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DIPARTIMENTO DI FISICA

Rivelatori di Particelle – Prof. A. Andreazza

Tracking systems

Sistemi traccianti: indice

- Lezione 1, 31/05: generalità sulle misure di posizione e momento
- Lezione 2, 05/06: sistemi di rivelatori, pattern recognition e track fit
- Esercitazioni pratiche (opzionali)
  - 14,16/06 alle 14 in Laboratorio Calcolo: simulazione di un rivelatore tracciante

- Appendice delle lezioni sui rivelatori a semiconduttore:
  12/06 Seminario rivelatori CMOS (E. Zaffaroni)

1. *What is the purpose of a tracking system*
2. *Helical trajectory and its approximations*
3. *Position measurements*
    1. intrinsic resolution
    2. multiple scattering effects
    3. vertex reconstrution
4. *Momentum measurements*
    1. intrinsic resolution
    2. multiple scattering effects

GENERALITY OF POSITION AND MOMENTUM MEASUREMENTS

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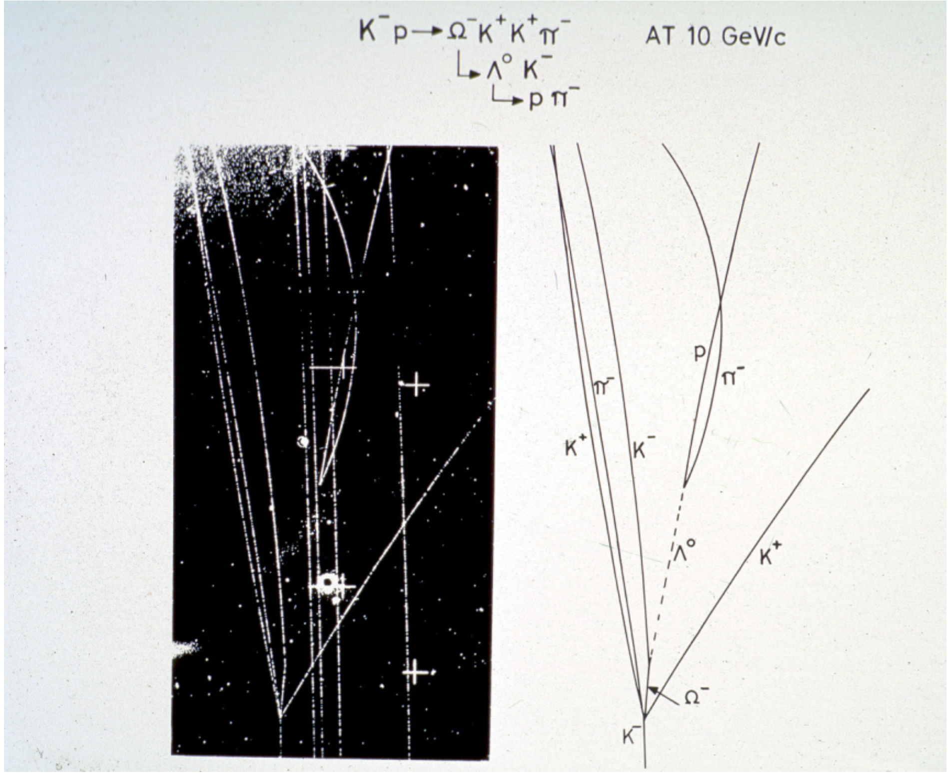
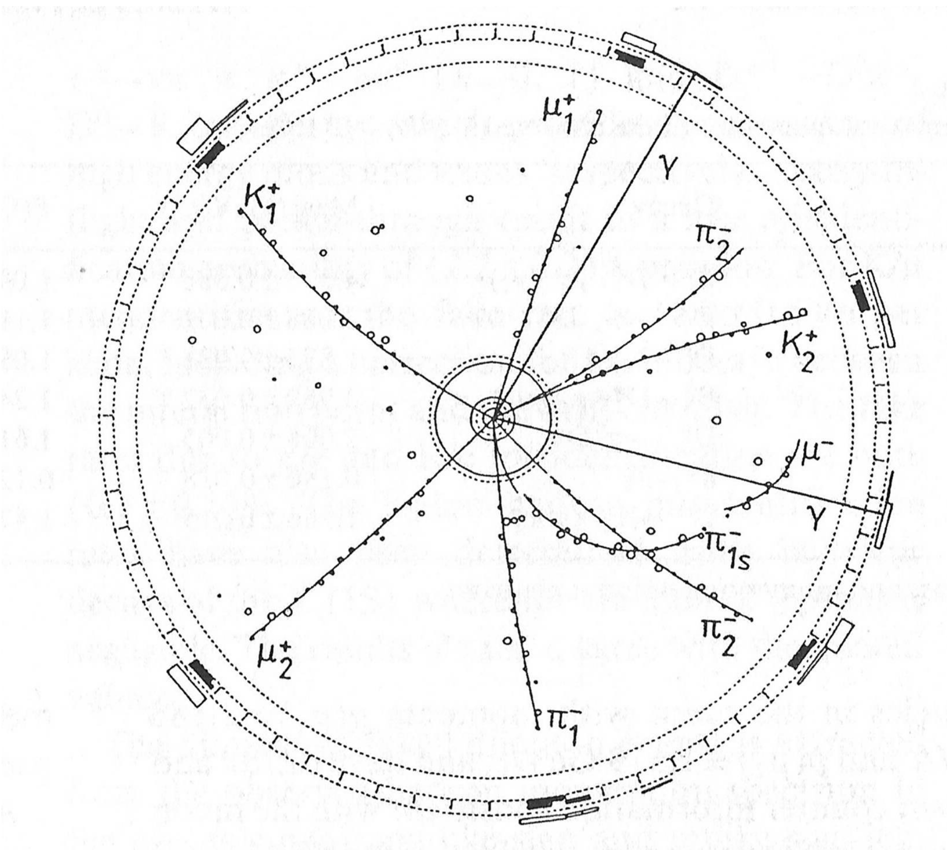
The legacy of visual detectors

- Many progress in particle physics by looking at events:
  - find particle trajectories,
  - find where they interact
  - find where they decay



The legacy of visual detectors

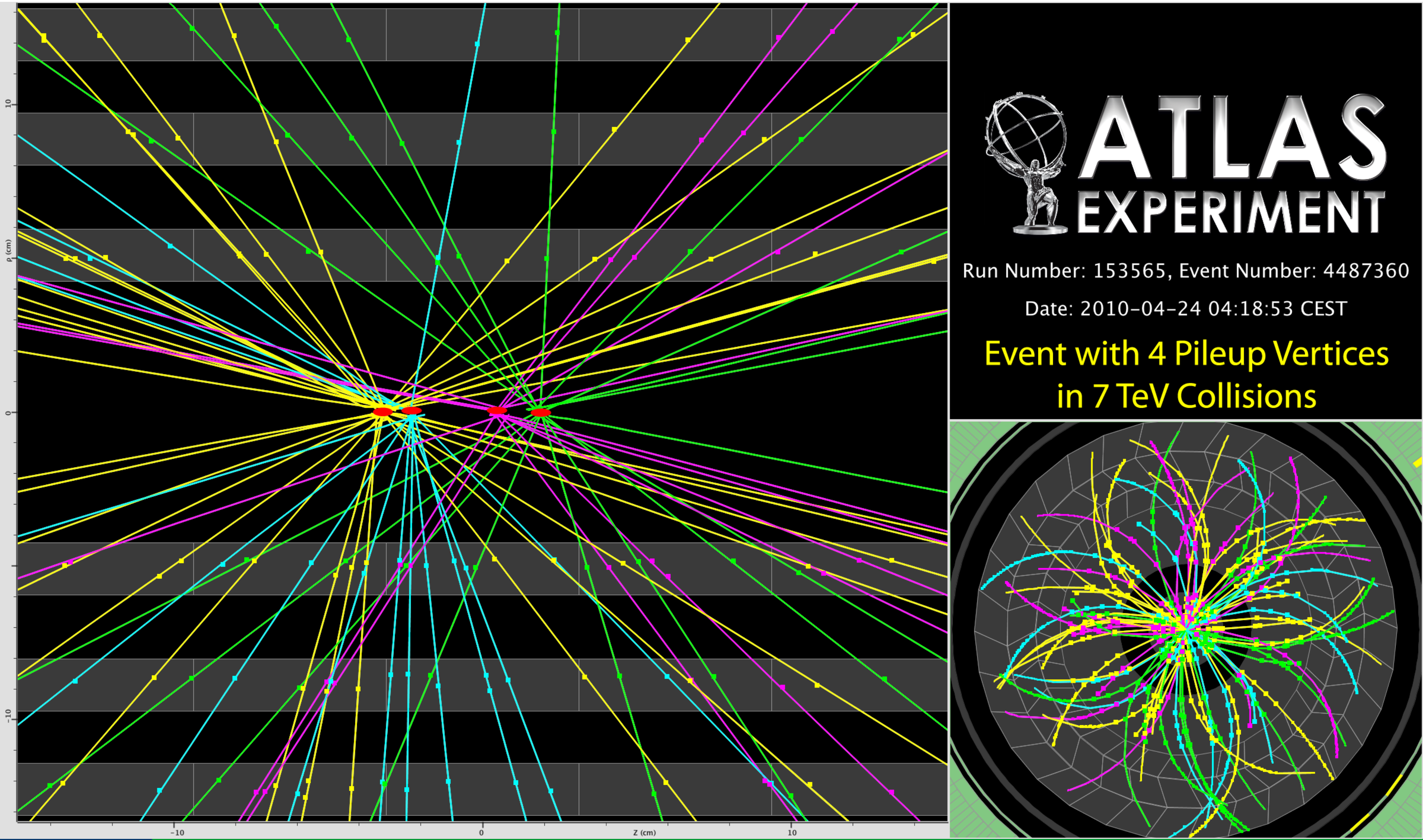
- Many progress in particle physics by looking at events:
  - find particle trajectories,
  - find where they interact
  - find where they decay
- Nothing has changed when moving to fully electronics readout
  - but density of measurements…
- Tracking is all about building **an image** of the particle interactions with **few observed points**.



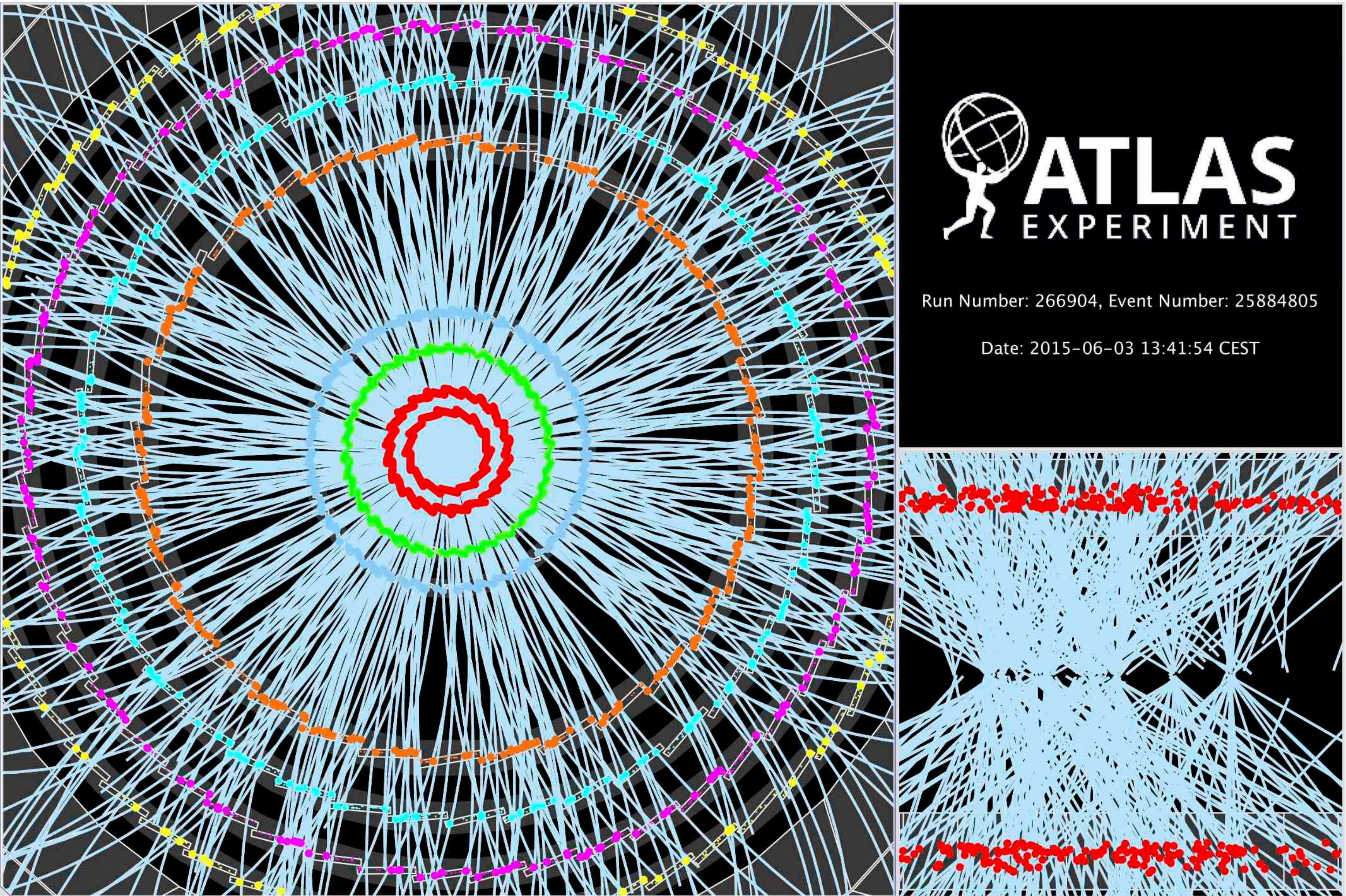
1) $B_1^0 \to D_1^{*-}\mu_1^+\nu_1,\ D_1^{*-} \to \pi_1^-\overline{D}^0,\ \overline{D}^0 \to K_1^+\pi_1^-$

2) $B_2^0 \to D_2^{*-}\mu_2^+\nu_2,\ D_2^{*-} \to \pi^0 D^-,\ D^- \to K_2^+\pi_2^-\pi_2^-$

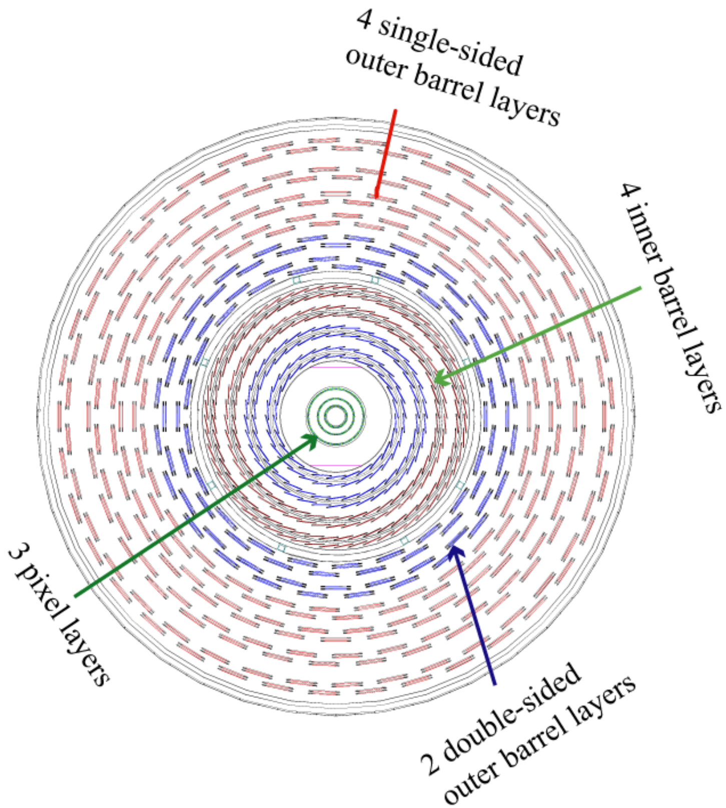
ATLAS event (low pile-up)



