health factor*, and the loan is said to be unhealthy if $h_t < 1$. The liquidators are paid in the collateral, and thus assume the market risk. To prevent the borrower from losing too much of their collateral, the fraction of the loan a liquidator can liquidate is bounded above by $F \in (0, 1)$, known as the *close factor*. Formally, the loan evolves as per the following steps:

1. The borrower starts the loan knowing contract parameters ℓ, F, λ, and the interest rates γ_C, γ_V.
2. At time t, if $c_t < V_t$, a liquidator is allowed to liquidate the unhealthy loan as follows:
   (a) The liquidator decides on a fraction $f \in (0, F)$ to liquidate, and pays $V_t f$ to the lender. The contract pays back the same value in collateral priced at $p_t/(1 + \lambda)$. The fraction f is said to be the "factor of liquidation". If $f = F$, we say that it is a "close-factor liquidation".
   (b) The loan now has debt $V_{t+dt} = V_t(1 - f)$ and collateral $\tilde{N}_{t+dt} = N_t - V_t f(1 + \lambda)/p_t$.

Note that the liquidator receives amount $V_t f(1 + \lambda)/p_t$ in collateral, and earns a profit of $\lambda V_t f$ if it can be sold at price p_t. This mechanism is thus referred to as *fixed spread liquidation*, λ being called the "liquidation spread". Collectively, we call F, ℓ, and λ as the contract parameters.

In the rest of the paper, we find it convenient to work with the health factor h_t, instead of working with the price. In particular, applying Itô's lemma, we can see that $\log(h_t)$ satisfies the stochastic differential equation $d\log(h_t) = \tilde{\mu}\,dt + \sigma\,dB_t$, where $\tilde{\mu} := r + \gamma_C - \gamma_V - \sigma^2/2$. In other words, $h_t = h_0 \exp(\sigma B_t + \tilde{\mu} t)$. Note that we assume that the liquidator can buy or sell arbitrary quantities of the asset without moving the price.

3 Cost of Liquidation

We assume the following condition is satisfied by the contract parameters.

Assumption 1 (Collateral Sufficiency). $\ell(1 + \lambda) < 1$.

We observe in Appendix A that this condition implies full recovery of the loan.

Let $\tau_1 := \inf\{t > 0 : h_t \leq 1\}$, denote the stopping time when the loan becomes available for liquidation. As discussed previously, because of sample path continuity, we have $h_{\tau_1} = 1$. At time τ_1, the liquidator removes collateral worth $\lambda V_{\tau_1} F$ by performing a close-factor liquidation. The loan then continues with debt amount $V_{\tau_1}(1-F)$ and health factor $\tilde{H}(1, F)$. This new loan is subject to the same liquidation risk and we can thus use a recursive formulation to compute the total cost of liquidation of the original loan. To make the calculations tractable, we shall assume that the monitoring horizon is exponentially distributed with rate ν. Due to the memoryless property of the exponential distribution, the cost of further liquidations is independent of the time passed. We can thus write the expected present value of the cost of liquidation as follows:

$$p_q(V, h) = \mathbb{E}_{\mathbb{Q}}\left[e^{-r\tau_1}\left(\lambda V e^{\gamma_V \tau_1} F + p_q\left(e^{\gamma_V \tau_1} V(1 - F), \tilde{H}(1, F)\right)\right)\mathbb{1}(\tau_1 \leq T)\;\middle|\; h_0 = h\right], \tag{1}$$

where, $\tilde{H}(1,F)$ is the health factor immediately after liquidation, given by equation (4). Before we can compute the above, we need some assumptions on the rate ν. In particular, we observe that, if there is no liquidation, the expected value of the debt amount after the monitoring horizon is given by

$$\mathbb{E}\left[e^{\gamma_V T} V\right] = \begin{cases} \frac{\nu}{\nu-\gamma_V}, & \nu - \gamma_V > 0 \\ \infty, & \nu - \gamma_V \leq 0. \end{cases}$$

To avoid the debt amount (similarly the collateral) diverging to infinity, we shall make the following assumptions about the monitoring frequency ν.

Assumption 2. $\nu > \gamma_V, \nu > \gamma_C$.

In practice (cf. Table 1), we find that extreme parameter values are required for Assumption 2 to be violated. Consider the following example.

Example 1. Let $\gamma_V = 30.0\%, \gamma_C = 25.0\%$ be the annualized interest rates. Then we need $\nu < 0.3$ for Assumption 2 to be violated. This is equivalent to the average monitoring horizon being over 3 years.

Note that the interest rates γ_V, γ_C in Example 1 are unusually high, making the divergence possible, and even then only when the monitoring horizon is very large in expectation as well. We can now prove the following theorem.

Theorem 1. *If monitoring horizon T is distributed exponentially with parameter ν that satisfies Assumption 2, then the borrower's expected discounted liquidation cost up to time T is given by*

$$p_q(V,h) = \lambda V F h^{-\kappa} \left(1 - \frac{1-F}{\tilde{H}(1,F)^{\kappa}}\right)^{-1},$$

where,

$$\kappa := \left(\sqrt{\tilde{\mu}^2 + 2\sigma^2(\nu + r - \gamma_V)} + \tilde{\mu}\right)/\sigma^2.$$

For convenience, we define $\pi(h) := p_q(V,h)/V$, to be the normalized expected discounted liquidation cost. Table 1 lists the values of contract parameters in various popular lending pools. As mentioned earlier, we observe that $r > \gamma_V > \gamma_C$ in all lending pools, except for Compound III. Thus, Assumption 2 is satisfied for any $\nu > 0$ in these pools. Example 1 shows that ν needs to be unrealistically small for it to be violated with Compound III parameters. We discuss the cost function in further detail in Section 4.1. We now consider a control problem where the borrower decides on an initial health factor for the loan. It costs the borrower αh to get a loan with health factor h per unit debt, for some $\alpha > 0$. Here, α determines the financing cost the borrower faces. The borrower aims to minimize the total cost along with the expected payment to the liquidator. In other words, the borrower faces the optimization problem of minimizing $C(h) := \alpha h + \pi(h)$ for $h \geq 1$. We can prove the following theorem.

