# Liquidation Cascade and Anticipatory Trading:

# Evidence from the Structured Equity Product Market \*

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#### **ABSTRACT**

We show that structured equity derivatives can cause significant price pressure of the underlying stock upon an event of dramatic payoff change. Moreover, one event causes another: the event cascade amplifies the magnitude of the impact. We find that a single event accounts for a -6.4% return on the event day, and it increases the probability of a subsequent event by 21.3%. Given the negative price impact, traders try to liquidate ahead of each other, exacerbating the degree of price pressure. Our results uncover the chain-reaction and (mis)coordination mechanism in complex derivatives markets that can provoke substantial price shocks.

Keywords: Delta-Hedging, Strategic Liquidation, Structured Equity Product, Over-the-Counter Derivatives Markets

JEL classification: G12, G14, G24

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Prespecified risk or portfolio management programs may cause a significant, albeit unintended, market impact. For example, a common stop-loss threshold incorporated into risk-management policy could provoke a vicious feedback cycle on asset prices via fire sale and contribute to a systematic market failure (Easley, López de Prado and O'Hara, 2011; Kirilenko, Kyle, Samadi and Tuzun, 2017). Also, the exclusion of assets from an index forces index-following investors to rebalance their portfolios, causing them to incur substantial losses on those assets (Harris and Gurel, 1986; Coval and Stafford, 2007; Dick-Nielsen and Rossi, 2018). In this paper, we document an additional channel for this disruption mechanism, using structured equity products (SEPs), packages of complex options that are marketed largely to retail investors. A predetermined payoff condition of SEPs can prompt an abrupt hedging portfolio change, which exerts acute price pressure on underlying stocks. We further show that the magnitude of this effect can be amplified through a cascade of events and traders' strategic execution.

For this task, we explore SEPs issued in Korea from 2006 through 2017. Several unique features of this market make it an ideal laboratory in which to study the asset-price implications of SEPs. First, the Korean SEP market is exceptionally big relative to its equity markets when compared with what occurs in other developed economies. Second, Korean SEPs widely share a payoff feature called "knock-in" that, once met, triggers sudden liquidity demand. Issuers of SEPs with this feature are exposed to negative delta when they issue such products. To neutralize this risk, the issuers accumulate long positions of the underlying stocks. A knock-in event, however, immediately makes the SEPs' deltas much

<sup>&</sup>lt;sup>1</sup>The SRP Global Structured Product Database shows that, as of 2019, the outstanding notional amount of SEPs in the Korean market is about 242 billion dollars. This accounts for 16% of the total market capitalization, whereas the corresponding ratio for the U.S. is 2% and for the European market it is 6%.

<sup>&</sup>lt;sup>2</sup>According to the SRP Global Structured Product Database, as of 2019, the payoff that features a 'knockin barrier' is the most popular type. Globally, it accounts for 27% of the entire outstanding notional as of 2019 (1.5 trillion dollars). Also, such SEPs are not concentrated in a particular region: the Asian market accounts for 38%, followed by the European market (36%) and the U.S. (27%). In particular, 16% of SEPs with the knock-in feature are issued in Korea, which makes Korea unarguably the largest country in terms of the outstanding balance per total market capitalization and the second-largest country in terms of issue volume during 2019, behind only the U.S. This particular SEP type accounts for 69% of the outstanding balance in the Korean SEP market.

less negative, leaving the issuers over-hedged. In the course of delta-hedging, the issuers are thus forced to unwind a substantial portion of the hedging positions to remain delta-neutral.

As is the case in the U.S. market, the issuance of SEPs in the Korean market is also driven predominantly by retail investors (Shin, 2018). To match the demand, financial intermediaries aggressively market complex high-yield products to retail investors (Celerier and Vallee, 2017; Egan, 2019). Such behaviors of both retail investors and issuers cause Korean SEPs on a particular underlying stock to be issued intensively over a short period, forming an issuance wave.

These peculiarities of the market allow us to answer the following questions. When a large number of SEPs with homogeneous payoffs are written on a certain underlying asset, would this circumstance exacerbate the price pressure? Which type of investor (retail versus institutional) provides liquidity upon such an event, entailing wealth transfer between them? Does one knock-in cause another? If so, would delta-hedging issuers trade strategically depending on whether the current liquidation affects them or their competitors?

We believe that answering all these questions is essential for understanding how complex derivatives could damage financial stability. To the best of our knowledge, ours is the first study to document the dynamic aspects of price pressure as well as the strategic behavior of SEP issuers that could contribute to systematic market disruption.

The structure of the SEPs we study involves a swap of two exotic options. The issuer sells a binary call option that pays the conditional coupon of the SEPs to investors in exchange for a knock-in barrier put option. Issuers compete for the coupon because it attracts retail investors. Therefore, if two SEPs on the same stock are issued at a similar time, the barriers levels of their puts are also likely to be similar. When a barrier is breached, a knock-in put suddenly becomes a vanilla put, dramatically shrinking the delta position. Consequently,

<sup>&</sup>lt;sup>3</sup>On the day when the most notes were issued in our sample, 179 SEPs were issued on 49 underlying stocks. To compare barriers on the same underlying asset, there must be multiple contracts on one issuance day. Requiring this results in 160 SEPs and 30 stocks. A comparison of barrier levels shows that the percentage of SEPs with barrier differences that are less than or equal to 5% is 73%.

acute liquidation is prompted by the delta-neutralization mechanism described above. We find that, on a knock-in day, financial institutions sell abnormally large amounts of the underlying stock. The abnormal sell volume peaks on the knock-in day with a cumulative magnitude as large as 17.8% of its average daily volume.

Korean stock data give us the advantage of being able to observe aggregate trading volume by investor type: institutional, retail, or foreign. Although it is not possible to identify transactions executed by a specific trader, this additional information allows us to examine potential wealth transfer between investor groups. We find that institutional traders' liquidity demand is absorbed to a remarkable extent by retail investors. Through liquidity provision, retail investors on average appear to enjoy a 4.2~6.4 percent premium. This finding may surprise those expecting a priori that retail investors are noise traders. Instead, this is consistent with prior studies that document rational liquidity provision by retail investors in a wide range of contexts (Barber, Odean and Zhu, 2008; Kaniel, Saar and Titman, 2008; Kelley and Tetlock, 2013; Barrot, Kaniel and Sraer, 2016).

Given the homogeneity of payoffs, multiple knock-ins tend to occur simultaneously: when an SEP's knock-in barrier is breached, other SEPs on the same underlying are also likely to provoke knock-ins, exacerbating the price pressure. To exploit this mechanism, we first show that a knock-in event imposes significant price pressure on the underlying stock price. Specifically, we observe a 'V'-shaped pattern of abnormal returns whose kink point coincides precisely with the timing of the events. It is not surprising to observe a series of negative returns prior to knock-in events because, by definition, those events occur as a result of the negative returns. To remove the plausible return drift, we apply the Hodrick and Prescott (1997) filter. The notable V-shape remains unchanged even with the detrended pattern, strongly suggesting that the price impact is caused by the knock-in. The pattern also indicates that the price does not revert immediately: full reversion takes four days after the event.

To formally distinguish the price pressure from a change in firm fundamentals, we construct a proxy variable: the ratio of the outstanding notional value of SEPs to the underlying stock's trading volume at issuance. We denote this variable as the 'notional-to-volume' (N2V) ratio. This ratio would be correlated with hedging intensity because the dollar amount of the position to be neutralized is proportional to the notional value. The ratio is, however, arguably uncorrelated with the change in firm fundamentals around the events because this ratio is predetermined at the issuance of SEPs, which is on average 15 months prior to knock-ins.

Our results indicate that knock-in events are associated with an abnormal return of about -5.2% at the time of the events when N2V falls into the low group by the sample median. When N2V is categorized as the high group, however, knock-in events impose an additional -2.3% of price pressure. This result is robust to unobserved heterogeneity regarding event time, industry, and their interaction, confirming that the liquidation driven by delta-hedging exerts sudden and significant price pressure.

In the presence of the aforementioned issuance wave, we find that one knock-in triggers another, producing a cascade of forced liquidation events. This result has an important implication for market destabilization. When a predetermined trading strategy exists (e.g. a stop-loss policy or a 5% Value-at-Risk rule) and it applies to a large portion of traders, an uncoordinated policy-driven execution can cause an undesirable market-wide impact via the chain-reaction mechanism.<sup>4</sup>

We next investigate the possibility that traders conduct strategic executions. Among SEP issuers who are commonly exposed to the price impact of a knock-in, an issuer can exhibit a

<sup>&</sup>lt;sup>4</sup>An intraday event in the E-mini S&P 500 futures market known as the 'Flash Crash' occurred on May 6, 2010. According to the CFTC-SEC (2010) report, the Flash Crash was initiated by a large number of fundamental traders who executed a program to sell E-Mini contracts to hedge their existing cash equity positions. This collective transaction drove futures prices down by approximately 3% in less than four minutes, from 2:41 p.m. to 2:44 p.m. The price impact in the futures market quickly spilled over to the underlying stock market. Even after the rapid recovery of the E-Mini, a prevailing stop-loss order triggered another crash, creating a systematic market disruption.

range of execution strategies depending on her position as well as others' positions. Suppose that she has to unwind a large position while hedging other SEPs on the same underlying whose knock-ins are probable. Since aggressive liquidation would increase the unwinding cost for the rest of her positions, she has an incentive to sell the current portion in a way that minimizes the market impact. On the other hand, if she expects competitors to imminently liquidate their positions, she may aggressively unwind hers first to avoid the negative market impact caused by other traders' behavior.

We first show that, upon a knock-in, the outstanding notional amount of SEPs whose barriers have not been touched contributes to the price pressure. To this end, we normalize the remaining notional amount by the trading volume of the underlying asset, constructing a 'remaining-to-volume' (R2V) ratio. Given that barriers associated with the remaining SEPs differ from each other, we show that the notional amount that is subject to the most imminent knock-in imposes the more severe price shock. The impact of R2V is significant even after controlling for N2V, revealing an additional channel for the price pressure other than the one associated with the current event.

To further distinguish the various incentives related to R2V, we separate R2V into the portion of SEPs issued by the same institution of the current knock-in  $(R2V_{in})$  and the remaining SEPs  $(R2V_{ex})$ . Our results show that the degree of price pressure is more severe with a higher  $R2V_{ex}$  but is not related significantly to  $R2V_{in}$ . This finding indicates that the trader involved in the current event neither smooths out the execution process nor imposes excessive price pressure beyond the degree associated with the current event. However, she appears to unwind her position, exerting greater price pressure when her competitors are likely to liquidate a substantial portion sometime soon.

This behavior is consistent with the 'rat race' mechanism documented in another context (Brunnermeier and Oehmke, 2013). When a trader expects a negative price shock caused by someone else, she has an incentive to move to the front of the line and conduct a preemptive

execution. Such 'anticipatory trading' aggravates the price pressure and prompts future knock-ins via the event cascade. This strategic behavior is more probable when information on other traders' positions is readily available. On April 25, 2013, the Korea Securities Depository (KSD) introduced a system in which a user can easily access SEP issuance history. We utilize this information shock and find that the price impact via  $R2V_{ex}$  becomes prominent after this service is implemented, supporting the 'rat race' explanation.

We contribute to research investigating the possibility of price pressure on stocks that underlie OTC derivatives. Prior studies demonstrate that issuers of OTC derivatives affect the underlying stock prices for reasons related to hedge rebalancing and price manipulation (Ahn, Choi, Kim and Liu, 2010; Henderson, Pearson and Wang, 2015; Ni, Pearson, Poteshman and White, 2020; Henderson, Pearson and Wang, 2020a). We explore knock-in events of SEPs, non-informational liquidity demand, and find consistent evidence that OTC derivatives cause a substantial impact on prices and stock market volatility.

Our paper is also related to research that documents the potential risk of predetermined trading strategies. For instance, a negative price shock can trigger a portfolio insurance program that liquidates the asset, potentially generating a vicious cycle (Gennotte and Leland, 1990; Easley and O'Hara, 1991; Jacklin, Kleidon and Pfeiderer, 1992). Osler (2005) argues that a stop-loss order provokes a cascade of negative price shocks in the currency market. Theoretically, Huang and Wang (2009) provide insight into our results by showing that an endogenous liquidity shock without a fundamental reason can crash the market. In this context, we find empirical evidence that the predetermined barrier associated with OTC derivatives prompts sudden liquidity demand, destabilizing the underlying stock market.

Our findings provide policy implications for market regulation and information transparency. In particular, we find it problematic that issuers competitively issue homogeneous SEPs

 $<sup>^5</sup>$ Ni et al. (2020) focus on expiration dates, Ahn et al. (2010) examine redemption or maturity dates, while Henderson et al. (2015) focus on pricing (issuance) dates or determination (redemption) dates, and Henderson et al. (2020a) investigate issuance dates.

on specific underlying stocks within a short time window. We show that, in spite of the likely individual risk budget, the collective issue quantity can be excessive, necessitating a coordination device or a regulation. Also, our finding pertaining to the introduction of the KSD system suggests that information transparency sometimes induces strategic execution that can destabilize the market.

Relatedly, our paper contributes to discussions related to the dark side of financial innovation. Existing studies have focused on financial intermediaries' exploitation of uninformed retail investors (Henderson and Pearson, 2011; Vokata, 2021). The opportunity for investor exploitation with a complex product such as SEPs incentivizes intermediaries to generate issuance waves that are highly correlated with a measure of investor sentiment (Henderson, Pearson and Wang, 2020b). We complement these arguments by showing that financial innovation can impair market stability, especially when the issuance wave interacts with issuers' own trading to manage the complex risk.

# 1. Institutional Background and Data Description

The Korean SEP market has experienced rapid growth over time. As shown in Panel (a) in Figure 1, the annual average growth in issue volume was 23% from 2003 through 2017.<sup>6</sup> Consequently, as of 2017, the outstanding notional amount of SEPs has grown to a 73 billion U.S. dollar-equivalent, accounting for almost 4.9% of total equity market capitalization.

Compared with the SEP markets in other countries, the Korean market exhibits exceptional concentration on one payoff type. Using the SRP Structured Product Database, Panel (b) of Figure 1 displays the share of outstanding balances across markets for SEPs whose

<sup>&</sup>lt;sup>6</sup>This figure uses issuance data from 2003 while our sample starts in 2006. Although the data from 2003 through 2005 has sufficient information for the aggregate pattern, it lacks specific payoff information, and consequently, we exclude this portion from the main sample. Also, while this figure incorporates SEPs on all types of underlying assets (e.g., domestic and foreign stocks as well as indices), our paper focuses on SEPs with domestic underlying stocks, as described in Table I.

payoffs feature a specific 'knock-in' barrier as of 2019.<sup>7</sup> The figure shows that open interest is uniformly distributed across the U.S., European, and Asian markets. However, Korea accounts for a predominant portion in Asia, making it the largest single country in terms of open interest (16% of the open interest in the world). Although the U.S. has the largest outstanding volume in terms of the absolute amount (108 billion dollars in the U.S. versus 64 billion dollars in Korea), Panel (a) of Table I reports that Korea has the highest proportion relative to the size of the underlying stock market (0.3% in the U.S. versus 4.3% in Korea).

[Insert Figure 1 and Table I here.]

## 1.1. Structured equity products (SEPs)

We first describe the typical payoff and structure of SEPs. An SEP investor would receive the principal and the fixed coupon if the underlying asset price is higher than the strike price of a digital call (e.g., 80% of the initial fixing) on a set of dates (observation dates). If an SEP has a basket of underlying assets, the worst performer determines whether this condition is met. The observation dates include the maturity date of the SEP as well as dates prior to maturity at a specific frequency (e.g., monthly or quarterly). This binary payoff makes the investor a buyer of the digital call option whose exercise dates coincide with the observation dates. Should this call option be exercised on any of these dates, the whole structure would be automatically called and all subsequent payoffs are cancelled.