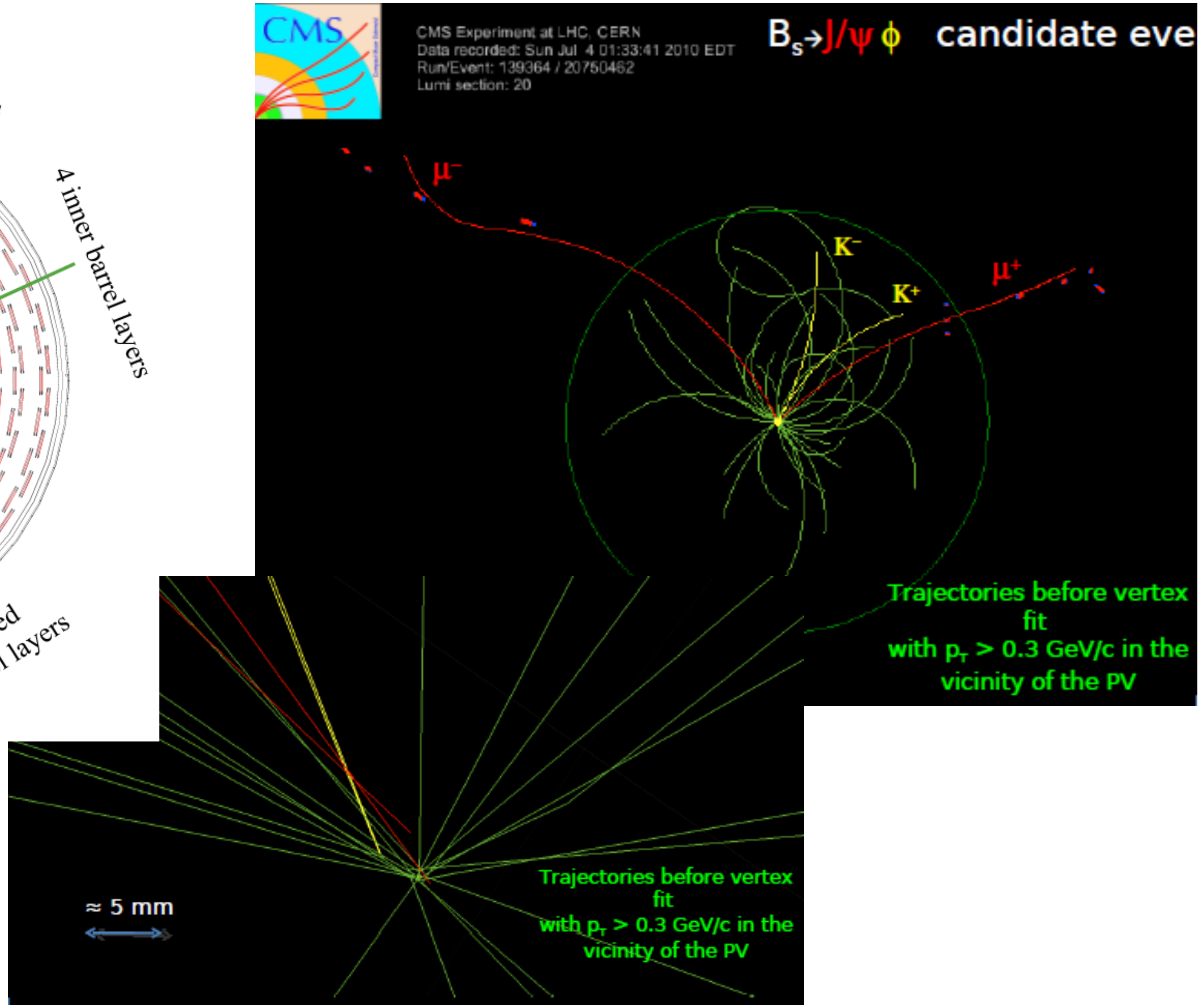
ATLAS event at $\sqrt{s}$=13 TeV



CMS event

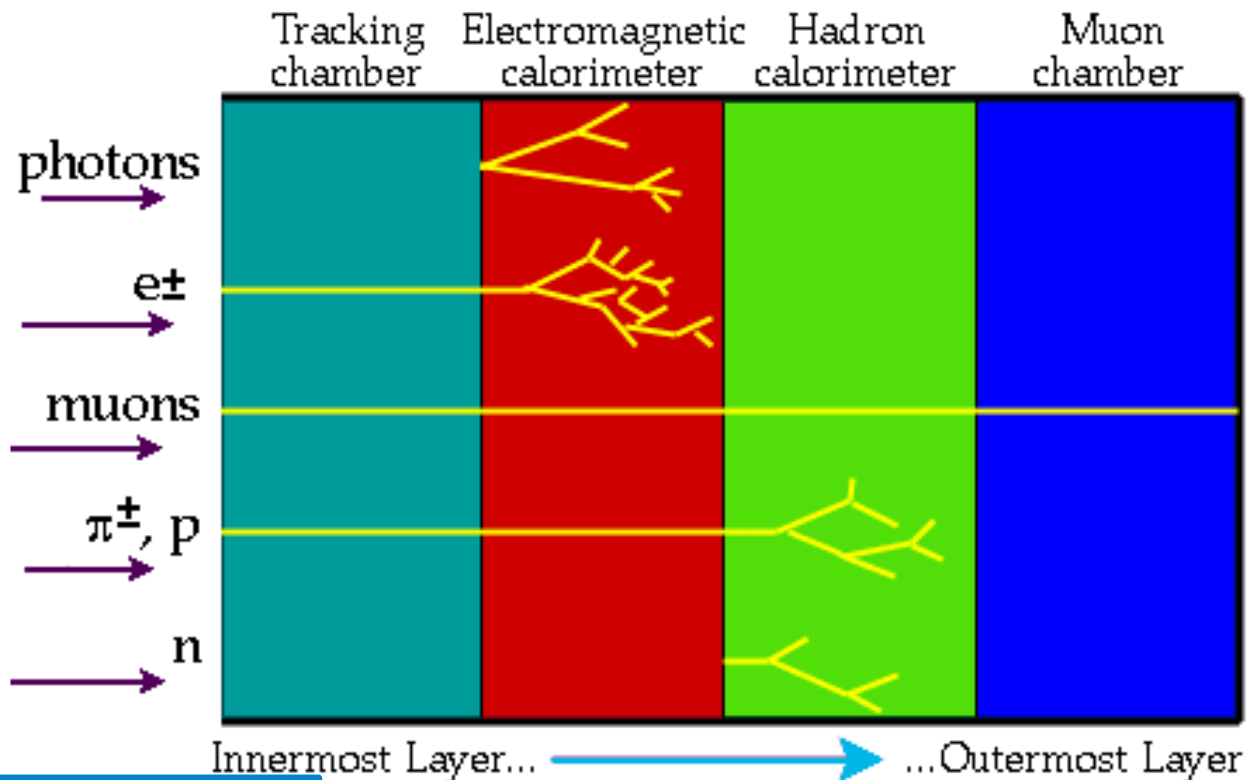


4 single-sided outer barrel layers
4 inner barrel layers
3 pixel layers
2 double-sided outer barrel layers



CMS
CMS Experiment at LHC, CERN
Data recorded: Sun Jul 4 01:33:41 2010 EDT
Run/Event: 139364 / 20750462
Lumi section: 20
$B_s \rightarrow J/\psi\ \phi$ candidate eve
μ^-
K^-
K^+
μ^+
Trajectories before vertex fit
with p_T > 0.3 GeV/c in the vicinity of the PV
≈ 5 mm
Trajectories before vertex fit
with p_T > 0.3 GeV/c in the vicinity of the PV

Particle detector structure



Momentum measurement
decay vertex reconstruction,
hadron identification
Low material (~1X_0)

Electron idenfication,
photon detection
~20X_0, 1-2λ_I

Charged hadron absorption,
neutral hadron detectionr
5-6λ_I

Identification of muons with enough energy to cross the calorimeters (>2-3 GeV)

Tracking system goals

1. Reconstruct charged-particle trajectories (tracks)

- join points to form a track (pattern recognition)
- measure direction and position
- measure momentum and charge (with magnetic field)
- Two major configurations:
  - inner spectrometers
  - muon systems

2. Reconstruct decay and interaction vertices

- “primary” vertex: collision point where most particle are produced
- “secondary” vertices:
  - decay of unstable particles
  - interaction with detector material
- evaluate compatibility of tracks with primary vertex
  - “weaker” way to detect interaction or unstable particles.

THE HELIX

AND ITS APPROXIMATIONS

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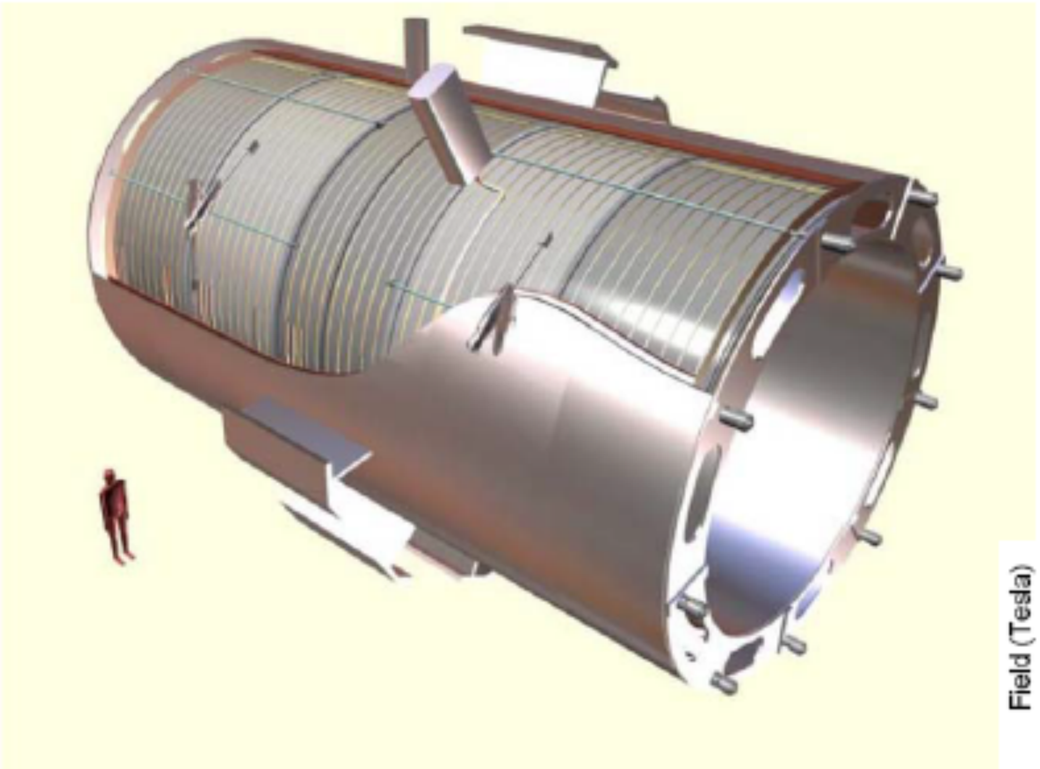
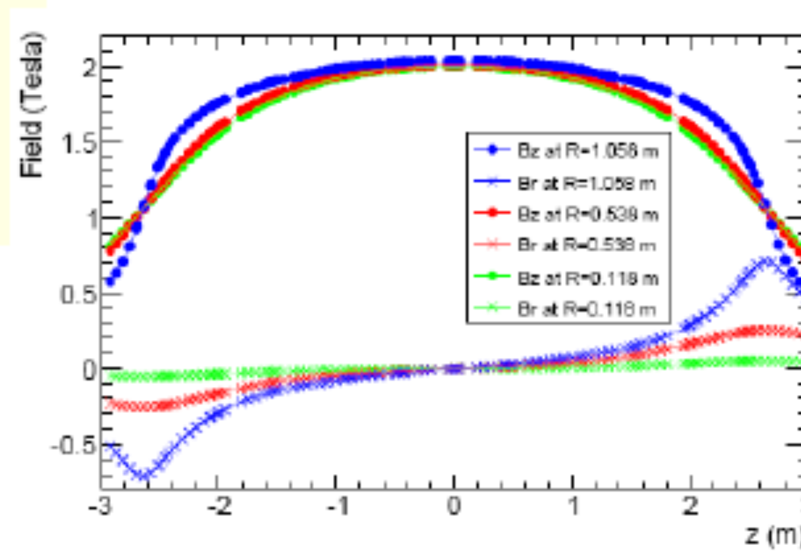
Superconducting magnets





Tracking systems are usually embedded in a magnetic field in order to measure particle momentum through curvature

- typical values 0.5-4 T
- solenoid, toroid, dipole configurations all used

Motion in a magnetic field

- The equation of motion of a charged particle in a magnetic field is:

$$\boxed{\frac{d\mathbf{p}}{dt} = e\,\mathbf{v} \times \mathbf{B}}$$

- The relativistic formula for the momentum is $pc = \varepsilon\, v/c$ with
  - ε particle energy
  - v particle velocity
- The equation of motion becomes:

$$\frac{d\mathbf{p}}{dt} = \frac{1}{c^2}\frac{d\varepsilon}{dt}\mathbf{v} + \frac{\varepsilon}{c^2}\frac{d\mathbf{v}}{dt} = e\,\mathbf{v} \times \mathbf{B}$$

- multiplying by **v** (**v** is perpendicular to both d**v** and **v**×**B**)

$$\boxed{\frac{d\varepsilon}{dt} = 0}$$

- This is the well-know feature the magnetic field does not perform work on the particle: energy if a constant of motion.

- The final equation of motions is:

$$\boxed{\frac{d\mathbf{v}}{dt} = \mathbf{v} \times \mathbf{\Omega}_v \qquad \mathbf{\Omega}_v = \frac{e\mathbf{B}}{\varepsilon}\; c^2}$$

- linear velocity precess with angular velocity Ω_v
- if **v** is normal to **B** the trajectory is a circumference with a revolution period $T = 2\pi/\Omega_v$
  - in the non-relativistic case: T=cost.
  - in the relativistic case: $T \propto \gamma$
- To find the circumference radius:

$$\frac{2\pi R}{T} = v \quad \rightarrow R = \frac{vT}{2\pi} = \frac{v}{\Omega_v}$$

$$R = \frac{v\varepsilon}{eBc^2} \qquad \beta\varepsilon = pc \qquad R = \frac{p}{eB}$$

- giving momentum in GeV, the radius in meters and magnetif field in Tesla

$$p = 0.299792458\, R\, B \Longrightarrow \boxed{p = 0.3\, R\, B}$$

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Motion in a magnetic field

- The equation of motion of a charged particle in a magnetic field is:

$$\boxed{\frac{d\mathbf{p}}{dt} = e\,\mathbf{v} \times \mathbf{B}}$$

- Since the B field does not change the energy of the particle:

$$m\gamma \frac{d\mathbf{v}}{dt} = e\,\mathbf{v} \times \mathbf{B}$$

$$m\gamma \frac{d^2\mathbf{r}}{dt^2} = e\frac{d\mathbf{r}}{dt} \times \mathbf{B}$$

- Using the length along the path

$$ds = v\,dt$$

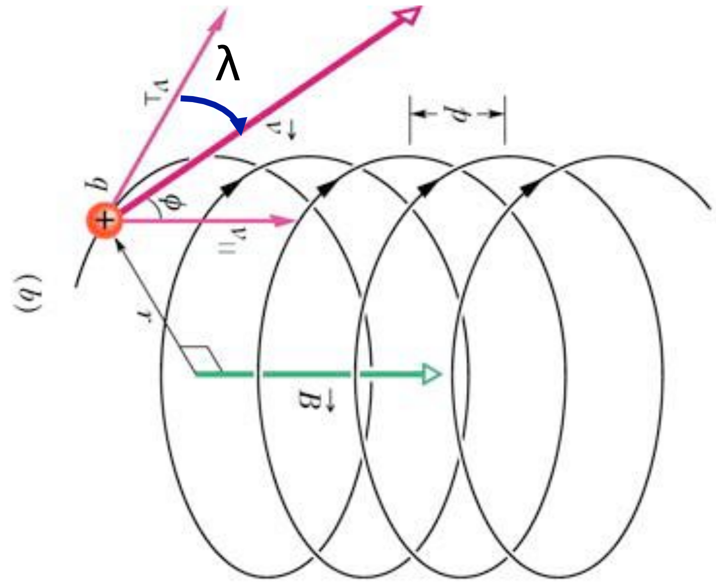
- One obtains:

$$m\gamma v \frac{d^2\mathbf{r}}{ds^2} = e\frac{d\mathbf{r}}{ds} \times \mathbf{B}$$

- Finally

$$\frac{d^2\mathbf{r}}{ds^2} = \frac{e}{p}\frac{d\mathbf{r}}{ds} \times \mathbf{B}$$

- In case of inhomogeneous magnetic field B(s) varies along the track, and to find the trajectory r(s) one needs to solve numerically the differential equation.
- In case of homogeneous magnetic field, the trajectory is given by an helix.

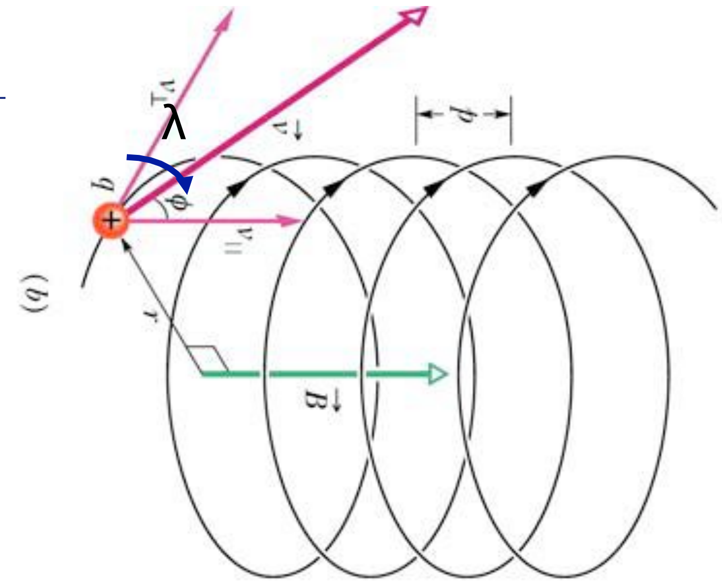


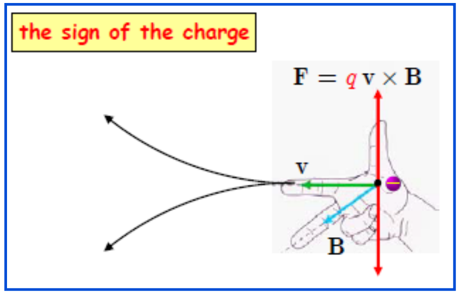
The helix equation

- The helix can be described in a parametric form

$$
\begin{aligned}
x(s) &= x_0 + R\left[\cos\left(\Phi_0 + \frac{hs\cos\lambda}{R}\right) - \cos\Phi_0\right] \\
y(s) &= y_0 + R\left[\sin\left(\Phi_0 + \frac{hs\cos\lambda}{R}\right) - \sin\Phi_0\right] \\
z(s) &= z_0 + s\sin\lambda
\end{aligned}
$$

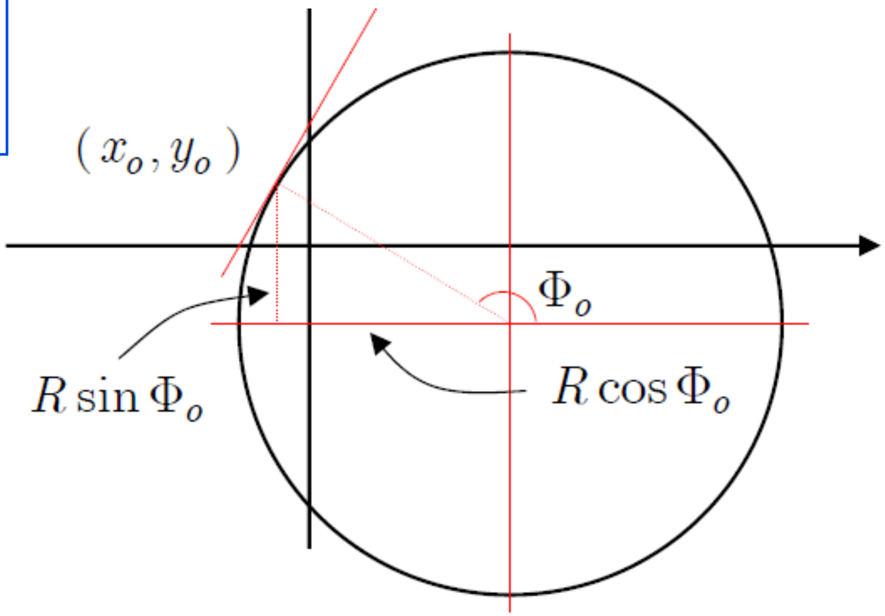
- λ is the dip-angle
- h=±1 is the sense of rotation of the helix





