Evaluating representations by the complexity of learning low-loss predictors

William F. Whitney **Min Jae Song** **David Brandfonbrener**

Jaan Altosaar **Kyunghyun Cho**

Courant Institute
New York University
wwhitney@cs.nyu.edu

Abstract

We consider the problem of evaluating representations of data for use in solving a downstream task. We propose to measure the quality of a representation by the complexity of learning a predictor on top of the representation that achieves low loss on a task of interest, and introduce two methods, surplus description length (SDL) and ε sample complexity (εSC). In contrast to prior methods, which measure the amount of information about the optimal predictor that is present in a specific amount of data, our methods measure the amount of information needed from the data to recover an approximation of the optimal predictor up to a specified tolerance. We present a framework to compare these methods based on plotting the validation loss versus training set size (the "loss-data" curve). Existing measures, such as mutual information and minimum description length probes, correspond to slices and integrals along the data-axis of the loss-data curve, while ours correspond to slices and integrals along the loss-axis. We provide experiments on real data to compare the behavior of each of these methods over datasets of varying size.

1 Introduction

One of the first steps in building a machine learning system is selecting a representation of data. Whereas classical machine learning pipelines often begin with feature engineering, the advent of deep learning has led many to argue for pure end-to-end learning where the deep network constructs the features (LeCun et al., 2015). However, huge strides in unsupervised learning (Hénaff et al., 2019; Chen et al., 2020; He et al., 2019; van den Oord et al., 2018; Bachman et al., 2019; Devlin et al., 2019; Liu et al., 2019; Raffel et al., 2019; Brown et al., 2020) have led to a reversal of this trend in the past two years, with common wisdom now recommending that the design of most systems start from a pretrained representation. With this boom in representation learning techniques, practitioners and representation researchers alike have the question: Which representation is best for my task?

Simple, traditional means of evaluating representations, such as the validation accuracy of linear probes (Ettinger et al., 2016; Shi et al., 2016; Alain & Bengio, 2016), have been widely criticized (Hénaff et al., 2019; Resnick et al., 2019). So, researchers have taken up a variety of alternatives such as the validation accuracy (VA) of nonlinear probes (Conneau et al., 2018; Hénaff et al., 2019), mutual information (MI) between representations and labels (Bachman et al., 2019; Pimentel et al., 2020), and minimum description length (MDL) of the labels conditioned on the representations (Blier & Ollivier, 2018; Yogatama et al., 2019; Voita & Titov, 2020). We propose two measures to resolve

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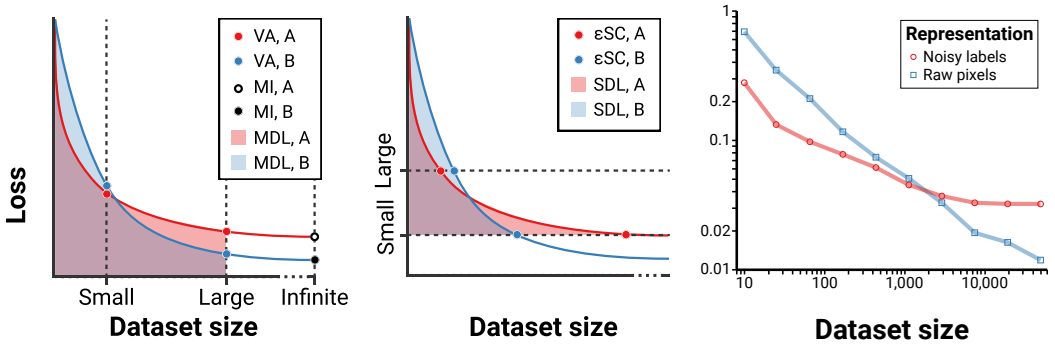
some of the limitations in prior work and propose a simple framework (shown in Figure 1) to contrast the different methods.



(a) Existing measures (b) Proposed measures (c) Illustrative experiment

Figure 1: Each measure for evaluating representation quality is a simple function of the "loss-data" curve shown here, which plots validation loss of a probe against training dataset size. **Left:** Validation accuracy (VA), mutual information (MI), and minimum description length (MDL) measure properties of a given dataset, with VA measuring the loss at a finite amount of data, MI measuring it at infinity, and MDL integrating it from zero to n. This dependence on dataset size can lead to misleading conclusions as the amount of available data changes. **Middle:** Our proposed methods instead measure the complexity of learning a predictor with a particular loss tolerance. ε sample complexity (εSC) measures the number of samples required to reach that loss tolerance, while surplus description length (SDL) integrates the surplus loss incurred above that tolerance. Neither depends on the dataset size. **Right:** A simple example task which illustrates the issue. One representation, which consists of noisy labels, allows quick learning, while the other supports low loss in the limit of data. Evaluating either representation at a particular dataset size risks drawing the wrong conclusion.

Answering the question of "which representation is best for my task?" in a principled manner requires precise notions of "task" and "best". In this paper, we consider the "task" to be finding a predictor that achieves low population risk on a downstream supervised learning problem, where "low" must be defined on a problem-dependent basis. We define the "best" representation as the one which allows for the most efficient learning of a predictor to solve the task, where we will argue for notions of efficiency in terms of either information or number of samples.

From this perspective, prior methods have clear limitations. VA and MDL measure the amount of information about the optimal predictor that is present in a *specific amount* of data instead of measuring the amount of information needed from the data to recover an approximation of the optimal predictor up to a *specified tolerance*. This difference is subtle, but we show (as can be seen in Figure 1) that this makes VA and MDL liable to choose different representations for the same task when given training sets of different sizes. On the other hand, MI measures the lowest loss achievable by any predictor irrespective of the complexity of learning such a function. It is important to note that while these methods do not correspond to our notion of measuring the best representation for a task, there may be different notions of "best" and "task" for which they measure the proper quantity.

To eliminate these issues, we propose two measures. In both of our measures, the user must specify a tolerance ε so that a population loss of less than ε qualifies as solving the task, i.e. approximating the optimal predictor to low error. The first measure is the *surplus description length* (SDL) which modifies the MDL to measure the complexity of learning an ε approximation of the optimal predictor rather than the complexity of the training dataset. The second is the *ε-sample complexity* (εSC) which measures the sample complexity of learning an ε approximation of the optimal predictor. We show that these measures resolve the issues with prior work.

We also propose a framework called the *loss-data framework*, illustrated in Figure 1, that plots the validation loss against the training set size (Talmor et al., 2019; Yogatama et al., 2019; Voita & Titov, 2020). This framework simplifies comparisons between methods. Prior work measures quantities corresponding to integrals (MDL) and slices (VA and MI) along the data-axis. Our work proposes instead measuring integrals (SDL) and slices (εSC) along the loss-axis. This clearly illustrates how prior work makes tacit assumptions about what quantity to measure based on the choice of dataset

size. Our work instead makes an explicit, interpretable choice of threshold ε and measures the complexity of solving the task to ε error.