On the other hand, if the SEP has not been called until its maturity, the investor can suffer a loss. Suppose that the knock-in barrier is set to 70% of the initial fixing. If the underlying asset has never had a return worse than -30% relative to the fixing (a 70% barrier implies a -30% return threshold) and thus the barrier has never been hit any time before maturity, there is no loss to the investor under any circumstances. If, however, the 70% barrier has

 $<sup>^7</sup>$ This type of payoff is classified as 'protected tracker' or 'reverse convertible' in SRP Global Structured Product Database.

<sup>&</sup>lt;sup>8</sup>The initial fixing refers to the reference price of the underlying assets. For the initial fixing, it is common to use the underlying assets' closing prices on the issuance date of SEPs.

been breached, the final payoff depends on the maturity price of the underlying asset. In this case, a negative maturity return relative to the fixing (e.g., -20%) would incur the same degree of loss (i.e., 20%) to the investor.

This payoff is identical to going short on a knock-in put option that has an at-the-money strike and a 70% knock-in barrier, whose payoff is conditional on whether the barrier has been touched by its maturity. The option becomes an at-the-money vanilla put when the barrier is touched (referred to as a "knock-in event"); otherwise, it has a zero maturity payoff regardless of the underlying price. Put differently, the knock-in condition is specified by the barrier, while the put strike and the underlying price determine the maturity payoff given knock-in. Taken together, the SEP forms a swap of the digital call (an investor is long) and the knock-in put (an investor is short), with the complication that the swap is path-dependent: once called, the entire swap terminates.

When the stock price is far from the barrier, the put option's expected payoff is close to zero regardless of the moneyness to its strike. This feature of the barrier option makes the sensitivity of the put value to the underlying price very low because the likelihood of a knockin event is remote. Unlike a vanilla put with the identical strike, the price of a knock-in put hardly changes even though the stock price becomes in-the-money (e.g., 95%) in this case. The sensitivity to the stock price (i.e., delta) is therefore close to zero.

As the stock price further declines and reaches a certain point closer to the barrier (e.g., 90%), however, the likelihood that the knock-in put becomes the vanilla one increases. As a result, the knock-in put option price begins converging on the price of the vanilla put. Such a 'catching-up' is possible via much greater sensitivity to the stock price (i.e., a high delta) than that of the vanilla put. Upon knock-in, the barrier put option becomes a vanilla put and, as a consequence, its delta dramatically shrinks (in absolute sense) because the vanilla put cannot have delta lower than -1.

Using the example SEP above (i.e., at-the-money put strike, 70% barrier, and 80% call

strike), Figure 2 shows the individual delta of the digital call and the barrier put of which the SEP is composed as well as the composite SEP delta from the issuer's point of view. Selling an SEP exposes the issuer to a negative delta, and it becomes substantially more negative in the run-up to the knock-in event (the shaded area). The delta-neutralizing issuer, therefore, needs to increasingly accumulate a long position of the underlying stock. Upon knock-in, however, the SEP's delta suddenly becomes much less negative, making the accumulated hedging position too large. The over-hedged issuer is then exposed to delta risk and it must immediately liquidate the excessive part of the position to remain delta-neutral.

Note that the composite delta variation (the solid line) is greater than that of stand-alone barrier put (the dotted line) because the SEP also consists of a digital call (the dashed line). In Appendix A, we provide a detailed illustration of the payoff with a specific example and analysis of the associated delta change with the explicit pricing formula.

[Insert Figure 2 here.]

# 1.2. Exposures to knock-in and price fragility

SEPs are largely marketed to retail investors, so the choices of underlying stocks are significantly influenced by investor attention. Once a product with an attractive payoff (typically a higher coupon rate) is initially issued, it draws investors' attention and leads them to demand more SEPs with the equivalent terms. This behavior creates an issuance-herding pattern: over a short period of time, a disproportionately large number of SEPs on a particular underlying stock are being issued. Given a set of underlying stocks, the competition evens out the coupon rates of SEPs across issuers. To keep the coupon rate competitive, the knock-in barriers of the SEPs tend to be largely identical to each other.

In practice, no issuer can supply SEPs on the particular stock indefinitely because issuing these notes is equivalent to writing option contracts, and each issuer is bound by its risk budget. Therefore, an issuer typically issues SEPs up to the maximum amount allowed. In aggregate, however, this mechanism can still create excessive issuance concentration at a certain time. Appendix A provides some empirical support for issuance herding.

This concentrated issuance of SEPs with homogeneous barrier levels excessively exposes a certain stock to knock-in events, making a certain price range of the stock substantially fragile. In Figure 3, we visualize the fragility using R2V, a variable that measures exposure to future knock-ins, which we later formally define in Section 4. The heat map indicates the degree of fragility (the darker the area, the more extreme the fragility) on two particular days as examples. We consider 12 stocks with the highest knock-in frequency displayed on the x-axis. On the y-axis, we normalize stock prices by moneyness. Panel (a) of the figure shows that all firms are exposed to knock-ins across a wide price range  $(40\% \sim 90\%)$  as of January 2014. For example, however, Stock 12's price range that corresponds to about 50% of the current price shows much greater fragility on this day. Seven months later, the fragile range of the same stock moves closer to the current price (100%) as a result of the recent price drop, as shown in Panel (b), indicating that the shock is imminent. In

[Insert Figure 3 here.]

### 1.3. Data construction

Given the non-transparent nature of OTC derivatives contracts, we exploit several information sources to construct a comprehensive data set for SEP issuance. We start with a commercial data provider called FnGuide that provides notes' issuance histories. For each issuance, we hand-collect contractual details such as knock-in barrier levels and maturities from individual prospectuses filed with the Korea Securities Depository (KSD) and Financial Supervisory Service (FSS). The collected information includes *all* 38,035 SEPs on a basket of stocks, both publicly and privately issued between 2006 and 2017.

First, we restrict our focus to SEPs whose underlying assets contain at least one domestic

<sup>&</sup>lt;sup>9</sup>Appendix B.2.1 provides the pattern of issuance herding.

<sup>&</sup>lt;sup>10</sup>The case study for this stock is described in Appendix B.2.2.

stock. Furthermore, we require SEPs to have the 'knock-in' feature in the payoff because we treat knock-in events as instruments that trigger dramatic changes in delta exposure. Often, such SEPs are written on a basket of stocks rather than a single stock. In this case, the worst performer triggers the knock-in event. For our purposes, this worst performer is recognized as the affected stock by the knock-in event. When a stock (typically the worst performer) triggers multiple knock-in events on the same day, we aggregate those events. This process converts our data structure to the stock  $\times$  day-of-event level. As a result, the final sample contains 1,292 unique observations (stock  $\times$  event days) on 489 unique days-of-event. Panel (b) of Table I illustrates the sample construction process in detail and Panel (c) of the same table presents the time trend for SEP issuance and knock-in events in our sample.

To analyze the dynamic price impact around each knock-in event, we use the performance of the corresponding stock for five days before and five days after the event day (i.e., for a total of 11 days). Unlike other studies that analyze synchronous events, our events could overlap with each other because SEPs of the same underlying assets may experience the knock-in event on separate days. Therefore, for a given stock, we select non-overlapping events where each event is unique during the event window.

The selection of the window involves a trade-off between the sample size of non-overlapping events and the number of post-event observations. For example, choosing the narrowest possible window (i.e., a 1-day window) would give us the largest samples because the fewest events would overlap with each other. Doing so would, however, fail to provide any post-event information. On the other hand, choosing a wide window (e.g., a 6-month window) would allow us to observe the post-event dynamics at the cost of restricting samples to those associated with a unique event during the 6-month period. In this trade-off, we choose an 11-day window to study the dynamic effects around the event with a resulting sample size of 3,300 observations (300 unique events × 11 days). To test robustness, in Section 5, we present the results with alternative windows to ensure that the dynamic results are not

sensitive to window size.

# 2. Price Impacts of Knock-In Events

## 2.1. Measuring the price impact of knock-in events

We first test whether the aforementioned hedging-driven liquidation imposes price pressure. The major challenge is to distinguish price changes caused by the hedging-driven liquidation from changes in fundamental value. The knock-in event is likely to be triggered when the fundamental value also falls. Our setup provides us with a particular advantage for addressing this challenge: the fundamental value does not necessarily differ much in the neighborhood of the barrier level. Therefore, should there be any price pressure, its magnitude would be most pronounced precisely at the moment of the knock-in event.

We next construct a variable that is associated with liquidation intensity but is unlikely to be correlated with changes in a firm's fundamental value. The dollar value of the delta is positively correlated with the size of the SEPs. Because it is likely that multiple SEPs on a common underlying are outstanding, there often exist multiple knock-ins from separate SEPs on the same underlying asset at the same time. Using this fact, we define the following 'notional-to-volume' or 'N2V' measure.

We aggregate the notional amount of a set of SEPs ks whose knock-in barrier is touched by their underlying stock i on day t. We then divide the aggregated notional amount by the average trading volume of stock i. Formally,

$$N2V_{i,t} = \frac{\sum_{k \in \mathbf{K}_{i,t}} Notional_k}{Volume_{i,\bar{t}(\mathbf{K}_{i,t})}},\tag{1}$$

where  $\mathbf{K}_{i,t}$  is a set of SEPs whose knock-in events are triggered at t by i,  $Notional_k$  is the notional size of SEP  $k \in \mathbf{K}_{i,t}$  divided by the total number of underlying assets, and  $Volume_{i,\bar{t}(\mathbf{K}_{i,t})}$  is the 6-month average dollar volume of i as of the earliest issue date among

SEPs in  $\mathbf{K}_{i,t}$  denoted by  $\bar{t}(\mathbf{K}_{i,t})$ .

We aggregate across multiple SEPs on the same knock-in day t as stock i triggers the event, so the observation unit for N2V naturally becomes event j, pinning down the affected stock i(j) at the knock-in event time t(j). In other words, when SEP k has a basket of underlying stocks, the affected stock i(j) in our analysis is the worst performer in the basket that triggers the knock-in event j, making the mapping of  $j \mapsto i$  unique. Therefore, for notational simplicity, we rewrite Equation (1) as follows:

$$N2V_j = N2V_{i(j),t(j)}. (2)$$

According to Equation (1), N2V is predetermined at the time of SEP issuance, which is long before the knock-in day t.<sup>11</sup> Table II presents summary statistics of all variables used in our paper.

## [Insert Table II here.]

The aggregation across SEPs also reflects the fact that multiple knock-ins often occur simultaneously. It is reasonable to expect that a higher N2V is associated with greater price pressure on the related underlying stocks. There is no obvious reason, though, that a higher N2V would be correlated with a bigger drop in fundamental value in the future, because it is difficult to envision that this predetermined variable in the OTC derivatives market changes fundamental values or that investors invest more actively in a firm whose fundamental value is expected to drop. One may argue that issuers can benefit by selling SEPs on overpriced stocks and leaving the position unhedged. If this is the case, however, knock-in events would not incur any changes in issuers' position, and therefore we should not observe such a price impact upon knock-in events.

<sup>&</sup>lt;sup>11</sup>The average time gap between issuance dates and knock-in event days is over 1 year (15 months). The trading volume around knock-in events can be a consequence of liquidations of hedging positions. In an unreported result, we find that trading volume increases significantly around such events. This endogeneity discourages the use of volume around knock-in and it can be circumvented by using the volume at the issue date of event-experiencing-notes.

It is possible, however, that N2V is correlated with future economic conditions. In particular, retail investors may chase a stock's returns. A stock that had recently generated a high return consequently draws investor attention and drives a large SEP issuance. When such issue-timing occurs near the peak of the economic cycle, multiple SEPs will be simultaneously knocked-in (resulting in high N2V) during a subsequent downturn. In this case, a high N2V may coincide with a more negative return because of the issuance timing. To circumvent this concern, we use N2V with interactive industry fixed effects and event calendar time fixed effects that control for industry-wide economic conditions.

## 2.2. Empirical design

In the first step of our empirical analysis, we examine movements of underlying stock prices around knock-in events. To this end, we use an event window that ranges from five trading days before through five trading days after each knock-in event. To separate out exposure to systematic risk, we use market-beta-adjusted cumulative abnormal returns (CARs) following MacKinlay (1997). This measure allows us to focus on an event-driven price and sidestep a potential selection issue as a result of which SEPs are likely to be written on high-beta stocks. Specifically, for event j that occurs at t(j), the CAR of affected stock i(j) on event day  $\tau \in [-5, +5]$  is calculated in the following way:

$$CAR_{j,\tau} = \sum_{q=-5}^{\tau} \left\{ \bar{R}_{i(j),t(j)+q} - \hat{\beta}_{i(j)} \bar{R}_{m,t(j)+q} \right\},$$
(3)

where  $\bar{R}_{i,t}$  and  $\bar{R}_{m,t}$  are excess returns on stock i and the market index (KOSPI Composite), respectively, on day t over the contemporaneous risk-free rate (91-day certificate of deposit rates). The reference point for CAR is the inception of each event window, i.e., 5 days before the event day (t-5). Market beta  $(\hat{\beta})$  is estimated over one year (252 business days) prior to each reference point using the canonical market model.

We next estimate the dynamic CAR pattern during the 5-day window of each event j via

the following regression specification:

$$CAR_{j,\tau} = \alpha_{I(i)\times M(t)} + \sum_{\tau=-5}^{5} \beta_{\tau} \cdot \mathbf{D}_{\tau} + \gamma X_{i,t-10} + \varepsilon_{j,\tau}, \tag{4}$$

where I(i) is the industry corresponding to stock i as per its SIC 2-digit classification, M(t) is the calendar month of event day t = t(j),  $\mathbf{D}_{\tau}$  is an indicator variable whose value is 1 only on event day  $\tau$  and 0 otherwise, and X is a vector of control variables comprising firm i's available information at t - 10.<sup>12</sup> Although we adjust for the market return via CAR, economic conditions across industries at the event time may vary. We address this by Industry  $\times$  Month fixed effects,  $\alpha_{I(i)\times M(t)}$ , that correspond to each event. Therefore, our results are robust to heterogeneity in industry-specific economic conditions or unobservable expectations around the event time. Our coefficients of interest are  $\beta_{\tau\in\{-5,\dots,+5\}}$ . The estimated dynamics of the abnormal return would suggest whether a knock-in event imposes price pressure.

Additionally, we use N2V in Equation (2) to further distinguish the unwinding effect from other factors that are unrelated to the hedging activity. For event j, we regress the CAR of affected underlying stock i = i(j) on the event day t = t(j) on N2V of the corresponding event:

$$CAR_{j} = \alpha_{I(i) \times M(t)} + \beta N2V_{j} + \gamma X_{i,t-10} + \varepsilon_{j}.$$
(5)

Other variable definitions are identical to those used in Equation (2) and (4). The coefficient  $\beta$  in Equation (5) would differentiate the CAR on stock i on the event day across the variation of predetermined N2V. Given that N2V measures hedging intensity, a negative  $\beta$  would provide strong evidence that hedging causes price pressure.

One valuable piece of information available in the Korean stock market is that trading volume is classified by type of trading entity (retail, institutional, and foreign investor). This

<sup>&</sup>lt;sup>12</sup>We lag the control variable to avoid an endogenous event-driven correlation between the control variable and the outcome variable. In particular, we use a 10-day lag because events are distanced from each other by more than 10 days in our non-overlapping sample.

advantage enables us to examine the unwinding impact from another perspective: trading quantity. In a similar spirit to CAR, we construct a measure of abnormal trading behavior on the part of retail investors and financial institutions. In particular, a pattern of abnormal selling of event-triggering stocks by institutions upon corresponding knock-ins would further confirm the delta-hedging mechanism deployed by SEP issuers.