Table 1: Values of contract parameters in leading lending platforms on Ethereum Mainnet assuming a loan of USDC against collateral of WETH. All rates are annualized, and all values are current as of Nov 30, 2023. Compound III does not pay any interest on the deposit being used as collateral.

Parameter	Aave V2	Aave V3	Compound III	JustLend
Liquidation threshold (ℓ)	86%	83%	93%	75%
Liquidation spread (λ)	4.5%	5%	3%	8%
Close factor (F)	50%	50%	100%	50%
Deposit rate (γ_C)	1.03%	1.84%	0.0%	0.72%
Borrow rate (γ_V)	5.06%	13.58%	2.86%	3.80%

Theorem 2. *Under the assumptions of Theorem 1, the optimal health factor is*

$$h^* = \max\left\{1, \left(\kappa\lambda F\alpha^{-1} \Big/ \left(1 - \frac{1-F}{\tilde{H}(1,F)^\kappa}\right)\right)^{\frac{1}{1+\kappa}}\right\}. \tag{2}$$

We discuss the optimal health factor in further detail in Section 4.1.

4 Empirical Analysis

4.1 Comparative Statics

Cost Function.
First, we consider the liquidation cost function $\pi(h) = p_q(V,h)/V$. It is instructive to write $\pi(h)$ as an infinite sum.

$$\pi(h) = \frac{\lambda F}{h^\kappa}\left(1 + \frac{1-F}{\tilde{H}(1,F)^\kappa} + \frac{(1-F)^2}{\tilde{H}(1,F)^{2\kappa}} + \dots\right). \tag{3}$$

Here, nth term in the sum can be interpreted as the expected cost from the nth liquidation. We can see that the cost decreases as κ increases. We would like to see how the cost changes with the horizon rate ν, and the volatility σ. Clearly, κ increases with ν. After some algebra, we can also see that, as long as $\nu + r - \gamma_V > 0$ as guaranteed by Assumption 2, κ decreases with σ. We now fix the contract parameters to their real world values. We use AAVE V2 lending pool to get our contract parameters because as of this writing, it is the lending pool with the highest TVL. For the risk-free rate, we use the 1 month yield curve rate published by US Treasury, which as of this writing is 5.56%. Fixing these parameters, we plot the normalized liquidation cost function, $\pi(h)$ against h for different values of σ and $\mathbb{E}[T] = 1/\nu$ in Figure 1. Here, both T and σ are in daily units. The figure shows that $\pi(h) = p_q(V,h)/V$ is a decreasing function of h, which is to be expected since it increases the likelihood of the first liquidation. We see that as volatility increases, the risk of liquidation increases, and hence so does the liquidation cost when everything else is held fixed. We also see that the cost decreases as expected horizon length decreases, and the borrower pays back the loan in a shorter length of time in expectation. Furthermore, as the expected horizon length decreases (ν increases), the curves for different σ come

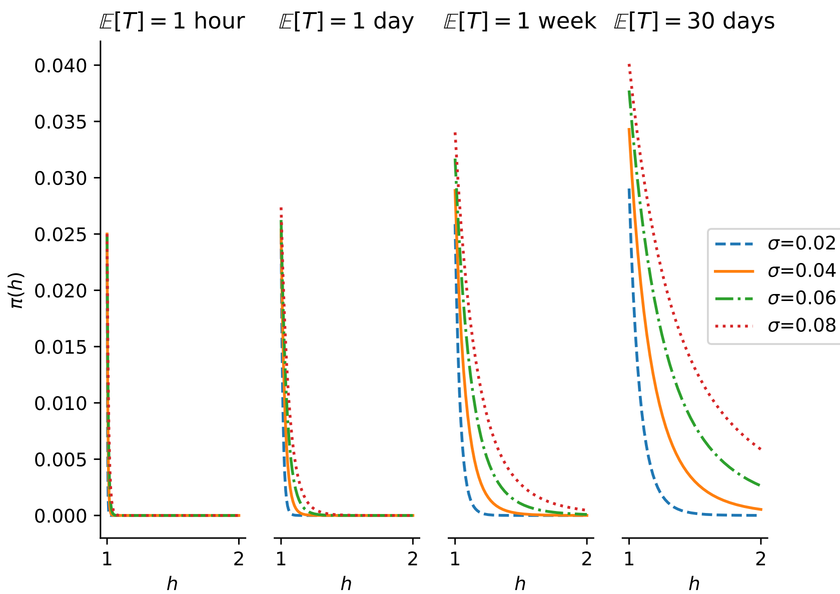


Fig. 1: Normalized expected discounted cost of liquidation against initial health factor for different values of volatility σ (daily) and expected horizon length.

closer indicating that the effect of volatility on the expected cost of liquidation decreases as the horizon becomes shorter in expectation.

Optimal Health Factor. Now, we would also like to know how h^* changes with $\mathbb{E}[T]$, σ, and other variables of interest. To that end, we fix all parameters from Table 1 and vary the expected length of monitoring horizon, the liquidation spread, and the financing rate in Figures 2, 3a, and 3b respectively for different values of σ. For all figures, the default financing rate is assumed to be the same as the risk-free rate, and the default expected monitoring horizon is assumed to be 1 day. Based on the financing rate, we set

$$\alpha = \frac{r'}{\ell(\nu - r')},$$

so that αh represents the expected cost of capital borne by the borrower in the monitoring horizon. Here r' is the financing rate. This is because getting a loan of V_0 amount and obtaining a health factor h requires one to deposit total collateral worth $N_0 b_0 = hV_0/\ell$. The cost of financing this amount of collateral with rate r' is given by,

$$\mathbb{E}\left[e^{r'T}\frac{hV_0}{\ell} - \frac{hV_0}{\ell}\right] = \frac{r'hV_0}{\ell(\nu - r')}.$$

Note that we assume $r' < \nu$ since otherwise, the cost of capital would diverge to ∞.