Finally, we experimentally investigate the behavior of these methods, confirming the sensitivity of VA and MDL to dataset size and illustrating the robustness of SDL and εSC.

2 The loss-data framework for representation evaluation methods

In this section we formally present the representation evaluation problem, define our loss-data framework, and show how prior work fits into the framework.

Notation. We use bold letters to denote random variables. A supervised learning problem is defined by a joint distribution $\mathcal{D}$ over observations and labels $(\mathbf{X}, \mathbf{Y})$ in the sample space $\mathcal{X} \times \mathcal{Y}$ with density denoted by p. Let the random variable $\mathbf{D}^n$ be a sample of n i.i.d. $(\mathbf{X}, \mathbf{Y})$ pairs, realized by $D^n = (X^n, Y^n) = \{(x_i, y_i)\}_{i=1}^n$. Let $\mathcal{R}$ denote a representation space and $\phi : \mathcal{X} \to \mathcal{R}$ a representation function. The methods we consider all use parametric probes, which are neural networks $\hat{p}_\theta : \mathcal{R} \to P(\mathcal{Y})$ parameterized by $\theta \in \mathbb{R}^d$ that are trained on D^n to estimate the conditional distribution $p(y \mid x)$. We often abstract away the details of learning the probe by simply referring to an algorithm $\mathcal{A}$ which returns a predictor: $\hat{p} = \mathcal{A}(\phi(D^n))$. Abusing notation, we denote the composition of $\mathcal{A}$ with ϕ by $\mathcal{A}_\phi$. Define the population loss and the expected population loss for $\hat{p} = \mathcal{A}_\phi(D^n)$, respectively as

$$L(\mathcal{A}_\phi, D^n) = \mathop{\mathbb{E}}_{(\mathbf{X},\mathbf{Y})} -\log \hat{p}(\mathbf{Y} \mid \mathbf{X}), \qquad L(\mathcal{A}_\phi, n) = \mathop{\mathbb{E}}_{\mathbf{D}^n} L(\mathcal{A}_\phi, \mathbf{D}^n). \tag{1}$$

In this section we will focus on population quantities, but note that any algorithmic implementation must replace these by their empirical counterparts.

The representation evaluation problem. The representation evaluation problem asks us to define a real-valued measurement of the quality of a representation ϕ for solving solving the task defined by $(\mathbf{X}, \mathbf{Y})$. Explicitly, each method defines a real-valued function $m(\phi, \mathcal{D}, \mathcal{A}, \Psi)$ of a representation ϕ, data distribution $\mathcal{D}$, probing algorithm $\mathcal{A}$, and some method-specific set of hyperparameters Ψ. By convention, minimizing the measure m corresponds to better representations. Defining such a measurement allows us to compare different representations.

2.1 Defining the loss-data framework.

The loss-data framework is a lens through which we contrast different measures of representation quality. The key idea, demonstrated in Figure 1, is to plot the loss $L(\mathcal{A}_\phi, n)$ against the dataset size n. Explicitly, at each n, we train a probing algorithm $\mathcal{A}$ using a representation ϕ to produce a predictor $\hat{p}$, and then plot the loss of $\hat{p}$ against n. Similar analysis has appeared in Voita & Titov (2020); Yogatama et al. (2019); Talmor et al. (2019). We observe that we can represent each of the prior measures as points on the curve at fixed x (VA, MI) or integrals of the curve along the x-axis (MDL). Our measures correspond to evaluating points at fixed y (εSC) and integrals along the y-axis (SDL).

2.2 Existing methods in the loss-data framework

Nonlinear probes with limited data. A simple strategy for evaluating representations is to choose a probe architecture and train it on a limited amount of data from the task and representation of interest (Hénaff et al., 2019; Zhang & Bowman, 2018). On the loss-data curve, this corresponds to evaluation at $x = n$, so that

$$m_{\text{VA}}(\phi, \mathcal{D}, \mathcal{A}, n) = L(\mathcal{A}_\phi, n). \tag{2}$$

Mutual information. Mutual information (MI) between a representation $\phi(\mathbf{X})$ and targets $\mathbf{Y}$ is another often-proposed metric for learning and evaluating representations (Pimentel et al., 2020; Bachman et al., 2019). In terms of entropy, mutual information is equivalent to the information gain about $\mathbf{Y}$ from knowing $\phi(\mathbf{X})$:

$$I(\phi(\mathbf{X}); \mathbf{Y}) = H(\mathbf{Y}) - H(\mathbf{Y} \mid \phi(\mathbf{X})). \tag{3}$$

In general mutual information is intractable to estimate for high-dimensional or continuous-valued variables (McAllester & Stratos, 2020), and a common approach is to use a very expressive model for $\hat{p}$ and maximize a variational lower bound:

$$I(\phi(\mathbf{X}); \mathbf{Y}) \geq H(\mathbf{Y}) + \mathop{\mathbb{E}}_{(\mathbf{X},\mathbf{Y})} \log \hat{p}(\mathbf{Y} \mid \phi(\mathbf{X})) \tag{4}$$

Since $H(\mathbf{Y})$ is not a function of the parameters, maximizing the lower bound is equivalent to minimizing the negative log-likelihood. Moreover, if we assume that $\hat{p}$ is expressive enough to represent p and take $n \to \infty$, this inequality becomes tight. As such, MI estimation can be seen a special case of nonlinear probes as described above, where instead of choosing some particular setting of n we push it to infinity. We formally define the mutual information measure of a representation as

$$m_{\mathrm{MI}}(\phi, \mathcal{D}, \mathcal{A}) = \lim_{n \to \infty} L(\mathcal{A}_\phi, n). \tag{5}$$

A decrease in this measure reflects an increase in the mutual information. On the loss-data curve, this corresponds to evaluation at $x = \infty$.

Minimum description length. Recent studies (Yogatama et al., 2019; Voita & Titov, 2020) propose using the Minimum Description Length (MDL) principle (Rissanen, 1978; Grünwald, 2004) to evaluate representations. These works use an online or prequential code (Blier & Ollivier, 2018) to encode the labels given the representations. The codelength ℓ of Y^n given $\phi(X^n)$ is then defined as

$$\ell(Y^n \mid \phi(X^n)) = -\sum_{i=1}^{n} \log \hat{p}_i(y_i \mid \phi(x_i)), \tag{6}$$

where $\hat{p}_i$ is the output of running a pre-specified algorithm $\mathcal{A}$ on the dataset up to element i: $\hat{p}_i = \mathcal{A}_\phi(X^n_{1:i}, Y^n_{1:i})$. Taking an expectation over the sampled datasets for each i, we define a population variant of the MDL measure (Voita & Titov, 2020) as

$$m_{\mathrm{MDL}}(\phi, \mathcal{D}, \mathcal{A}, n) = \mathbb{E}\left[\ell(\mathbf{Y}^n \mid \phi(\mathbf{X}^n))\right] = \sum_{i=1}^{n} L(\mathcal{A}, i). \tag{7}$$

Thus, m_{MDL} measures the area under the loss-data curve on the interval $x \in [0, n]$.