To examine this possibility, we define the net buying amount by reference to the difference between the buying and selling amount of event-triggering stock i on the event day t. We then subtract its past 6-month average to calculate the abnormal net buying amount, and normalize it by the average trading volume of stock i. Formally,

$$nb_{i,t} = \frac{NB_{i,t} - \overline{NB}_{i,t-10}}{Volume_{i,t-10}},\tag{6}$$

where  $NB_{i,t}$  is the net buying amount of stock i on the day t,  $Volume_{i,t}$  and  $\overline{NB}_{i,t}$  are the average trading volume and  $NB_{i,t}$  over the past 6 months from t, respectively. The average numbers are referenced at t-10 to avoid overlapping with any variables during the 11-day event window.

We then use a specification identical to Equation (4) to estimate a dynamic pattern of abnormal net buying around the knock-in event  $(\tau \in [-5, 5])$ , using nb instead of CAR:

$$nb_{j,\tau} = \alpha_{I(i)\times M(t)} + \sum_{\tau=-5}^{5} \beta_{\tau} \cdot \mathbf{D}_{\tau} + \gamma X_{i,t-10} + \varepsilon_{j,\tau}, \tag{7}$$

where all other variable definitions are identical to those in Equation (4).

# 2.3. Dynamic price impact

The results reported in Panel (a) of Figure 4 illustrate the dynamics of CAR as defined in Equation (3). Using Equation (4), we plot the point estimates of each  $\beta_{\tau}$  and their 90% confidence intervals. Because our events occur at different times and with different stocks,

our outcome variables are subject to time-series as well as cross-sectional heterogeneity. To address potentially confounding effects that this might cause, we construct a control variable vector X based on firm-specific variables (size and book-to-market) as well as past market information (past returns, the volatility of the affected stock, and market returns) in addition to Industry  $\times$  Time fixed effects.

We note that, on the knock-in day (the vertical line), the CAR becomes noticeably negative at 6.4 percent. This magnitude is highly significant, both economically and statistically. We first observe that the CAR is already negative prior to the event. This result may simply reflect the fact that a knock-in event occurs as a result of the negative return. It is also possible that the issuer foresees the knock-in and begins unwinding the position in advance. Such a preemptive liquidation possibly contributes to the declining CAR pattern. On the other hand, the negative CAR pattern flips over immediately after the knock-in. This notable "V"-shape yields an implication regarding the source of the negative CAR prior to the knock-in. Although CAR is constructed based on the return adjusted for the exposure to systematic risk, the negative CAR itself cannot be automatically attributed to the hedging activity. The pattern of which the negative CAR peaks precisely at the moment of the knock-in event and recovers immediately after, however, provides compelling evidence that such dramatic movement is caused primarily by knock-in events.

It is worth noting that SEPs typically have baskets of underlying stocks. Because the final payoffs are likely to be determined by the worst performer at knock-in, however, the delta exposure of the SEPs is dominated by this stock (Wallmeier and Diethelm, 2012; Detemple and Kitapbayev, 2021). Consistently, in unreported results, we find that the remaining underlying stocks exhibit no significant price pressure around the knock-in events.

## [Insert Figure 4 here.]

<sup>&</sup>lt;sup>13</sup>The impact of the hedging might vary each year. We find, however, that the market-adjusted price impact is qualitatively similar during the sample period. Although there is a slight attenuation over time, our general results are not driven by a particular year.

In fact, a knock-in event occurs when the underlying stock price declines. Although CAR is adjusted for the market return, a downward secular trend that reflects a worsening idiosyncrasy of the firm may coexist with the price pressure. To decompose the extent of the abnormal return, we further detrend CAR using the Hodrick-Prescott filter (Hodrick and Prescott, 1997).

The Hodrick-Prescott (HP) filter is a tool that is widely used to isolate a trend component from a time series (Gorton, Hayashi and Rouwenhorst, 2012; Imrohoroglu and Tuzel, 2014; Schularick, ter Steege and Ward, 2021). We adopt this approach to disentangle the firm-specific trend from the market-adjusted CAR and construct the detrended series of abnormal returns denoted as filtered CAR or fCAR. Appendix C offers detailed explanations of this method. The pattern with fCAR presented in Panel (b) of Figure 4 exhibits a more symmetric "V"-shape, which implies a full reversion of the price pressure. This result further supports the notion that the observed "V" shape of the abnormal return is driven by price pressure, not by fundamental changes around the event.

The magnitude of the price impact is smaller with fCAR than with the unfiltered CAR. This finding indicates that the removed trend is on average running downward, reflecting the idea that knock-ins occur in the period of declining stock prices. fCAR estimates -4.2% of 5-day abnormal return on the event day, as opposed to -6.4% in the unfiltered case, providing a more conservative estimate of the price pressure.

# 2.4. Price pressure on the knock-in day

Table III presents our baseline results derived from Equation (5). Unlike the analysis of the dynamic pattern, this test estimates the price impact only on the event day, which allows

<sup>&</sup>lt;sup>14</sup>The filtering specification relies on  $\lambda$ , a smoothing parameter of a positive number. A higher  $\lambda$  imposes a larger penalty on the time-series variation. If  $\lambda$  increases to infinity, the time trend to be filtered becomes linear. Following Hodrick and Prescott (1997) and Ravn and Uhlig (2002), we use  $1600(252/4)^4$  (a daily standard). However, the "V" shape of fCAR still holds with other common choices of  $\lambda$ , such as 129,600 (a monthly standard) or 1,600 (a quarterly standard).

us to use the entire sample of events. For Columns (1)-(5), we use varying combinations of control variables with Industry  $\times$  Time fixed effects. Across these specifications, the coefficient  $\beta$ s are consistently negative and highly significant, measuring the price pressure upon a knock-in.

An SEP generates yield by going short on a put option, so it is plausible that stocks with more skewed return distributions (i.e., fatter left tails) are popular underlying assets. If this is the case, N2V could be negatively correlated with CAR for reasons not related to knock-ins. Although the immediate reversion of CAR upon the knock-in strongly implies hedging-driven price pressure, we estimate the results reported in Column (6) with Stock  $\times$  Time fixed effects to further rule out this possibility. With these fixed effects, the coefficient on N2V is estimated within the stock and calendar month in which the heterogeneity of return skewness is reasonably controlled for. The results are consistent with our baseline patterns, suggesting that the underlying stock selection does not drive our findings.

We proceed to examine how N2V is related to CAR, and to do so we repeat the analysis with quartile indicators of N2V. Column (7) of Table III implies that the relationship is weakly convex. Moving up from the first quartile (the omitted category), the incremental impact on prices becomes monotonically larger. However, the price impact is not driven solely by the top quartile; an N2V value that is above the median (i.e., in the third and fourth quartiles) appears to have a statistically significant effect.

Given that the sample average of N2V is 9.96%, our result indicates that the average price impact is about 35 basis points on a knock-in day. Considering that CAR begins turning significantly negative three days prior to the event day, as indicated in Figure 4, this one-day estimate is the lower bound of the full impact that lasts for more than 5 days around each event. It is worth noting that the standard deviation of N2V is large: a one-standard-deviation rise in N2V corresponds to a 68 basis point more negative CAR on a knock-

in day.<sup>15</sup> The aforementioned convex relationship suggests that the price impact becomes disproportionately large with the liquidation intensity (N2V).

### [Insert Table III here.]

Overall, our results show that, when knock-ins are clustered in time, the price pressure will be severe. Multiple and contemporaneous events increase the aggregate intensity of liquidation, corresponding to a high N2V. Such a large-scale unwinding event would result in greater price pressure, implying that issuance herding contributes to price fragility.

## 2.5. Liquidity provision

In this subsection, we present the result of Equation (7) in which we estimate abnormal net buying by retail and institutional investors around knock-in events. <sup>16</sup> Figure 5 presents the results of this test. Panel (a) shows the abnormal net-buying volume of financial institutions by coefficients  $\beta_{\tau}$  in Equation (6), while Panel (b) displays the same estimates for retail investors. The figures show a clear pattern: starting at t-3, institutions exhibit significant abnormal selling behavior and its magnitude that peaks on event day t. This abnormal selling pattern notably mimics the pattern of abnormal returns shown in Figure 4, confirming that the price pressure is caused by SEP issuers' liquidity demand. On the other hand, the pattern of retail flows (Panel b) shows that retail investors absorb the sudden liquidity demand by financial institutions. Using the abnormal returns (CAR) and the abnormal trading volume (nb), we estimate the size of the value transfer from institutions to retail investors during [t-5, t] for each event at t. According to our unreported result, retail investors appear to enjoy a benefit of 164 million USD per year, whereas financial investors suffer an 86 million

 $<sup>^{15}</sup>$ The standard deviation of N2V is 0.1948. The estimated coefficient on N2V is -0.035. Therefore, the magnitude of a one-standard-deviation change in N2V is about -0.68% (-0.0068=0.1948\*(-0.035)).

<sup>&</sup>lt;sup>16</sup>In some cases, SEPs are hedged by foreign institutions. However, they are also classified as institutional instead of foreign investors because they are registered and licensed as onshore market makers. Also, we exclude pension funds from the institution type because they do not issue SEPs.

USD fire sale loss.<sup>17</sup>

## [Insert Figure 5 here.]

This finding may not appear to be consistent with the priori expectation that retail investors are noise traders. However, recent studies document that retail investors are capable of making short-term predictions (Kaniel et al., 2008). To further test whether the observed retail flow is driven by active liquidity provision, we exploit analysts' forecast data in I/B/E/S and construct the following variable that measures the degree of valuation uncertainty at the firm level. Following Avramov, Chordia, Jostova and Philipov (2009) and Banerjee (2011), we calculate:

$$Dispersion_{j} \equiv Dispersion_{i,t} = \frac{EPS_{i,t}^{Max} - EPS_{i,t}^{Min}}{|EPS_{i,t}^{Med}|}$$
(8)

for all stocks associated with knock-in events, where  $EPS_{i,t}^{Max}$ ,  $EPS_{i,t}^{Med}$ , and  $EPS_{i,t}^{Min}$  are the maximum, median, and minimum values, respectively, among all EPS forecasts for firm i at time t.<sup>18</sup> The higher Dispersion implies the greater uncertainty regarding firm valuation. Based on this measure, we create tercile indicator variables  $Uncertainty^{H,M,L}$ , and estimate the following specification using  $Uncertainty^H$  as the omitted variable:

$$nb_{i} = \alpha_{I(i) \times M(t)} + \beta_{1} Uncertainty_{i}^{M} + \beta_{2} Uncertainty_{i}^{L} + \gamma X_{i,t-10} + \varepsilon_{i}, \tag{9}$$

where nb is the normalized abnormal net buying amount as defined in Equation (6).

The results are presented in Table IV. We find that retail investors' net buying is more pronounced in the case of lower EPS uncertainty. Compared with the high uncertainty case, here retail investors exhibit 11.1% higher abnormal net buying amounts relative to the trading volume associated with the case of low uncertainty. This finding suggests that

<sup>&</sup>lt;sup>17</sup>The institutional investors' loss is subject to the netting problem, so we observe only the aggregate trading volume involving financial institutions, rather than the set of SEP issuers only. This data limitation understates the loss and contributes to the discrepancy between these two values.

<sup>&</sup>lt;sup>18</sup>We use the 1-year-ahead forecast horizon. When it is not available, we pick a horizon that best corresponds to that point. Also, we use the absolute value of the corresponding EPS because some median estimates are negative.

retail investors' liquidity provision is opportunistic upon the temporal price pressure and complements prior studies that document liquidity provision of retail investors in a range of contexts (Barber et al., 2008; Kaniel et al., 2008; Kelley and Tetlock, 2013). Our finding is also aligned with Barrot et al. (2016) in that retail investors act as primary liquidity providers, especially when institutional liquidity dries up. One difference between ours and their results is that retail investors do not benefit from liquidity provision because of their extended holding period.

On the other hand, financial institutions' trading behaviors generally are not related to this variable, indicating that the dominant driver of atypical net selling is the mechanical delta-hedging and has little to do with (mis) valuation. Although it is possible that a third-party financial institution also engages in liquidity provision, the hedging-related flow seems to be the dominant force. A similar mechanism has been previously documented in other contexts. For instance, Coval and Stafford (2007) show that stocks commonly held by underperforming funds that experience fund outflows are subject to price pressure. To mention another example, exclusion from an index imposes substantial price pressure via index-tracking funds' rebalancing (Harris and Gurel, 1986; Dick-Nielsen and Rossi, 2018). We complement this strand of the literature with the additional channel of intermediary-driven price pressure from the abrupt hedging activity. As in the papers cited above, we also show that investors who provide liquidity enjoy considerable economic gain.

## [Insert Table IV here.]

<sup>&</sup>lt;sup>19</sup>The unreported constant term in this regression is the unconditional knock-in effect on net buying. While it is insignificant for retail investors, it is significantly negative for institutional investors, indicating abnormal net selling on their part, as in Figure 5.

## 3. A Cascade of Events

## 3.1. Knock-in cascade

As previewed in Figure 3, a certain price range for a stock is exposed to a high density of knock-in barriers. The concentrated exposure can make this price range highly vulnerable to a shock. In particular, the price pressure from a single knock-in can be exacerbated if this event triggers another.

To conduct the formal analysis, we construct a variable that allows us to screen SEPs whose barriers fall within a certain proximity. Specifically, for knock-in event j, its corresponding stock i = i(j) and event time t = t(j), we construct the proximity variable as follows:

$$Z_j \equiv Z_{i,t} = \frac{1}{\operatorname{n}[\mathbf{K}'_{i,t}]} \sum_{k \in \mathbf{K}'_{i,t}} \mathbb{P}(s \le T \mid \mathcal{F}_t)_{i,k,t}, \tag{10}$$

where  $\mathbf{K}'_{i,t}$  is a set of outstanding SEPs underlain by stock i whose barriers have never been touched as of t,  $\mathbf{n}[\mathbf{K}'_{i,t}]$  is the number of SEPs in  $\mathbf{K}'_{i,t}$ , and finally  $\mathbb{P}(s \leq T \mid \mathcal{F}_t)_{i,k,t}$  is the probability of hitting the barrier before the SEP's maturity T (see Appendix D for detail). Z is therefore the average of this probability across SEPs in  $\mathbf{K}'_{i,t}$ . A high  $Z_j = Z_{i(j),t(j)}$  implies that, on average, outstanding SEPs on stock i = i(j) have barriers close to being touched at times subsequent to t = t(j).

# 3.2. Empirical design

We propose a model for testing whether a contemporaneous knock-in event increases the future knock-in probability. To this end, for each event j, we construct an indicator variable  $KI_{i,(t+1,t+10)}$  that yields 1 if there is any knock-in on the same underlying i = i(j) within the 10 days immediately following the event time t = t(j) and 0 otherwise. With this variable,

we specify a Probit model using N2V as the measure of hedging activity intensity as follows:

$$KI_{i,(t+1,t+10)} = \Phi\left(\alpha_{I(i)\times M(t)} + \beta N2V_j + \gamma X_{i,t-10} + \varepsilon_j\right),\tag{11}$$

where  $\Phi(\cdot)$  is the standard normal cumulative density function,  $KI_{i,(t+1,t+10)}$  is as defined above, and all other variable definitions are identical to those used in Equation (2) and Equation (4). The coefficient  $\beta$  of the above equation would reveal whether the price shock driven by one event would lead another to occur, underpinning the chain-reaction mechanism.