We see from Figure 2 that the borrower needs to start with a larger initial health factor if the asset is more volatile or if the monitoring horizon is longer. This is because in both cases the loan is more likely to get liquidated. We also observe that the rate of increase in the optimal health factor is larger when the asset has a larger volatility.

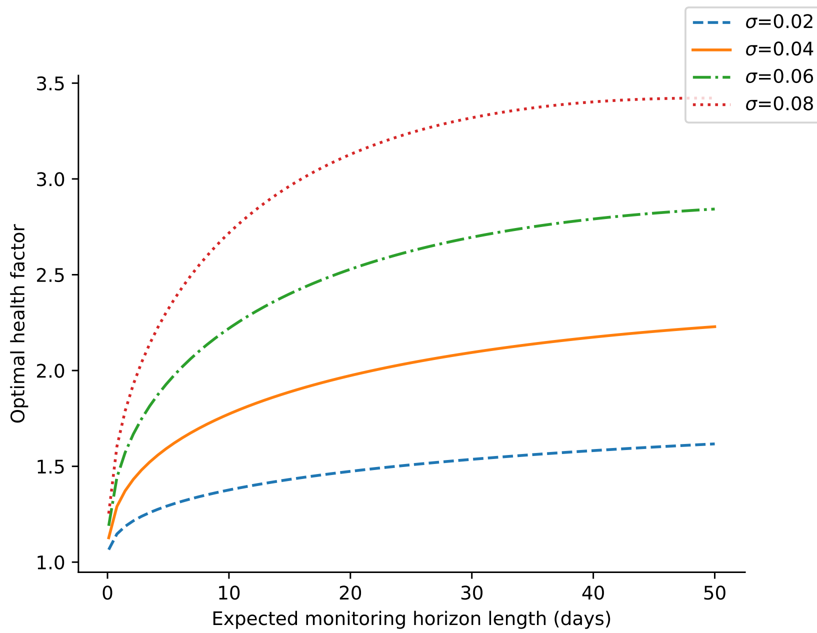


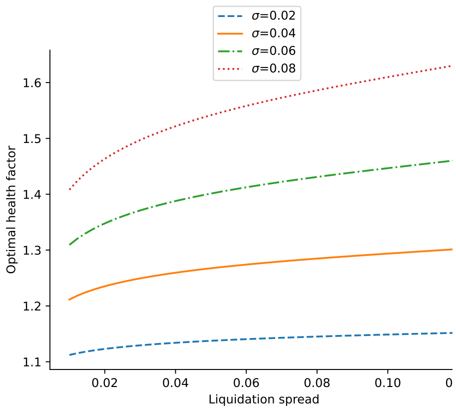
Fig. 2: h^* vs $\mathbb{E}[T]$

Figure 3a shows that the optimal health factor increases with the liquidation spread, and that this increase is steeper for a more volatile asset. This is because the borrower loses more in each liquidation when the spread is large, and the probability of such a liquidation increases with σ.

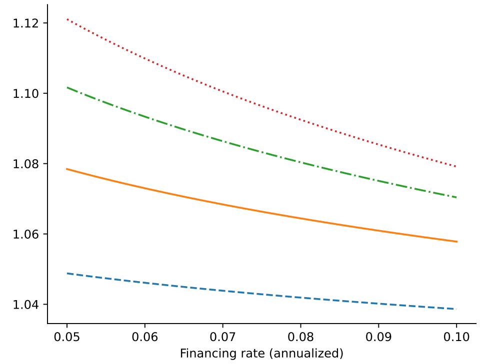
Figure 3b shows that the optimal health factor decreases with the financing rate since it costs more to obtain the necessary capital to get a loan with any given health factor. This decline is also steeper for riskier assets. This is because the optimal health factor of a riskier asset is larger than that of a less risky asset to begin with, and so the incremental cost due to rising financing rate is higher for a risky asset held in a loan at an optimal health factor.

4.2 Data

We get lending data from AAVE V2 lending pool on Ethereum Mainnet [1], risk free interest rates from US treasury daily par yield curve rates for 1 month



(a) h^* vs liquidation spread



(b) h^* vs financing rate

Fig. 3: h^* vs liquidation threshold and financing rate.

maturity [23], and minutely prices from Binance Public Data [4]. All data spans from March 15, 2021 through Jan 31, 2024.

From the lending pool, we fetch all deposit, borrow, repay and withdraw events, and filter them so that only users that have borrowed USDC against a collateral of WETH remain. For each event, we ensure that this condition is satisfied for two consecutive blocks: the block in which the event occurred (current block), and the immediately preceding block (prior block). For these two blocks, we get user health factors. We also get user's debt, collateral amount, borrow rates, and the pool's liquidation threshold and liquidation spread for the current block. We further filter this data to remove rows with a debt less than 100 USDC, or a health factor less than 1. We call this the `health-factors` table. This table has 28,876 rows.

For all the users in the `health-factors` table, we separately fetch all their interactions with the pool. These may include setting or unsetting one of their deposits as collateral for their loan, borrowing or depositing currencies other than USDC or WETH, etc. We use this data to figure out the lifetime of a loan and the average time elapsed between user interactions with the lending pool when a particular loan was active. Lifetime of a loan is defined to be from the first block with a non-zero debt to the last block with a non-zero debt. We assign each distinct loan a unique loan id. We call this the `user-interactions` table. We then join this table with the health factors table by figuring out the loan id from the block number of the current block, so that each row in the health factor table has the loan id, as well as the average intervention time.