- The projection on the xy-plane is a circle:

$$
(x - x_0 + R\cos\Phi_0)^2 + (y - y_0 + R\sin\Phi_0)^2 = R^2
$$

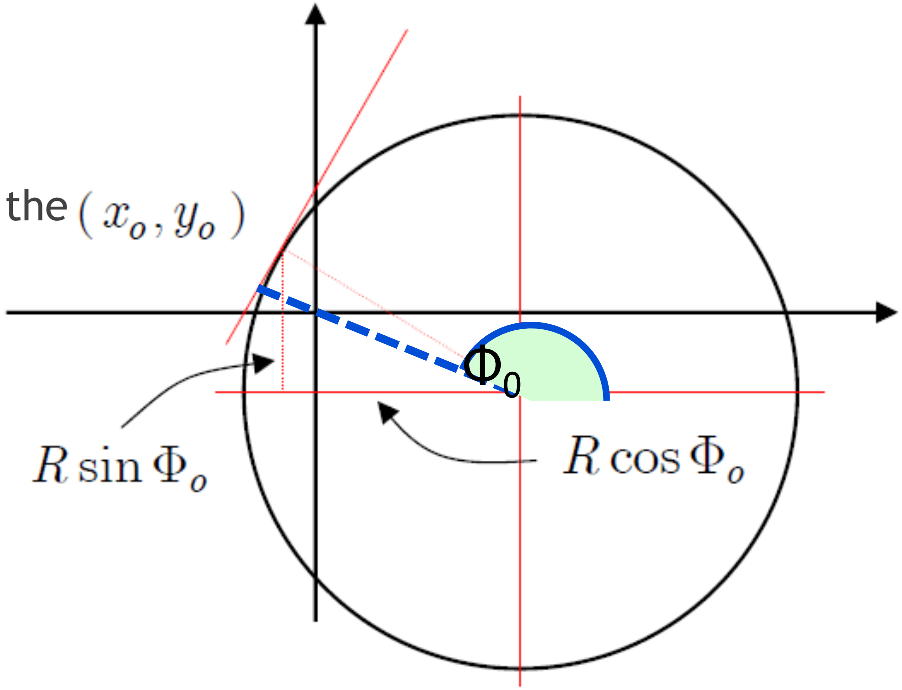


- x_0 and y_0 are the coordinates at *s*=0
- Φ_0 is also related to the slope of the tangent to the circle at *s*=0

High-p_T parabolic approximation

- In the helix equation:
  - The s=0 point is an arbitrary choice
  - A common use case is when the track is reconstructed in a region of size ≪R
    - p_T=1 GeV, B=2 T, R=1.7 m
    - radius of ATLAS tracking system is 1.05 m
    - …or if interested in the proximity of the interaction region: b, c, τ decay in few mm
- Choose as reference point the closest one to the origin of the reference frame (i.e. detector center)
  - “perigee”
- Write as a Taylor expansion in s/R
  - this is an approximation!
    - error ~s^3/R^2
  - but it will be very useful for future examples

$$p_T\,[\mathrm{GeV}] = 0.3\,B\,[\mathrm{T}]\,R\,[\mathrm{m}]$$



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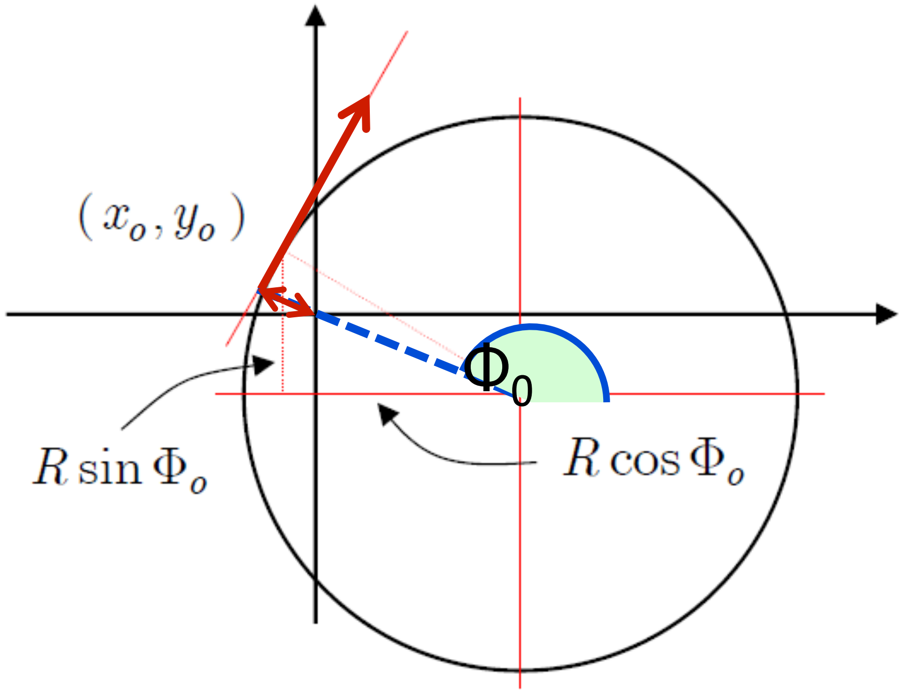
High-p_T parabolic approximation

- Development in s/R:

$$
\begin{aligned}
x(s) &= x_0 - hs\cos\lambda\sin\Phi_0 - \frac{1}{2}\frac{s^2\cos^2\lambda}{R}\cos\Phi_0 \\
y(s) &= y_0 + hs\cos\lambda\cos\Phi_0 - \frac{1}{2}\frac{s^2\cos^2\lambda}{R}\sin\Phi_0 \\
z(s) &= z_0 + s\sin\lambda
\end{aligned}
$$

- we can now introduce the “perigee” parameters:
  - impact parameter d_0:
    $$x_0 = d_0 h\cos\Phi_0,\quad y_0 = d_0 h\sin\Phi_0$$
    notice it has a sign!
  - the direction of the track at the perigee φ_0:
    $$\cos\varphi_0 = h\sin\Phi_0,\quad \sin\varphi_0 = -h\cos\Phi_0$$
  - the curvature $\kappa = \dfrac{h}{R}$

    which includes the sign of the charge
  - and the polar angle $\vartheta = \dfrac{\pi}{2} - \lambda$



$$
\begin{aligned}
x(s) &= -d_0\sin\varphi_0 + s\sin\vartheta\cos\varphi_0 + \tfrac{1}{2}\kappa s^2\sin^2\vartheta\sin\varphi_0 \\
y(s) &= d_0\cos\varphi_0 + s\sin\vartheta\sin\varphi_0 - \tfrac{1}{2}\kappa s^2\sin^2\vartheta\cos\varphi_0 \\
z(s) &= z_0 + s\cos\vartheta
\end{aligned}
$$

High-p_T parabolic approximation

- Starting from the parametric trajectory

$$
\begin{aligned}
x(s) &= -d_0 \sin\phi_0 + s\sin\vartheta\cos\phi_0 + \tfrac{1}{2}\kappa s^2 \sin^2\vartheta \sin\phi_0 \\
y(s) &= d_0 \cos\phi_0 + s\sin\vartheta\sin\phi_0 - \tfrac{1}{2}\kappa s^2 \sin^2\vartheta\cos\phi_0 \\
z(s) &= z_0 + s\cos\vartheta
\end{aligned}
$$

- It is now interesting to define a change of coordinates with the x-axis directed along the track direction:

$$
\begin{aligned}
\rho &= x\cos\varphi_0 + y\sin\varphi_0 \\
y' &= -x\sin\varphi_0 + y\cos\varphi_0
\end{aligned}
$$

$$
\begin{aligned}
\rho(s) &= s\sin\vartheta \\
y'(s) &= d_0 - \tfrac{1}{2}\kappa s^2 \sin^2\vartheta \\
z(s) &= z_0 + s\cos\vartheta
\end{aligned}
$$

- In these coordinates the trajectory has a simple expression in the “transverse” and “longitudinal” planes:

$$
\boxed{
\begin{aligned}
z &= z_0 + \rho\cot\vartheta \\
y' &= d_0 - \tfrac{1}{2}\kappa\rho^2
\end{aligned}
}
$$

  - Often $r=\surd(x^2+y^2)$ is used instead of ρ:
    - this is a “double” approximation valid for rd_0
  - If rotating to an axis “near” to the particle direction (the jet-axis for example)

$$
\boxed{y' = d_0 + \rho\tan(\varphi_0 - \varphi_{\mathrm{jet}}) - \tfrac{1}{2}\kappa\rho^2}
$$

    - This will be our “workhorse”

The region near the interaction vertex

POSITION MEASUREMENTS

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Decay vertex reconstruction

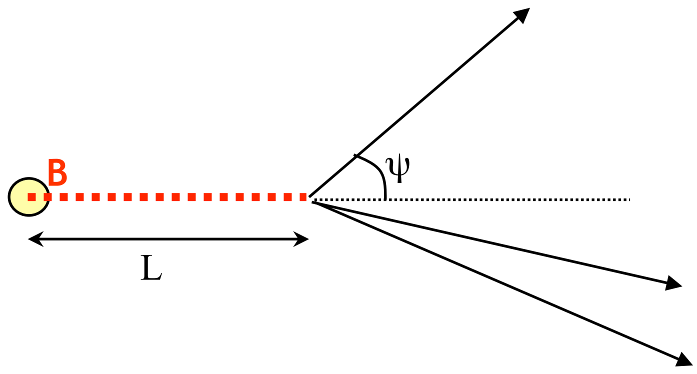
- In proximity of the interaction region, at first order, it is possible to neglect the curvature:
  - focus on position and direction.
  - Example: detection of short-lived particles
- There is a group of particles with lifetimes of ~1 ps
- The flight length L can be measurable: $L = \gamma\beta c t$
- To estimate the order of magnitued of the **decay angle ψ**, let's consider at which angle, with respect to the flight direcfion, a particle, emitted at an angle θ* in the rest frame of the mother particle, is observed in the laboratory frame:

$$p^{*\mu} = (\ p^* \quad p^*\sin\theta^* \quad 0 \quad p^*\cos\theta^*\ )$$

- in the lab frame:

$$p^{\mu} = [\ \gamma p^*(1+\beta\cos\theta^*) \quad p^*\sin\theta^* \quad 0 \quad \gamma p^*(\cos\theta^* + \beta)\ ]$$

- therefore the decay angle is: $\sin\psi = \frac{1}{\gamma}\frac{\sin\theta^*}{1+\beta\cos\theta^*} = O\left(\frac{1}{\gamma}\right)$



- Some typical examples:
  - Symmetric B-factory: ϒ(4S) at rest
    γ=1.002, β=0.06, L~30 μm, ψ~1
  - Asymmetric B-factory: e^- 9 GeV, e^+ 3.1 GeV
    γ=1.15, β=0.5, L~290 μm, ψ~1
  - High energy collisions (LEP, Tevatron, LHC)
    γ=5-10, β=1, L=2-3 mm, ψ~0.1

Impact parameter

- It is useful to introduce the **impact parameter *d***, defined at the distance between the daugther particle trajectory and the mother particle production point:

$$d = L\sin\psi = O(\gamma\beta c\tau)\times O\left(\frac{1}{\gamma}\right) = O(c\tau)$$

- for relativistic particles is approximately independent of the boost.
- An experimental apparatus with decay vertex capabilities must be able to separate the production and devay vertices:

$$\sigma_L/L \ll 1$$

- As a practical examples, let's consider a relativistic situation, where we can approximate:

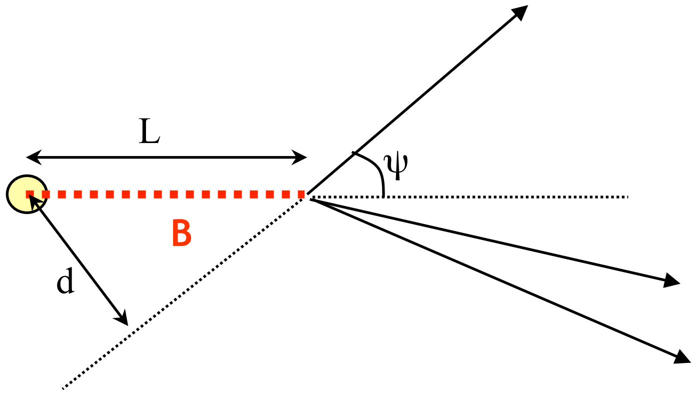
$$\tan\psi \approx \psi \approx \sin\psi$$

  and set the x-axis direction along the mother particle flight direction.
- This apparatus reconstructs trajectories

$$y=\tan\psi_i x+d_i$$

  with measurement uncertainty σ_d

  (σ_ψ is negligible in most practical cases)



- The decay vertex position is given by the intersection of two trajectories:

$$\begin{cases} y = \tan\psi_1 x + d_1 \\ y = \tan\psi_2 x + d_2 \end{cases}$$

$$L = \frac{d_2 - d_1}{\tan\psi_1 - \tan\psi_2} \approx \frac{d_2 - d_1}{\psi_1 - \psi_2}$$

$$\frac{\sigma_L}{L} = \frac{\sqrt{2}\sigma_d}{d_2 - d_1} = O\left(\frac{\sigma_d}{d}\right) = O\left(\frac{\sigma_d}{c\tau}\right)$$

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Impact parameter resolution

- Let's assume a pair of detector layers
  - at positions x_1 and x_2,
  - with a y coordinate measurement resolution σ_{y1} and σ_{y2}.
- The reconstructed trajectory is

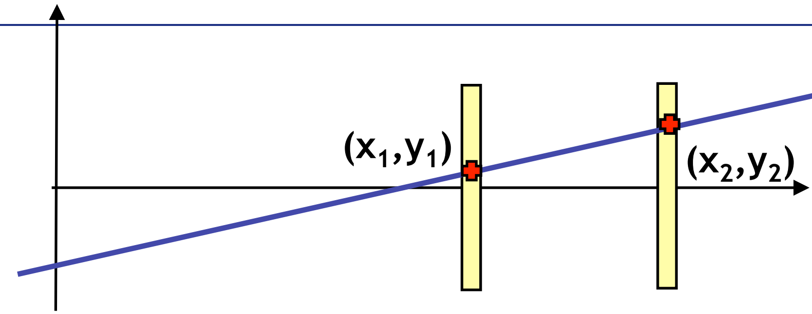
$$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$$

$$m=\frac{y_2-y_1}{x_2-x_1} \qquad d_0=\frac{x_2y_1-x_1y_2}{x_2-x_1}$$

- If $\sigma_{y1}=\sigma_{y2}=\sigma_y$ the impact parameter uncertainty is given by

$$\boxed{\sigma_d=\sqrt{\frac{n^2+1}{(n-1)^2}}\sigma_y}$$

where $n=x_2/x_1$ is referred to "lever arm"



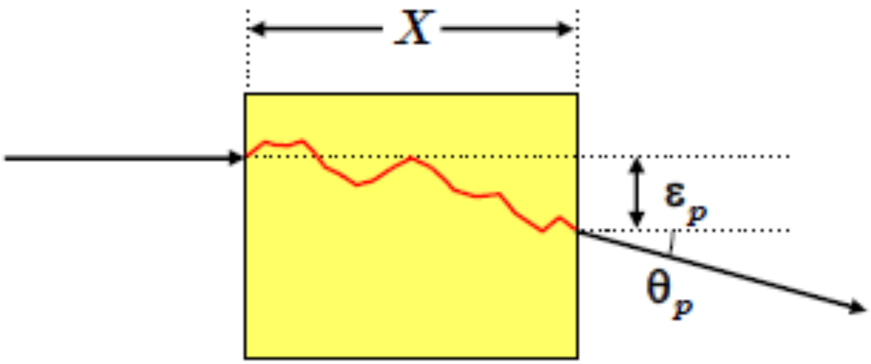
$$C_{d_0,m}=\begin{pmatrix}\frac{\partial d_0}{\partial y_1} & \frac{\partial d_0}{\partial y_2}\\ \frac{\partial m}{\partial y_1} & \frac{\partial m}{\partial y_2}\end{pmatrix}\begin{pmatrix}\sigma_{y1}^2 & 0\\ 0 & \sigma_{y2}^2\end{pmatrix}\begin{pmatrix}\frac{\partial d_0}{\partial y_1} & \frac{\partial m}{\partial y_1}\\ \frac{\partial d_0}{\partial y_2} & \frac{\partial m}{\partial y_2}\end{pmatrix}$$

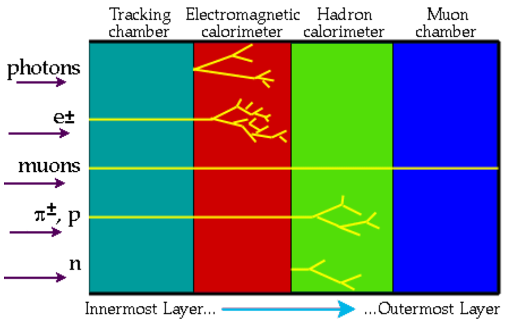
$$\boxed{\frac{1}{(x_2-x_1)^2}\begin{pmatrix}x_2^2\sigma_{y1}^2+x_1^2\sigma_{y2}^2 & -\left(x_2\sigma_{y1}^2+x_1\sigma_{y2}^2\right)\\ -\left(x_2\sigma_{y1}^2+x_1\sigma_{y2}^2\right) & \sigma_{y1}^2+\sigma_{y2}^2\end{pmatrix}}$$

- The geometrical coefficient n front of σ_y is is always greater than 1.
- x_1 should be as small as possible. Usually limited by the accelerator beam pipe.
- x_2 is limited by cost

Multiple scattering

- All'uscita in di un materiale una particella subisce:
  - una deviazione θ_p
  - uno spostamento ε_p
- spesso ε_p è trascurabile.
- Non è sempre il caso per grandi spessori:
  - es.: nell'attraversamento dei calorimetri tra tracciatore interno e e sistema a muoni

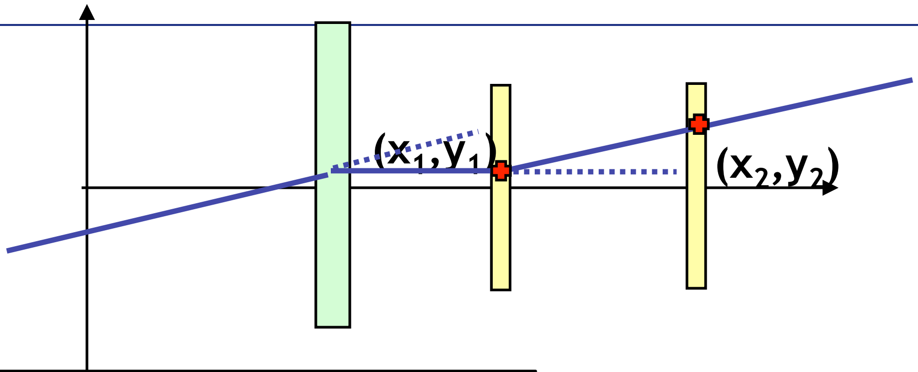




- Si può dimostrare che:
  - dato: $$\langle \theta_p^2 \rangle = KX, \quad K = \left(\frac{13.6\ \text{MeV}}{\beta c p} z \right)^2 \frac{1}{X_0}$$
  - vale: $$\langle \varepsilon_p^2 \rangle = \frac{1}{3} K X^3$$ $$\langle \varepsilon_p \theta_p \rangle = \frac{1}{2} K X^2$$

- **Il coefficiente di correlazione tra θ_p e ε_p è $\sqrt{3}/2$=0.87**
- Hint: scrivere la formula per spessori discreti e passare al continuo.