3 Limitations of existing methods

Each of the prior methods, VA, MDL, and MI, have limitations that we attempt to solve with our methods. In this section we present these limitations. In Section 3.1, we describe a toy example which demonstrates why evaluation metrics that depend on the training set size like VA and MDL can be misleading. Then in Section 3.2 we argue that MI, which does not depend on the training set size, can be misleading as well since it is insensitive to the quality of the representation.

3.1 Sensitivity to dataset size in VA and MDL

As seen in Section 2.2, the representation quality measures of VA and MDL both depend on n, the size of the training set. Because of this dependence, the ranking of representations given by these evaluation metrics can change as n increases. As a result, favoring one representation over others by comparing these metrics at arbitrary n may lead to premature decisions in the machine learning pipeline since evaluating on a larger training set could give a different representation ranking.

A theoretical example. Let $s \in \{0,1\}^d$ be a fixed binary vector and consider a data generation process where the $\{0,1\}$ label of a data point is given by the parity on s, i.e., $y_i = \langle x_i, s \rangle \bmod 2$ where $y_i \in \{0,1\}$ and $x_i \in \{0,1\}^d$. Let $Y^n = \{y_i\}_{i=1}^n$ be the given labels and consider the following two representations.

1. Noisy label: $z_i = \langle x_i, s \rangle + e_i \bmod 2$, where $e_i \in \{0,1\}$ is a random bit with bias $\alpha < 1/2$.
2. Raw data: x_i.

For the noisy label representation, guessing $y_i = z_i$ achieves validation accuracy of $1 - \alpha$ for any n, which, is information-theoretically optimal. On the other hand, the raw data representation will achieve perfect validation accuracy once the training set contains d linearly independent x_i's. In this case, Gaussian elimination will exactly recover s. The probability that a set of $n > d$ random vectors in $\{0, 1\}^d$ does not contain d linearly independent vectors decreases exponentially in $n - d$. Hence, the expected validation accuracy for n sufficiently larger than d will be exponentially close to 1. As a result, the representation ranking given by validation accuracy and description length favors the noisy label representation when $n \ll d$, but the raw data representation will be much better in these metrics when $n \gg d$. This can be misleading.

Although this is a concocted example for illustration purposes, our experiments in Section 5 show that dependence of representation rankings on n does occur in practice.

3.2 Insensitivity to representation quality and computational complexity of MI

MI considers the lowest validation loss achievable with the given representation. This ignores any concerns about statistical or computational complexity of achieving such accuracy which leads to some counterintuitive properties:

1. MI is insensitive to statistical complexity. Two random variables which are perfectly predictive of one another have maximal MI, though their relationship may be sufficiently complex that it requires exponentially many samples to verify (McAllester & Stratos, 2020).
2. MI is insensitive to computational complexity. For example, the mutual information between an intercepted encrypted message and the enemy's plan is high (Shannon, 1948; Xu et al., 2020), despite the extreme computational cost required to break the encryption.
3. MI is insensitive to representation. By the data processing inequality (Cover & Thomas, 2006), *any* ϕ applied to $\mathbf{X}$ can only decrease its mutual information with $\mathbf{Y}$; no matter the query, MI always reports that the raw data is at least as good as the best representation.

As a result, we believe that in most settings MI is an undesirable metric for evaluating representations.

3.3 Lack of a predefined notion of success

All three prior methods lack a predefined notion of successfully solving the task. They will return an ordering of representations regardless of whether or not this order is meaningful. Ultimately, we care about achieving high predictive accuracy on the given task. We would not even care about the rankings of representations if all gave terrible validation loss. That is, there is often an implicit minimum requirement for the validation loss a representation should achieve for it to be considered meaningful. As we will see in the next section, our methods makes this requirement explicit.

4 Our methods: surplus description length and ε sample complexity

The methods discussed above measure a property of the data, such as the attainable accuracy on n points, by learning an unspecified function. Instead, we propose to precisely define the function of interest and measure its complexity using data. Fundamentally we shift from making a statement about the inputs of an algorithm, like validation accuracy and MDL do, to imposing a constraint on the outputs of the algorithm.

4.1 Surplus description length (SDL)

Imagine trying to efficiently encode a large number of samples of a random variable $\mathbf{e}$ which takes values in $\{1 \dots K\}$ with probability $p(\mathbf{e})$. An optimal code for these events has expected length[1] $\mathbb{E}[\ell(\mathbf{e})] = \mathbb{E}_{\mathbf{e}}[-\log p(\mathbf{e})] = H(\mathbf{e})$. If this data is instead encoded using a probability distribution $\hat{p}$, the expected length becomes $H(\mathbf{e}) + D_{\mathrm{KL}}\big(p \,||\, \hat{p}\big)$. We call $D_{\mathrm{KL}}\big(p \,||\, \hat{p}\big)$ the *surplus description length* (SDL) from encoding according to $\hat{p}$ instead of p:

$$D_{\mathrm{KL}}\big(p \,||\, \hat{p}\big) = \mathop{\mathbb{E}}_{\mathbf{e}\sim p}\left[\log p(\mathbf{e}) - \log \hat{p}(\mathbf{e})\right]. \tag{8}$$

[1] in nats

When the true distribution p is a delta mass, the entire length of a code under $\hat{p}$ is surplus since $\log 1 = 0$.

Recall that the prequential code for estimating MDL computes the description length of the labels given observations in a dataset by iteratively creating tighter approximations $\hat{p}_1 \dots \hat{p}_n$ and integrating the area under the curve. Examining Equation (7), we see that

$$m_{\text{MDL}}(\phi, \mathcal{D}, \mathcal{A}, n) = \sum_{i=1}^{n} L(\mathcal{A}_\phi, i) \geq \sum_{i=1}^{n} H(\mathbf{Y} \mid \phi(\mathbf{X})) \tag{9}$$

If $H(\mathbf{Y} \mid \phi(\mathbf{X})) > 0$, the description length grows without bound as n increases.

We instead propose to measure the complexity of an approximate labeling function $p(\mathbf{Y} \mid \phi(\mathbf{X}))$ by computing the surplus description length of encoding an infinite stream of data according to the online code instead of the true conditional distribution.

