6/18/21, 1:39 PM

## A Hybrid Genetic Algorithm for Three-Index Assignment Problem

Gaofeng Huang and Andrew Lim

{gfhuang, iealim}@ust.hk Clear Water Bay, Kowloon, Hong Kong Hong Kong University of Science & Technology Dept of Industrial Engineering & Engineering Management

al(2003). and Spieksma(1992), Burkard et al(1996), and Aiex et those proposed by Balas and Saltzman(1991), Crama be superior to all previous heuristic methods including experimental results indicate that our hybrid method to heuristic with the Genetic Algorithm (GA). Extensive it to the assignment problem. We further hybridize our mization (FO), which solves the problem by simplifying pose a new iterative heuristic, called Fragmental Opti-Inspired by the classical assignment problem, we proact and heuristic methods have been proposed to solve it. This problem has been studied extensively, and many exknown problem which has been shown to be  $N\mathcal{P}$ -hard. Abstract- Three-Index Assignment Problem (AP3) is well-

## 1 Introduction

ject of extensive research since the sixties[10, 11]. as the 3-Dimensional Assignment Problem, has been a sub-The Three-Index Assignment Problem (AP3), also known

The 0-1 programming model for AP3 is

$$\min \sum_{i \in I, j \in J, k \in K} c_{ijk} x_{ijk}$$

subject to

$$\sum_{j \in J, k \in K} x_{ijk} = 1 \quad , \quad \forall i \in I$$

$$\sum_{j \in J, k \in K} x_{ijk} = 1 , \forall i \in I$$

$$\sum_{i \in I, j \in J} x_{ijk} = 1 , \forall k \in K$$
(2)

$$\sum_{i \in I, k \in K} x_{ijk} = 1 \quad \forall j \in J$$
(3)

$$x_{ijk} \in \{0,1\}$$
 ,  $\forall \{i,j,k\} \in I \times J \times K$  (4)

where I, J, K are three disjoint sets with |I| = |J| = |K| =

entries are chosen. minimized. As a result, of all the  $n^3$  entries of  $c_{i,j,k}$ , only nchoose n disjoint triangles (i, j, k) so that the total cost is  $(i, j, k) \in I \times J \times K$  is  $c_{i,j,k}$ . The objective of AP3 is to the three disjoint vertex sets. The cost of choosing triangle K;  $(I \cup J) \times (I \cup K) \times (J \cup K)$ ), where I, J and K are lem on a complete tripartite graph  $K_{n,n,n} = (I \times J \times$ In fact, AP3 can be considered as an optimization prob-

permutation based AP3 formulation: be also represented by 3 permutations. Below provides the tions of 1, 2, ..., n. This means that the solution to AP3 can If we line up these n chosen triples, we get 3 permuta-

$$\min \sum_{i=1}^n c_{I(i),p(i),q(i)} \qquad \text{with } I,p,q \in \pi_N$$

0-7803-7804-0 /03/\$17.00 © 2003 IEEE

IEEE Xplore Full-Text PDF:

integers  $N = \{1, 2, ..., n\}$ . where  $\pi_N$  denotes the set of all permutations on the set of

can be represented by using a pair of permutations (p, q). Once this "index permutation" is fixed, the solution to AP3 can reorder them such that I(i) = i, or I = (1, 2, 3, ..., n). Since we do not care the order of these n triples, we

AP3 can be formulated as:

: (p,q),  $p,q \in \pi_N$ Solution a matrix  $C = \{c_{i,j,k}\}_{n \times n \times n}$ 

**Objective** : to minimize  $C(p,q) = \sum_{i=1}^{n} c_{i,p(i),q(i)}$ 

(AP2) defined below: sion of the classical two-dimensional assignment problem It is quite obvious that AP3 is a straightforward exten-

:  $q = (q_1, q_2, ..., q_n), q \in \pi_N$ Solution : matrix $D = \{d_{i,j}\}_{n \times n}$ (bipartite graph) Instance

Objective : to minimize  $D(q) = \sum_{i=1}^{n} d_{i.q(i)}$ 

hard problems[7] is one of the special case of AP3. the 3-D Matching Problem, which is one of the basic NPciently in polynomial time[8, 9], the AP3 is NP-hard since Although it is well-known that AP2 can be solved effi-

for AP3 and obtained better results than all other existing Aiex et al.(2003)[1] applied GRASP with Path Relinking for these two special cases, AP3 is still NP-hard. Recently, AP3 with decomposable cost coefficients. However, even of triangle inequalities. Burkard et al(1996)[2] focused on restricting the cost of edges in any triangle to obey the rule and Spieksma(1992)[4] studied a special case of AP3 by VARIABLE DEPTH INTERCHANGE heuristics. Crama and Saltzman(1991)[5] proposed the MAX-REGRET and to solve AP3[1, 2, 3, 4, 5, 6, 10, 11]. Among these, Balas Both exact and heuristic algorithms have been proposed

that GA benefited from this hybridization. We test our albridize FO with Genetic Algorithm. Experiments indicate problem-specific knowledge to solve AP3. We then hyrithm, named Fragmental Optimization (FO), which utilizes In this paper, we first propose an iterative heuristic algoknowledge into GA, the results can be further improved. mization problems. If we can incorporate problem-specific competitive technique to solve general combinatorial optivival of the fittest". Genetic algorithms have shown to be a Darwin's evolution theory of natural selection and "surof the most successful evolutionary algorithms based on netic algorithm for AP3. Genetic Algorithm (GA) is one The purpose of this paper is to present a hybrid ge-

2762

Authorized licensed use limited to: Temple University. Downloaded on June 18,2021 at 17:39:39 UTC from IEEE Xplore. Restrictions apply.