From Binance, we get the minutely candles of WETH-USDC prices. Unfortunately, these prices are not available for the entire period as of this writing. Notably, they are missing for the period between September 29, 2022 through March 12, 2023. We use WETH-USDT prices as a proxy for this period. We verify that for the period where both prices are available, the difference between the two is always less than 10%, and is less than 0.2% in 95% of the times. We resample this data to a 15 minute frequency and compute daily volatility for each calendar day, and use it to compute expected liquidation costs and the optimal health factors on that day.

From US Treasury, we get the daily par yield curve rates for 1 month maturity. We use this rate as the risk free rate on that day. These rates are not published on weekends. We use the most recent available rates as proxy for the rates on days on which they are unavailable.

We also get liquidation data from the lending pool. This consists of all loans that covered a USDC debt by grabbing a WETH collateral. For each liquidation, we get the debt liquidated, and the block number. This liquidation table has 1,213 rows. We use this table to compute the liquidation costs between user interactions in the following way: for each user interaction in the user-interaction table, we discount and add up the liquidated amount until the next interaction on that same loan. For discounting, we use the risk-free rate at the interaction time. For interactions that are common between the user-interaction and health-factor tables, we get the liquidated debt. We multiply this by the liquidation spread and divide by the debt in the current block to get the empirical liquidation cost function $\hat{\pi}(h)$.

To summarize, we have the following tables:

1. `health-factors`: Containing health factors immediately before and after a deposit/withdraw/borrow/repay event. Also contains borrow and deposit interest rates, debt and collateral amounts, liquidation cost and loan id.
2. `prices`: Contains WETH-USDC prices at 15 minute intervals. Also contains daily return volatility.
3. `liquidations`: For each liquidation event, contains the amount of debt repaid.
4. `user-interactions`: Each lending pool transaction for each user in health-factors table.
5. `interest-rates`: daily risk-free rates.

4.3 Why do Liquidations Occur?

A rational borrower who observes that their loan is close to being liquidated would never allow such a liquidation. They could instead get a flashloan, repay part of their debt, free up and sell some collateral, and repay the flash-loan. Since we observe liquidations, one of two things must be happening:

1. The price process has downward "jumps", leading to liquidations.
2. Users do not monitor their loans at all times.

In this section, we validate our hypothesis that the majority of liquidations occur due to users not monitoring their loans. To do this, we construct a hypothesis test to detect jumps in the price process. First, we resample our price process to a period Δt. Let the price at hour t be p_t. We have for each t:

H0: There is no jump from $t - \Delta t$ to t.
H1: There is a jump from $t - \Delta t$ to t.

To conduct this test, we need an estimate of the volatility at time $t - \Delta t$, and we use returns from n preceding periods to compute it. This methodology is described by [17]. Specifically, we compute the returns $r_t = \log(p_t/p_{t-1})$, and the realized bipower variation in the past twenty periods. We obtain a square root and scale it by $\sqrt{\pi/2}$ to get an estimate of volatility $\sigma^{(j)}_{t-1}$ that is robust to jumps. Formally,

$$\sigma^{(j)}_{t-1} = \sqrt{\frac{\pi}{18 \cdot 2} \sum_{i=1}^{18} |r_{t-i} r_{t-i-1}|}$$

We then compute the t-statistic $s_t = r_t/\sigma^{(j)}_{t-1}$. If $|s_t| > 1.96$, we refute the null hypothesis at a 95% confidence level. If $s_t > 1.96$, we say that there was an upward jump, and if $s_t < -1.96$, we say that there was a downward jump at time t.

Thus, we identify the specific hours in which downward jumps occur. For each liquidation in the `liquidations` table, we check whether it occurred in an hour with a downward jump. That is, if the liquidation happened at time $t^{(l)}$, we find i such that $t_i < t^{(l)} \leq t_{i+1}$, and check if there was a downward jump at hour t_{i+1}. Table 2 summarizes the results that show a vast majority of liquidations occurring in time periods without any downward jumps.

Table 2: The fraction of liquidations occurring in periods without downward jumps computed for different period lengths, and different window sizes.

Δt	$n = 20$	$n = 30$	$n = 40$	$n = 50$	$n = 60$
30 mins	0.81	0.81	0.82	0.81	0.79
1 hour	0.78	0.77	0.79	0.79	0.79
2 hours	0.77	0.76	0.77	0.76	0.79
12 hours	0.69	0.71	0.71	0.70	0.70
1 day	0.72	0.73	0.75	0.75	0.75

4.4 Empirical Liquidation Cost

Now, we compute the empirical liquidation cost and compare it with our closed form. To get the expected liquidation cost for a loan k at time t using our closed form, we need the following parameters: $h = h^{(k,t)}, F = F^{(t)}, \ell = \ell^{(t)}, \lambda = \lambda^{(t)}, \gamma_C = \gamma_C^{(k,t)}, \gamma_V = \gamma_V^{(k,t)}, r = r^{(t)}, \sigma = \sigma^{(t)}, \nu = \nu^{(k)}$. All of these except for $\nu^{(k)}$ can be found from the `health-factors` and the `prices` tables. Unfortunately, there is no way for us to determine the average monitoring frequency ν for a user. Instead, we use the average interaction frequency $\hat{\nu}$, which is a lower bound on ν. Assuming that users monitor more frequently than $\hat{\nu}$, we expect to overestimate the liquidation costs when we use the closed form to compute them.