Definition 1 (Surplus description length of online codes)**.** *Given random variables $\mathbf{X}, \mathbf{Y} \sim \mathcal{D}$, a representation function ϕ, and a learning algorithm $\mathcal{A}$, define*

$$m_{\text{SDL}}(\phi, \mathcal{D}, \mathcal{A}) = \sum_{i=1}^{\infty} \mathop{\mathbb{E}}_{\mathbf{X},\mathbf{Y}} \Big[-\log \hat{p}_i(\mathbf{Y} \mid \phi(\mathbf{X})) + \log p(\mathbf{Y} \mid \mathbf{X}) \Big] \tag{10}$$

$$= \sum_{i=1}^{\infty} \Big[L(\mathcal{A}_\phi, i) - H(\mathbf{Y} \mid \mathbf{X}) \Big] \tag{11}$$

We generalize this definition to measure the complexity of learning an approximating conditional distribution with loss ε, rather than the true conditional distribution only:

Definition 2 (Surplus description length of online codes with an arbitrary baseline)**.** *Given random variables $\mathbf{X}, \mathbf{Y} \sim \mathcal{D}$, a representation function ϕ, a learning algorithm $\mathcal{A}$, and a loss tolerance $\varepsilon \geq H(\mathbf{Y} \mid \mathbf{X})$, define*

$$m_{\text{SDL}}(\phi, \mathcal{D}, \mathcal{A}, \varepsilon) = \sum_{i=1}^{\infty} \Big[L(\mathcal{A}_\phi, i) - \varepsilon \Big]_+ \tag{12}$$

where $[c]_+$ denotes $\max(0, c)$.

In our framework, the surplus description length corresponds to computing the area between the loss-data curve and a baseline set by $y = \varepsilon$. Whereas MDL measures the complexity of a sample of n points, SDL measures the complexity of a function which solves the task to ε tolerance.

Estimating the SDL. Naively computing SDL would require unbounded data and the estimation of $L(\mathcal{A}_\phi, i)$ for every i. However, if we assume that algorithms are monotonically improving so that $L(\mathcal{A}, i+1) \leq L(\mathcal{A}, i)$, SDL only depends on i up to the first point where $L(\mathcal{A}, n) \leq \varepsilon$. Approximating this integral in practice can be done efficiently by taking a log-uniform partition of the dataset size and computing the Riemann sum as in Voita & Titov (2020). Crucially, if the tolerance ε is set too low or the maximum amount of available data is insufficient, an implementation may report that the given complexity estimate is only a lower bound.

In Appendix A we provide a detailed algorithm for estimating surplus description length, along with a theorem proving its data requirements as a function of the sample complexity and desired confidence.

4.2 ε sample complexity (εSC)

In addition to the surplus description length, we suggest the use of a second, simpler complexity measure: sample complexity.

Definition 3 (Sample complexity of an ε-loss predictor)**.** *Given random variables $\mathbf{X}, \mathbf{Y} \sim \mathcal{D}$, a representation function ϕ, a learning algorithm $\mathcal{A}$, and a loss tolerance $\varepsilon \geq H(\mathbf{Y} \mid \phi(\mathbf{X}))$, define*

$$m_{\varepsilon\text{SC}}(\phi, \mathcal{D}, \mathcal{A}, \varepsilon) = \min \Big\{ n \in \mathbb{N} : L(\mathcal{A}_\phi, n) \leq \varepsilon \Big\}. \tag{13}$$

Sample complexity measures the complexity of learning an ε-loss predictor by the number of samples it takes a given algorithm to find it. In our framework, sample complexity corresponds to taking a horizontal slice of the loss-data curve at $y = \varepsilon$, and in this sense it is analogous to VA. Whereas VA makes a statement about the data (by setting n) and reports the accuracy of some function given that data, sample complexity specifies the desired function and determines its complexity by how many samples are needed to learn it.

Estimating the εSC. Given an assumption that algorithms are monotonically improving such that $L(\mathcal{A}, n+1) \leq L(\mathcal{A}, n)$, εSC can be estimated efficiently. With n finite samples of training data, an algorithm may estimate εSC by splitting the data into k uniform-sized bins and estimating $L(\mathcal{A}, {}^{ik}/_{n})$ for $i \in \{1 \dots k\}$. By recursively performing this search on the interval which contains the transition from $L > \varepsilon$ to $L < \varepsilon$, an algorithm can rapidly reach a precise estimate or report that $m_{\varepsilon\text{SC}}(\phi, \mathcal{D}, \mathcal{A}, \varepsilon) > n$.

A more detailed examination of the algorithmic considerations of estimating εSC can be found in Appendix B.

Using objectives other than negative log-likelihood. Our exposition of εSC uses negative log-likelihood for consistency with other methods, such as MDL, which require it. However, it is straightforward to extend εSC to work with whatever objective function is desired under the assumption that said objective is monotone with increasing data when using algorithm $\mathcal{A}$.

5 Experiments

n	Representation	CIFAR	Pixels	VAE
60	Val loss	0.12	0.22	**0.10**
	MDL	21.85	23.82	**13.42**
	SDL, ε=0.1	> 15.85	> 17.82	**7.43**
	SDL, ε=0.02	> 20.65	> 22.62	> 12.22
	εSC, ε=0.1	> 60.0	> 60.0	**60**
	εSC, ε=0.02	> 60.0	> 60.0	> 60.0
20398	Val loss	**0.01**	0.02	0.02
	MDL	**246.68**	499.45	600.32
	SDL, ε=0.1	15.85	21.31	**7.43**
	SDL, ε=0.02	**32.21**	138.28	198.48
	εSC, ε=0.1	94	360	**60**
	εSC, ε=0.02	**2168**	13029	20398

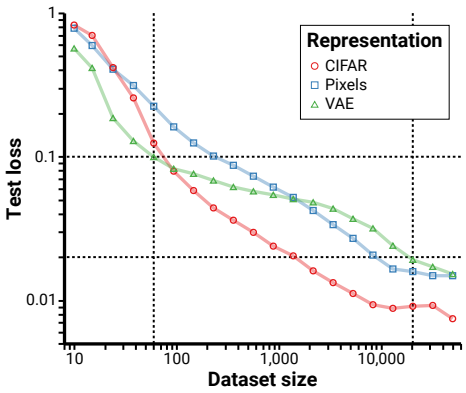


Figure 2: Results using three representations on the MNIST dataset. We omit MI as for any finite amount of data, the MI measure is the same as the validation loss.

We empirically demonstrate the issue of sensitivity to dataset size by two experiments on real data.