Further, to observe whether such a shock is propagated to other SEPs with barriers close by, we divide total events into terciles of N2V and Z. The SEPs that belong to the top tercile of Z are likely those in close proximity to knock-ins. Therefore, interactions between these two terms allow us to observe how a single instance of price pressure is propagated via the event cascade. We propose the following categorical Probit specification:

$$KI_{i,(t+1,t+10)} = \Phi\left(\alpha_{I(i)\times M(t)} + \beta_1 N 2 V_j^M + \beta_2 N 2 V_j^H + \beta_3 Z_j^M + \beta_4 Z_j^H + \beta_5 N 2 V_j^M \cdot Z_j^M + \beta_6 N 2 V_j^M \cdot Z_j^H + \beta_7 N 2 V_j^H \cdot Z_j^M + \beta_8 N 2 V_j^H \cdot Z_j^H + \gamma X_{i,t-10} + \varepsilon_j\right),$$
(12)

where Z is defined as it is for Equation (10), and  $N2V_j^{H,M}$  and  $Z_j^{H,M}$  are dummy variables that indicate whether i belongs to the top tercile (H) or middle tercile (M) of the respective measure.  $N2V_j^L$  and  $Z_j^L$  are the omitted categories. All other variable definitions are identical to those used in Equation (4). The coefficients of interest are those on the interaction terms  $(\beta_{5-8})$ , as they would reveal the chain-reaction mechanism.

### 3.3. Results of event chain reactions and related discussion

Table V presents the results of Equation (11). For ease of interpretation, we report the marginal effects of each variable at the mean. The coefficients, therefore, measure the in-

stantaneous rate of change at their mean. Our results show that a higher N2V increases the knock-in probability over the following 10 days, implying that the liquidity demand that results from the current event can trigger another knock-in in the future. According to Column (5) of the same table, a one-standard-deviation rise in N2V is associated with a 5.3% higher future knock-in probability.<sup>20</sup>

#### [Insert Table V here.]

Table VI presents the results of Equation (12) and shows that price pressure caused by a current event increases the knock-in probability for other SEPs with adjacent barriers, as measured by Z. Neither N2V nor Z alone appears to induce future knock-ins. However, a high N2V (indicated by  $N2V^H$ ) is more likely to lead to subsequent knock-ins when their barriers are closer to the barrier of the current event (indicated by  $Z^H$ ). According to the results reported in Column (5), for example, an event corresponding to the high-Z, high-N2V group is 21.3% more likely to trigger future knock-ins than an event corresponding to the high-Z, low-N2V group.<sup>21</sup>

These results not only provide additional support for the chain-reaction mechanism, but they also further alleviate the concern that N2V correlates with a market condition in a particular way. In the previous analysis, we address this with Industry  $\times$  Month fixed effects. As we examine future events, however, Z helps us rule out the possibility that there is a spurious relationship between N2V and future knock-ins. We are concerned in particular that N2V could be larger in the period of negative returns for reasons unrelated to the liquidity demand measured by N2V. N2V would then be positively correlated with the future knock-ins but not because of liquidity demand. On the other hand, during the period of negative returns, Z

 $<sup>^{20}</sup>$ The mean of N2V is 0.0996 and the standard deviation is 0.1948. The predicted probability of N2V at the mean level is 0.7291 and the predicted probability at a one-standard-deviation increase above the mean level (0.2944 = 0.0996+0.1948) is 0.7825. Therefore, the marginal effect of a one-standard-deviation increment of N2V at the mean is 5.3% (0.0534=0.7825-0.7291).

 $<sup>^{21} \</sup>mathrm{In}$  the high-Z group  $(Z^H),$  the predicted probability associated with the high-N2V group  $(N2V^H)$  is 0.8617, and the predicted probability associated with the low-N2V group  $(N2V^L)$  is 0.6487. Therefore, the marginal effect of  $N2V^H$  on the dependence of the future knock-in probability  $Z^H$  is 21.3% (0.2130=0.8617-0.6487).

would decrease continuously as more SEPs from the same issue vintage experience knock-ins and eventually disappear from the subsequent calculation of Z.

To illustrate this point, suppose that there is only one SEP of a specific vintage whose barrier remains untouched. Also suppose that a latent variable that is orthogonal to liquidity demand drives N2V higher and makes the underlying asset's return more negative, as we have surmised. Once the SEP is knocked-in, the corresponding Z would become small because it is based entirely on SEPs from another vintage with a substantially lower barrier level. The positive correlation of N2V and the future knock-in probability must then be stronger when Z is small.

Our results show the opposite: the N2V is positively correlated with future knock-ins only when Z is higher. This finding bolsters the explanation based on the chain-reaction mechanism as a result of which a knock-in leads to another knock-in and rules out the spurious relationship.

[Insert Table VI here.]

# 4. Strategic Hedging

In the presence of event-driven price pressure and event cascading, changes in an expected shock can alter hedging behavior upon a knock-in. Also, how a trader conducts the liquidation can differ depending on her incentives. In this section, we examine this possibility.

#### 4.1. Measures of a future knock-in

We first construct a 'remaining-to-volume,' or 'R2V' ratio. This variable, like N2V, measures the price impact of a contemporaneous knock-in, except that R2V is related to the price impact of future knock-ins. Specifically, for underlying stock i and event time t corresponding to event j (i.e., i = i(j), t = t(j)), we aggregate the notional amount of SEP k in the relevant set  $\mathbf{K}_{i,t}^{''},$  and then normalize it by the average volume of stock i:

$$R2V_j \equiv R2V_{i(j),t(j)} = \frac{\sum_{k \in \mathbf{K}_{i,\mathbf{t}}^{"}} Notional_k}{Volume_{i,\bar{t}(\mathbf{K}_{i,t}^{"})}},$$
(13)

where  $\mathbf{K}''_{i,t}$  is a set of outstanding SEPs on stock i whose barriers have never been touched as of t,  $Notional_k$  is the notional size of SEP  $k \in \mathbf{K}''_{i,t}$  divided by the total number of underlying assets, and  $Volume_{i,\bar{t}(\mathbf{K}''_{i,t})}$  is the 6-month average dollar volume of stock i as of the earliest issue date among SEPs in  $\mathbf{K}''_{i,t}$  denoted by  $\bar{t}(\mathbf{K}''_{i,t})$ .

From this baseline measure, we construct three additional sub-R2V variables.  $R2V_j^1$  uses only SEPs for which the distance between their barriers and the barrier associated with event j is the smallest among  $\mathbf{K}''_{i,t}$ . In other words,  $R2V^1$  is based on SEPs whose barriers are immediately below the currently touched barrier.  $R2V^2$  and  $R2V^3$  are created in the same way with the second and third immediate barriers, respectively. Given the strict knock-in order among  $R2V^1$ ,  $R2V^2$ , and  $R2V^3$ , we conjecture that  $R2V^1$  provides the most relevant information while the three ratios may have differential impacts on the hedging behavior.

To study strategic hedging behaviors, we further decompose  $R2V^1$  based on issuer identity. Specifically, we split  $R2V^1$  into an in-house portion and an external portion. The in-house portion  $(R2V_{j,in}^1)$  exploits only SEPs issued by the same issuer of event j in the corresponding set  $\mathbf{K}''_{i,t}$  with i=i(j) and t=t(j), whereas the external portion  $(R2V_{j,ex}^1)$  uses the remaining SEPs in the same set issued by different institutions.

# 4.2. Incentives for strategic execution

Rational delta-hedging traders can respond to numerous incentives that affect how they execute a transaction triggered by a knock-in event. We first discuss three distinct incentives and explore their testable implications for execution behavior.

1. Price pressure: An SEP issuer is typically long on the down-and-in barrier put

option. If the trader is not sufficiently delta-neutralized, she can benefit from a knockin event because it would activate the barrier option and the trader then becomes a holder of the vanilla put. If she has a large interest subject to an imminent knock-in (high  $R2V_{in}^1$ ), she may unwind the current hedging so as to increase the probability of a knock-in for her remaining SEP position.

- 2. Execution smoothing: If a trader at the time of the current event has additional positions subject to subsequent knock-ins with a high likelihood (high  $R2V_{in}^1$ ), she wants to avoid the price pressure from the current unwinding. In this case, she would liquidate the portion related to the current event in a smoother way to minimize the market impact.
- 3. Liquidation 'rat race': Suppose a trader needs to unwind a part of her position in response to the current knock-in and sees that unaffiliated traders have large positions that are subject to imminent knock-ins (high  $R2V_{ex}^1$ ). In this case, beyond the position related to the current event, she has an incentive to go in front of the line and preemptively liquidate the remaining position before the third-party trader imposes negative price pressure. This collective behavior that can destabilize the market resembles the 'rat race' mechanism that Brunnermeier and Oehmke (2013) analyze.

# 4.3. Empirical design

We first examine whether expected future knock-ins have any effect on the price shock when the current event occurs. In the spirit of Equation (5), we regress the CAR of underlying stock i = i(j) on event day t = t(j) upon knock-in event j on the future hedging-intensity measure  $R2V_j$ :

$$CAR_{j} = \alpha_{I(i) \times M(t)} + \beta R2V_{j}^{q} + \gamma X_{i,t-10} + \varepsilon_{j} \qquad (q = 1, 2, 3),$$
 (14)

where  $R2V_j$  follows Equation (13) and all other variables are identically defined as for Equation (4). Our coefficients of interest are  $\beta$ , as they would indicate whether the future knockins influence the abnormal return in response to the current event as well as the relative importance depending on the immediacy.

Next, we decompose the contribution to the price pressure into a portion related to the contemporaneous liquidity demand and a portion related to the expected future unwinding. For this task, we simultaneously exploit the measures of contemporaneous effects (N2V) and future effects  $(R2V^1)$  of knock-in events as follows:

$$CAR_j = \alpha_{I(i) \times M(t)} + \beta_1 N 2V_j + \beta_2 R 2V_j^1 + \gamma X_{i,t-10} + \varepsilon_j, \tag{15}$$

where N2V and  $R2V^1$  are defined as they are for Equations (2) and (13), respectively. Also, the definitions of all other variables are identical to those for Equation (4). Comparing  $\beta_1$  and  $\beta_2$  should reveal the relative importance of the two sources of the negative price impact.

Using issuer identity, we further discriminate the effects of future knock-ins when the next immediate position to be liquidated is an in-house position and when it is an external position. Given event j, its underlying stock i = i(j) and event time t = t(j), we use intensity measures of future knock-ins by issuer type (in versus ex):

$$CAR_{i} = \alpha_{I(i) \times M(t)} + \beta_{1}N2V_{i} + \beta_{2}R2V_{i,in}^{1} + \beta_{3}R2V_{i,ex}^{1} + \gamma X_{i,t-10} + \varepsilon_{i}.$$
 (16)

 $R2V_{in}$  and  $R2V_{ex}$  are defined as they are for Equation (13) and the description following the equation. Controlling for liquidity demand related to the current event (N2V), comparing  $\beta_2$  and  $\beta_3$  should show the heterogeneous effects depending on issuer type on the price pressure upon the current event. All other variable definitions are identical to those used in Equation (4).

The external position measure  $R2V_{ex}$  is subject to information accessibility. Practically

speaking, the opaqueness of OTC derivatives markets makes it difficult for any issuer to discover its competitors' outstanding positions. Under the rat-race explanation, the effects of  $R2V_{ex}$  would be pronounced when the unwinding trader has an easy way to learn of the unaffiliated traders' outstanding SEPs and their terms. To test this possibility, we exploit an institutional shock to the information accessibility. On April 25, 2013, information on SEP issuance became electronically accessible to market participants. The database was prepared by the Korea Securities Depository to enhance market transparency. Previously, such information had to be hand-collected from each institution's issuance report.

To formally test this possibility, we use the following specification. Given event j, for affected stock i = i(j) and at event time t = t(j):

$$CAR_{j} = \alpha_{I(i) \times M(t)} + \beta_{1}N2V_{j} + \beta_{2}R2V_{j}^{1} + \beta_{3}Ex + \beta_{4}Post$$

$$+ \beta_{5}R2V_{j}^{1} \cdot Ex + \beta_{6}R2V_{j}^{1} \cdot Post + \beta_{7}Ex \cdot Post$$

$$+ \beta_{8}R2V_{j}^{1} \cdot Ex \cdot Post + \gamma X_{i,t-10} + \varepsilon_{j},$$

$$(17)$$

where N2V and  $R2V^1$  are defined as they are for Equations (2) and (13). Ex is an indicator variable whose value is 1 if the dominant portion of  $R2V^1$  is competitors' and 0 otherwise. In a simple case, Ex equals 1 when R2V is composed entirely of external positions. When R2V is composed of both in-house and external positions, we calculate the median of the external portion of these samples and assign the value of 1 when the external portion is higher than the calculated median. Post is also an indicator variable that equals 1 if t is on or after the inception time of the data service (April 25, 2013) and 0 otherwise. All other variable definitions are identical to those in Equation (4). The coefficient on the triple interaction term,  $\beta_8$ , should reveal the differential effects of R2V when it corresponds to mostly external positions and after competitors' information becomes readily available.

## 4.4. Results and discussion of strategic hedging behavior

Table VII presents estimation results derived from Equation (14). We find strong evidence that the imminent risk of price pressure is priced in across all R2Vs; however, the magnitudes of the R2Vs vary. Specifically,  $R2V^1$  as shown in Column (1) has the largest negative coefficient at the 1% significance level. As the degree of immediacy drops ( $R2V^2$  and  $R2V^3$ ), both the magnitude and statistical significance decrease. A one-standard-deviation increase of  $R2V^1$  corresponds to a CAR that is 67 basis points more negative. The economic magnitude is comparable to that associated with N2V. Our results reveal the pre-execution behavior and indicate that  $R2V^1$  is of first-order importance regarding price pressure.

### [Insert Table VII here.]

To compare the price impact of a current knock-in with that of future knock-ins, we estimate Equation (15) and report the results in Table VIII. We find that the economic magnitudes of  $R2V^1$  are in line with the results reported in Table VII, even after controlling for liquidity demand the current event stimulates. This result confirms that those channels are independent. A one-standard-deviation increase in  $R2V^1$  corresponds to a 57 basis point decrease in CAR, which is almost identical to the 56 basis point decrease that is associated with N2V, supporting the conclusion that, in terms of price impact, the expectation of a future knock-in is as important as the intensity of liquidation as a result of the current event.

#### [Insert Table VIII here.]

We next estimate Equation (16) to reveal traders' strategic hedging behavior in the context of the incentives we have discussed. The results are presented in Table IX. While the magnitude of N2V remains qualitatively unchanged, we find that the coefficients on  $R2V_{in}^1$  are negative but not statistically different from zero. These results suggest that traders do not try to minimize the market impact of the knock-in event even if they have a substantial position

The standard deviation of  $R2V^1$  is 0.0542 and the coefficient on  $R2V^1$  is -0.124. Therefore, the marginal effect of a one-standard-deviation increment of  $R2V^1$  is -0.67% (-0.0067 = 0.0542\*(-0.124)).

to liquidate imminently. Also, we do not find sufficient evidence of intentional price pressure to trigger future knock-ins. Our interpretation of this result is that issuers are largely deltaneutralized and therefore do not have a particular incentive to trigger a knock-in event: a potential gain from the put is likely to be offset by their hedging positions.

On the other hand, the coefficients on  $R2V_{ex}^1$  are consistently negative and highly significant. This result supports the conclusion that a trader employs a preemptive hedging strategy. As she unwinds the position related to the current event (as measured by N2V), she appears to jump in front of the line and liquidates excess positions before competitors impose negative price pressure, exhibiting liquidation 'rat race' behavior (Brunnermeier and Oehmke, 2013). Our findings suggest that such a strategic trading behavior exacerbates the price pressure.

## [Insert Table IX here.]

Lastly, in Table X we report the estimation results derived from Equation (17). We find that the coefficients on N2V are comparable to previous outcomes. Interestingly, the effect on  $R2V^1$  is concentrated when the outstanding notional amount is composed predominantly of external positions and traders have easy access to information about outstanding SEPs issued by their competitors  $(R2V^1 \times Ex \times Post)$ . In this case, we estimate that a one-standard-deviation change in  $R2V^1$  corresponds to a 2.6% decrease in CAR.

This finding further supports the rat-race explanation. Allowing traders easy access to competitors' positions induces strategic hedging behavior. Given the previous finding that one knock-in can trigger another, the strategic incentive to engage in liquidation front-running can create a significant market disruption. We find that improving the transparency of the market can sometimes serve as a mis-coordination device.