To compute the empirical liquidation costs, we divide the `health-factors` table into twenty bins, so that each bin contains an equal number of rows. We then compute an average liquidation cost, as well as the average of the expected liquidation cost for each bin. Figure 4 shows the plot of these two costs against

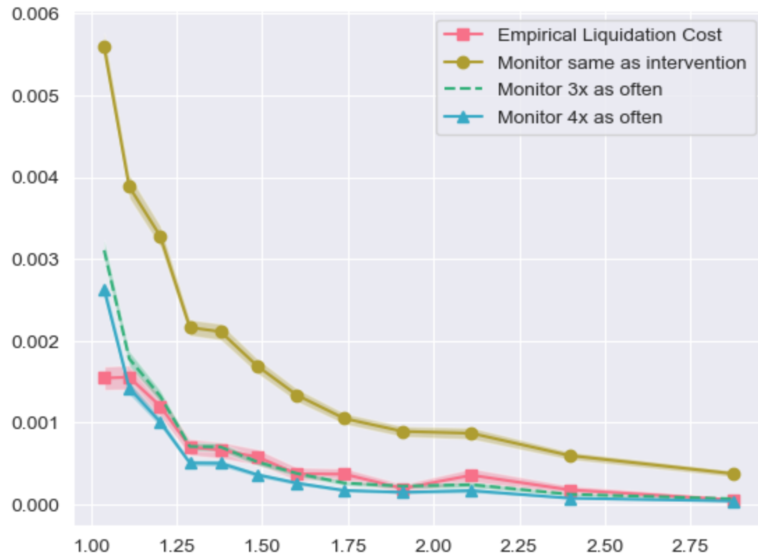


Fig. 4: The empirical and expected liquidation cost versus health factor.

the centers of the health factor bins. As we can see, the liquidation cost is indeed overestimated due to our usage of $\hat{\nu}$ instead of ν. However, if we assume that the monitoring frequency is three or four times the intervention frequency, then the two costs match in most bins. Note that to compute this, we discarded the few ($< 5\%$) cases where the liquidation took place at a health factor less than one. We also discarded the last two health factor bins since they did not have any liquidations. We note a mismatch in the very first bin, and Figure 5, which shows the medians of intervention intervals for the same health factor bins, explains the reason: The intervention interval in the first bin is small (=0.25 days), indicating a monitoring frequency of 12–16 times a day. The fact that oracle updates are hourly can not be ignored when computing the liquidation cost for this bin, and this is the reason our model overestimates the cost.

4.5 Observed Health Factors

We now turn to our empirical findings. As before, we use the intervention frequency $\hat{\nu}$ in lieu of the monitoring frequency ν. We therefore expect to overestimate the optimal health factors. Let our estimates be $\hat{h}^*$. Another parameter we need is the financing rate. We use the borrow rate for the collateral (WETH) as the financing rate r'. We are interested in knowing how $\hat{h}^*$ compare with those held

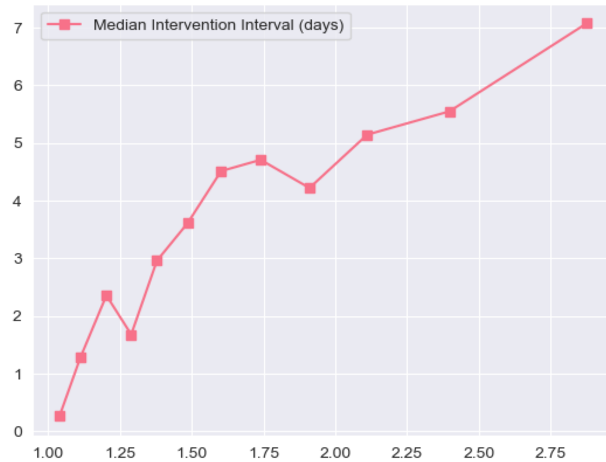


Fig. 5: Medians of intervention intervals in each health factor bin.

by the users. For the user health factors ($\hat{h}$), we only consider the times when a new loan was opened, that is, we only consider cases where debt amount of the user went from 0 in the previous block to >0 in the current block. After removing the rows with health factors in the top 0.5% percentile as outliers, this table `firsts` consists of 5,285 rows.

To compare the observed health factors with the optimal ones, we use a similar strategy as we used for the liquidation costs. First, we compute $\hat{h}^*_k$ using equation (2), assuming the monitoring frequency to be k times the intervention frequency of the borrower. Then, We divide the `firsts` table into 10 bins of equal sizes based on the prevailing volatility on the day the loan was opened. We then compute the medians of $\hat{h}$ and $\hat{h}^*_k$.

Figure 6 shows the graph of the medians of $\hat{h}$ and those of $\hat{h}^*_k$ against the centers of the bins of daily volatility. We observe that, empirically, the health factors are slightly larger than what we predict, especially when the volatility is small, and that the difference becomes smaller as the volatility increases. The plot indicates that borrowers are prepared for a higher volatility than observed.

We formalize our observation by doing a regression analysis. Starting with the `firsts` table, we first remove all entries where $\hat{h}^*_3$ was undefined, and then we also remove the rows with values of $\hat{h}^*_3$ in the top 0.5% percentile. On the remaining table with 4,018 rows, we regress $\hat{h}$ against $\hat{h}^*_3$. Formally, we estimate the model $\hat{h} = \beta_1 \hat{h}^*_3$. On performing OLS regression, we get $\beta_1 = 1.1786$. The standard error and the t-value in the coefficient is 0.017 and 69.066 respectively. The positivity of β_1 indicates that borrowers do indeed respond to the macroeconomic conditions in a way similar to what is predicted by our model.