For the first, shown in Figure 2, we train probes to solve MNIST from three representations: (1) the last hidden layer of a small convolutional network pretrained on CIFAR-10; (2) raw pixels; and (3) a variational autoencoder (VAE) (Kingma & Welling, 2014; Rezende et al., 2014) trained on MNIST. At small n, the VAE representation would appear to be the best according to VA and MDL, but as the amount of data grows, both prefer the CIFAR representation. By contrast, SDL and εSC are aware when they do not have sufficient data to estimate complexity. Even with very little data available, they estimate the complexity of an 0.1-loss function; however, they only bound the complexity of a 0.02-loss function until they have more data. Once SDL and εSC estimate a quantity, such as the complexity of learning an 0.1-loss predictor from the VAE representation, it does not change with n; we suggest that this may be a useful property for an evaluation benchmark.

For the second experiment, we use the part-of-speech task introduced by Hewitt & Liang (2019) as implemented by Voita & Titov (2020) with the same probe architecture and other hyperparameters as those works. We compare the representations given by different layers of a pretrained ELMo (Peters et al., 2018) model. As shown in Figure 3, we find the ranking of the three representations according to VA and MDL changes as the amount of data increases. Meanwhile the measures computed by SDL and εSC reflect the insufficient data in the low-data regime, since the sample complexity is greater than the amount of available training data.

Details of the experiments, including representation training, probe architectures, and hyperparameters, are available in Appendix C, and code is available in the supplement.

n	ELMo layer	0	1	2
461	Val loss	0.73	**0.72**	0.85
	MDL	**1213.98**	1313.75	1320.52
	SDL, ε=0.5	> 283.75	> 334.43	> 366.35
	SDL, ε=0.1	> 472.15	> 522.83	> 554.75
	εSC, ε=0.5	> 461	> 461	> 461
	εSC, ε=0.1	> 461	> 461	> 461
474838	Val loss	0.17	**0.08**	0.09
	MDL	92403.41	**52648.50**	65468.54
	SDL, ε=0.5	**338.48**	365.25	450.84
	SDL, ε=0.1	> 42162.47	**2922.04**	8734.11
	εSC, ε=0.5	1256	**854**	1446
	εSC, ε=0.1	> 474838	**149946**	474838

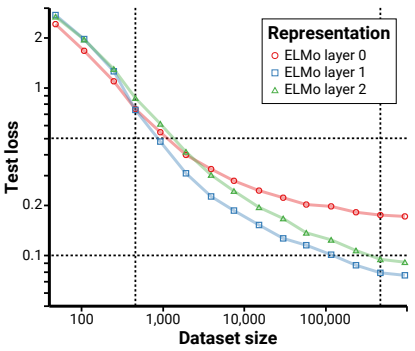


Figure 3: Results using three representations on a part of speech task. We omit MI as for any finite amount of data, the MI measure is the same as the validation loss.

6 Related work

Zhang & Bowman (2018) and Hewitt & Liang (2019) propose random baselines for linguistic tasks to provide context for how much linguistic structure is readily accessible in representations. To show separation between the validation accuracy achieved by these random baselines and representations pretrained on genuine linguistic labels, they had to limit the amount of training data or restrict the capacity of probes. As an alternative, Voita & Titov (2020) propose using the MDL framework, which accounts for the "effort of learning" required by the probes to achieve high validation accuracy, to demonstrate the separation between pretrained representations and random baselines. An earlier work by Yogatama et al. (2019) also uses prequential codes to evaluate representations for linguistic tasks. Talmor et al. (2019) look at the loss-data curve (called "learning curve" in their work) and use a weighted average of the validation loss at various training set sizes to evaluate representations.

7 Discussion

In this work we have introduced the loss-data framework for comparing representation evaluation measures and used it to diagnose the issue of sensitivity to dataset size in the validation accuracy and minimum description length measures. We proposed two measures, surplus description length and ε sample complexity, which eliminate this issue by measuring the complexity of learning a predictor which solves the task of interest to ε tolerance. Empirically we showed that sensitivity to dataset size occurs in practice for VA and MDL, while SDL and εSC are robust to the amount of available data and are able to report when it is insufficient to make a judgment.

Each of these measures depends on a choice of algorithm $\mathcal{A}$, including hyperparameters such as probe architecture, which could make the evaluation procedure less robust. To alleviate this, future

work might consider a set of algorithms $A = \{\mathcal{A}_i\}_{i=1}^{K}$ and a method of combining them, such as the model switching technique of Blier & Ollivier (2018); Erven et al. (2012) or a Bayesian prior.

Finally, while existing measures such as VA, MI, and MDL do not measure our notion of the best representation for a task, under other settings they may be the correct choice. For example, if only a fixed set of data will ever be available, selecting representations using VA might be a reasonable choice; and if unbounded data is available for free, perhaps MI is the most appropriate measure. However, in many cases the robustness and interpretability offered by SDL and εSC make them a practical choice for practitioners and representation researchers alike.

Broader Impact

Improving the tools for evaluating representations could lead to better pretrained representations and increasing use of those pretrained models. Given that large-scale learning from scratch is expensive in terms of energy and thus also carbon output (Strubell et al., 2019), making the use of small models on top of pretrained representations more appealing could reduce the carbon footprint of machine learning. However, the training of such large-scale unsupervised models is itself costly.

Work on evaluating representations can be applied not only in the service of building better models, but also inspecting the contents of their learned representations. The field of machine learning fairness focuses on questions of bias in learned models. Recent work (Kurita et al., 2019; Bordia & Bowman, 2019) aims to examine whether the representations in deep learning models contain protected information which might be used in downstream decision-making. Whether or not it would be the correct choice, measures such as ours might be used to compare how readily such protected information can be accessed from a given representation. A major downside is that just like any other metric that relies on the test accuracy/loss, our measures do not reveal any subtle, undesirable aspects of representation, such as gender bias. We caution users of the proposed approach that the proposed measures need to be complemented with in-depth qualitative analysis in order to avoid any undesirable outcome of representation choice.

As always, we note that machine learning improvements come in the form of "building machines to do X better". For a sufficiently malicious or ill-informed choice of X, such as surveillance or recidivism prediction, almost any progress in machine learning might indirectly lead to a negative outcome, and our work is not excluded from that.

References

Alain, G. and Bengio, Y. Understanding intermediate layers using linear classifier probes. *International Conference on Learning Representations*, 2016.

Bachman, P., Hjelm, R. D., and Buchwalter, W. Learning representations by maximizing mutual information across views. In *Advances in Neural Information Processing Systems*, 2019.

Blier, L. and Ollivier, Y. The description length of deep learning models. In *Advances in Neural Information Processing Systems*, 2018.