[Insert Table X here.]

## 5. Robustness Tests

## 5.1. Price dynamics depends on sorted N2Vs

We repeat the estimation of the dynamic pattern following Equation (4) separately for events whose N2Vs are higher than the sample median and for the rest. Figure 6 presents the dynamic abnormal returns of the bottom N2V group (the triangle marker) and the top N2V group (the circle marker). In both unfiltered and filtered CAR, top N2V groups exhibit more significant price pressure. The pre-trends of these two groups are remarkably similar, suggesting that the N2V measure is not strongly correlated with variations other than the intensity of the immediate hedging.

[Insert Figure 6 here.]

### 5.2. Robustness with various event windows

In the presence of the trade-off between sample size and the observable length of the dynamics, we select an 11-day window in our main analysis (i.e., [-5,+5]). To check whether our 'V'-shaped pattern depends on this choice, we repeat the dynamic analysis with three alternative event windows: [-3,+3], [-10,+10], and [-15,+15].

On the one hand, in an effort to ensure that we collect non-overlapping events, we reduce our window size to [-3,+3] and increase the number of events from 300 to 373 at the expense of the event period. In Panel (a) of Figure 7, we plot the point estimates of each  $\beta_{\tau}$  and their 90% confidence intervals, applying this event window to Equation (4). The CAR pattern still shows the "V" shape, with a magnitude of -5.3% on the event day t. The confidence interval in this case is noticeably tighter.

On the other hand, to check the post-event reversion pattern, we expand the event window to two weeks ([-10,+10]) and three weeks ([-15,+15]), and report the results in Panel (b) and (c) of Figure 7, respectively. Widening the window reduces the total number of non-

overlapping events from 300 events to 206 events for the two-week window and 179 for the three-week window. This exercise, however, enables us to better observe a recovery pattern. Both analyses demonstrate a similar "V"-shaped CAR, confirming that our main finding is robust to our choice of event window.

[Insert Figure 7 here.]

#### 5.3. Robustness with stocks with comparable past returns

Finally, we conduct a placebo test in which we examine the fCAR of stocks with similar negative past returns but that are not associated with the knock-in event. For each event, we match a set of control stocks whose past 1-month returns are within 1% of the event stocks, but with zero N2V (i.e., not associated with any events). We repeat our analysis specified in Equation (4) with this set and present the results in Figure 8. The patterns in these two comparable figures demonstrate a stark contrast. We do not find any similar pattern in this placebo test, ruling out the possibility that the "V"-shaped pattern is driven by past returns.

[Insert Figure 8 here.]

## 6. Conclusion

In this paper, we document a salient case in which financial innovation can sometimes entail price disruption. In particular, a certain type of SEP triggers a sudden and dramatic change in the delta position, forcing hedgers to immediately unwind a large position. This event occurs when the stock price breaches a pre-determined boundary. Such liquidity demand imposes pricing pressure on the underlying stock by a significant degree and for a considerable time period. Upon knock-ins, we find that retail investors actively provide liquidity.

This mechanism can be amplified in two ways. When a large number of products with similar

triggering conditions have been issued in the market, one event can cause another, creating a cascade of price pressure. In this case, the impact of a shock can last substantially longer. On the other hand, when a trader can observe her competitors' outstanding positions, she has an incentive to front-run others. This strategic hedging behavior can lead to coordination failure.

We believe that our results imply that issuance coordination is necessary. In practice, every financial institution imposes a risk budget on itself. We show, however, that it is not sufficient to keep an aggregated quantity under a desirable level. Also, in the presence of pre-determined execution conditions, we find that a policy that requires other market participants' interests to be revealed does not necessarily enhance market stability.

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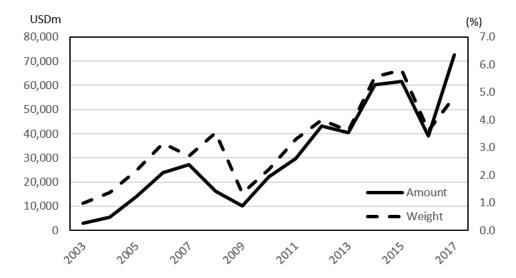
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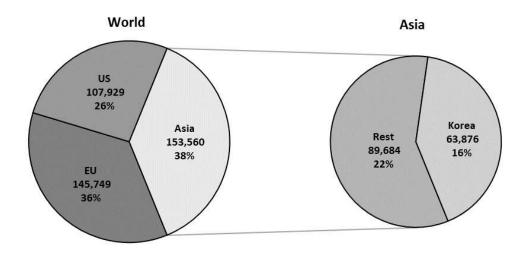
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### Figure 1. Overview of Structured Equity Products (SEPs) Market

Panel (a) shows the aggregate pattern of SEP issuance in Korea. The solid line represents the aggregate issuance amount of SEPs and the dotted line illustrates the portion of the aggregated issuance amount of SEPs of Korean equity market capitalization. Panel (b) illustrates the share of outstanding balances across markets for SEPs whose payoffs feature specific 'knock-in' barriers as of 2019. The pie chart on the left shows the portion of products distributed across the U.S., European, and Asian markets, and the right chart presents the share of the Korean market within Asian markets.



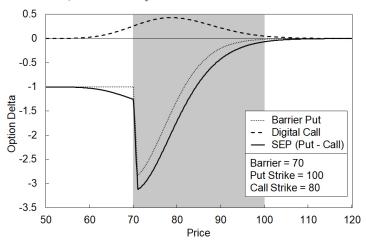
(a) SEPs Issuance in the Korean Market



(b) Distribution of 'Knock-in' Type SEPs in the Global Market (USDm)

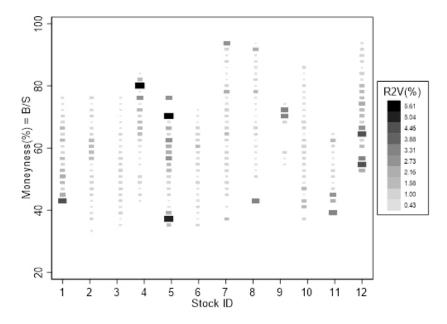
## Figure 2. Delta Variation of SEPs

This figure presents the delta of SEP, where SEP can be decomposed into two: a short position of a digital call option and a long position of a knock-in down-in put option. The horizontal axis indicates the underlying stock price, and the vertical axis represents the option delta. This figure is plotted based on the assumption that the SEP's call strike price is \$80, the pre-determined coupon rate is 6%, the put strike price and the knock-in barrier are \$100 and \$70, and maturity is 3 months.

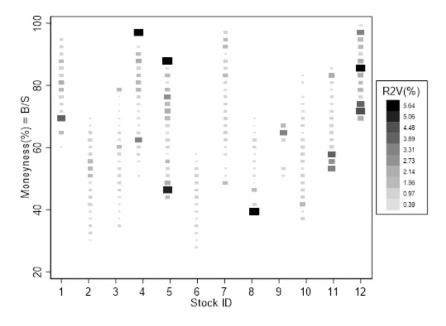


#### Figure 3. Knock-In Exposure

This figure visualizes fragility using R2V, a variable that measures exposure to future knock-ins. In this heat map, the grey spectrum indicates the degree of fragility (the darker the square, the more severe the fragility) on a given day. We select the top 12 stocks from our sample (out of total 89) based on the number of associated SEPs that experienced knock-ins and place them along the horizontal axis (in descending order). Panel (a) and Panel (b) show the measures of the top 12 stocks on January 16 and August 14 in 2014.



(a) Knock-In Exposure Measurement on January 16, 2014



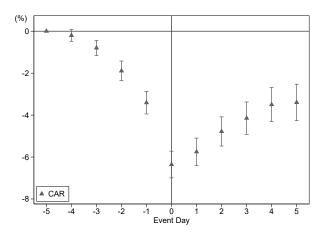
(b) Knock-In Exposure Measurement on August 14, 2014

#### Figure 4. Cumulative Abnormal Returns around Knock-in Events

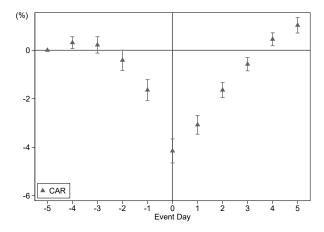
This figure plots the event-time pattern of CARs and filtered CARs (fCAR), where a potential non-linear trend is isolated from the CAR using the Hodrick and Prescott (1997) filter. For event j that occurs at t = t(j), we regress the CAR of affected underlying stock i = i(j) on event-day-dummy variables  $\mathbf{D}$  following Equation (4):

$$CAR_{j,\tau} = \alpha_{I(i) \times M(t)} + \sum_{\tau = -5}^{5} \beta_{\tau} \cdot \mathbf{D}_{\tau} + \gamma X_{i,t-10} + \varepsilon_{j,\tau},$$

where I(i) is the industry of stock i as per the Korean SIC 2-digit classification, M(t) is the calendar month of day t,  $\mathbf{D}_{\tau}$  is an indicator variable whose value is 1 only on event day  $\tau$  and 0 otherwise, and X is a vector of control variables using the most recent information on firm i known at t-10.  $\alpha_{I(i)\times M(t)}$  are Industry × Month fixed effects corresponding to each event. The figure presents the point estimates of each  $\beta_{\tau\in\{-5,\ldots,5\}}$  and their 90% confidence intervals. Panel (a) and Panel (b) show the average knock-in effects of non-overlapping samples on CAR. The total sample for Panel (a) and Panel (b) consists of 3,300 observations (300 events × 11 days). The vertical line indicates the knock-in day, and  $\mathbf{D}_{t-5}$  is an omitted category.



(a) Knock-in Effects on CAR



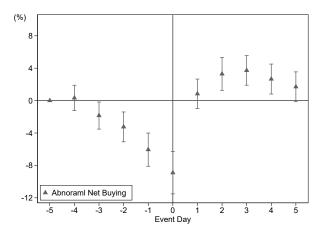
(b) Knock-in Effects on fCAR

#### Figure 5. Abnormal Net Buying

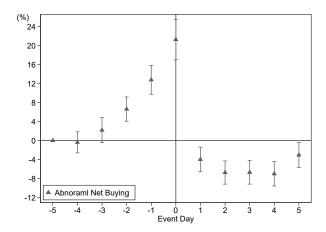
This figure plots the event-time pattern of abnormal net buying (nb). For event j that occurs at t = t(j), we regress the nb of affected underlying stock i = i(j) on event-day-dummy variables  $\mathbf{D}$  following Equation (7):

$$nb_{j,\tau} = \alpha_{I(i) \times M(t)} + \sum_{\tau = -5}^{5} \beta_{\tau} \cdot \mathbf{D}_{\tau} + \gamma X_{i,t-10} + \varepsilon_{j,\tau},$$

where I(i) is the industry of stock i as per the Korean SIC 2-digit classification, M(t) is the calendar month of day t,  $\mathbf{D}_{\tau}$  is an indicator variable whose value is 1 only on event day  $\tau$  and 0 otherwise, and X is a vector of control variables using the most recent information on firm i known at t-10.  $\alpha_{I(i)\times M(t)}$  are Industry  $\times$  Month fixed effects corresponding to each event. The figure presents the point estimates of each  $\beta_{\tau\in\{-5,\dots,5\}}$  and their 90% confidence intervals. Panel (a) shows the point estimates of financial institutions (FIs), while Panel (b) presents those of retail investors. Both of the samples consist of 3,300 observations (300 events  $\times$  11 days). The vertical line indicates the knock-in day, and  $\mathbf{D}_{t-5}$  is an omitted category.



(a) Knock-in Effects on Net Buying: FIs



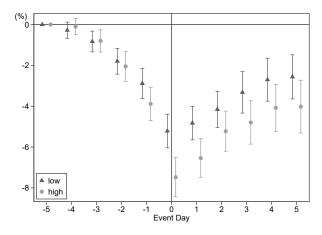
(b) Knock-in Effects on Net Buying: Retail

#### Figure 6. Cumulative Abnormal Returns by N2V

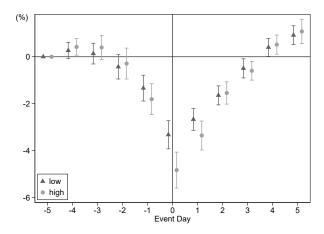
This figure plots the event-time pattern of CARs and filtered CARs (fCAR), where a potential non-linear trend is isolated from the CAR using the Hodrick and Prescott (1997) filter. For event j that occurs at t = t(j), we regress the CAR of affected underlying stock i = i(j) on event-day-dummy variables  $\mathbf{D}$  following Equation (4):

$$CAR_{j,\tau} = \alpha_{I(i) \times M(t)} + \sum_{\tau = -5}^{5} \beta_{\tau} \cdot \mathbf{D}_{\tau} + \gamma X_{i,t-10} + \varepsilon_{j,\tau},$$

where I(i) is the industry of stock i as per the Korean SIC 2-digit classification, M(t) is the calendar month of day t,  $\mathbf{D}_{\tau}$  is an indicator variable whose value is 1 only on event day  $\tau$  and 0 otherwise, and X is a vector of control variables using the most recent information on firm i known at t-10.  $\alpha_{I(i)\times M(t)}$  are Industry  $\times$  Month fixed effects corresponding to each event. We categorize our sample events into two groups using the median of N2V. Both panels display the point estimates of each  $\beta_{\tau\in\{-5,\dots,5\}}$  and their 90% confidence intervals when the events fall into the bottom N2V group (the triangle marker) versus the top N2V group (the circle marker). The samples for the bottom N2V and the top N2V groups consist of 1,650 observations (150 events  $\times$  11 days). The vertical line indicates the knock-in day, and  $\mathbf{D}_{t-5}$  is an omitted category.



(a) Knock-in Effects on CAR



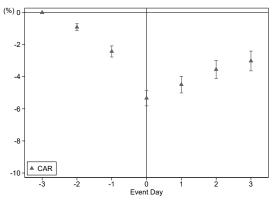
(b) Knock-in Effects on fCAR

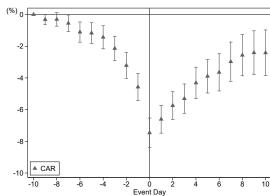
#### Figure 7. Cumulative Abnormal Returns with Various Event Windows

The figure describes the event-time pattern of CARs. For event j that occurs at t = t(j), we regress the CARs of affected underlying stock i = i(j) on event-day-dummy variables **D** following Equation (4):

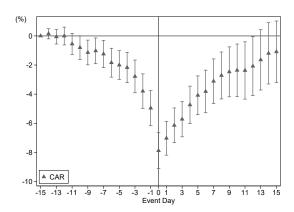
$$CAR_{j,\tau} = \alpha_{I(i)\times M(t)} + \sum_{\tau=-T}^{T} \beta_{\tau} \cdot \mathbf{D}_{\tau} + \gamma X_{i,t-(2*T)} + \varepsilon_{j,\tau},$$

where I(i) is the industry of stock i as per the Korean SIC 2-digit classification, M(t) is the calendar month of day t,  $\mathbf{D}_{\tau}$  is an indicator variable whose value is 1 only on event day  $\tau$  and 0 otherwise, and X is a vector of control variables using the most recent information on firm i known at t - (2\*T).  $\alpha_{I(i) \times M(t)}$  are Industry  $\times$  Month fixed effects corresponding to each event. This figure presents the point estimates of each  $\beta_{\tau}$  and their 90% confidence intervals. The window for Panel (a) is [-3,+3], T=3. Similarly, T=10 is set for Panel (b) and T=15 is set for Panel (c). The total sample for Panel (a) consists of 2,611 observations (373 events  $\times$  7 days), for Panel (b) it consists of 4,326 observations (206 events  $\times$  21 days), and for Panel (c) it consists of 5,549 observations (179 events  $\times$  31 days). The vertical line indicates the knock-in day, and  $\mathbf{D}_{t-T}$  is an omitted category.