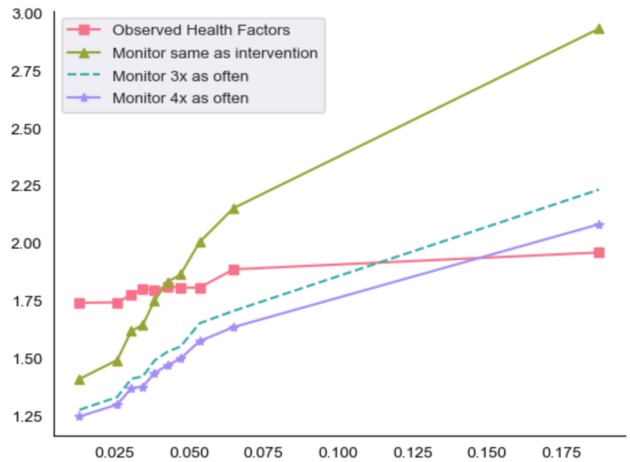


Fig. 6: Medians of Empirical and Optimal health factors versus volatility

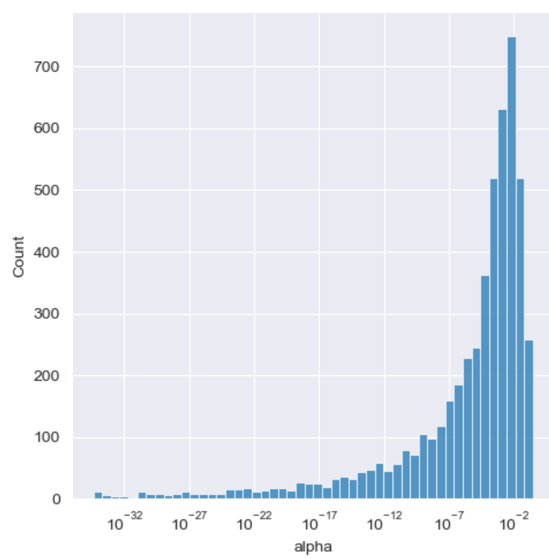


Fig. 7: Histogram of implied α, the weight given to the cost of capital.

We also compute the implied α from observed health factors. Figure 7 shows the histogram of this implied α in log scale after removing the bottom 5% percentiles.

A Contract Parameters and Lender Recovery

First, we establish what effect a liquidation has on the health of a loan. Clearly, it would be desirable to the borrower and the lender that the loan becomes healthier after liquidation. Lemma 1 establishes such a condition. We note that this condition was also observed by [21] and [24].

Lemma 1. *If $h_t > \ell(1+\lambda)$, then the health factor improves upon any liquidation.*

Proof. Let $\tilde{H}(h_t, f)$ denote the post-liquidation health factor when a loan having a health factor $h_t \leq 1$ is liquidated at factor f. We have,

$$\tilde{H}(h_t, f) := \frac{p_t \ell}{V_t(1-f)} \left(N_t - \frac{f V_t(1+\lambda)}{p_t} \right) = \frac{h_t - f(1+\lambda)\ell}{1-f}. \tag{4}$$

Then, we have,

$$\tilde{H}(h_t, f) > h_t \iff h_t > \ell(1+\lambda).$$

Let us now consider liquidator incentives and establish their optimal behavior. Assuming that the liquidation occurs at time t_0, a profit-maximizing liquidator faces the following optimization problem (M):

$$\max_{0 \leq f \leq F} \quad \lambda V_{t_0} f \tag{5}$$

$$\text{subject to} \quad V_{t_0} f(1+\lambda)/p_{t_0} \leq N_{t_0}. \tag{6}$$

Here, f is factor of liquidation, and is bounded above by the close factor. The constraint (6) limits the payment to the liquidator by the available collateral.

Lemma 2. *It is optimal to liquidate the maximum amount possible.*

Proof. Since the objective function increases with f, the profit is maximized when f is set as large as possible.

Optimal liquidator behavior was also considered by [21], but their setting allowed for jumps in the price process, and thus health factors could become strictly less than 1 at the time of liquidation. Consequently, their setting allowed for multiple successive liquidations on the same loan, which would not be possible here.

We now assume that there are a large number of liquidators competing to liquidate unhealthy loans. Due to Lemma 2, this means that an unhealthy loan gets liquidated as soon as its health factor drops to 1. We are now ready to prove that whenever the "collateral sufficiency" condition (Assumption 1) is satisfied, the lender is able to recover the loan.

Theorem 3. *If the collateral sufficiency condition is satisfied, then*

1. *$h_t \geq 1$ for all t.*
2. *Lender never loses any money, and*
3. *Each liquidation takes place at the close factor.*

Proof. For part 1, it suffices to prove that each time there is a liquidation, the health factor becomes larger than 1. To that end, suppose that the first liquidation takes place at time t. Then, $\ell(1+\lambda) < 1 = h_t$, and hence by Lemma 1, the post-liquidation health factor is greater than 1. Combined with the fact that the initial health-factor is greater than 1, the liquidators have an incentive to liquidate, and that the prices are continuous, we get the result.

For part 2, we note that as long as $N_t p_t > V_t$, the borrower has no endogenous incentive to default. This is equivalent to the condition that $h_t > \ell$. Since $\ell < 1$, it is always satisfied due to part 1.

For part 3, we note that constraint (6) is redundant due to our assumption, i.e., it is satisfied by any $f \in [0, F]$. To see this, we observe that

$$\begin{aligned} \frac{N_{t_0} p_{t_0}}{V_{t_0}(1+\lambda)} &= \frac{h_{t_0}}{\ell(1+\lambda)}, \\ &= \frac{1}{\ell(1+\lambda)}, && (h_{t_0} = h_{\tau_1} = 1) \\ &\geq 1. && \text{(due to the assumption.)} \end{aligned}$$

Thus, constraint (6) is satisfied by any $f \leq F \leq 1$. The maximum allowable factor of liquidation as prescribed by Lemma 2 then must be the close factor.

