Statistical Inference Project Part1 Simulation Exercise

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Basic Setting

```
echo = TRUE # always make code visible
error = FALSE
library(ggplot2)
library(plyr)
```

Simulations

The exponential distribution can be simulated in R with rexp(n, lambda) where lambda(\(\lambda\)) is the rate parameter. The mean of exponential distribution is \(1/\lambda\) and the standard deviation is also \(1/\lambda\). For this simulation, we set \(\lambda=0.2\). In this simulation, we investigate the distribution of averages of 40 exponential(0.2)s. Lets start by doing a thousand simulated averages of 40 exponentials.

```
lambda = 0.2; n = 40; simulation=1000; mu = 1/lambda; sigma = 1/lambda;
```

Then, create a sample distribution using rexp, and calculate the sample means and sample standard deviation.

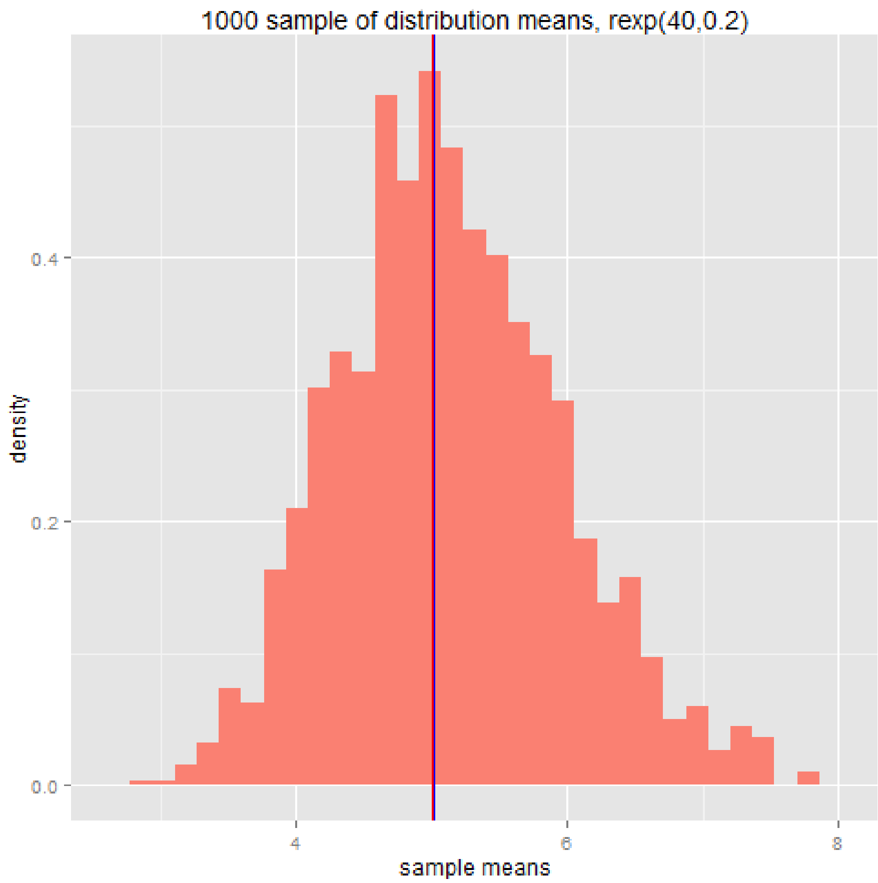
```
sample_dist <- sapply(1:simulation, FUN=function(x) { mean(rexp(n, lambda))})
sd_mean <- mean(sample_dist)
sd_stddev <- sd(sample_dist)
df <- data.frame(mns=sample_dist) #gglot need to input dataframe
```

Now we answering the following question

1.Show where the distribution is centered at and compare it to the theoretical center of the distribution.

```
g <- ggplot(df, aes(x=mns, weight=mns/sum(mns)))
g + geom_histogram(aes(y=..density..),fill = "salmon") +
  geom_vline(xintercept = sd_mean, colour = "blue") +
  geom_vline(xintercept = 5, colour = "Red" ) +
  labs(title = "1000 sample of distribution means, rexp(40,0.2)",x ="sample means",y="densit
```

```
## stat_bin: binwidth defaulted to range/30. Use 'binwidth = x' to adjust this.
```



We show the theoretical mean in red, and the mean of the sample distribution in blue. They are nearly identical.