- (a) Knock-in Effects on CAR [-3,+3]
- (b) Knock-in Effects on CAR [-10,+10]



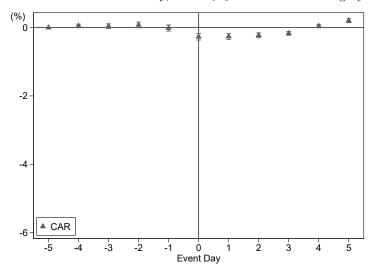
(c) Knock-in Effects on CAR [-15,+15]

#### Figure 8. Cumulative Abnormal Returns with a Placebo Sample

This figure displays the result of a placebo test in which we examine the fCAR of stocks with similar negative past returns but that are not associated with the knock-in event. We use the Hodrick and Prescott (1997) filter for the fCAR adjustments. For each event, we match a set of control stocks whose past 1-month returns are within 1% of the event stocks, but with zero N2Vs (i.e., not associated with any events). For event j that occurs at t = t(j), we regress the fCAR of affected underlying stock i = i(j) on event-day-dummy variables  $\mathbf{D}$  following Equation (4):

$$fCAR_{j,\tau} = \alpha_{I(i)\times M(t)} + \sum_{\tau=-5}^{5} \beta_{\tau} \cdot \mathbf{D}_{\tau} + \gamma X_{i,t-10} + \varepsilon_{j,\tau},$$

where I(i) is the industry of stock i as per the Korean SIC 2-digit classification, M(t) is the calendar month of day t,  $\mathbf{D}_{\tau}$  is an indicator variable whose value is 1 only on event day  $\tau$  and 0 otherwise, and X is a vector of control variables using the most recent information on firm i known at t-10.  $\alpha_{I(i)\times M(t)}$  are Industry  $\times$  Month fixed effects corresponding to each event. The figure presents the point estimates of each  $\beta_{\tau\in\{-5,\ldots,5\}}$  and their 90% confidence intervals. The total sample consists of 89,551 observations (8,141 events  $\times$  11 days). The vertical line indicates the knock-in day, and  $\mathbf{D}_{t-5}$  is an omitted category.



#### Table I. Data Description

In Panel (a), the first two rows show the amounts and shares of outstanding balances across markets for structured equity products (SEPs) whose payoffs feature a specific 'knock-in' barrier as of 2019. The following two rows include total stock market capitalization and the ratio of the outstanding balance of SEPs to total stock market capitalization as of 2019. Amounts are presented in millions of U.S. Dollars and ratios are in percentages. Panel (b) illustrates the process of sample selection from 2006 through 2017. The results reported in Panel (c) indicate the time trends of SEP issuance and knock-in events. The issuance columns' include the total number of SEPs, the aggregate notional monetary amounts, and the unique numbers of underlying stocks by year of issuance. The knock-in columns present the comparable quantities, conditioned on the assumption that the knock-in event has been triggered in the corresponding year.

(a) Distribution of 'Knock-in' Type SEPs by Region

	US	EU	Asia	Korea
SEPs (USDm)	107,929	145,749	153,560	63,876
SEPs/Gobal SEPs (%)	26.50%	35.79%	37.71%	15.69%
Exchange ME (USDm)	37,689,256	8,078,749	23,820,189	1,489,611
SEPs/Exchange ME (%)	0.29%	1.80%	0.64%	4.29%

#### (b) Sample Selection

. , , , ,		
Filter	Sample Size	No. of Stock
Initial Sample: SEPs between 1/2006 - 12/2017 Excluding SEPs (w/ foreign underlying stocks, non-downside knock-in barrier, missing barrier, and issue amount info)	38,035 21,273	141
Filtered SEPs	16,736	122
Excluding SEPs with un-knock-in event	8,562	
Final Sample: SEPs with knock-in event	8,174	89
Final Sample: SEPs knock-in event days (Aggregate the events when a stock triggers multiple knock-in events on the same day)	1,292	89

#### (c) Time Trend of SEPs in Korea

		Issuance			Knock-In	
Year	N.Obs (1 unit)	Amount (bil. KRW)	Stock (1 unit)	N.Obs (1 unit)	Amount (bil. KRW)	Stock (1 unit)
2006	237	1,544	43	•		
2007	882	5,808	62	16	83	4
2008	1,228	5,736	61	1,604	9,342	65
2009	1,456	2,900	55	14	19	3
2010	2,551	3,878	73	25	27	6
2011	3,231	$4,\!105$	78	2,152	3,028	59
2012	3,339	3,307	72	386	481	27
2013	2,106	1,630	79	756	832	28
2014	796	377	59	1,915	1,784	28
2015	135	54	38	1,186	814	29
2016	139	135	27	109	53	11
2017	636	499	52	11	3	3
Total	16,736	29,974	122	8,174	16,466	90
			40			

#### Table II. Summary Statistics

In this table, we report the summary statistics for selected variables. N2V ('notional-to-volume') is associated with the extent of liquidation intensity, where we normalize the notional amounts of SEPs related to knock-ins by trading volume of the underlying stocks. Dispersion measures the degree of valuation (EPS) uncertainty at the firm level. Z measures the average future knock-in probability across SEPs on the same underlying stocks of a given knock-in event. R2V ('remaining-to-volume') is related to the price impact of unrealized future knock-ins, where we normalize the remaining notional amount by trading volume of the underlying stocks. From this baseline measure, we construct three additional sub-R2V variables.  $R2V^1$  is based on SEPs whose barriers fall immediately below the currently touched barrier (the first immediate barrier).  $R2V^2$  and  $R2V^3$  are created in the same way with the second and the third immediate barriers, respectively. We further decompose  $R2V^1$  based on issuer identity. Specifically, we split  $R2V^1$  into an in-house portion and an external portion. The in-house portion  $(R2V_{in})$  indicates only the portion of R2V when an issuer who has knocked-in SEPs still has remaining SEPs under the same underlying stock, whereas the external portion  $(R2V_{ex})$  uses the remaining SEPs in the same set issued by different institutions. Detailed functional definitions of N2V, Dispersion, Z, and R2Vs are given for Equations (1), (8), (10), and (13), respectively. For N2V, Z, and R2V, we aggregate the structured note information based on stock level to convert the data structure to event-day  $\times$  stock-level information. Size is the logarithm of the book value of assets (values given in thousands). Book-to-market is calculated by dividing the book value of equity by the market value of equity. RVol is annualized volatility using 1-month stock return. The sample for N2V, Size, Book-tomarket, Average 1-month stock returns, and Average 6-month index returns consist of 1,292 observations, which is the same as the total sample represented in Table I, while the sample for Z and R2Vs consists of 1,225 observations because we rule out cases in which R2V equals zero.

	Mean	St.Dev	Q1	Median	Q3	N
N2V	0.0996	0.1948	0.0096	0.0288	0.1035	1,292
Dispersion	2.0257	5.9698	0.3679	0.7137	1.5957	992
Z	0.7994	0.2139	0.7149	0.8705	0.9661	1,225
$R2V^1$	0.0273	0.0542	0.0039	0.0100	0.0252	1,225
$R2V_{in}^1$	0.0045	0.0215	0.0000	0.0000	0.0017	1,225
$R2V_{out}^1$	0.0229	0.0483	0.0027	0.0077	0.0202	1,225
$R2V^2$	0.0527	0.0953	0.0106	0.0235	0.0528	1,225
$R2V_{in}^2$	0.0086	0.0312	0.0000	0.0010	0.0055	1,225
$R2V_{out}^{2}$	0.0441	0.0843	0.0081	0.0189	0.0439	1,225
$R2V^3$	0.0776	0.1275	0.0186	0.0373	0.0798	1,225
$R2V_{in}^3$	0.0117	0.0351	0.0000	0.0021	0.0094	1,225
$R2V_{out}^{3}$	0.0659	0.1154	0.0146	0.0303	0.0688	1,225
Size	23.8164	1.1355	23.0511	23.5989	24.4018	1,292
Book-to-Market	1.2289	0.8116	0.7367	1.0509	1.5371	1,292
Avg. 1M Ret	-0.0805	0.0954	-0.1368	-0.0775	-0.0204	1,292
RVol	0.4245	0.2487	0.2521	0.3616	0.5321	1,292
Avg. 6M Index Ret	-0.0381	0.1145	-0.0758	-0.0179	0.0356	1,292

### Table III. Impact of Knock-in Events on Cumulative Abnormal Returns

This table shows the results of regressing CAR on N2V following Equation (5):

$$CAR_{j} = \alpha_{I(t) \times M(t)} + \beta N2V_{j} + \gamma X_{i,t-10} + \varepsilon_{j},$$

where we regress the CAR of underlying stock i=i(j) of the event on the event day t(j) on the N2V of the same event. N2V is defined in Table II. I(i) is the industry of stock i as per the Korean SIC 2-digit classification, M(t) is the calendar month of day t, and X is a vector of control variables using the most recent information on firm i known at t-10. The definitions of control variables are identical to those given in Table II. From Column (1) through Column (5), we regress the specification with Industry  $\times$  Month fixed effects ( $\alpha_{I(i)\times M(t)}$ ) corresponding to each event. For Column (6), we repeat the regression with Stock  $\times$  Time fixed effects. Column (7) shows results with quartile indicators of N2V instead of continuous N2V. The t-statistics with standard errors clustered at the event level are reported below the coefficients. The superscripts \*\*\*, \*\*, and \* refer to the 1%, 5%, and 10% levels of statistical significance, respectively.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
N2V	-0.036**	-0.036***	-0.035**	-0.036**	-0.035***	-0.022**	
	(-2.53)	(-2.67)	(-2.40)	(-2.56)	(-2.60)	(-1.98)	
q.N2V=2							-0.002
							(-0.43)
q.N2V=3							-0.010*
- NOV 4							(-1.72)
q.N2V=4							-0.025*** (-3.54)
Size		0.016***			0.017***	-0.066***	0.017***
Size		(3.17)			(3.14)	(-3.86)	(3.21)
Book-to-Market		0.003			0.003	0.102***	0.003
		(0.64)			(0.48)	(4.93)	(0.56)
Avg. 1M Ret		, ,	-0.048	-0.002	-0.026	-0.109**	-0.035
			(-1.36)	(-0.05)	(-0.62)	(-2.36)	(-0.88)
RVol			-0.044	-0.042	-0.040	-0.002	-0.046*
			(-1.62)	(-1.52)	(-1.45)	(-0.05)	(-1.67)
Avg. 6M Index Ret				-0.189**	-0.159*	0.194**	-0.162*
				(-2.12)	(-1.73)	(2.03)	(-1.76)
$\mathrm{Ind}\;\mathrm{FE}\times\mathrm{Time}\;\mathrm{FE}$	Y	Y	Y	Y	Y	N	Y
Stock FE $\times$ Time FE	N	N	N	N	N	Y	N
N.Obs	1,292	1,292	1,292	$1,\!292$	1,292	1,052	1,292
Adj. $R^2$	0.353	0.373	0.357	0.363	0.383	0.570	0.386

### Table IV. Abnormal Net Buying and Earning Per Share Uncertainty

This table shows the results of regressing nb on Uncertainty following Equation (9):

$$nb_j = \alpha_{I(i) \times M(t)} + \beta_1 Uncertainty_j^M + \beta_2 Uncertainty_j^L + \gamma X_{i,t-10} + \varepsilon_j,$$

where nb is the normalized abnormal net buying amount as defined in Equation (6).  $Uncertainty_j^{H,M,L}$  are tercile indicator variables that indicate whether i belongs to the high tercile (H), the middle tercile (M), or the bottom tercile (L) of the  $Dispersion_j$ , which is defined in Equation (8).  $Uncertainty_j^H$  is an omitted category. I(i) is the industry of stock i as per the Korean SIC 2-digit classification, M(t) is the calendar month of day t, and X is a vector of control variables using the most recent information on firm i known at t-10. The definitions of control variables are identical to those used for Table II.  $\alpha_{I(i)\times M(t)}$  are Industry  $\times$  Month fixed effects corresponding to each event. The t-statistics with standard errors clustered at the event level are reported below the coefficients. The superscripts \*\*\*, \*\*, and \* refer to the 1%, 5%, and 10% levels of statistical significance, respectively.

	(1) Retail	(2) FIs	
$Uncertainty^{M}$	0.086*	0.002	
	(1.74)	(0.11)	
$Uncertainty^L$	0.111*	0.009	
	(1.90)	(0.35)	
Size	0.001	0.015*	
	(0.07)	(1.66)	
Book-to-Market	-0.052**	0.010	
	(-2.35)	(1.04)	
Avg. 1M Ret	0.251	-0.131*	
	(1.51)	(-1.80)	
RVol	0.197**	-0.163***	
	(2.01)	(-3.25)	
Avg. 6M Index Ret	0.537	-0.021	
·	(1.57)	(-0.13)	
$Ind FE \times Time FE$	Y	Y	
N.Obs	992	992	
$Adj. R^2$	0.249	0.217	

#### Table V. Impact of Knock-in Events on Future Knock-in Probability

This table shows the results of Probit regressions using N2V following Equation (11): for each event j, its corresponding stock i = i(j) and event time t = t(j),

$$KI_{i,(t+1,t+10)} = \Phi(\alpha_{I(i)\times M(t)} + \beta N2V_j + \gamma X_{i,t-10} + \varepsilon_j),$$

where  $\Phi(\cdot)$  is the standard normal cumulative density function, N2V is defined as in Table II,  $KI_{i,(t+1,t+10)}$  is an indicator variable that yields 1 if there is any knock-in on the same underlying i in the next 10 days from the event time t and 0 otherwise. I(i) is the industry of stock i as per the Korean SIC 2-digit classification, M(t) is the calendar month of day t, and X is a vector of control variables using the most recent information on firm i known at t-10. The definitions of control variables are identical to those used for Table II.  $\alpha_{I(i)\times M(t)}$  are Industry  $\times$  Month fixed effects corresponding to each event. The t-statistics with standard errors clustered at the event level are reported below the coefficients. The superscripts \*\*\*, \*\*, and \* refer to the 1%, 5%, and 10% levels of statistical significance, respectively.

	(1)	(2)	(3)	(4)	(5)
N2V	0.218***	0.217**	0.261***	0.287***	0.290***
	(2.62)	(2.56)	(2.90)	(3.09)	(3.06)
Size		0.036***			0.036***
		(2.83)			(2.71)
Book-to-Market		-0.033*			-0.033*
		(-1.90)			(-1.86)
Avg. 1M Ret			0.182	0.146	0.120
			(1.38)	(1.10)	(0.90)
RVol			-0.179***	-0.109*	-0.094
			(-3.51)	(-1.67)	(-1.40)
Avg. 6M Index Ret				0.255*	0.295*
				(1.69)	(1.88)
$\mathrm{Ind}\;\mathrm{FE}\times\mathrm{Time}\;\mathrm{FE}$	Y	Y	Y	Y	Y
N.Obs	1,292	1,292	1,292	1,292	1,292
LRChi2	7.70	15.23	23.31	25.85	30.98
ProbChi2	0.021	0.004	0.000	0.000	0.000
Pseudo R-squared	0.006	0.013	0.018	0.020	0.026

# Table VI. Interaction Effects of Knock-in Intensity and Proximity Contributed to the Future Knock-in Probability

In this table, we report the results of Probit regressions using N2V as an indicator of knock-in intensity and Z as the average knock-in probability (proximity) following Equation (12). We divided total events into terciles for N2V and Z. We propose the following categorical Probit specification:

$$\begin{split} KI_{i,(t+1,t+10)} = & \Phi \left( \alpha_{I(i) \times M(t)} + \beta_1 N 2 V_j^M + \beta_2 N 2 V_j^H + \beta_3 Z_j^M + \beta_4 Z_j^H + \beta_5 N 2 V_j^M \cdot Z_j^M \right. \\ & + \beta_6 N 2 V_j^M \cdot Z_j^H + \beta_7 N 2 V_j^H \cdot Z_j^M + \beta_8 N 2 V_j^H \cdot Z_j^H + \gamma X_{i,t-10} + \varepsilon_j \right), \end{split}$$

where Z is defined as in Table II, and  $KI_{i,(t+1,t+10)}$  is an indicator variable that yields 1 if there is any knock-in on the same underlying i in the next 10 days from the event time t and 0 otherwise.  $N2V_j^{H,M}$  and  $Z_j^{H,M}$  are dummy variables that indicate whether i belongs to the top tercile (H) or the middle tercile (M) of the respective measures.  $N2V_j^L$  and  $Z_j^L$  are omitted categories. I(i) is the industry of stock i as per the Korean SIC 2-digit classification, M(t) is the calendar month of day t, and X is a vector of control variables using the most recent information on firm i known at t-10. The definitions of control variables are identical to those used for Table II.  $\alpha_{I(i)\times M(t)}$  are Industry  $\times$  Month fixed effects corresponding to each event. The t-statistics with standard errors clustered at the event level are reported below the coefficients. The superscripts \*\*\*, \*\*\*, and \* refer to the 1%, 5%, and 10% levels of statistical significance, respectively.