B Proof of Theorem 1

Proof. First, we note by a sample path argument that $p_q(\alpha V, h) = \alpha p_q(V, h)$ for $\alpha \geq 0$. This is because, in any sample path, keeping h constant, multiplying V by α results in each payoff to the liquidator being multiplied by α. Thus, we can rewrite equation (1) as

$$p_q(V, h) = (1-F)p_q(V, \tilde{H}(1, F))E_1(h) + \lambda V F E_1(h),$$

where,

$$E_1(h) = \mathbb{E}_{\mathbb{Q}}\left[e^{(\gamma_V - r)\tau_1}\mathbb{1}(\tau_1 \leq T) \,\middle|\, h_0 = h\right].$$

We observe that,

$$E_1(h) = \mathbb{E}_{\mathbb{Q}}\left[e^{(\gamma_V - r - \nu)\tau_1}\mathbb{1}(\tau_1 < \infty) \,\middle|\, h_0 = h\right],$$

by conditioning the expectation on τ_1 and applying tower law. Define

$$\begin{aligned} X_t(h) &:= \frac{\log(h_0) - \log(h_t)}{\sigma}, \\ &= (-B_t) + \left(-\frac{\tilde{\mu}}{\sigma}\right)t, \\ &\overset{d}{=} B_t + ct, \end{aligned} \tag{7}$$

where $c := -\frac{\tilde{\mu}}{\sigma}$, and consider again the expression,

$$E_1(h) = \mathbb{E}_{\mathbb{Q}}\left[e^{(\gamma_V - r - \nu)\tau_1} \mathbb{1}(\tau_1 < \infty) \,\middle|\, h_0 = h \right].$$

We can redefine τ_1 to be the first hitting time of the drifted Brownian motion to the level $a := \frac{\log(h_0)}{\sigma} > 0$. By Girsanov Theorem, the process X_t is a standard Brownian motion on $0 \leq t \leq \tau_1$ under measure $\tilde{\mathbb{Q}}$, where

$$\frac{d\tilde{\mathbb{Q}}}{d\mathbb{Q}} = \exp\left(-cB_{\tau_1} - c^2\tau_1/2\right).$$

Denoting by $\mathbb{E}_{\tilde{\mathbb{Q}}}$ the expectation with respect to measure $\tilde{\mathbb{Q}}$, we can write,

$$\begin{aligned} E_1(h) &= \mathbb{E}_{\tilde{Q}}\left[\exp\left(B_{\tau'_a} c + \frac{c^2}{2}\tau'_a + (\gamma_V - r - \nu)\,\tau'_a \right) \mathbb{1}(\tau'_a < \infty) \,\middle|\, h_0 = h \right] \\ &= \mathbb{E}_{\tilde{Q}}\left[\exp\left(X_{\tau'_a} c - \frac{c^2}{2}\tau'_a + (\gamma_V - r - \nu)\,\tau'_a \right) \mathbb{1}(\tau'_a < \infty) \,\middle|\, h_0 = h \right] \\ &= e^{ac}\mathbb{E}_{\tilde{Q}}\left[\exp\left(\left(-\frac{c^2}{2} + \gamma_V - r - \nu \right) \tau'_a \right) \mathbb{1}(\tau'_a < \infty) \,\middle|\, h_0 = h \right], \end{aligned}$$

where τ'_a is the hitting time of a standard Brownian motion (X_t under measure $\tilde{\mathbb{Q}}$) to the level $a > 0$. Since τ'_a is not integrable, and finite with probability 1, $E_1(h)$ is infinite whenever,

$$\begin{aligned} &-\frac{c^2}{2} + \gamma_V - r - \nu > 0 \\ \iff\ &\tilde{\mu}^2 + 2\sigma^2(\nu + r - \gamma_V) < 0. \end{aligned}$$

Note that due to Assumption 2, we have

$$\tilde{\mu}^2 + 2\sigma^2(\nu + r - \gamma_V) > 0.$$

Now, we can use the fact that,

$$M_t = \exp\left(\eta X_t - \eta^2 t/2\right),$$

is a Martingale for $\eta \in \mathbb{R}$ with respect to $(\mathcal{F}_t)_{t\geq 0}$ under the measure $\tilde{\mathbb{Q}}$ to prove that,

$$\mathbb{E}_{\tilde{Q}}\left[\exp\left(-\frac{\eta^2}{2}\tau'_a \right) \right] = \exp\left(-\left|\eta\right| a\right). \tag{8}$$

Then, we get that,

$$\begin{aligned} E_1(h) &= e^{ac}e^{-a\sqrt{c^2+2(\nu+r-\gamma_V)}} \\ &= h^{-\kappa}. \end{aligned}$$

To summarize, we have

$$E_1(h) = \begin{cases} h^{-\kappa}, & \left(\tilde{\mu} + \sigma^2\right)^2 + 2\sigma^2(\nu - \gamma_C) \geq 0, \\ \infty, & \left(\tilde{\mu} + \sigma^2\right)^2 + 2\sigma^2(\nu - \gamma_C) < 0. \end{cases} \tag{9}$$

Finally, we have

$$\begin{aligned} p_q(V,h) &= h^{-\kappa}\left(\lambda VF + (1-F)p_q(V,\tilde{H}(1,F))\right), \\ \implies p_q(V,\tilde{H}(1,F)) &= \tilde{H}(1,F)^{-\kappa}\left(\lambda VF + (1-F)p_q(V,\tilde{H}(1,F))\right), \\ \implies p_q(V,\tilde{H}(1,F)) &= \frac{\lambda VF\tilde{H}(1,F)^{-\kappa}}{\left(1-(1-F)\tilde{H}(1,F)^{-\kappa}\right)}, \\ \implies p_q(V,h) &= \frac{\lambda VF}{h^{\kappa}\left(1-\dfrac{1-F}{\tilde{H}(1,F)^{\kappa}}\right)}. \end{aligned}$$

C Proof of Theorem 2

Proof. We note that the function $C(h)$ is convex and therefore, apply first order conditions to get the maxima. We further note that if the maxima is less than 1, then the cost functions is increasing beyond 1, and $h = 1$ is optimal feasible.

D Price Manipulation

We consider the case of *predatory borrowing*, where an agent can manipulates the price of an asset in the spot market so as to inflate the value borrow against it at the inflated value. The agent can then walk away with the borrowed amount. First, we need to model price impact. For this, we use the very general model introduced by [11]. Let $B(p,q)$ denote the expected price of trade when quantity $q > 0$ is bought in the market at the starting price p. After the trade, let the expected final price be denoted by $U(p,q) \geq p$. Now consider the following sequence of events: (1) a borrower buys quantity q of an asset at starting price p, and moves it up; (2) then, the borrower uses the asset as collateral and borrows against it. To maximize their profits, the needs to find $q > 0$ that maximizes $qU(p,q)\ell - qB(p,q)$. In this scenario, we can prove the following theorem.