Bordia, S. and Bowman, S. R. Identifying and reducing gender bias in word-level language models. In *North American Chapter of the Association for Computational Linguistics: Student Research Workshop*, pp. 7–15, 2019.

Brown, T. B., Mann, B., Ryder, N., Subbiah, M., Kaplan, J., Dhariwal, P., Neelakantan, A., Shyam, P., Sastry, G., Askell, A., Agarwal, S., Herbert-Voss, A., Krueger, G., Henighan, T., Child, R., Ramesh, A., Ziegler, D. M., Wu, J., Winter, C., Hesse, C., Chen, M., Sigler, E., Litwin, M., Gray, S., Chess, B., Clark, J., Berner, C., McCand lish, S., Radford, A., Sutskever, I., and Amodei, D. Language Models are Few-Shot Learners. *arXiv preprint arXiv:2005.14165*, 2020.

Chen, T., Kornblith, S., Norouzi, M., and Hinton, G. E. A simple framework for contrastive learning of visual representations. *arXiv preprint arXiv:2002.05709*, 2020.

Conneau, A., Kruszewski, G., Lample, G., Barrault, L., and Baroni, M. What you can cram into a single $&!#* vector: Probing sentence embeddings for linguistic properties. In *Annual Meeting of the Association for Computational Linguistics*, pp. 2126–2136, 2018.

Cover, T. M. and Thomas, J. A. *Elements of information theory*. Wiley, 2006.

Devlin, J., Chang, M.-W., Lee, K., and Toutanova, K. BERT: Pre-training of deep bidirectional transformers for language understanding. In *North American Chapter of the Association for Computational Linguistics*, pp. 4171–4186, 2019.

Erven, T., Grünwald, P., and Rooij, S. Catching up faster by switching sooner: A predictive approach to adaptive stimation with an application to the aic-bic dilemma. *Journal of the Royal Statistical Society. Series B (Statistical Methodology)*, 74, 06 2012.

Ettinger, A., Elgohary, A., and Resnik, P. Probing for semantic evidence of composition by means of simple classification tasks. In *Workshop on Evaluating Vector-Space Representations for NLP*, pp. 134–139, 2016.

Grünwald, P. A tutorial introduction to the minimum description length principle. *arXiv preprint math:0406077*, 2004.

He, K., Fan, H., Wu, Y., Xie, S., and Girshick, R. Momentum contrast for unsupervised visual representation learning. *arXiv preprint arXiv:1911.05722*, 2019.

Hénaff, O. J., Razavi, A., Doersch, C., Eslami, S. M. A., and van den Oord, A. Data-efficient image recognition with contrastive predictive coding. *arXiv preprint arXiv:1905.09272*, 2019.

Hewitt, J. and Liang, P. Designing and interpreting probes with control tasks. In *Empirical Methods in Natural Language Processing and International Joint Conference on Natural Language Processing*, pp. 2733–2743, 2019.

Kingma, D. P. and Ba, J. Adam: A method for stochastic optimization. *CoRR*, abs/1412.6980, 2015.

Kingma, D. P. and Welling, M. Auto-encoding variational Bayes. *International Conference on Learning Representations*, 2014.

Kurita, K., Vyas, N., Pareek, A., Black, A. W., and Tsvetkov, Y. Measuring bias in contextualized word representations. In *Workshop on Gender Bias in Natural Language Processing*, pp. 166–172, 2019.

LeCun, Y., Bengio, Y., and Hinton, G. Deep learning. *Nature*, 521(7553):436–444, 2015.

Liu, Y., Ott, M., Goyal, N., Du, J., Joshi, M., Chen, D., Levy, O., Lewis, M., Zettlemoyer, L., and Stoyanov, V. RoBERTa: A robustly optimized BERT pretraining approach. *arXiv preprint arXiv:1907.11692*, 2019.

McAllester, D. and Stratos, K. Formal limitations on the measurement of mutual information. *International Conference on Artificial Intelligence and Statistics*, 2020.

Paszke, A., Gross, S., Massa, F., Lerer, A., Bradbury, J., Chanan, G., Killeen, T., Lin, Z., Gimelshein, N., Antiga, L., Desmaison, A., Köpf, A., Yang, E., DeVito, Z., Raison, M., Tejani, A., Chilamkurthy, S., Steiner, B., Fang, L., Bai, J., and Chintala, S. Pytorch: An imperative style, high-performance deep learning library. In *NeurIPS*, 2019.

Peters, M., Neumann, M., Iyyer, M., Gardner, M., Clark, C., Lee, K., and Zettlemoyer, L. Deep contextualized word representations. In *North American Chapter of the Association for Computational Linguistics*, pp. 2227–2237, 2018.

Pimentel, T., Valvoda, J., Maudslay, R. H., Zmigrod, R., Williams, A., and Cotterell, R. Information-theoretic probing for linguistic structure. *arXiv preprint arXiv:2004.03061*, 2020.

Raffel, C., Shazeer, N., Roberts, A., Lee, K., Narang, S., Matena, M., Zhou, Y., Li, W., and Liu, P. J. Exploring the limits of transfer learning with a unified text-to-text transformer. *arXiv preprint arXiv:1910.10683*, 2019.

Resnick, C., Zhan, Z., and Bruna, J. Probing the state of the art: A critical look at visual representation evaluation. *arXiv preprint arXiv:1912.00215*, 2019.

Rezende, D. J., Mohamed, S., and Wierstra, D. Stochastic backpropagation and approximate inference in deep generative models. In *International Conference on Machine Learning*, 2014.

Rissanen, J. Modeling by shortest data description. *Automatica*, 14:465–471, 1978.

Shannon, C. E. A mathematical theory of communication. 1948.

Shi, X., Padhi, I., and Knight, K. Does string-based neural MT learn source syntax? In *Empirical Methods in Natural Language Processing*, pp. 1526–1534, 2016.

Strubell, E., Ganesh, A., and McCallum, A. Energy and policy considerations for deep learning in nlp. In *ACL*, 2019.

Talmor, A., Elazar, Y., Goldberg, Y., and Berant, J. oLMpics–on what language model pre-training captures. *arXiv preprint arXiv:1912.13283*, 2019.

van den Oord, A., Li, Y., and Vinyals, O. Representation learning with contrastive predictive coding. *arXiv preprint arXiv:1807.03748*, 2018.

Voita, E. and Titov, I. Information-theoretic probing with minimum description length. *arXiv preprint arXiv:2003.12298*, 2020.

Xu, Y., Zhao, S., Song, J., Stewart, R., and Ermon, S. A theory of usable information under computational constraints. In *International Conference on Learning Representations*, 2020.

Yogatama, D., d'Autume, C. d. M., Connor, J., Kocisky, T., Chrzanowski, M., Kong, L., Lazaridou, A., Ling, W., Yu, L., Dyer, C., et al. Learning and evaluating general linguistic intelligence. *arXiv preprint arXiv:1901.11373*, 2019.