2.Show how variable it is and compare it to the theoretical variance of the distribution.

standard deviation of distribution of averages of 40 exponentials

```
sd(sample_dist)
```

```
## [1] 0.8085
```

standard deviation from analytical expression

```
(1/lambda)/sqrt(n)
```

```
## [1] 0.7906
```

Variance of the sample mean

```
var(sample_dist)
```

```
## [1] 0.6537
```

Theoritcal variance of the distribution

```
1/((0.2*0.2) * 40)
```

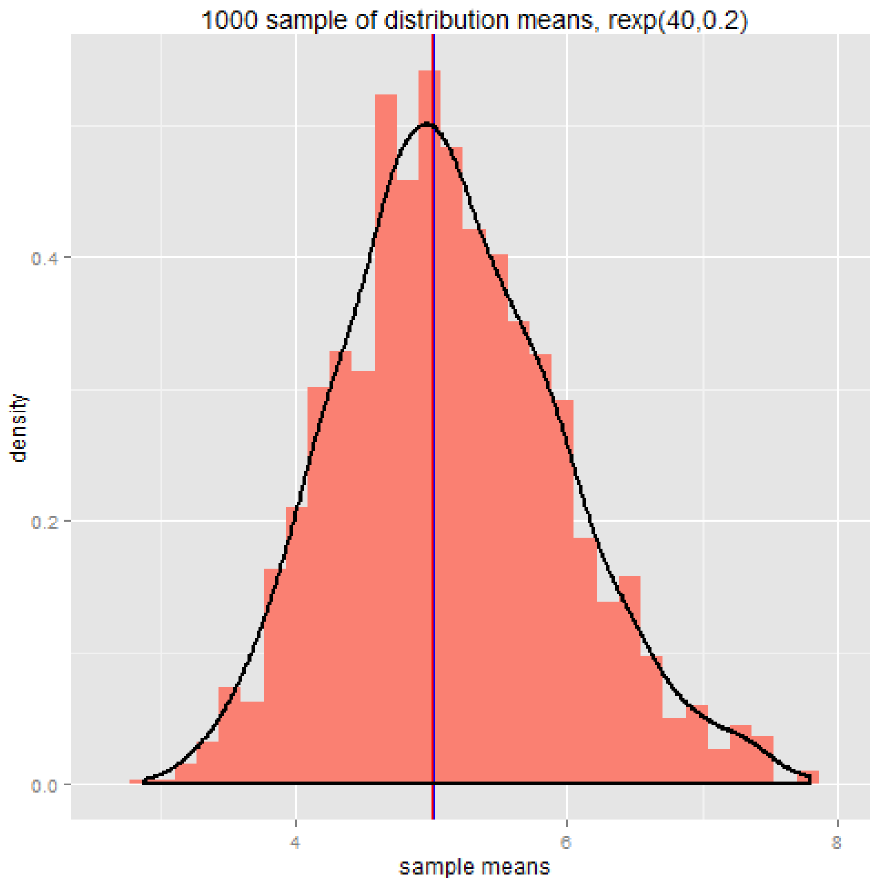
```
## [1] 0.625
```

Therefore, the variability in distribution of averages of 40 exponentials is close to the theoretical variance of the distribution.

3.Show that the distribution is approximately normal.

```
g <- ggplot(df, aes(x=mns, weight=mns/sum(mns)))
g + geom_histogram(aes(y=..density..),fill = "salmon") +
  geom_vline(xintercept = sd_mean, colour = "blue") +
  geom_vline(xintercept = 5, colour = "Red") +
  labs(title = "1000 sample of distribution means, rexp(40,0.2)",x ="sample means",y="densit
  geom_density(fill=NA,colour="black",size = 1)
```

```
## stat_bin: binwidth defaulted to range/30. Use 'binwidth = x' to adjust this.
```



The distribution of sample means approximate normal distribution line

4.Evaluate the coverage of the confidence interval for \(1/\lambda = \bar{X} \pm 1.96 \frac{S}{\sqrt{n}}\).

The 97.5% confidenct interval for sample means is

```
mean(sample_dist) + c(-1,1)*qnorm(0.975)*sd(sample_dist)/sqrt(n)
```

```
## [1] 4.764 5.265
```