Impact parameter resolution: multiple scattering



- Multiple scattering has a critical role in determining the impact parameter resolution.
- Every material layer crossed by the particle before reaching the detector introduce a random deviations with r.m.s.

$$\theta_p = \frac{13.6\,\mathrm{MeV}}{\beta c p} z \sqrt{\frac{X}{X_0}} \left(1 + 0.038 \ln \frac{X}{X_0}\right)$$

- This devation contributes to the track parameter uncertainties:

$$\delta d_0 = -R \cdot \theta_p \qquad \delta m = \theta_p$$

where *R* is the distance between the interaction point and the material layer.
- Summing all contribution in quadrature:

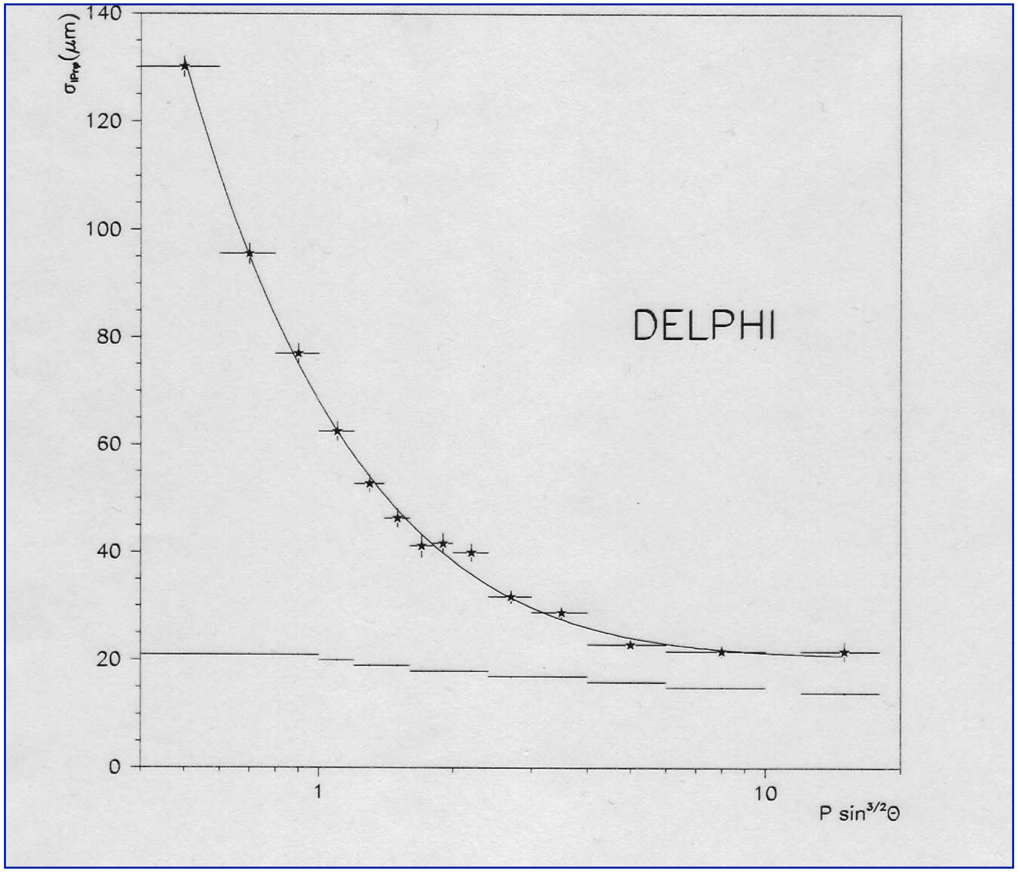
$$\boxed{C_{d_0,m} = \begin{pmatrix} \sum_i R_i^2 \theta_{p,i}^2 & -\sum_i R_i\, \theta_{p,i}^2 \\ -\sum_i R_i\, \theta_{p,i}^2 & \sum_i \theta_{p,i}^2 \end{pmatrix}} \approx \frac{1}{p \sin^{3/2}\theta}$$

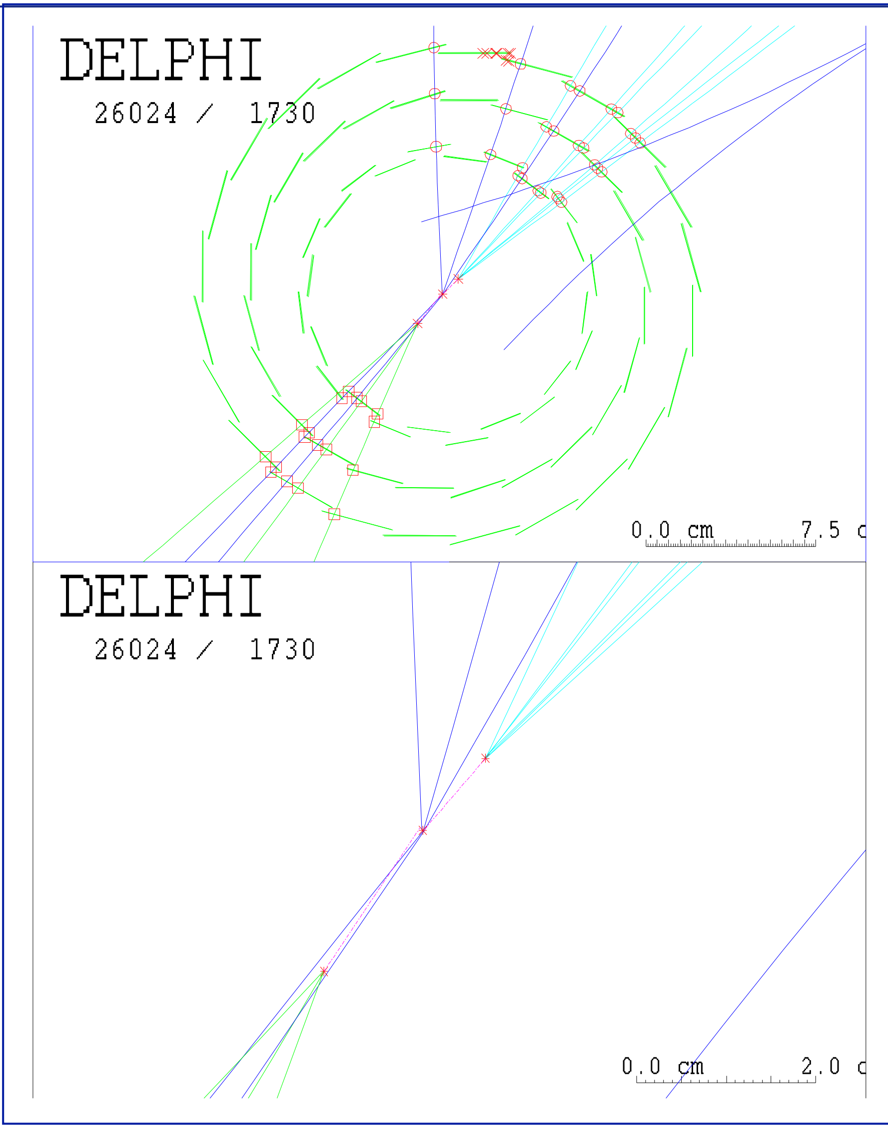
- The sum runs on all material layers till the first measured point (included).
- The formula for θ_p is valid only in a plane perpendicular to the track. If it is inclined by an angle ϑ with respect to the *xy* plane, the scattering angle is amplified by a geometrical factor $1/\sin\vartheta$.
- Also the thickness *X* of the crossed material layer increases by an additional factor $1/\sin^{1/2}\vartheta$.

Impact parameter and vertex reconstruction

- Often the impact parameter resolution is parameterized as

$$\sigma_d = \sigma_{\text{tracking}} \oplus \frac{\sigma_{\text{ms}}\left[\mu\text{m}\cdot\text{GeV}\right]}{p\sin^{3/2}\vartheta}$$





Some practical examples

	DELPHI	SLD
detector type	microstrips	CCD
pitch	25 μm (lettura 50 μm)	20 μm x 20 μm
beam pipe	R=5.25 cm, l=0.4% X_0	R=2.35 cm, l=0.5% X_0
first detector layer	R=6.3 cm, l=0.5% X_0	R=2.80 cm, l=0.4% X_0
last detector layer	R=10.7 cm, l=0.5% X_0	R=4.83 cm, l=0.4% X_0
point resolution	8 μm	4.4 μm
$\sigma_{tracking}$	20 μm	11 μm
σ_{ms}	65 μm×GeV	33 μm×GeV

measured values

$$\sigma_d = \frac{\sqrt{x_2^2 + x_1^2}}{x_2 - x_1}\sigma_y \rightarrow \sigma_d = \sigma_{\text{tracking}} \oplus \frac{\sigma_{\text{ms}}}{p \sin^{3/2}\vartheta} \leftarrow \sigma_d = \sqrt{\sum_i R_i^2 \theta_{0,i}^2}$$

Determining curvature

MOMENTUM MEASUREMENT

Momentum measurement (sagitta)

- A widespread method, if it is possible to insert detectors inside the magnetic field, consists of measuring the sagitta of the particle trajectory:

$$s = R\left(1-\cos\frac{\theta}{2}\right) \approx R\frac{\theta^2}{8}$$

$$= \frac{qBL^2}{8p}$$

- Numerically:

$$s[\mathrm{m}] = \frac{0.3B[\mathrm{T}]L^2[\mathrm{m}]}{p[\mathrm{GeV/c}]}$$

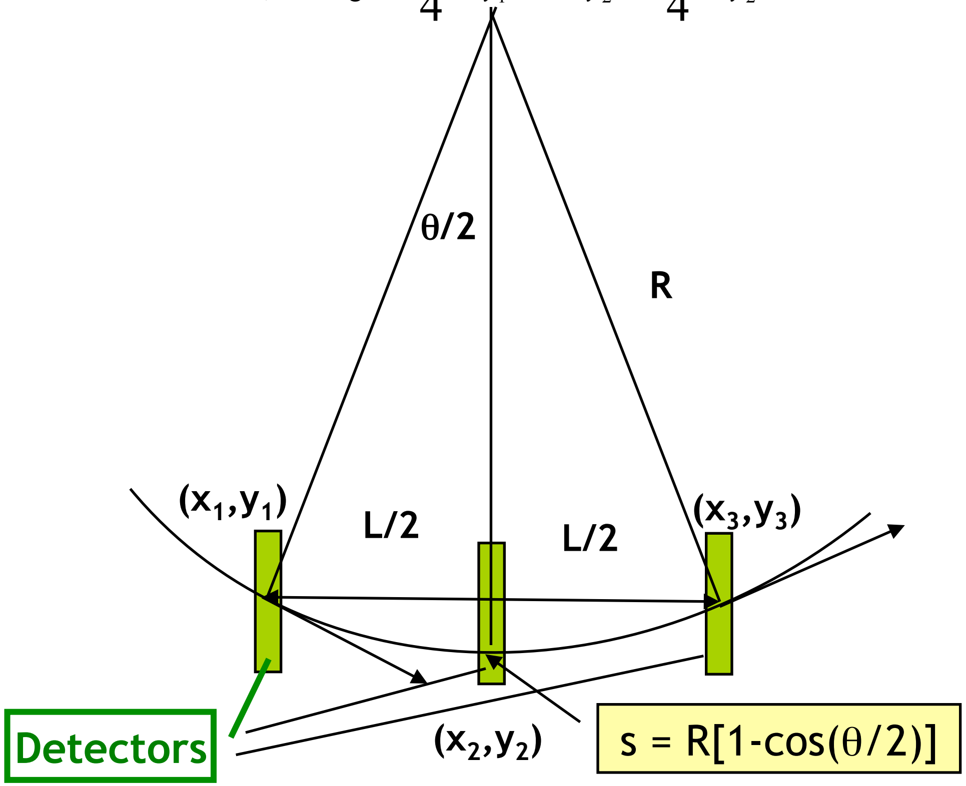
- And the relative momentum resolution is:

$$\frac{\sigma_p}{p} = \frac{\sigma_s}{s} = \frac{8p}{0.3BL^2}\sigma_s$$

- In the case the sagitta is measured by only three detectors:

$$s = y_2 - \frac{1}{2}(y_1 + y_3)$$

$$\sigma^2_{s,\mathrm{tracking}} = \frac{1}{4}\sigma^2_{y_1} + \sigma^2_{y_2} + \frac{1}{4}\sigma^2_{y_2}$$



Momentum measurement: multiple scattering

- In case of deflection in the detector material:

$$\delta y_2 = \frac{L}{2}\delta\theta_1 \quad \delta y_3 = L\delta\theta_1 + \frac{L}{2}\delta\theta_2 \quad \Rightarrow \quad \delta s = \delta y_2 - \frac{1}{2}\delta y_3 = -\frac{L}{2}\delta\theta_2$$

and therefore a multiple scattering contribution of $\sigma_s = \frac{L}{2}\theta_{p,2}$

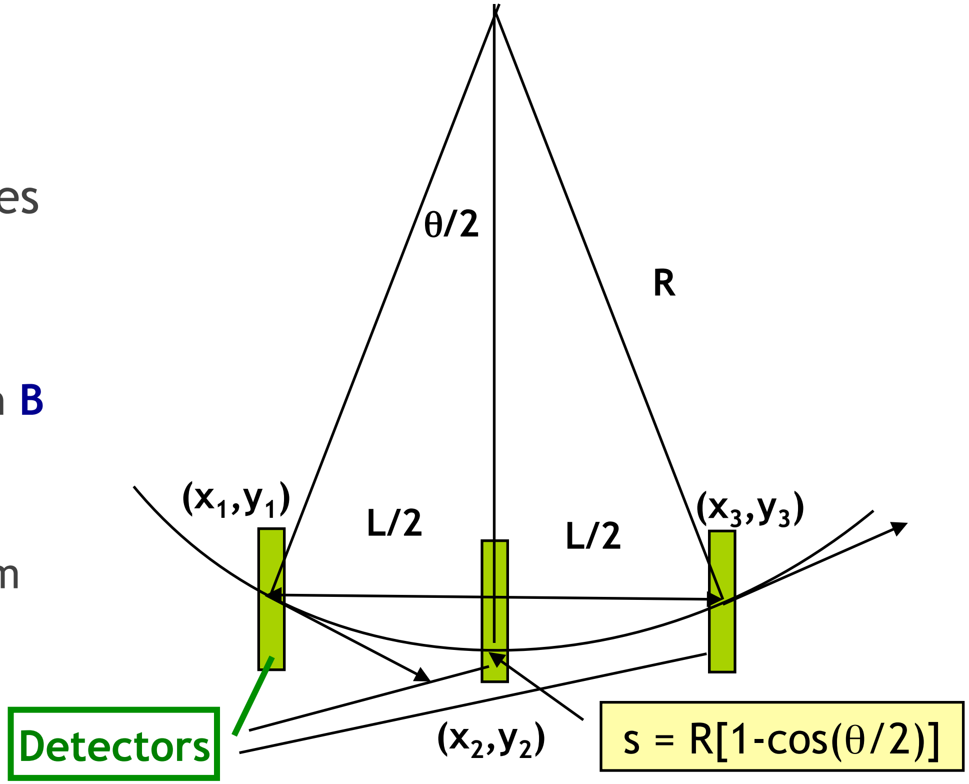
- Adding the two contributions:

$$\sigma_s = \sigma_{\text{tracking}} \oplus \frac{\sigma_{\text{MS}}}{p}$$

- And the momentum resolution becomes

$$\frac{\sigma_p}{p} = \frac{\sigma_s}{s} = \frac{8}{0.3BL^2}\left(p\sigma_{\text{tracking}} \oplus \sigma_{\text{MS}}\right)$$

  - The resolutions improves **linearly** with **B** and with the **detector resolution**.
  - **It improves quadratically with L**
  - The relative uncertainty on momentum is constant at low momentum (MS), but **increases linearly with increasing momentum**.



Momentum measurement (bending)

- A widely used method consists of the measurements of the bending of the track direction after crossing a magnetic field.

- A particle moving across a region with a constant magnetic field will get a pulse of

$$\Delta p_T \approx pL/R = qBL$$

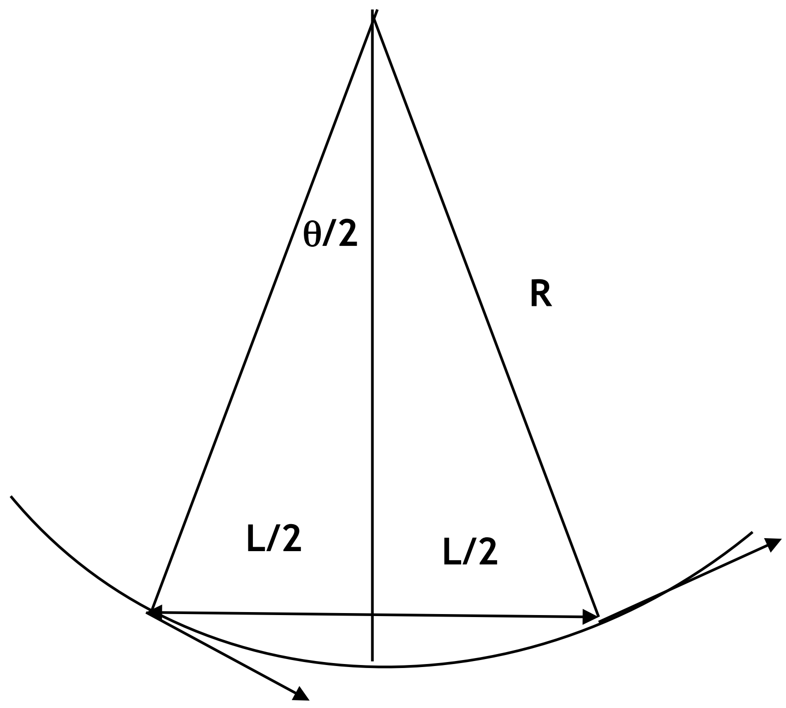
- The more general formula for non-constant magnetic field is:

$$\Delta p_T \approx q\boxed{\int Bdl}$$

bending power

- It is therefore possible to determine the momentum of a particle by the angular deviation after crossing a magnetic field:

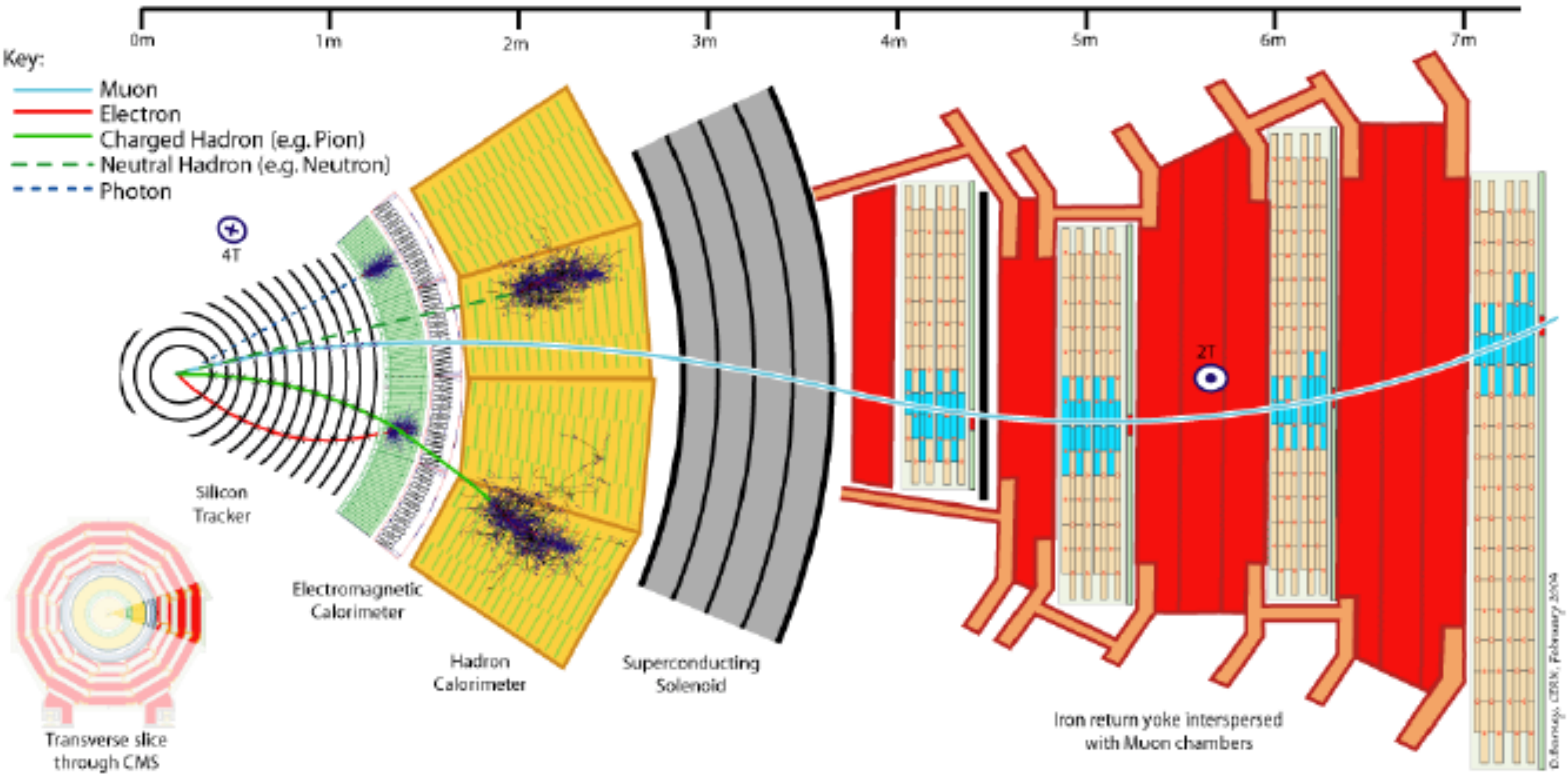
$$\theta \approx \frac{\Delta p_T}{p_T} = \frac{q\int Bdl}{p_T}$$



- Also in this case, the relative uncertainty worsen with increasing momentum:

$$\frac{\sigma_{p_T}}{p_T} = \frac{\sigma_\theta}{\theta} = \frac{p_T \sigma_\theta}{q\int Bdl}$$

CMS: Compact Muon Solenoid

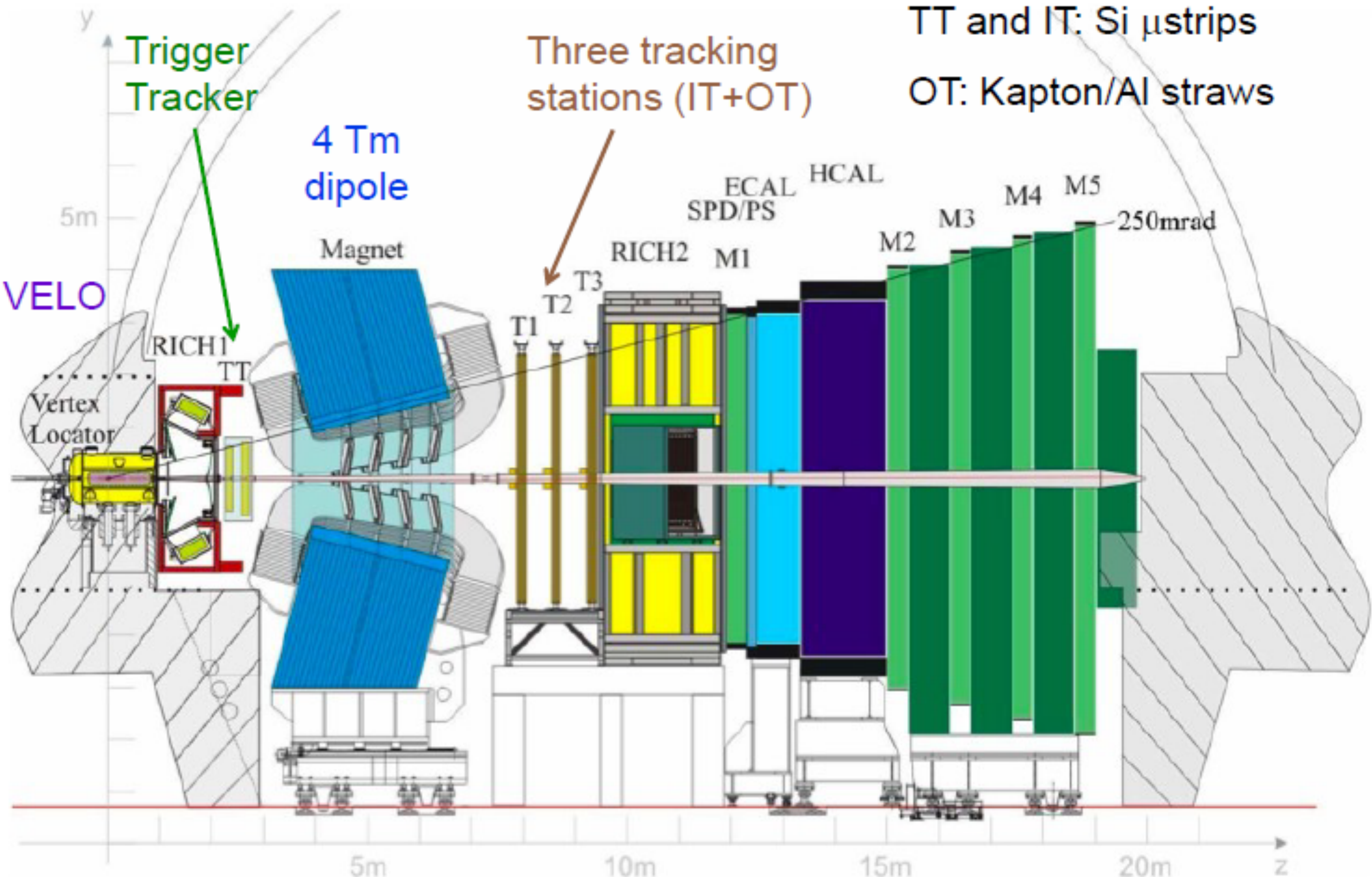


LHCb tracking

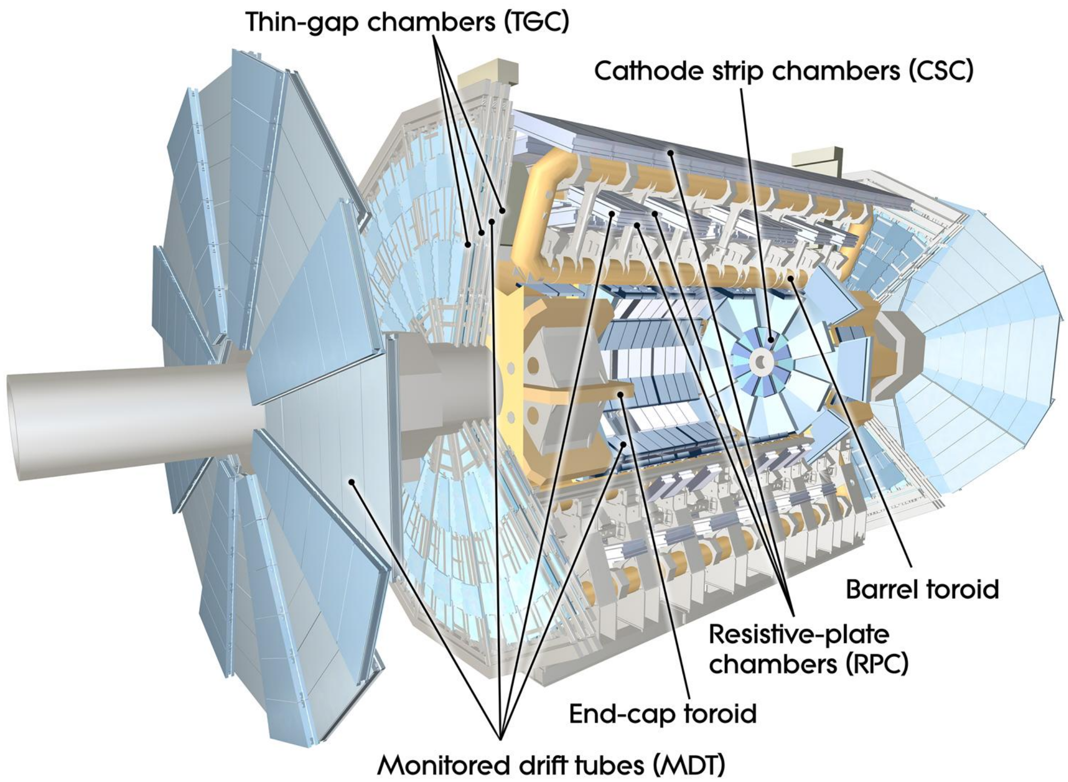
VELO: rφ Si strips

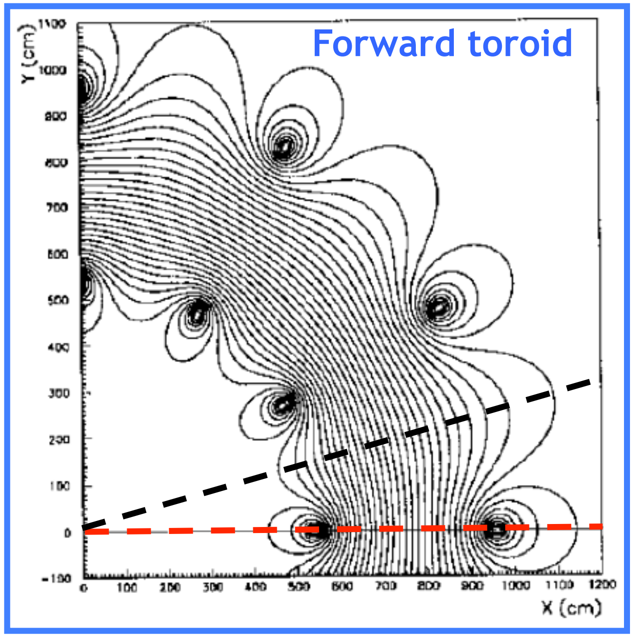
TT and IT: Si μstrips

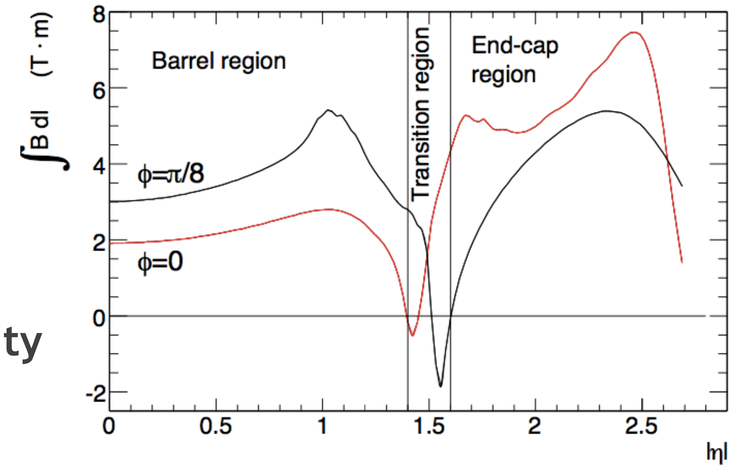
OT: Kapton/Al straws



ATLAS Muon Spectrometer







- **Instrumented air-core toroid system:**
  - bending power 1-7.5 Tm
  - standalone momentum reconstruction capability σ_{p_T}/p_T = 10% at p_T=1 TeV

1. *Kalman filter*
2. *Other approaches to fitting*
3. *Elements of track finding*
4. *Examples of tracking systems*

TRACK FITTING AND FINDING

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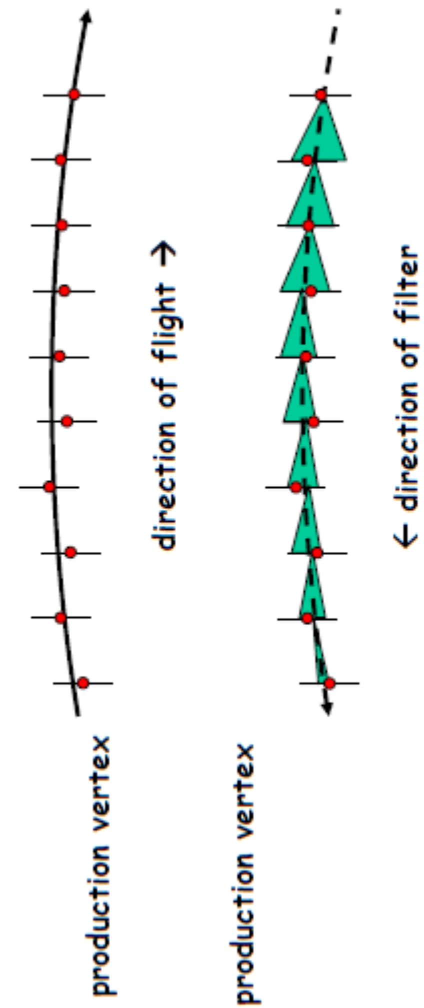
Kalman filter

- Step-by-step updating procedure:
  - use initial estimation of track parameters
  - extrapolate to next measured point
  - compare extrapolation with measurement
  - derive updated track parameters
- Continue adding all points one at the time.
- For each point invert a matrix of size equal to the track parameters
  - computation time is Nd^3 instead of N^3
  - N=number of measurements
  - d=number of track paramenters
- Comparison allows for rejection of outliers
  - can also be used during pattern recognition



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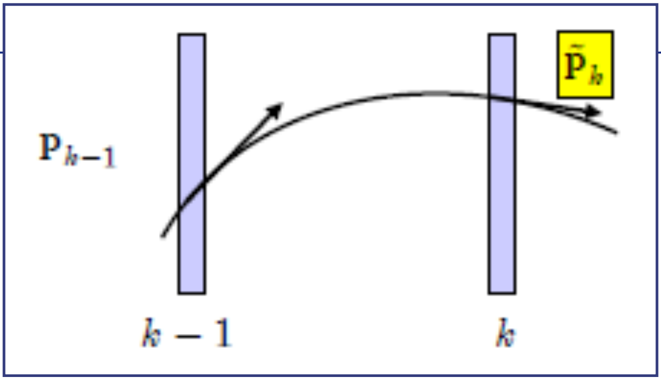
Kalman filter



- Only providing basic idea of Kalman filtering
  - one iteration of the fit, from detector plane *k*-1 to *k*
  - see bibliography(*) for more details
- At plane *k*-1 we have:
  - $\mathbf{p}_{k-1}$: estimation of the track parameters
  - $\mathbf{C}_{k-1}$: covariance matrix
- Extrapolation to plane *k*:
  - new parameters: $\tilde{\mathbf{p}}_k = \mathbf{f}(\mathbf{p}_{k-1})$
  - Jakobian matrix: $\mathbf{F} = \frac{\partial \mathbf{f}}{\partial \mathbf{p}}(\mathbf{p}_{k-1})$
  - covariance matrix of the extrapolated parameters:
$$\tilde{\mathbf{C}}_k = \mathbf{F}\mathbf{C}_{k-1}\mathbf{F}^{\mathrm{T}} + \mathbf{M}_{ms}$$
- $\mathbf{M}_{ms}$: includes the multiple scattering uncertainty in the extrapolation.

- On surface *k* we have some measurements $\mathbf{m}_k$ with covariance $\mathbf{V}_k$.
- The updated parameters $\mathbf{p}_k$ are obtained my minimizing a χ^2 including:
  - comparison of $\mathbf{m}_k$ with expectations $\mathbf{y}_k(\mathbf{p}_k)$ from the track model
  - the extrapolated parameters
$$\chi^2 = \left(\mathbf{m}_k - \mathbf{y}_k(\mathbf{p}_k)\right)^{\mathrm{T}} \mathbf{V}_k^{-1} \left(\mathbf{m}_k - \mathbf{y}_k(\mathbf{p}_k)\right) + \left(\tilde{\mathbf{p}}_k - \mathbf{p}_k\right)^{\mathrm{T}} \tilde{\mathbf{C}}_k^{-1} \left(\tilde{\mathbf{p}}_k - \mathbf{p}_k\right)$$

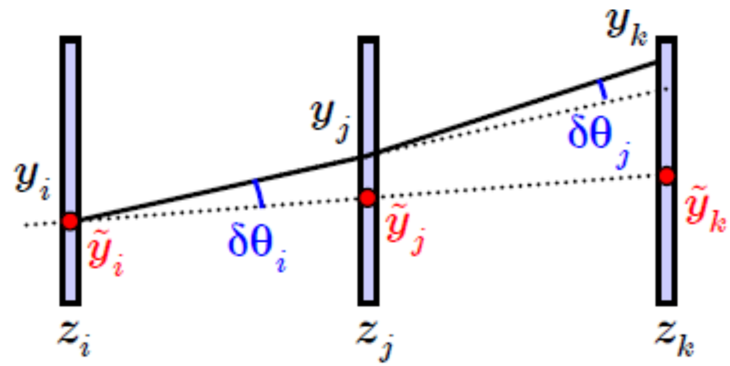
(*) Boch, Grote, Notz, Regler, *Data analysis techniques in high energy physics experiments*, Cambridge University Press 1990

Kalman filter: straight line example

- Initial estimate of track parameters using *j*, *k*

$$y = a_j + b_j(z - z_j)$$

$$\mathbf{p}_j = \begin{pmatrix} a_j \\ b_j \end{pmatrix} = \begin{pmatrix} y_j \\ \dfrac{y_k - y_j}{z_k - z_j} \end{pmatrix} \quad \mathbf{C}_j = \begin{pmatrix} \sigma_{y_j}^2 & -\dfrac{\sigma_{y_j}^2}{z_k - z_j} \\ -\dfrac{\sigma_{y_j}^2}{z_k - z_j} & \dfrac{\sigma_{y_j}^2 + \sigma_{y_k}^2}{(z_k - z_j)^2} \end{pmatrix}$$



- Extrapolate to point *i*: $y = a_j + b_j(z - z_j) \Rightarrow y = a_i + b_i(z - z_i)$

$$\tilde{\mathbf{p}}_i = \begin{pmatrix} \tilde{a}_i \\ \tilde{b}_i \end{pmatrix} = \begin{pmatrix} a_j - b_j(z_j - z_i) \\ b_j \end{pmatrix}$$

$$\tilde{\mathbf{C}}_j = \frac{1}{(z_k - z_j)^2} \begin{pmatrix} (z_k - z_i)^2\sigma_{y_j}^2 + (z_j - z_i)^2\sigma_{y_k}^2 & -(z_k - z_i)\sigma_{y_j}^2 - (z_j - z_i)\sigma_{y_k}^2 \\ -(z_k - z_i)\sigma_{y_j}^2 - (z_j - z_i)\sigma_{y_k}^2 & \sigma_{y_j}^2 + \sigma_{y_k}^2 \end{pmatrix} + \theta_{p,j}^2 \begin{pmatrix} (z_j - z_i)^2 & z_j - z_i \\ z_j - z_i & 1 \end{pmatrix}$$

which gives contribution to the χ^2 for the parameters at *i*:

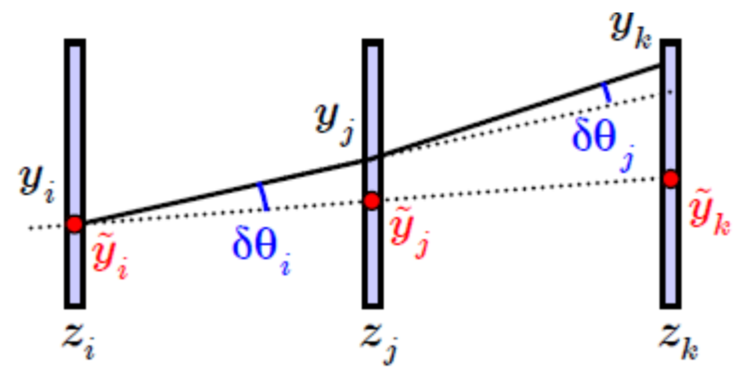
$$\chi^2 = (\tilde{\mathbf{p}}_i - \mathbf{p}_i)^T \tilde{\mathbf{C}}^{-1} (\tilde{\mathbf{p}}_i - \mathbf{p}_i) \qquad \mathbf{p}_i = \begin{pmatrix} a_i \\ b_i \end{pmatrix}$$

Kalman filter: straight line example

- The measurement at i is the y position at z_i:
  $$y = a_i + b_i(z - z_i)$$
- in this case the term $\mathbf{m}_i - \mathbf{y}_i(\mathbf{p}_i)$ has the form:

$$\mathbf{m}_i = (y_i) \quad \mathbf{y}_i(\mathbf{p}_j) = \mathbf{H}_i\mathbf{p}_i \quad \mathbf{H}_i = \begin{pmatrix} 1 & 0 \end{pmatrix}$$

$$\mathbf{m}_i - \mathbf{y}_i(\mathbf{p}_i) = y_i - \mathbf{H}_i\mathbf{p}_i = y_i - a_i \quad \Rightarrow \chi^2 = \frac{(y_i - a_i)^2}{\sigma_{y_i}^2}$$



- The new parameters are obtained by the minimization of:

$$\chi^2 = (\tilde{\mathbf{p}}_i - \mathbf{p}_i)^T \tilde{\mathbf{C}}^{-1} (\tilde{\mathbf{p}}_i - \mathbf{p}_i) + \frac{(y_i - a_i)^2}{\sigma_{y_i}^2}$$

- Which can be put in the general x^2 form: $\chi^2 = (\mathbf{Y} - \mathbf{A}\mathbf{p})^T \mathbf{V}^{-1} (\mathbf{Y} - \mathbf{A}\mathbf{p})$

$$\mathbf{p} = \begin{pmatrix} a_i \\ b_i \end{pmatrix} \quad \mathbf{Y} = \begin{pmatrix} \tilde{a}_i \\ \tilde{b}_i \\ y_i \end{pmatrix} \quad \mathbf{A} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \mathbf{V} = \begin{bmatrix} \tilde{\mathbf{C}}_i & \mathbf{0} \\ \mathbf{0} & \sigma_{y_i}^2 \end{bmatrix}$$

- whose solution is:

$$\mathbf{p} = (\mathbf{A}^T\mathbf{W}\mathbf{A})^{-1}\mathbf{A}^T\mathbf{W}\mathbf{Y} \quad \mathbf{C}_i = (\mathbf{A}^T\mathbf{W}\mathbf{A})^{-1} \quad \mathbf{W} = \mathbf{V}^{-1} = \begin{bmatrix} \tilde{\mathbf{C}}_i^{-1} & \mathbf{0} \\ \mathbf{0} & 1/\sigma_{y_i}^2 \end{bmatrix}$$

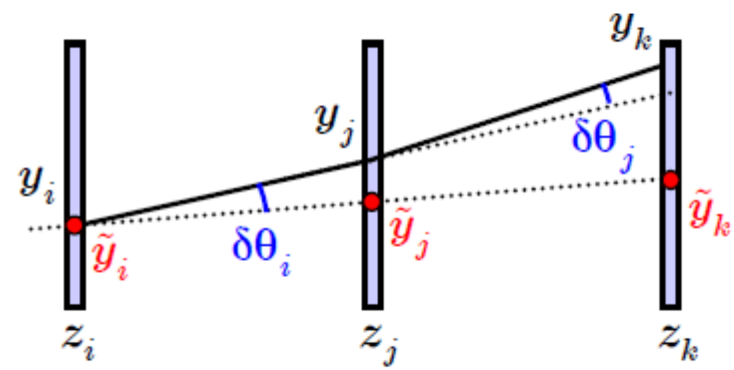
Kalman filter: straight line example

- Finally, going to the interaction point:

$$y = a_0 + b_0 z$$

$$\mathbf{p}_0 = \begin{pmatrix} a_0 \\ b_0 \end{pmatrix} = \begin{pmatrix} a_i - b_i z_i \\ b_i \end{pmatrix}$$

$$\mathbf{C}_0 = \begin{pmatrix} 1 & -z_i \\ 0 & 1 \end{pmatrix} \mathbf{C}_i \begin{pmatrix} 1 & 0 \\ -z_i & 1 \end{pmatrix} + \theta_{p,i}^2 \begin{pmatrix} z_i^2 & -z_i \\ -z_i & 1 \end{pmatrix}$$



The first and second features are general characteristics of all track fitter

- **Some important concepts:**
  - Track parameters depend on the point at which we are looking at the track:
    $\mathbf{p}_0 \neq \mathbf{p}_i \neq \mathbf{p}_j$
  - Even if at each point we are measuring $y_i \approx a_i$, the χ^2 minimization also changes the other track parameter b_i due to the **correlations** in $\mathbf{C}_i$
  - This **predictor-corrector** method can be used to reject measurements which are not consistent with the extrapolation, **for example defining requirements on the** χ^2:

$$\chi^2 = \left(\mathbf{m}_i - \mathbf{y}_i(\tilde{\mathbf{p}}_i)\right)^T \tilde{\mathbf{V}}_i^{-1} \left(\mathbf{m}_i - \mathbf{y}_i(\tilde{\mathbf{p}}_i)\right) \Rightarrow \chi^2 = \frac{(y_i - \tilde{a}_i)^2}{\sigma_{y_i}^2 + \sigma_{a_i}^2}$$

Kalman filter: exercises

- Exercise 1 (needed for Geant4 practical exercises)
  - repeat the same computation for a parabolic trajectory
  - In case multiple scattering can be neglected, compute the initial parameters and their covariance matrix, using 3 measurement planes, spaced by a length l and with a measurement uncertainty σ

- Exercise 2
  - apply the Kalman filter method to a sequence of measurements y_i, i=0...N of a physics quantity a, each with its own uncertainty σ_i
  - show this method is equivalent to the weigthed mean

- Exercise 3
  - the best estimation of the parameters at a point i, $\mathbf{p}_i$, can be obtained performing a forward Kalman filter (points from 1 to i) and a backward one (points from i to N). Indicating with $\mathbf{p}_f$, $\mathbf{C}_f$, $\mathbf{p}_b$, $\mathbf{C}_b$, respectively, the parameters and their covariance matrices of these two fits, try to give an expression for $\mathbf{p}_i$ and $\mathbf{C}_i$.

Global χ^2 fit

- Il Kalman filter is the common choice if many measurements
- In modern experiments often few, N=O(10), measurements with high resolution:
  - inverting the correlation matrix is computationally feasible
  - minimization of a global χ^2 including all measurments at once
- It is interesting to see how multiple scattering is taken into account.
  - No multiple scattering:
    - N measurements
    - 2 parameters
    - N-2 degrees of freedom
  - With multiple scattering
    - N additional parameters: scattering angles
    - N constraints: $\langle\varphi_{ms,k}\rangle=0$ $\langle\varphi^2_{ms,k}\rangle=\theta^2_{ms,k}$
    - 2N-(N+2) degrees of freedom

No Multiple scattering

$$y(z) = a + bz$$

$$\chi^2 = \sum_{k=1,N} \frac{\left(y_k - a - bz_k\right)^2}{\sigma_k^2}$$

With Multiple scattering

$$y(z) = \begin{cases} a + bz & z \le z_1 \\ a + bz + b_{\mathrm{ms},1}(z - z_1) & z_1 < z \le z_2 \\ a + bz + b_{\mathrm{ms},1}(z - z_1) + b_{\mathrm{ms},2}(z - z_2) & z_2 < z \le z_3 \\ \vdots & \\ a + bz + \sum_{k=1,N} b_{\mathrm{ms},k}(z - z_k) & z > z_N \end{cases}$$

$$b_{\mathrm{ms},k} = \tan\varphi_{\mathrm{ms},k}$$

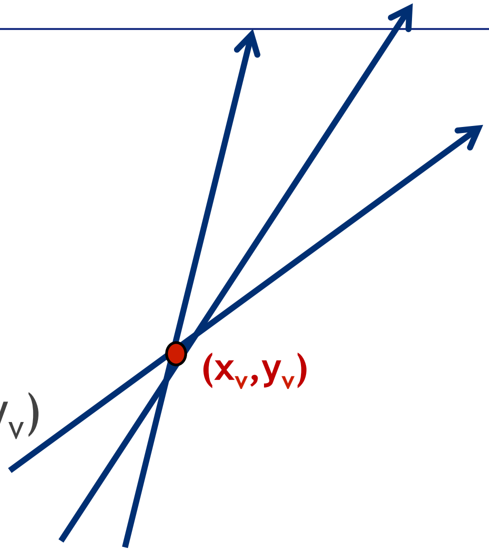
$$\chi^2 = \sum_{k=1,N} \frac{\left(y_k - y(z_k)\right)^2}{\sigma_k^2} + \frac{\varphi^2_{\mathrm{ms},k}}{\theta^2_{\mathrm{ms},k}}$$

Vertex fit (trivial)

- Take a set of tracks fitted as straight lines:

$$\begin{cases} x = \quad d_{0,i}\sin\varphi_i + s\cos\varphi_i \\ y = -d_{0,i}\cos\varphi_i + s\sin\varphi_i \end{cases}$$

$$d_{0,i} - x\sin\varphi_i + y\cos\varphi_i = 0$$



- Assume they have been produced at a single point (x_v,y_v)
- Determine the point that minimizes:

$$\chi^2 = \sum_{\text{tracks}} \frac{\left(d_{0,i} - x_v\sin\varphi_i + y_v\cos\varphi_i\right)^2}{\sigma^2_{d,i}}$$

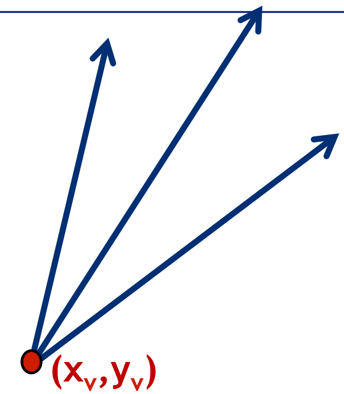
- The problem is only apparently linear:
  the uncertainty on the distance depends from vertex position:

$$\sigma^2_{d,i} = \begin{pmatrix} 1 & -x_v\cos\phi_i - y_v\sin\phi_i \end{pmatrix} \begin{bmatrix} \sigma^2_{d_{0,i}} & \sigma_{d_{0,i},\phi_i} \\ \sigma_{d_{0,i},\phi_i} & \sigma^2_{\phi_i} \end{bmatrix} \begin{pmatrix} 1 \\ -x_v\cos\phi_i - y_v\sin\phi_i \end{pmatrix}$$

  - it is linear only when the uncertainty on direction can be neglected.

Vertex fit

- The real model consists of N_{tracks}+2 parameters
  - the 2 vertex coordinates: x_v, y_v
  - the N_{tracks} directions at the vertex: $\varphi_{v,i}$
- And there are $2N_{tracks}$ observables
  - the trajectory parameters: $d_{0,i}$, φ_i
- resulting in the more complex



$$\chi^2 = \sum_{\text{tracks}} \begin{pmatrix} d_{0,i} - x_v \sin\phi_{v,i} + y_v \cos\phi_{v,i} & \phi_j - \phi_{v,i} \end{pmatrix} \begin{bmatrix} \sigma^2_{d_{0,i}} & \sigma_{d_{0,i},\phi_j} \\ \sigma_{d_{0,i},\phi_j} & \sigma^2_{\phi_j} \end{bmatrix}^{-1} \begin{pmatrix} d_{0,i} - x_v \sin\phi_{v,i} + y_v \cos\phi_{v,i} \\ \phi_j - \phi_{v,i} \end{pmatrix}$$

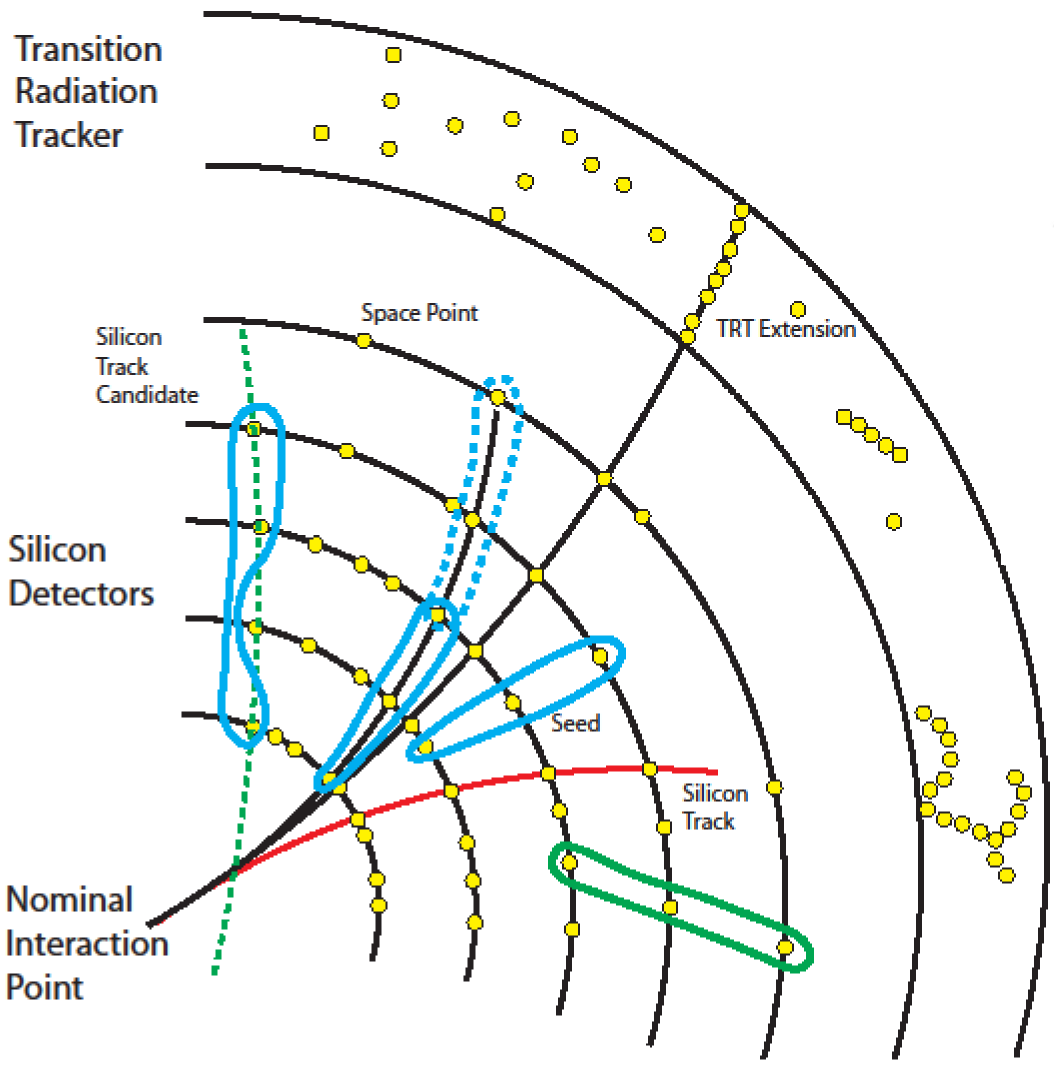
- Fitting a vertex is not simply “adding” something to a set of tracks
  - also track properties are modified:
    - the fact that all tracks come from a vertex actually constrains track parameters
    - basically the vertex is an additional measured point in the track fit
  - it is important to be sure tracks originate from the same vertex
    - iterative procedures removing tracks giving too high contributions to the χ^2
    - sometimes further constraints can be added (for example invariant mass of tracks associated to the vertex)

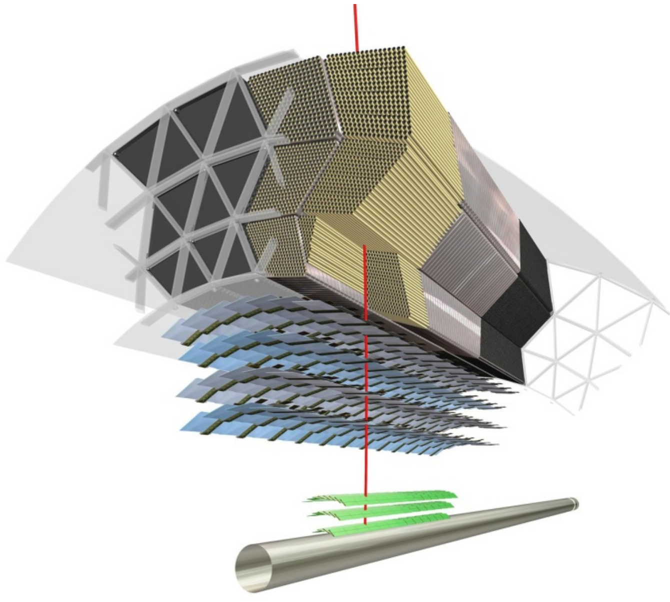
Element of track finding (pattern recognition)

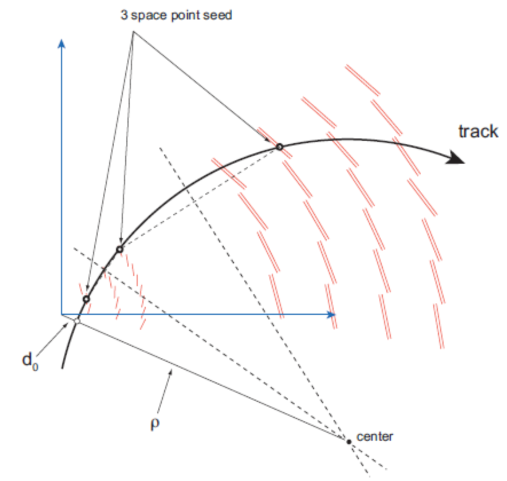
- Problem:
  - we have many individual measurements
  - which of them come from a particle?