Zhang, K. and Bowman, S. Language modeling teaches you more than translation does: Lessons learned through auxiliary syntactic task analysis. In *EMNLP Workshop BlackboxNLP: Analyzing and Interpreting Neural Networks for NLP*, pp. 359–361, 2018.

Appendix A Algorithmic details for estimating surplus description length

Recall that the SDL is defined as

$$m_{\mathrm{SDL}}(\phi, \mathcal{D}, \mathcal{A}, \varepsilon) = \sum_{n=1}^{\infty} \Big[L(\mathcal{A}_\phi, n) - \varepsilon \Big]_+ \tag{14}$$

For simplicity, we assume that L is bounded in $[0, 1]$. Note that this can be achieved by truncating the cross-entropy loss.

Algorithm 1: Estimate surplus error

Input: tolerance ε, max iterations M, number of datasets K, representation ϕ, data distribution $\mathcal{D}$, algorithm $\mathcal{A}$

Output: Estimate $\hat{m}$ of $m(\phi, \mathcal{D}, \varepsilon, \mathcal{A})$ and indicator I of whether this estimate is tight or lower bound

Sample K datasets $D_M^k \sim \mathcal{D}$ of size $M + 1$
for $n = 1$ **to** M **do**
  For each $k \in [K]$, run $\mathcal{A}$ on $D_M^k[1:n]$ to produce a predictor $\hat{p}_n^k$
  Take K test samples $(x_k, y_k) = D_M^k[M+1]$
  Evaluate $\hat{L}_n = \frac{1}{K} \sum_{k=1}^{K} \ell(\hat{p}_n^k, x_k, y_k)$
Set $\hat{m} = \sum_{n=1}^{M} [\hat{L}_n - \varepsilon]_+$
if $\hat{L}_M \leq \varepsilon/2$ **then** Set $I =$ `tight` **else** Set $I =$ `lower bound`;
return $\hat{m}, I$

In our experiments we replace $D_M^k[1:n]$ of Algorithm 1 with sampled subsets of size n from a single training set. Additionally, we use between 10 and 20 values of n instead of evaluating $L(\mathcal{A}_\phi, n)$ at every integer between 1 and M. This strategy, also used by Blier & Ollivier (2018) and Voita & Titov (2020), corresponds to the description length under a code which updates only periodically during transmission of the data instead of after every single point.

Theorem 4. *Let the loss function L be bounded in $[0, 1]$ and assume that it is decreasing in n. With $(M + 1)K$ datapoints, if the sample complexity is less than M, the above algorithm returns an estimate $\hat{m}$ such that with probability at least $1 - \delta$*

$$|\hat{m} - m(\phi, \mathcal{D}, \varepsilon, \mathcal{A})| \leq M \sqrt{\frac{\log(2M/\delta)}{2K}}. \tag{15}$$

If $K \geq \frac{\log(1/\delta)}{2\varepsilon^2}$ and the algorithm returns `tight` *then with probability at least $1 - \delta$ the sample complexity is less than M and the above bound holds.*

Proof. First we apply a Hoeffding bound to show that each $\hat{L}_n$ is estimated well. For any n, we have

$$P\left(\left| \hat{L}_n - L(\mathcal{A}_\phi, n) \right| > \sqrt{\frac{\log(2M/\delta)}{2K}} \right) \leq 2 \exp\left(-2K \frac{\log(2M/\delta)}{2K} \right) = 2\frac{\delta}{2M} = \frac{\delta}{M} \tag{16}$$

since each $\ell(\hat{p}_n^k, x_k, y_k)$ is an independent variable, bounded in [0,1] with expectation $L(\mathcal{A}_\phi, n)$.

Now when sample complexity is less than M, we use a union bound to translate this to a high probability bound on error of $\hat{m}$, so that with probability at least $1-\delta$:

$$|\hat{m} - m(\phi, \mathcal{D}, \varepsilon, \mathcal{A})| = \left| \sum_{n=1}^{M} [\hat{L}_n - \varepsilon]_+ - [L(\mathcal{A}_\phi, n) - \varepsilon]_+ \right| \tag{17}$$

$$\leq \sum_{n=1}^{M} \left| [\hat{L}_n - \varepsilon]_+ - [L(\mathcal{A}_\phi, n) - \varepsilon]_+ \right| \tag{18}$$

$$\leq \sum_{n=1}^{M} \left| \hat{L}_n - L(\mathcal{A}_\phi, n) \right| \tag{19}$$

$$\leq M \sqrt{\frac{\log(2M/\delta)}{2K}} \tag{20}$$

This gives us the first part of the claim.

We want to know that when the algorithm returns `tight`, the estimate can be trusted (i.e. that we set M large enough). Under the assumption of large enough K, and by an application of Hoeffding, we have that

$$P\bigg(L(\mathcal{A}_\phi, M) - \hat{L}_M > \varepsilon/2 \bigg) \leq \exp\bigg(-2K\varepsilon^2 \bigg) \leq \exp\bigg(-2\frac{\log(1/\delta)}{2\varepsilon^2}\varepsilon^2 \bigg) = \delta \tag{21}$$

If $\hat{L}_M \leq \varepsilon/2$, this means that $L(\mathcal{A}_\phi, M) \leq \varepsilon$ with probability at least $1-\delta$. By the assumption of decreasing loss, this means the sample complexity is less than M, so the bound on the error of $\hat{m}$ holds. □

Appendix B Algorithmic details for estimating sample complexity

Recall that ε sample complexity (εSC) is defined as

$$m_{\varepsilon\mathrm{SC}}(\phi, \mathcal{D}, \mathcal{A}, \varepsilon) = \min \Big\{ n \in \mathbb{N} : L(\mathcal{A}_\phi, n) \leq \varepsilon \Big\}. \tag{22}$$

We estimate $m_{\varepsilon\mathrm{SC}}$ via recursive grid search. To be more precise, we first define a search interval $[1, N]$, where N is a large enough number such that $L(\mathcal{A}_\phi, N) \ll \varepsilon$. Then, we partition the search interval in to 10 sub-intervals and estimate risk of hypothesis learned from $D^n \sim \mathcal{D}^n$ with high confidence for each sub-interval. We then find the leftmost sub-interval that potentially contains $m_{\varepsilon\mathrm{SC}}$ and proceed recursively. This procedure is formalized in Algorithm 2 and its guarantee is given by Theorem 5.