	(1)	(2)	(3)	(4)	(5)
$\mathrm{N}2\mathrm{V}^{M}$	0.064	0.097	0.061	0.062	0.099
	(0.43)	(0.64)	(0.41)	(0.41)	(0.65)
$\mathrm{N}2\mathrm{V}^H$	-0.254	-0.229	-0.251	-0.245	-0.215
	(-1.45)	(-1.29)	(-1.43)	(-1.40)	(-1.21)
$\mathrm{Z}^M$	-0.048	0.006	-0.040	-0.036	0.026
	(-0.31)	(0.04)	(-0.25)	(-0.23)	(0.17)
$\mathrm{Z}^H$	-0.084	0.000	-0.041	-0.041	0.058
	(-0.51)	(0.00)	(-0.24)	(-0.24)	(0.33)
$N2V^M \times Z^M$	0.183	$0.14\overset{\circ}{5}$	0.193	0.193	0.152
	(0.80)	(0.64)	(0.85)	(0.85)	(0.67)
$N2V^M \times Z^H$	0.117	0.060	0.125	0.127	0.067
	(0.50)	(0.25)	(0.53)	(0.54)	(0.28)
$N2V^H \times Z^M$	0.602**	$0.\dot{5}66*\dot{*}$	0.627**	$0.\dot{6}30*\dot{*}$	0.593**
	(2.46)	(2.30)	(2.56)	(2.57)	(2.40)
$N2V^H \times Z^H$	0.787***	0.733***	0.818***	0.834***	0.779***
	(3.17)	(2.92)	(3.30)	(3.35)	(3.10)
Size	, ,	0.045	, ,	, ,	0.054
		(1.04)			(1.23)
Book-to-Market		-0.112**			-0.122**
		(-1.99)			(-2.14)
Avg. 1M Ret		, ,	0.193	0.156	0.084
			(0.44)	(0.35)	(0.19)
RVol			-0.301*	-0.212	-0.249
			(-1.69)	(-0.96)	(-1.11)
Avg. 6M Index Ret			. ,	0.367	0.389
				(0.74)	(0.76)
and FE × Time FE	Y	Y	Y	Y	Y
N.Obs	1,225	1,225	1,225	1,225	1,225
LRChi2	23.07	27.73	27.02	28.14	33.40
ProbChi2	0.006	0.004	0.005	0.005	0.003
Pseudo R-squared	0.018	0.020	0.020	0.021	0.024

# Table VII. Impact of Expected Future Knock-ins on Cumulative Abnormal Returns

In this table, we report the results of an analysis in which we examine whether expected future knock-ins have any effect on the price shock upon the current event. We regress CAR of underlying stock i = i(j) on day t = t(j) upon knock-in event j on the future hedging intensity measure  $R2V_j$  following Equation (14):

$$CAR_{j} = \alpha_{I(i) \times M(t)} + \beta R2V_{j}^{q} + \gamma X_{i,t-10} + \varepsilon_{j} \qquad (q = 1, 2, 3),$$

where R2V is defined as in Table II. I(i) is the industry of stock i as per the Korean SIC 2-digit classification, M(t) is the calendar month of day t, and X is a vector of control variables using the most recent information on firm i known at t-10. The definitions of control variables are identical to those in Table II.  $\alpha_{I(i)\times M(t)}$  are Industry  $\times$  Month fixed effects corresponding to each event. The t-statistics with standard errors clustered at the event level are reported below the coefficients. The superscripts \*\*\*, \*\*, and \* refer to the 1%, 5%, and 10% levels of statistical significance, respectively.

	(1)	(2)	(3)	
$R2V^1$	-0.124***			
	(-3.10)			
$R2V^2$		-0.052**		
		(-2.31)		
$R2V^3$			-0.042**	
			(-2.17)	
Size	0.017***	0.017***	0.017***	
	(3.45)	(3.50)	(3.45)	
Book-to-Market	-0.001	-0.000	-0.000	
	(-0.16)	(-0.07)	(-0.03)	
Avg. 1M Ret	-0.053	-0.053	-0.052	
	(-1.29)	(-1.30)	(-1.27)	
RVol	-0.037	-0.036	-0.036	
	(-1.35)	(-1.30)	(-1.29)	
Avg. 6M Index Ret	-0.069	-0.071	-0.068	
	(-0.77)	(-0.79)	(-0.76)	
$\mathrm{Ind}\;\mathrm{FE}\times\mathrm{Time}\;\mathrm{FE}$	Y	Y	Y	
N.Obs	1,225	1,225	$1,\!225$	
R-squared	0.401	0.398	0.397	

### Table VIII. Decomposition of the Impact of Knock-in Events

In this table, we decompose contributions to price pressure into a portion related to the contemporaneous liquidity demand and a portion related to expected future unwinding. For this task, we simultaneously exploit measures of contemporaneous effects (N2V) and future effects  $(R2V^1)$  of knock-in events following Equation (15):

$$CAR_{j} = \alpha_{I(i) \times M(t)} + \beta_{1}N2V_{j} + \beta_{2}R2V_{j}^{1} + \gamma X_{i,t-10} + \varepsilon_{j}.$$

N2V and  $R2V^1$  are defined as in Table II. I(i) is the industry of stock i as per the Korean SIC 2-digit classification, M(t) is the calendar month of day t, and X is a vector of control variables using the most recent information on firm i known at t-10. The definitions of control variables are identical to those given in Table II.  $\alpha_{I(i)\times M(t)}$  are Industry  $\times$  Month fixed effects corresponding to each event. The t-statistics with standard errors clustered at the event level are reported below the coefficients. The superscripts \*\*\*, \*\*, and \* refer to the 1%, 5%, and 10% levels of statistical significance, respectively.

	(1)	(2)	(3)	(4)	(5)
N2V	-0.030**	-0.029**	-0.030*	-0.031**	-0.029**
	(-2.08)	(-2.14)	(-1.95)	(-2.08)	(-2.06)
$R2V^1$	-0.130***	-0.103**	-0.138***	-0.132***	-0.106***
	(-3.13)	(-2.39)	(-3.39)	(-3.31)	(-2.59)
Size		0.016***			0.016***
		(3.48)			(3.38)
Book-to-Market		0.001			-0.000
		(0.12)			(-0.02)
Avg. 1M Ret			-0.053	-0.027	-0.045
			(-1.42)	(-0.65)	(-1.08)
RVol			-0.046*	-0.045	-0.038
			(-1.66)	(-1.61)	(-1.37)
Avg. 6M Index Ret				-0.112	-0.079
				(-1.28)	(-0.88)
$\overline{\text{Ind FE} \times \text{Time FE}}$	Y	Y	Y	Y	Y
N.Obs	1,225	1,225	1,225	1,225	1,225
R-squared	0.385	0.401	0.389	0.391	0.407

# Table IX. Decomposition of the Impact of Future Knock-ins and the Liquidity Effect

In this table, we discriminate the effects of future knock-ins when the next immediate position to be liquidated is an in-house position and when it is an external position. Given event j, for its underlying stock i = i(j) and at event time t = t(j), we use intensity measures of future knock-ins by issuer type (in versus ex) following Equation (16):

$$CAR_{j} = \alpha_{I(i) \times M(t)} + \beta_{1}N2V_{j} + \beta_{2}R2V_{j,in}^{1} + \beta_{3}R2V_{j,ex}^{1} + \gamma X_{i,t-10} + \varepsilon_{j},$$

where N2V,  $R2V_{in}$ , and  $R2V_{ex}$  are defined as in Table II. I(i) is the industry of stock i as per the Korean SIC 2-digit classification, M(t) is the calendar month of day t, and X is a vector of control variables using the most recent information on firm i known at t-10. The definitions of control variables are identical to those used for Table II.  $\alpha_{I(i)\times M(t)}$  are Industry  $\times$  Month fixed effects corresponding to each event. The t-statistics with standard errors clustered at the event level are reported below the coefficients. The superscripts \*\*\*, \*\*\*, and \* refer to the 1%, 5%, and 10% levels of statistical significance, respectively.

	(1)	(2)	(3)	(4)	(5)
N2V	-0.030**	-0.029**	-0.030*	-0.031**	-0.029**
	(-2.08)	(-2.14)	(-1.95)	(-2.08)	(-2.06)
$R2V_{in}^1$	-0.164*	-0.110	-0.162*	-0.160*	-0.109
	(-1.68)	(-1.18)	(-1.68)	(-1.67)	(-1.18)
$R2V_{ex}^1$	-0.123***	-0.102**	-0.133***	-0.127***	-0.106**
	(-2.87)	(-2.23)	(-3.15)	(-3.05)	(-2.43)
Size		0.016***			0.016***
		(3.47)			(3.37)
Book-to-Market		0.001			-0.000
		(0.12)			(-0.02)
Avg. 1M Ret			-0.053	-0.026	-0.045
			(-1.41)	(-0.64)	(-1.08)
RVol			-0.045*	-0.045	-0.038
			(-1.65)	(-1.60)	(-1.37)
Avg. 6M Index Ret				-0.113	-0.079
				(-1.28)	(-0.88)
$\text{Ind FE} \times \text{Time FE}$	Y	Y	Y	Y	Y
N.Obs	1,225	1,225	1,225	1,225	1,225
R-squared	0.384	0.401	0.389	0.390	0.406

# Table X. The Impact of Expectation Effects depends on Information Accessibility

In this table, we insert two dummies—Ex and Post—in the regression to identify the impact of the exterior R2V subjected to information accessibility following Equation (17): given event j, for affected stock i = i(j) and at event time t = t(j),

$$CAR_{j} = \alpha_{I(i) \times M(t)} + \beta_{1}N2V_{j} + \beta_{2}R2V_{j}^{1} + \beta_{3}Ex + \beta_{4}Post + \beta_{5}R2V_{j}^{1} \cdot Ex + \beta_{6}R2V_{j}^{1} \cdot Post + \beta_{7}Ex \cdot Post + \beta_{8}R2V_{j}^{1} \cdot Ex \cdot Post + \gamma X_{i,t-10} + \varepsilon_{j},$$

where N2V and  $R2V^1$  are defined as in Table II. Ex equals 1 if the predominant portion of  $R2V^1$  is competitors' and 0 otherwise. Specifically, Ex equals 1 if  $R2V^1$  is composed entirely of external positions or if the external portion of  $R2V^1$  is higher than the calculated median across cases in which in-house and external portions are mixed. Post is also an indicator variable that equals 1 if t is on or after the inception point of the data service (April 25, 2013) and 0 otherwise. I(i) is the industry of stock i as per the Korean SIC 2-digit classification, M(t) is the calendar month of day t, and t is a vector of control variables using the most recent information on firm t known at t-10. The definitions of control variables are identical to those given in Table II.  $\alpha_{I(i)\times M(t)}$  are Industry t Month fixed effects corresponding to each event. The t-statistics with standard errors clustered at the event level are reported below the coefficients. The superscripts \*\*\*, \*\*\*, and \* refer to the 1%, 5%, and 10% levels of statistical significance, respectively.

	(1)	(2)	(3)	(4)	(5)
N2V	-0.028**	-0.027**	-0.028*	-0.029**	-0.027*
	(-1.97)	(-2.02)	(-1.86)	(-1.98)	(-1.95)
$R2V^1$	-0.241***	-0.195**	-0.243***	-0.236***	-0.192**
	(-2.74)	(-2.24)	(-2.80)	(-2.81)	(-2.31)
Ex	-0.013*	-0.012*	-0.013*	-0.013*	-0.012
	(-1.74)	(-1.67)	(-1.75)	(-1.68)	(-1.64)
Post	-0.003	0.005	-0.001	0.004	0.007
	(-0.16)	(0.26)	(-0.04)	(0.17)	(0.31)
$R2V^1 \times Ex$	$0.160^{*}$	0.135	0.150	0.149	0.126
	(1.68)	(1.41)	(1.58)	(1.61)	(1.36)
$R2V^1 \times Post$	0.245	0.244	0.251	0.249	0.249
	(1.45)	(1.47)	(1.52)	(1.52)	(1.55)
$Ex \times Post$	0.010	0.010	0.010	0.010	0.010
	(0.95)	(1.00)	(0.97)	(0.94)	(1.02)
$R2V^1 \times Ex \times Post$	-0.456**	-0.490**	-0.442**	-0.446**	-0.476**
	(-2.21)	(-2.44)	(-2.15)	(-2.18)	(-2.40)
Size		0.016***			0.016***
		(3.49)			(3.40)
Book-to-Market		0.001			-0.000
		(0.09)			(-0.04)
Avg. 1M Ret			-0.051	-0.025	-0.043
			(-1.36)	(-0.60)	(-1.03)
RVol			-0.044	-0.044	-0.037
			(-1.59)	(-1.55)	(-1.32)
Avg. 6M Index Ret			, ,	-0.109	-0.076
				(-1.24)	(-0.85)
$\overline{\text{Ind FE} \times \text{Time FE}}$	Y	Y	Y	Y	Y
N.Obs	1,225	1,225	1,225	1,225	1,225
R-squared	0.386	0.403	0.391	0.392	0.408

# Appendix A. Structured Equity Products

## Appendix 1.1. Payoff Profile of an SEP

In this appendix, we explain the payoff of a typical SEP. For the sake of simplicity, we assume that its underlying is a single asset and that it can be called only at maturity.<sup>23</sup> Also, we assume that the initial fixing (the reference price of the underlying stock) is \$100, the SEP's call strike price  $K^{Call}$  is \$80, the pre-determined coupon rate C is 6%, the knock-in barrier B is \$70 and the put strike price  $K^{Put}$  is set to the initial fixing \$100 (i.e., the put is at-the-money).

Panel (a) of Figure A.1 demonstrates the payoff (the vertical axis) with respect to the underlying price at maturity (the horizontal axis). The investor receives one of the following three payoffs: 1) a 6% coupon, 2) no loss or gain, or 3) a loss depending on the maturity stock price, conditional on the entire path of the underlying prices during the SEP's life. Panel (b) explains this path-dependency via a flowchart of payoff determination. At maturity, which is the only call date, we first examine whether the stock price  $S_T$  is greater than or equal to  $K^{Call}$ . If  $S_T \geq K^{Call}$ , the investor receives C and the principal.

If the former condition is not satisfied  $(S_T < K^{Call})$ , an investor cannot receive the coupon. In this case, we determine whether the stock price during the life of the SEP has ever touched the barrier B (70%). If a knock-in event has not occurred  $(\min_{t \in [0,T]} \{S_t\} > B)$ , neither a gain nor a loss is incurred and the investor receives the entire principal.