- Local methods
  - Start from seeds build out of a small set of points:
    - enough to provide initial set of track parameters
  - Use the track model to add points and form a track
    - clearly not all seeds will become a track
    - Examples: road method, kalman filter
  - Very flexible, but complexity may increases like N^2-N^3
- Global methods
  - Process all hits at the same time
    - Examples: histogramming method, pattern matching
  - Processing time scales better with N, but, for practical reason:
    - acceptance in parameter space may be restricted
    - not the whole tracker granularity can be exploited

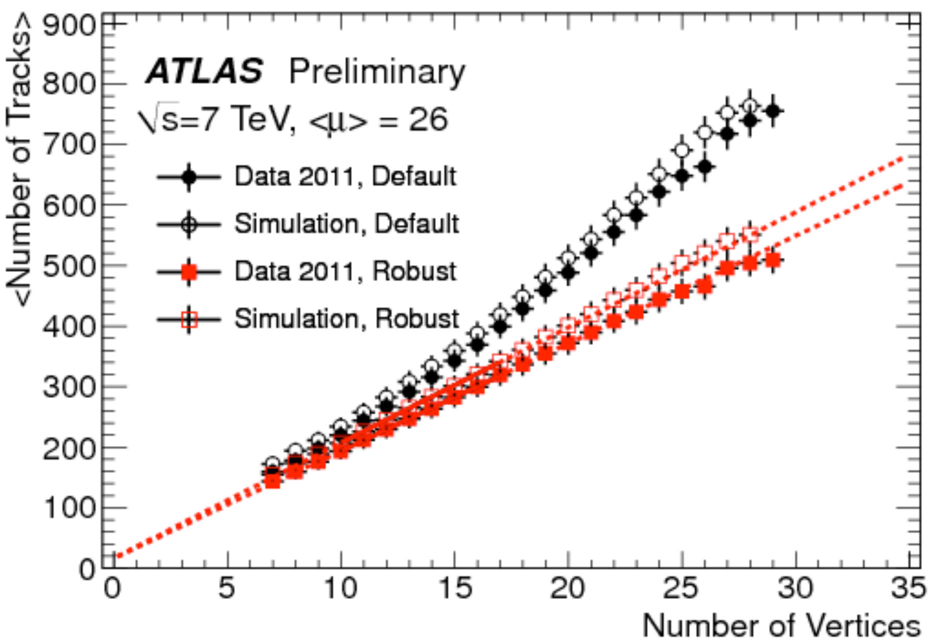
Seeding in ATLAS

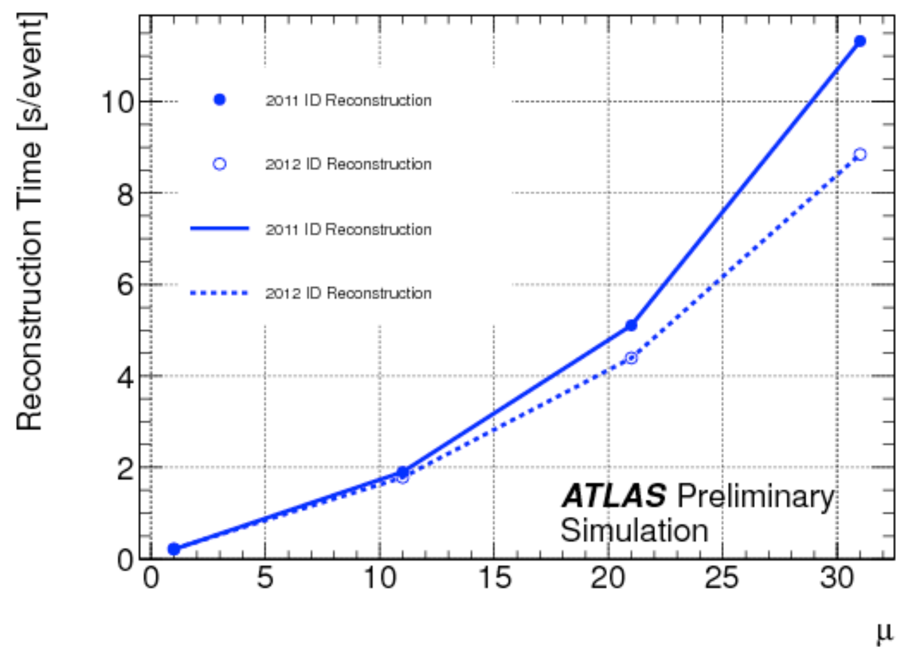






Dependence on particle density

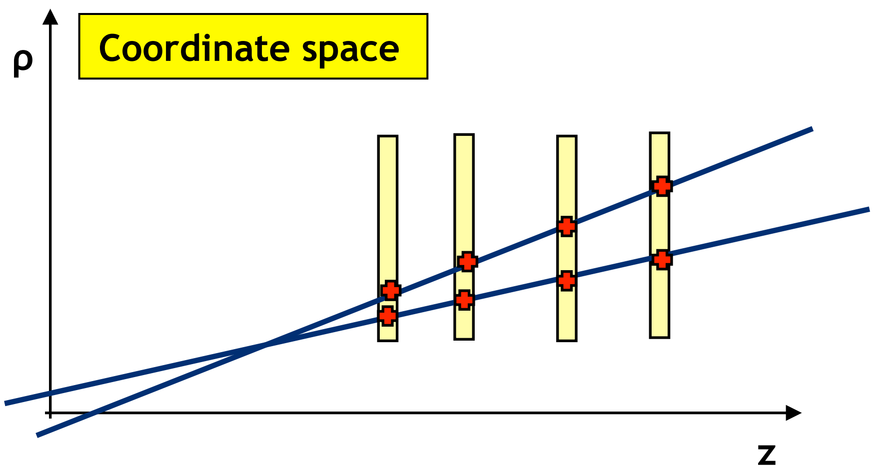


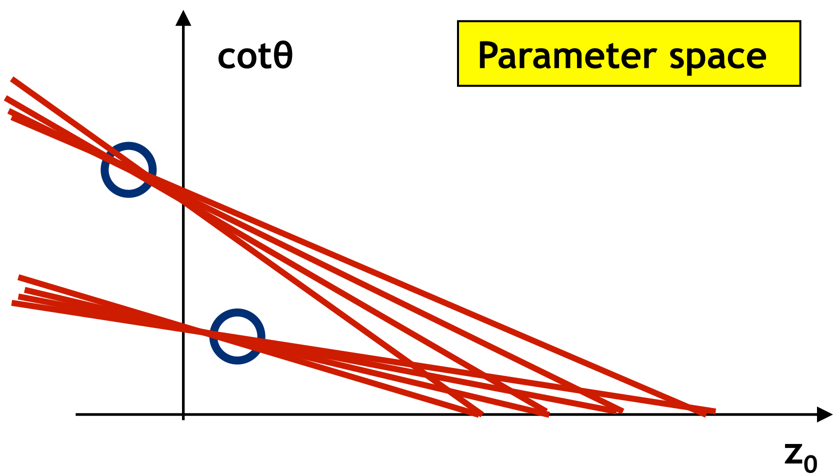


- Method is sensitive to the detector occupancy (=pile-up)
  - increase of combinatorial background
    - number of reconstructed tracks non linear with number of events: fake tracks
    - can be reduces by tightening acceptance criteria
  - increase of needed CPU resources
    - optimization/parallelization of reconstruction software

Hough transform

- The Hough transform is a method widely used in image processing.
- It is basically a transform between
  - the coordinate space, in which mesurements are performed
  - the parameter space space in which tracks are defined
- The track model $z = z_0 + \rho \cot \vartheta$
- Given a measured point $\begin{pmatrix} \rho_1 & z_1 \end{pmatrix}$
  - can be interpreted as a family of trajectories across this point $\cot \vartheta = \dfrac{z_1 - z_0}{\rho_1}$





“Linear” Hough transform

- The above parameterization cannot be directly applied to the tranverse plane:
  - it has a singularity for 0/π rad angles
  - there are three parameters

$$
\begin{aligned}
x(s) &= -d_0 \sin\phi_0 + s\sin\vartheta\cos\phi_0 + \tfrac{1}{2}\kappa s^2 \sin^2\vartheta\sin\phi_0 \\
y(s) &= d_0 \cos\phi_0 + s\sin\vartheta\sin\phi_0 - \tfrac{1}{2}\kappa s^2 \sin^2\vartheta\cos\phi_0 \\
z(s) &= z_0 + s\cos\vartheta
\end{aligned}
$$

- Let’s consider two simplified cases:
  - straight lines κ=0
    - multiplying first equation for -sinφ_0 and second for cosφ_0:
      $$-\sin\phi_0 x + y\cos\phi_0 = d_0$$
    - φ_0 and d_0 are better behaved parameters;
      one measured point corresponds to a sinusoid in the parameter space.
  - particles coming from the interaction point d_0=0:
    - trajectory is:
      $$
      \begin{aligned}
      (x - R\sin\phi_0)^2 + (y + R\cos\phi_0)^2 &= R^2 \\
      x^2 + y^2 - 2xR\sin\phi_0 + 2yR\cos\phi_0 &= 0
      \end{aligned}
      $$
    - using conformal coordinates $X = 2x/(x^2+y^2)$ $Y = 2y/(x^2+y^2)$
      $$(1/R) - X\sin\phi_0 + Y\cos\phi_0 = 0$$
    - In conformal space circles become straight lines, intercept is the curvature

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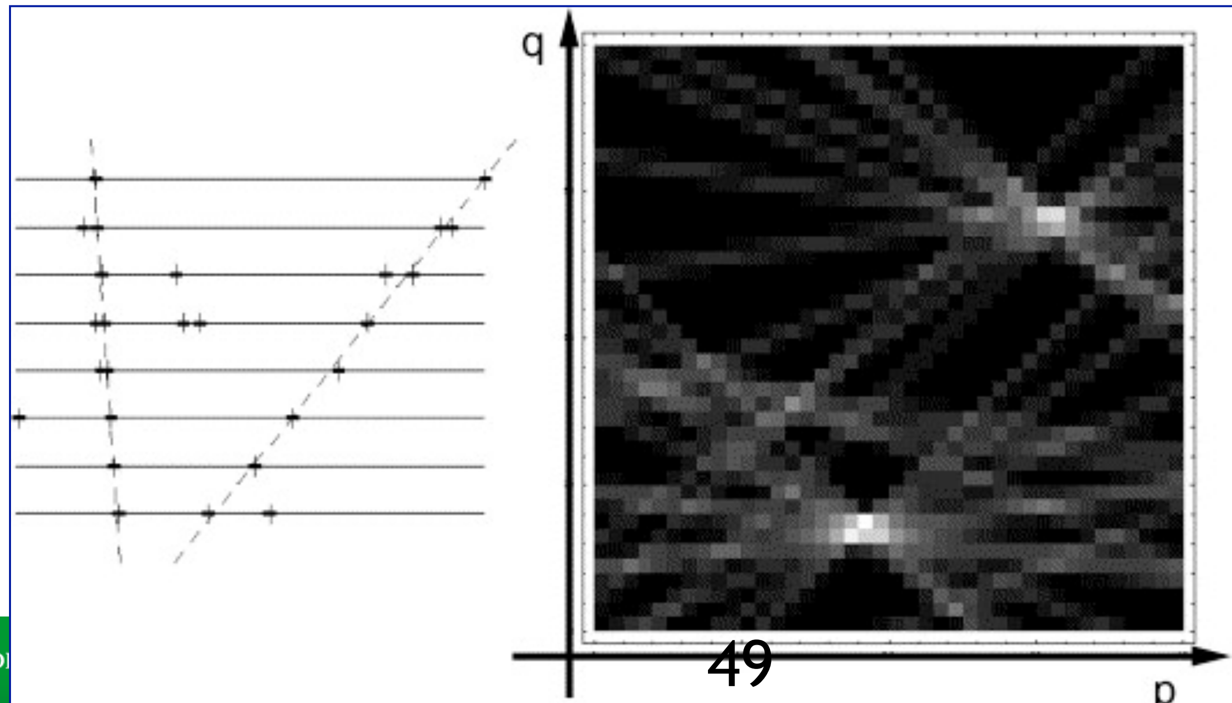
Histogramming and "retina" tracking

- Histogramming method:
  - patition parameter space (p,q) in bins of the possible track parameteres
  - fill each bin with a "vote" for each measurement compatible with the parameters
  - look for majorities in the matrics.
- Retina tracking
  - A refinement, inspirated by pattern recognition in the visual cortex.
  - Define a set of neurons excitated by shapes,
  - interpolate around local maxima.
- $$R_{ij} = \sum_{k,r} \exp\left(-\frac{s_{ijkr}^2}{2\sigma^2}\right)$$

  $$s_{ijkr} = x_r^{(k)} - y_k(p_i, q_j)$$

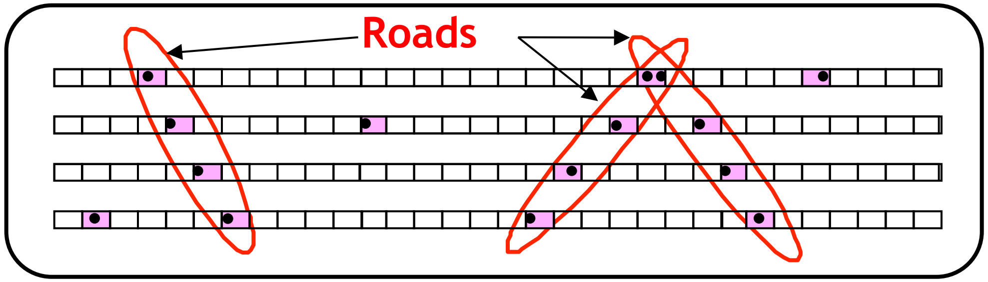
measure ment

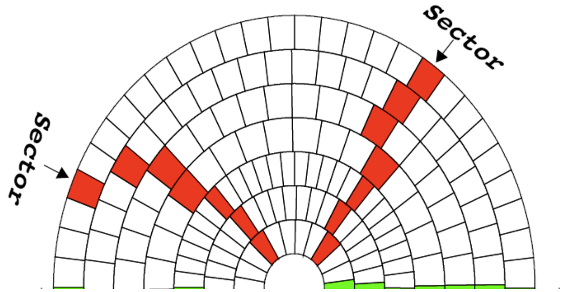
expectation from track model



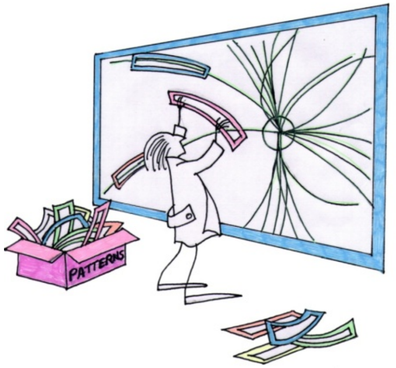
Pattern matching

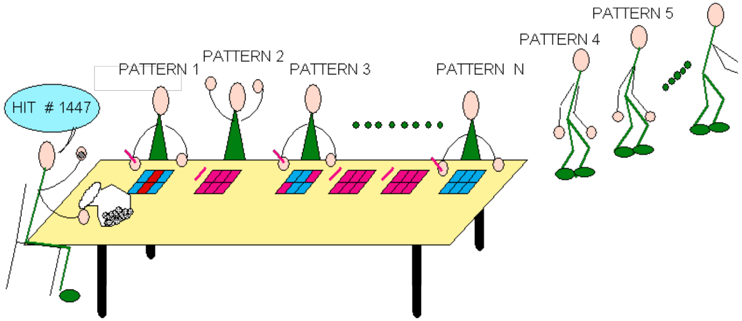
- Predefine a set of predefined hit pattern corresponding to a sample of possible trajectories.





- Find if observed hit match one of the templates

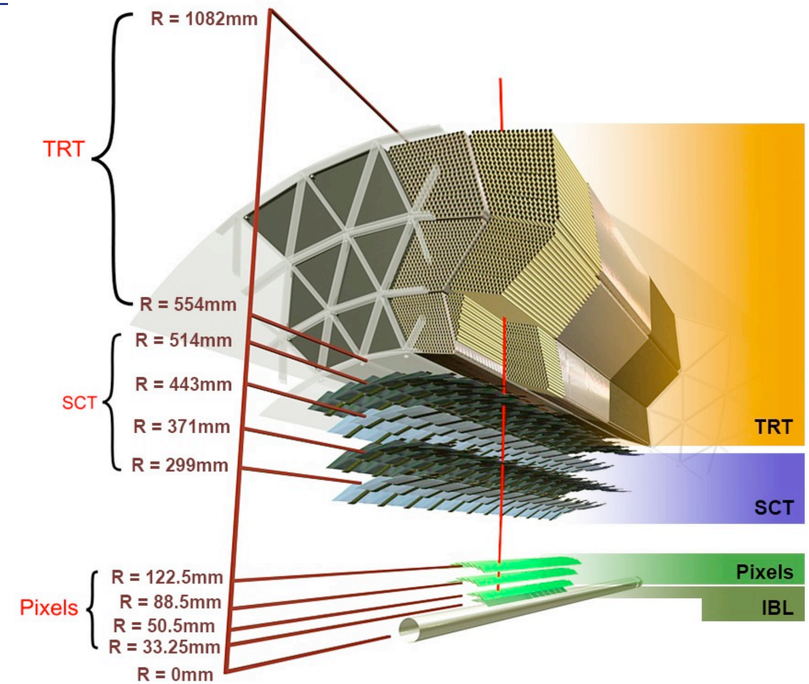


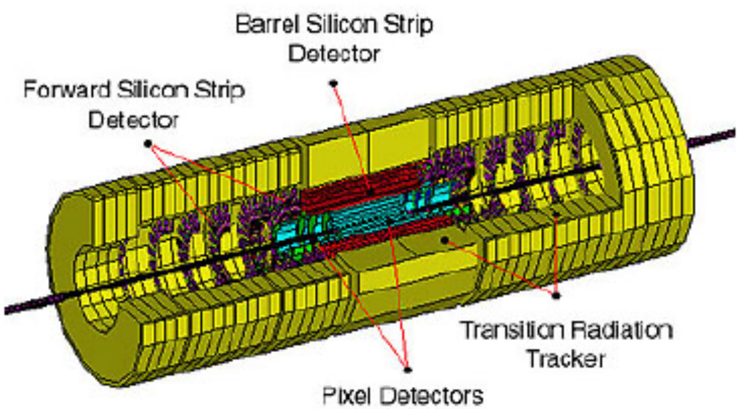


  - Fast decision: can be used at trigger level
  - Coarse granularity of patterns

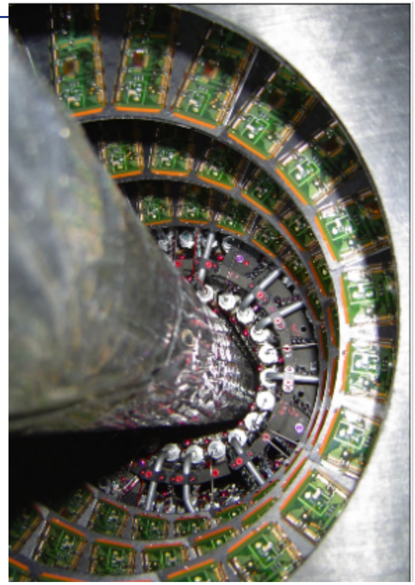
Examples of tracking systems: ATLAS

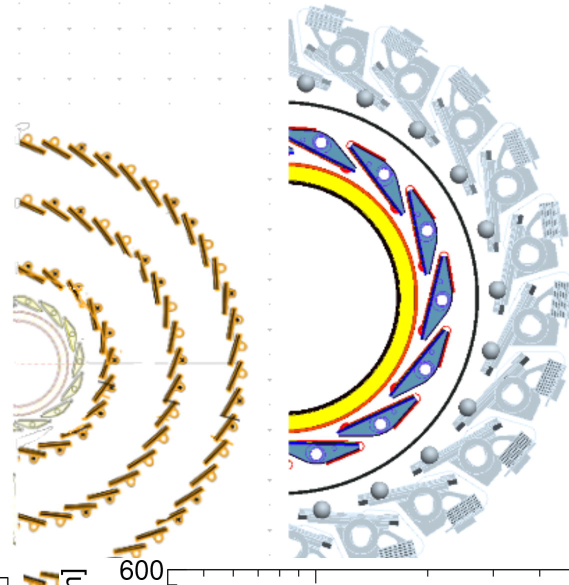
- Pixel Detector
  - 4 barrel layers, 3+3 disks, 10^8 pixels
  - barrel radii: 33, 50, 85, 122 mm
  - pixel size 50×400 μm^2 (50×250 IBL)
  - $\sigma_{R\Phi}$=6-10 μm, σ_z~66 μm
- SemiConductor Tracker
  - 4 barrel layers,7+7 disks, 6×10^6 strips
  - barrel radii: 30, 37, 44, 51 cm
  - strip pitch 80 μm (40 mrad stereo)
  - $\sigma_{R\Phi=}$16 μm, σ_z=580 μm
- Transition Radiation Tracker
  - barrel 55<R<102 cm
  - 36 layers, 400000 drift tubes
  - 4 mm diameter
  - $\sigma_{R\Phi=}$170 μm