Theorem 5. *Let the loss function L be bounded in $[0, 1]$ and assume that it is decreasing in n. Then, Algorithm 2 returns an estimate $\hat{m}$ that satisfies $m_{\varepsilon\mathrm{SC}}(\phi, \mathcal{D}, \mathcal{A}, \varepsilon) \leq \hat{m}$ with probability at least $1-\delta$.*

Proof. By Hoeffding, the probability that $|\hat{L}_n - L(\mathcal{A}_\phi, n)| \geq \varepsilon/2$, where $\hat{L}$ is computed with $S = 2\log(20k/\delta)/\varepsilon^2$ independent draws of $D^n \sim \mathcal{D}^n$ and $(x, y) \sim \mathcal{D}$, is less than $\delta/(10k)$. The algorithm terminates after evaluating $\hat{L}$ on at most $10k$ different n's. By a union bound, the probability that $|\hat{L}_n - L(\mathcal{A}_\phi, n)| \leq \varepsilon/2$ for all n used by the algorithm is at least $1-\delta$. Hence, $\hat{L}_n \leq \varepsilon/2$ implies $L(\mathcal{A}_\phi, n) \leq \varepsilon$ with probability at least $1-\delta$. □

Appendix C Experimental details

In each experiment we first estimate the loss-data curve using a fixed number of dataset sizes n and multiple random seeds, then compute each measure from that curve. Reported values of SDL correspond to the estimated area between the loss-data curve and the line $y = \varepsilon$ using Riemann sums with the values taken from the left edge of the interval. This is the same as the chunking procedure of Voita & Titov (2020) and is equivalent to the code length of transmitting each chunk of data using a

Algorithm 2: Estimate sample complexity via recursive grid search

Input: Search upper limit N, parameters ε, confidence parameter δ, data distribution $\mathcal{D}$, and learning algorithm $\mathcal{A}$.
Output: Estimate $\hat{m}$ such that $m_{\varepsilon\text{SC}}(\phi, \mathcal{D}, \mathcal{A}, \varepsilon) \leq \hat{m}$ with probability $1 - \delta$.

let $S = 2\log(20k/\delta)/\varepsilon^2$, and let $[\ell, u]$ be the search interval initialized at $\ell = 1, u = N$.
for $r = 1$ **to** k **do**
  Partition $[\ell, u]$ into 10 equispaced bins and let Δ be the length of each bin.
  for $j = 1$ **to** 10 **do**
    Set $n = \ell + j\Delta$.
    Compute $\hat{L}_n = \frac{1}{S}\sum_{i=1}^{S} \ell(\mathcal{A}(D_i^n), x_i, y_i)$ for S independent draws of D^n and test sample (x, y).
    if $\hat{L}_n \leq \varepsilon/2$ **then**
      Set $u = n$ and $\ell = n - \Delta$.
      break
return $\hat{m} = u$, which satisfies $m_{\varepsilon\text{SC}}(\phi, \mathcal{D}, \mathcal{A}, \varepsilon) \leq \hat{m}$ with probability $1 - \delta$, where the randomness is over independent draws of D^n and test samples (x, y).

fixed model and switching models between intervals. Reported values of εSC correspond to the first measured n at which the loss is less than ε.

All of the experiments were performed on a single server with 4 NVidia Titan X GPUs, and on this hardware no experiment took longer than an hour. All of the code for our experiments, as well as that used to generate our plots and tables, is included in the supplement.

C.1 MNIST experiments

For our experiments on MNIST, we implement a highly-performant vectorized library in JAX to construct loss-data curves. With this implementation it takes about one minute to estimate the loss-data curve with one sample at each of 20 settings of n. We approximate the loss-data curves at 20 settings of n log-uniformly spaced on the interval $[10, 50000]$ and evaluate loss on the test set to approximate the population loss. At each dataset size n we perform the same number of updates to the model; we experimented with early stopping for smaller n but found that it made no difference on this dataset. In order to obtain lower-variance estimates of the expected risk at each n, we run 8 random seeds for each representation at each dataset size, where each random seed corresponds to a random initialization of the probe network and a random subsample of the training set.

Probes consist of two-hidden-layer MLPs with hidden dimension 512 and ReLU activations. All probes and representations are trained with the Adam optimizer (Kingma & Ba, 2015) with learning rate 10^{-3}.

Each representation is normalized to have zero mean and unit variance before probing to ensure that differences in scaling and centering do not disrupt learning. The representations of the data we evaluate are implemented as follows.

Raw pixels. The raw MNIST pixels are provided by the Pytorch `datasets` library (Paszke et al., 2019). It has dimension $28 \times 28 = 784$.

CIFAR. The CIFAR representation is given by the last hidden layer of a convolutional neural network trained on the CIFAR-10 dataset. This representation has dimension 784 to match the size of the raw pixels. The network architecture is as follows:

```
nn.Conv2d(1, 32, 3, 1),
nn.ReLU(),
nn.MaxPool2d(2),
nn.Conv2d(32, 64, 3, 1),
nn.ReLU(),
nn.MaxPool2d(2),
```

```
    nn.Flatten(),
    nn.Linear(1600, 784)
    nn.ReLU()
    nn.Linear(784, 10)
    nn.LogSoftmax()
```

VAE. The VAE (variational autoencoder; Kingma & Welling (2014); Rezende et al. (2014)) representation is given by a variational autoencoder trained to generate the MNIST digits. This VAE's latent variable has dimension 8. We use the mean output of the encoder as the representation of the data. The network architecture is as follows:

```
self.encoder_layers = nn.Sequential(
    nn.Linear(784, 400),
    nn.ReLU(),
    nn.Linear(400, 400),
    nn.ReLU(),
    nn.Linear(400, 400),
    nn.ReLU(),
)
self.mean = nn.Linear(400, 8)
self.variance = nn.Linear(400, 8)

self.decoder_layers = nn.Sequential(
    nn.Linear(8, 400),
    nn.ReLU(),
    nn.Linear(400, 400),
    nn.ReLU(),
    nn.Linear(400, 784),
)
```

C.2 Part of speech experiments

We follow the methodology and use the official code[2] of Voita & Titov (2020) for our part of speech experiments using ELMo (Peters et al., 2018) pretrained representations. In order to obtain lower-variance estimates of the expected risk at each n, we run 4 random seeds for each representation at each dataset size, where each random seed corresponds to a random initialization of the probe network and a random subsample of the training set. We approximate the loss-data curves at 10 settings of n log-uniformly spaced on the range of the available data $n \in [10, 10^6]$. To more precisely estimate εSC, we perform one recursive grid search step: we space 10 settings over the range which in the first round saw $L(\mathcal{A}_\phi, n)$ transition from above to below ε.

Probes consist of the MLP-2 model of Hewitt & Liang (2019); Voita & Titov (2020) and all training parameters are the same as in those works.

[2] https://github.com/lena-voita/description-length-probing