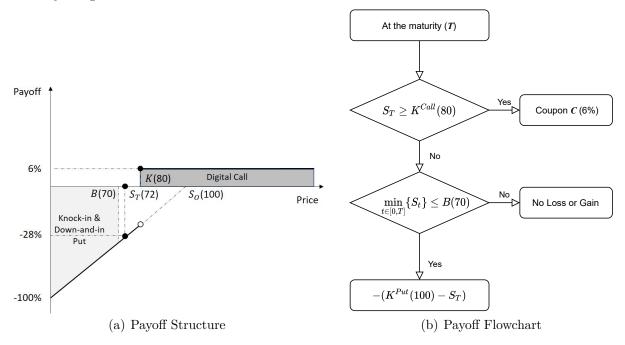
If, however, the knock-in event has occurred, the investor would lose the principal. The loss will be determined by  $S_T$ . For example, if  $S_T = \$72$ , the investor would lose \$28 (\$100-\$72) as specified by the payoff of an at-the-money vanilla put whose strike  $K^{Put}$  is \$100.

An important implication of this payoff is that, in spite of the same maturity price (\$72),

<sup>&</sup>lt;sup>23</sup>SEPs are generally associated with underlying assets (a basket), where the worst performer determines the call condition as well as multiple observation dates. This simplified structure is, however, sufficient for the payoff illustration.

#### Figure A.1. Payoff Profile of an SEP

These two figures are plotted based on the assumption that the SEP's call strike price  $(K \text{ or } K^{Call})$  is \$80, the pre-determined coupon rate (C) is 6%, the put strike price  $(K^{Put} = S0)$  is \$100, the knock-in barrier (B) is \$70, and the underlying stock price at maturity  $(S^T)$  is \$72. Panel (a) shows the SEP payoff structure from the investor perspective, where SEP can be decomposed into two: a long position of a digital call option and a short position of a knock-in down-in put option. The horizontal axis indicates the underlying stock price, and the vertical axis represents the payoff. Panel (b) illustrates the process of determining the payoff at maturity using a flowchart.



the payoff to the investor can be very different depending on the knock-in occurrence (0% gain vs. 28% loss). This dramatic payoff change also heavily alters the risk profile of the derivatives upon the knock-in event.

## Appendix 1.2. Decomposition of an SEP

The SEP payoff is identical to a situation in which the investor goes long on a digital call with an \$80 strike and a 6% payoff and short on a knock-in put with an at-the-money strike and a \$70 barrier. Therefore, the value of SEP  $\psi(t)$  with maturity T can be expressed as the following transaction:

$$\psi(t) = Call_t - Put_t, \tag{A1}$$

where Call is the digital call option with a strike  $K^{Call}(80)$  and a digital payoff C(6%). Put is the down-and-in barrier put option with a strike  $K^{Put}(100)$  and a knock-in barrier B(70).<sup>24</sup>

Given the underlying stock price  $S_t$ , the price of the down-and-in barrier put option is (Hull, 2012):

$$Put_{t} = K^{Put} * e^{-r(T-t)} \left[ \Phi(-x_{1} + \sigma\sqrt{T-t}) - \left(\frac{B}{S_{t}}\right)^{2\lambda-2} \left\{ \Phi(y - \sigma\sqrt{T-t}) - \Phi(y_{1} - \sigma\sqrt{T-t}) \right\} \right]$$

$$- S_{t} \left[ \Phi(-x_{1}) - \left(\frac{B}{S_{t}}\right)^{2\lambda} \left\{ \Phi(y) - \Phi(y_{1}) \right\} \right],$$
(A2)

where  $\Phi(\cdot)$  is the standard normal cumulative distribution function and

$$\lambda = \frac{r + \sigma^2/2}{\sigma^2},$$

$$y = \frac{\ln(B^2/(S_t K^{Put}))}{\sigma\sqrt{T - t}} + \lambda\sigma\sqrt{T - t},$$

$$x_1 = \frac{\ln(S_t/B)}{\sigma\sqrt{T - t}} + \lambda\sigma\sqrt{T - t},$$

$$y_1 = \frac{\ln(B/S_t)}{\sigma\sqrt{T - t}} + \lambda\sigma\sqrt{T - t}.$$

How sensitively the put value responds to the stock price movement (i.e., delta) depends on where  $S_t$  is relative to B. To see this, suppose that  $S_t \gg B$ . In this case where the knock-in is unlikely, y and  $y_1 \to -\infty$ , giving us  $\Phi(y)$  and  $\Phi(y_1) \to 0$ . Also,  $x_1 \to \infty$ , resulting in  $\Phi(-x) \to 0$ . As these near-0 values are multiplied to otherwise negative terms, Put becomes insensitive to  $S_t$ , yielding near-0 delta regardless of the moneyness  $(S_t/K)$ . However, when  $S_t \downarrow B$ , all these negative terms become activated and start creating extra kicks, making the delta much larger than that of the comparable vanilla put.

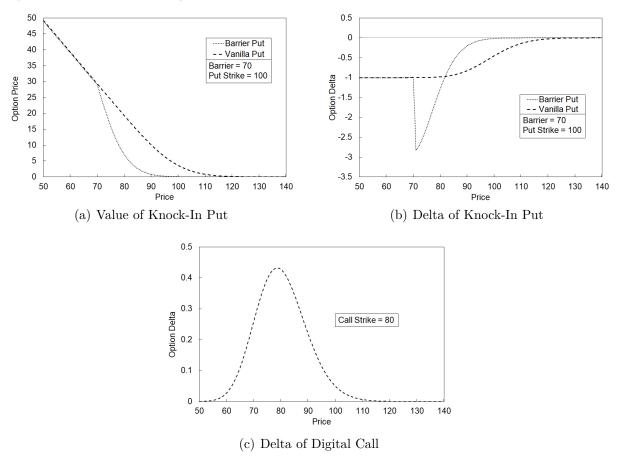
 $<sup>^{24}</sup>$ The numbers in parentheses are the values used in the example in Section A.1.1

Figure A.2 shows the price and delta of the barrier put option according to Equation (A2). For comparison, those of a vanilla put option with the same strike are overlaid in this figure. In Panel (b) in Figure A.2, our numerical example shows that the delta of the barrier option can be substantially lower than -1.

Upon the knock-in, however, the barrier put option becomes a vanilla put whose delta cannot be lower than -1, creating the acute delta change. Since the SEP issuer goes long on this put, the sign of the delta is negative from its perspective. To neutralize the negative delta, the issuer buys the underlying stock. When delta shrinks upon knock-in, however, the hedging

#### Figure A.2. Delta Variation of the Knock-In Put and Digital Call Option

In this figure, Panel (a) displays the option price of the knock-in down-in barrier put option and vanilla put option, and Panel (b) illustrates their deltas. Panel (C) shows the delta of the digital call option. The horizontal axis indicates the underlying stock price, and the vertical axis represents the option price (Panel a) or option delta (Panel b and Panel c). These figures are plotted based on the assumption that the SEP's put strike price and the knock-in barrier are \$100 and \$70, the call strike price is \$80, the pre-determined coupon rate is 6%, and maturity is 3 months.



position becomes too large, forcing the issuer to liquidate the excessive position immediately.

On the other hand, the digital call in Equation (A1) is defined as follows:

$$Call_t = Ce^{-r(T-t)}\Phi(-d2), \tag{A3}$$

where  $\Phi(\cdot)$  is the standard normal cumulative distribution function and

$$d_2 = \frac{\ln(S_t/K^{Call}) + (r - \sigma^2/2)(T - t)}{\sigma\sqrt{T - t}}.$$

As we decompose the SEP in Equation (A1), the total delta change to the issuer consists of the delta from going long the put (Panel b) and the delta from going short the call (Panel c). This composite delta is presented in Figure 2.

## Appendix B. Issuance and Knock-In Crowding

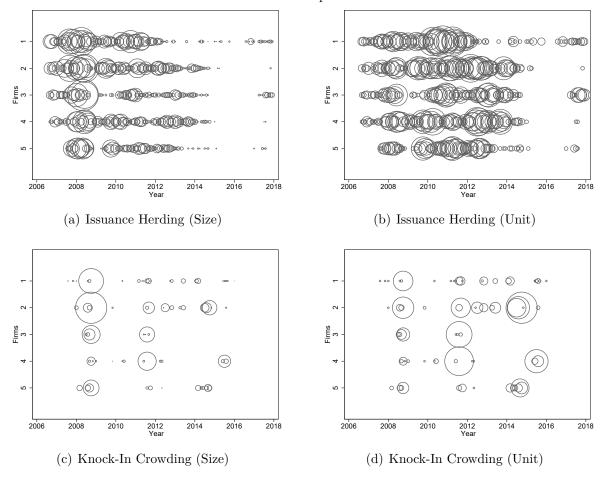
## Appendix 2.1. Issuance Herding and Knock-in Crowding

This section shows that the issuance of SEPs and knock-in events are concentrated at a particular moment. Initially, Panel (a) and Panel (b) in Figure B.1 visualize the issuance herding. We select the top five stocks from our sample (out of a total of 89 stocks) based on the total number of SEPs whose barriers were breached during our sample period and place them along the vertical axis (descending order). Circle size represents the notional amount (Panel a) or the number (Panel b) of newly issued notes each month.

We expect these circles to be comparable in size and spread proportionally across time in the absence of issuance herding. However, the figure shows the contrary: we observe that larger circles are concentrated at specific times and are also asynchronous for different stocks, illustrating issuance herding. As a result of the concentrated issuance of SEP with nearly

#### Figure B.1. Issuance Herding and Knock-In Crowding

This figure visualizes how issuance and knock-in events are concentrated at a certain time. We select the top 5 stocks from our sample (a total of 89 stocks) based on the total amount of SEPs whose barriers are breached during our sample period and place them along the vertical axis (ranging from the biggest stock to the fifth-biggest stock). Circle size on the horizontal axis represents the notional size (Panel a) or the number (Panel b) of newly issued notes corresponding to each stock in each month on the horizontal axis. Likewise, circle size represents the notional size (Panel c) or the number (Panel d) of SEPs that are subject to knock-in events. We match the same firms across the panels.



identical barriers, the knock-in events are also concentrated during specific periods. Panel (c) and Panel (d) display this pattern using the same set of firms as in Panel (a). Similarly, the sizes of circles are particularly large in some specific months.

## Appendix 2.2. Anecdotal Case: S-Oil Corporation

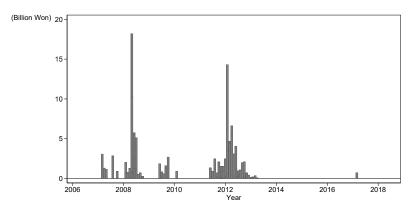
We exploit a case of a firm called "S-Oil." Panel (a) of Figure B.2 shows the issuance amount of SEPs on S-Oil and indicates that the issuance is concentrated in the years 2008 and 2012.

Their barrier levels expressed in price-level are plotted in Panel (b). This figure clearly visualizes that knock-in triggers are severely clustered by issuance vintage. For example, most SEPs issued in 2012 have barrier levels located between 40,000 and 60,000 KRW.

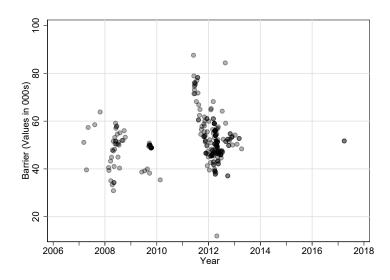
This figure implies that this price range of this stock is fragile: a small shock within this range can cause a substantial price drop via an event cascade. When one knock-in is triggered, it would induce price pressure and make another knock-in more probable because barrier levels are close to each other.

#### Figure B.2. Issuance Information of S-Oil

The bar graph in Panel (a) shows monthly issuance amounts (in billions of Korean Won) of SEPs on S-Oil, and the circle markers in Panel (b) present the barrier levels (in Korean Won) of SEPs from 2006 through 2017.



(a) Issuance Herding

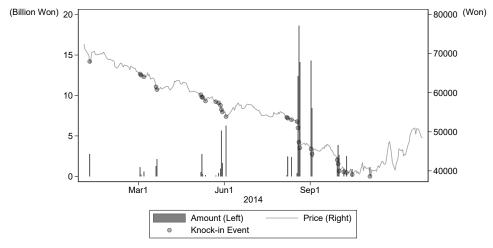


(b) Distribution of Barrier Level

Figure B.3 presents the knock-in events of these SEPs and demonstrates this domino effect. Caused by the price decline of S-Oil, the 2012-vintage SEPs experienced frequent knockins during 2014 (139 out of a total of 174 events). As the price entered the clustered range (40,000-60,000 KRW), we see that knock-ins typically occurred in series rather than in single events' (the circle marker). Further, the series of events is associated with a notable price drop (the solid line) and a large spike of triggered notionals (the bar), conforming to the amplification mechanism described in Section 3.

#### Figure B.3. Knock-In Crowding of S-Oil

This chart illustrates the knock-in events and the stock price of S-Oil company in 2014. Circle markers present the knock-in events, the solid line presents the stock price, and the bar graph presents the sum of the notional amounts of knocked-in SEPs on each day.



# Appendix C. The Hodrick-Prescott (HP) Filter

We start with the premise that firm i's cumulative abnormal return  $CAR_{i,t}$  consists of a time-trend component  $(d_{i,t})$  and a transient component  $(fCAR_{i,t})$ :

$$CAR_{i,t} = d_{i,t} + fCAR_{i,t}. (C1)$$

The Hodrick and Prescott (HP) filter is commonly used to remove the time trend from the series of observations. Specifically, Hodrick and Prescott (1997) suggest disentangling the

time trend  $(d_{i,t})$  from the original series  $(CAR_{i,t})$  by the following minimization problem:

$$\min_{d_i} \left\{ \sum_{t=1}^{t=T} (CAR_{i,t} - d_{i,t})^2 + \lambda \sum_{t=1}^{t=T} [(d_{i,t} - d_{i,t-1}) - (d_{i,t-1} - d_{i,t-2})]^2 \right\},$$
(C2)

where  $\lambda$  is a positive number that penalizes variability in the time-trend component.

In our context, the HP filter extracts a smooth price trend from the cumulative abnormal return  $(CAR_{i,t})$  with a chosen smoothing parameter  $\lambda$ , leaving us only the transient series  $(fCAR_{i,t})$ . Note that, as  $\lambda$  approaches 0, the trend component becomes equivalent to the original series. However, as  $\lambda$  increases to infinity,  $d_{i,t}$  becomes a linear trend.

# Appendix D. First-passage Time Probability

The first-passage time probability extends the probability of the original Merton model by finding the probability that a stock price touches the barrier until maturity. Bielecki and Rutkowski (2004) define the first-passage time probability as follows:

$$\mathbb{P}(s \leq T \mid \mathcal{F}_t) = \Phi\left(\frac{-\ln(\frac{S(t)}{B}) - \nu(T - t)}{\sigma\sqrt{T - t}}\right) + e^{-\frac{2\nu}{\sigma^2}\ln\left(\frac{S(t)}{B}\right)}\Phi\left(\frac{-\ln(\frac{S(t)}{B}) + \nu(T - t)}{\sigma\sqrt{T - t}}\right), \tag{D1}$$

where  $\mathcal{F}_t$  is filtration within time t, representing the information known at time t.  $\Phi(\cdot)$  is the standard normal cumulative density function, the stock price S(t) follows a standard Geometric Brownian Motion with the Wiener process  $W: dS(t) = \mu S(t)dt + \sigma S(t)dW(t)$ , and B is the barrier of interest. Also, s is the first-passage time, i.e.,  $s = \inf\{t \geq 0 : S(t) \leq B\}$  and  $\nu = \mu - \frac{1}{2}\sigma^2$ . The parameter set  $\{\mu, \sigma\}$  is estimated via Maximum Likelihood Estimation (MLE) using prior 252-day (12-month) returns at the reference point S(t).