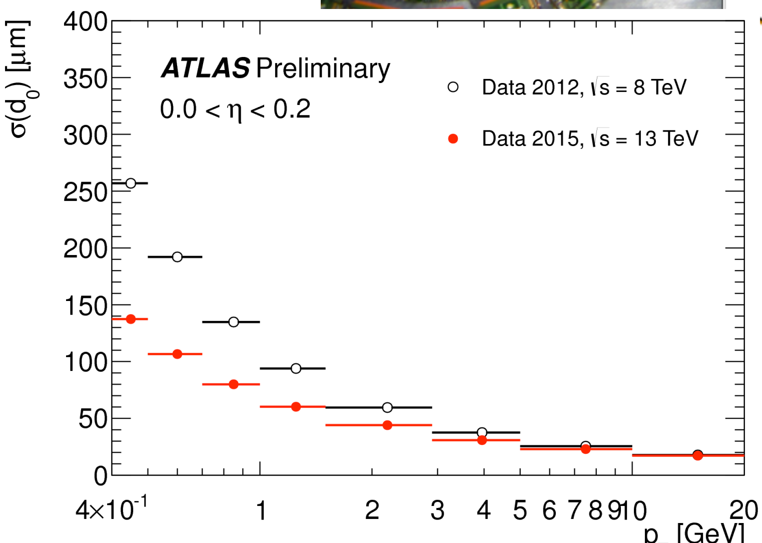


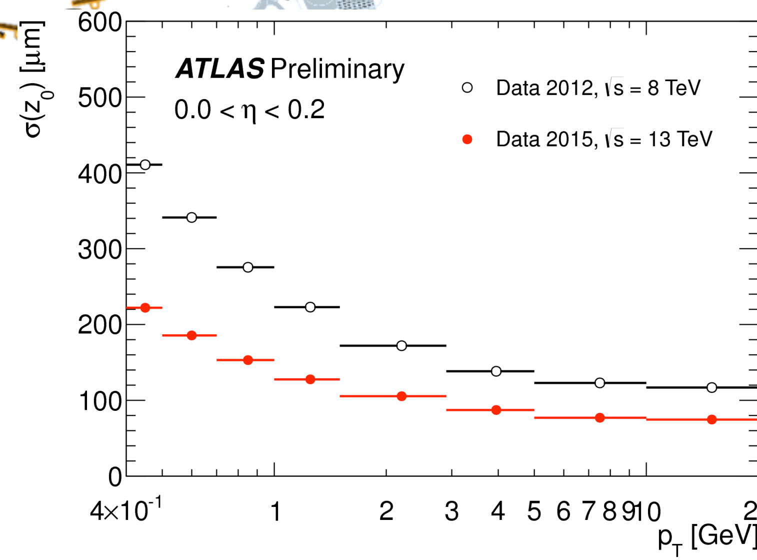


Examples of tracking systems: ATLAS



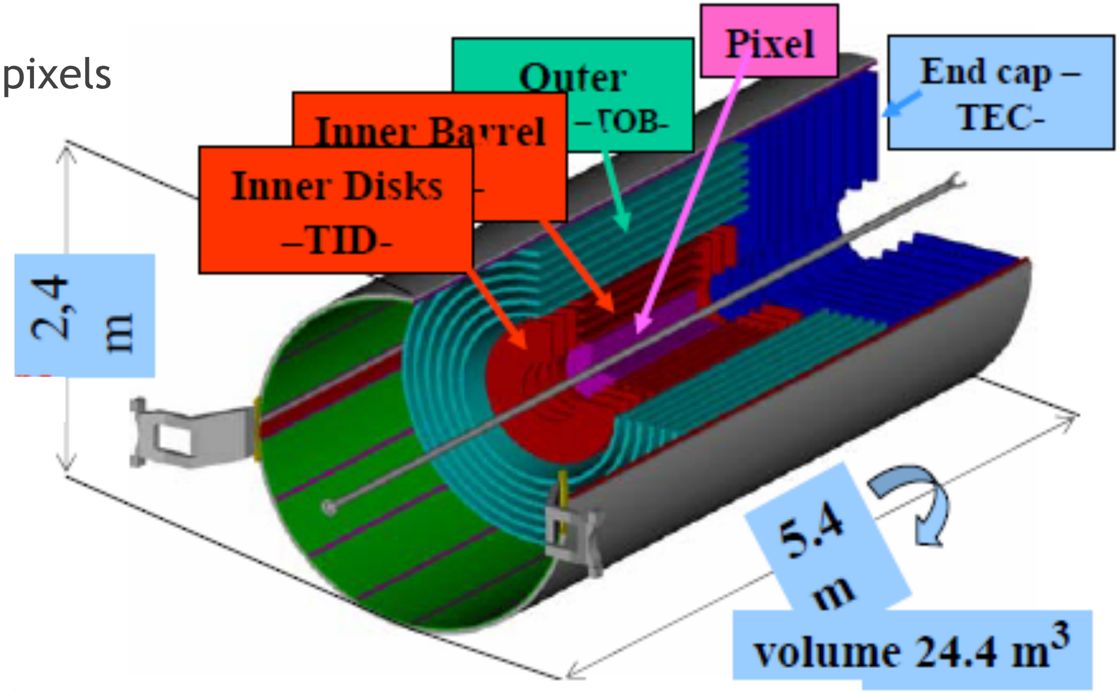


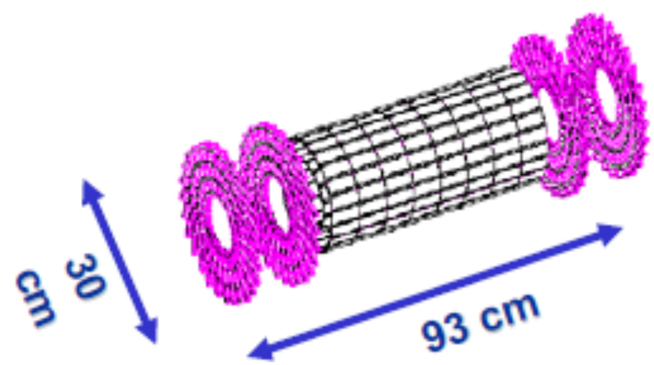




Examples of tracking systems: CMS

- Pixel Detector
  - 2 barrel layers, 2+2 disks, 10^7 pixels
  - barrel radii: 41, 100 mm
  - pixel size 100×150 μm^2
  - $\sigma_{R\Phi}$~10 μm, σ_z~10 μm
- Internal Silicon Strip Tracker
  - 4 barrel layers, many disks, 2×10^6 strips
  - barrel radii: 20-55 cm
  - strip pitch 80-120 μm
  - $\sigma_{R\Phi}$~23-35 μm
- External Silicon Strip Tracker
  - 6 barrel layers, many disks, 7×10^6 strips
  - barrel radii: max 116 cm
  - strip pitch 120-180 μm
  - $\sigma_{R\Phi}$~35-53 μm



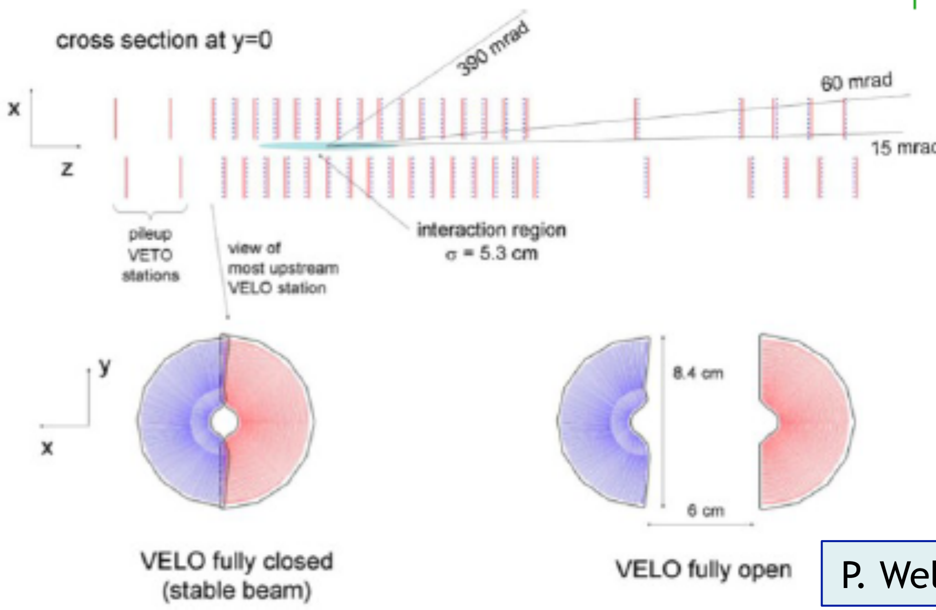


Examples of tracking systems: vertex detectors

	ALICE	ATLAS	CMS
Radii (mm)	39 – 76	50.5 – 88.5 – 122.5	44 – 73 – 102
Pixel size $r\phi$ x z (μm^2)	50 x 425	40 x 400	100 x 150
Thickness (μm)	200	250	285
Resolution $r\phi$ / z	12 / 100	10 / 115	~15-20
Channels (million)	9.8	80.4	66
Area (m^2)	0.2	1.8	1

The LHCb VELO: forward geometry strip detector with 42 stations along, inner radius of 7 mm.

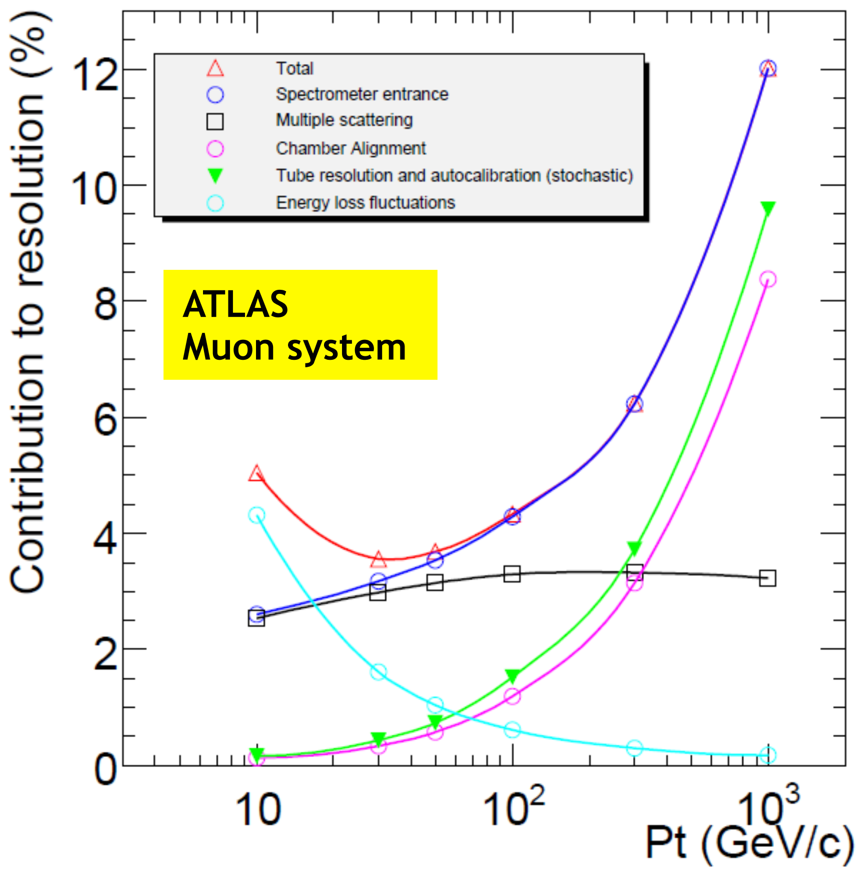
Moves close to beam when it is stable.

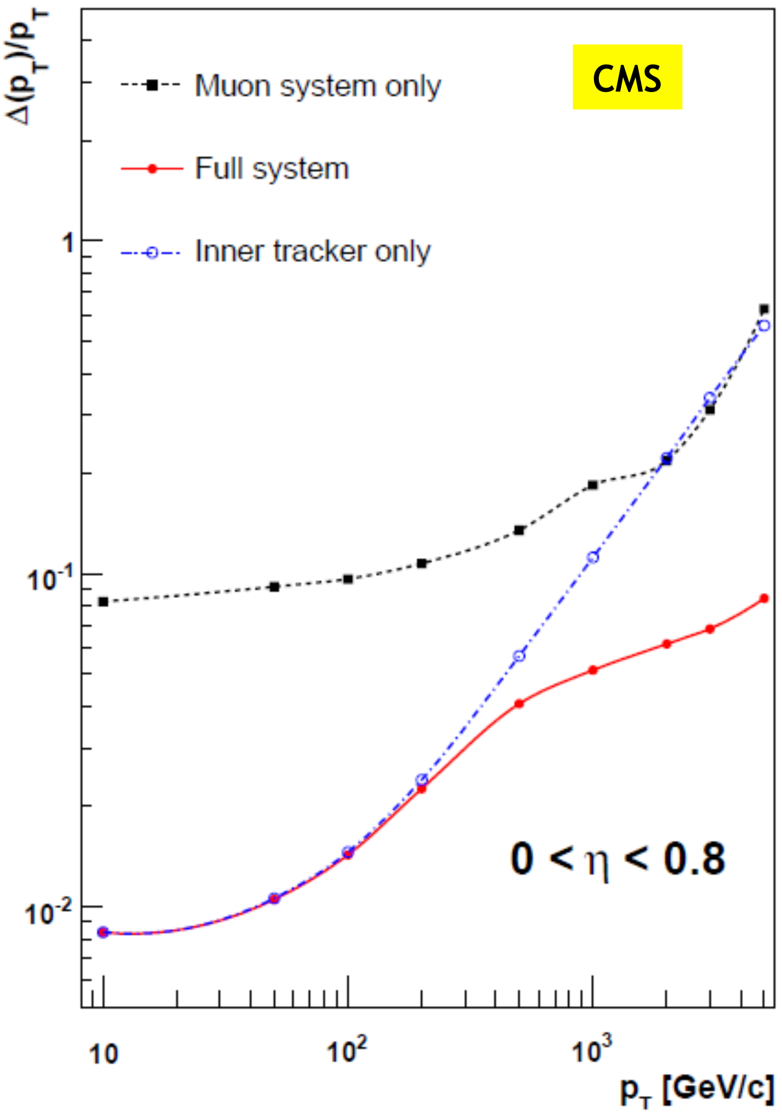


5 February 2011

P. Wells, ERDIT 2011

Examples of tracking systems: momentum resolution





BACKUP

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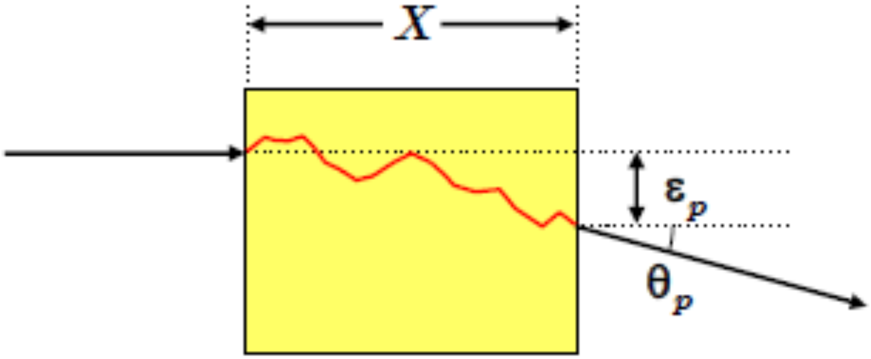
Multiple scattering

- Particles moving through the detector material suffer innumerable EM collisions which alter the trajectory in a random fashion (stochastic process)



$$\boxed{\langle \theta_p^2 \rangle = K \frac{X}{X_0}}$$

$$\boxed{K = z^2 \left(\frac{0.0136}{p\beta} \right)^2}$$

$$\langle \varepsilon_p^2 \rangle = \frac{1}{3} K \frac{X}{X_0} X^2$$

$$\langle \varepsilon_p \theta_p \rangle = \frac{1}{2} K \frac{X}{X_0} X$$

strongly correlated

$$\rho = \frac{\langle \varepsilon_p \theta_p \rangle}{\sqrt{\langle \varepsilon_p^2 \rangle \langle \theta_p^2 \rangle}} = 0.87$$

- Few examples:
  - Argon $X_o = 110\ m$
  - Silicon $X_o = 9.4\ cm$
- consider a $10\ GeV$ pion

	θ_p	ε_p
Argon: $1\ m$	0.10×10^{-3}	$80\ \mu m$
Silicon: $300\ \mu m$	0.08×10^{-3}	$0.01\ \mu m$

- The effect goes as $1/p$: for a pion of $1\ GeV$ the effect is 10 times larger
- The lateral displacement is proportional to the thickness of the detector: usually can be neglected for thin detectors
- In what follow we will consider only thin detectors
- For thick detectors (for example large volume gas detectors) see [Gluckstern 63 Blum-Rolandi 93, Block et al. 90]