Theorem 4. *The above strategy is profitable in expectation to the borrower for some q only if $\ell > \min_q B(p,q)/U(p,q)$.*

Proof. We can rewrite the profit of the borrower as

$$qU(p,q)\left(\ell - \frac{B(p,q)}{U(p,q)}\right).$$

This is positive for some q if and only if

$$\ell > \min_q \frac{B(p,q)}{U(p,q)}.$$

Theorem 4 does not assume anything about the buying process, but only about the price evolution as a function of the quantity being bought. In fact, we can microfound the functions B and U by assuming that the quantity q is traded over a finite interval $[0, t]$ and that the price process follows a price impact function. As an example, consider the model proposed by [7] (the Gatheral model). This is the simplest model in that the price process follows a random walk and its drift is influenced by the trades. We need only consider the expected price process. According to this model, if the price at time 0 is p_0, and the asset is bought at a determinstic rate ν_t, then the price evolves as per the following equation:

$$\mathbb{E}\left[p_t\right] = p_0 + \int_0^t f(\nu_s)G(t-s)\,ds \tag{10}$$

where $f(\nu_t)$ represents the immediate impact of trading at rate ν_t and the transience function $G(\tau)$ represents decay of that price impact after time τ.

Corollary 1. *Under the Gatheral model, if $f : \mathbb{R} \to \mathbb{R}$ is an increasing continuous surjective function, and $G : \mathbb{R}^+ \to \mathbb{R}^+$ is a decreasing continuous function such that $G(0) = 1$, then a predatory borrowing attack with a constant purchase rate is profitable in expectation for some purchase rate if*

$$\ell > \int_0^T tG(T-t)\,dt \bigg/ \left(T\int_0^T G(T-t)\,dt\right).$$

Proof. The last term in equation (10) signifies the noise due to external market participants. For this discussion, we only consider the effect of price impact in expectation, and so ignore this term. Let the constant rate of trading by the borrower be ν. In this scenario, we have $q = \nu T$, the average trading price is given by

$$B(p_0, q) = \frac{\int_0^T \nu p_t dt}{\nu T} = \frac{p_0 T + f(\nu)\int_0^T\int_0^t G(t-s)dsdt}{T} = p_0 + f(\nu)\int_0^T \frac{(T-t)G(t)}{T}dt,$$

and the final price is given by,

$$U(p_0, q) = p_T = p_0 + f(\nu)\int_0^T G(T-t)dt.$$

Finally, we have,

$$\begin{aligned}\min_{q=\nu T}\frac{B(p_0,q)}{U(p_0,q)} &= \min_{\nu}\frac{p_0 + f(\nu)\int_0^T \frac{(T-t)G(t)}{T}dt}{p_0 + f(\nu)\int_0^T G(T-t)dt} \\ &= \frac{\int_0^T tG(T-t)dt}{T\int_0^T G(T-t)dt}.\end{aligned}$$

Thus, the Gatheral model allows for the predatory borrowing attack if

$$\ell > \frac{\int_0^T tG(T-t)dt}{T\int_0^T G(T-t)dt}.$$

We note that the existence of a profitable rate of purchase does not depend on the price impact function, but only on the decay function. How large the rate should be for profitability does, however, depend on the price impact function.

We now consider specific examples of the impact and transience functions.

Example 2. The canonical functional form of the transience function $G(\tau)$ as proposed by [7] is $G(\tau) = \tau^{-\gamma}$. In this case we need $\ell > 1/(2-\gamma)$ for the existence of a constant purchase rate that would be profitable to the borrower. Note that such a purchase rate can be high and might require access to a large amount of capital. This result can be used by lending pool designers to decide contract parameters. For example, if the price impact is known to follow the Gatheral model with $G(\tau) = \tau^{-0.4}$, as suggested by [7], then the designers should ensure that $\ell < 0.625$ to disallow such an attack.

Example 3. Another interesting possibility is that $G(\tau) = e^{-\rho\tau}$ for some $\rho > 0$ as suggested by [19]. In this scenario, we need $\ell > 1 + \frac{1}{e^{\rho T}-1} - \frac{1}{\rho T}$. We can also get a similar, but more complicated result when G is exponential. Picking a suitable T is necessary in this case. We note that the borrower needs to be able to sustain trading at a constant rate for the entire time horizon T, and so the designer need to not worry about T being arbitrarily large. For example, if ρ is estimated to be 0.5 per day, and we set $T = 1$ day, then $\ell < 0.541$ disallows the predatory borrowing attack. With the same ρ, $T = 0.5$ days require that $\ell < 0.521$.

Example 4. When ℓ satisfies the condition in Theorem 4, we can use our model to compute the quantity needed for such an attack. For example, let $\ell = 0.93$ as seen in Compound III. Suppose that the price impact function follows the square-root law, which is widely used in practice (see [15]). For simplicity, suppose that buying is instantaneous and that the price impact is permanent ($G(\tau) = 1$). To compute the transaction cost, we use the square-root model as calibrated by [16]. In this model, we have

$$B(p, q) = p + 1.208 \cdot 10^{-3} p\sigma\sqrt{q/V},$$

where σ is the daily returns volatility and V is the daily traded volume of the asset. This model corresponds to the price update function

$$U(p, q) = p + 1.812 \cdot 10^{-3} p\sigma\sqrt{q/V}.$$

Now suppose we have $\ell = 0.93$ as in Compound III. For attacker profitability, we need to find q such that $\ell U(p, q) > B(p, q)$. Taking $p = 2000$ for a WETH-USDC loan and solving, we find that q needs to be larger than $34 \cdot 10^{12}$ times the daily traded volume for this attack to be profitable to the attacker. Such an attack does not seem feasible in practice. However, this example illustrates the utility of our methodology.

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