An Approximate Muscle Guided Global Optimization Algorithm for the Three-Index Assignment Problem

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***Abstract*—The Three-Index Assignment Problem (AP3) is a famous NP-hard problem with wide applications. Since it's intractable, many heuristics have been proposed to obtain near optimal solutions in reasonable time. In this paper, a new meta-heuristic was proposed for solving the AP3. Firstly, we introduced the conception of muscle (the union of optimal solutions) and proved that it is intractable to obtain the muscle under the assumption that $P \neq NP$. Moreover, we showed that the whole muscle can be approximated by the union of local optimal solutions. Therefore, the Approximate Muscle guided Global Optimization (AMGO) is proposed to solve the AP3. AMGO employs a global optimization strategy to search in a search space reduced by the approximate muscle, which is constructed by a multi-restart scheme. During the global optimization procedure, the running time can be dramatically saved by detecting feasible solutions and extracting poor partial solutions. Extensive experimental results on the standard AP3 benchmark indicated that the new algorithm outperforms the state-of-the-art heuristics in terms of solution quality. Work of this paper not only provides a new meta-heuristic for NP-hard problems, but shows that global optimization can provide promising results in reasonable time, by restricting it to a fairly reduced search space.**

I. Introduction

The Three-Index Assignment Problem (AP3) is a well-known NP-hard problem with wide applications including scheduling capital investments, military troop assignment, satellite coverage optimization, and production of printed circuit boards (Pierskalla, 1967, 1968; Frieze and Yadegar, 1981; Crama, Kolen, Oerlemans and Spieksma, 1990). Since it is intractable for NP-hard problems to obtain optimal solutions, both exact and heuristic algorithms have been developed for solving AP3 instances, including Balas and Saltzman (1991), Burkard and Rudolf (1993), Crama and Spieksma (1992), Pardalos and Pitsoulis (2000), Voss(2000), Aiex, Resende, Pardalos and Toraldo (2005), Huang and Lim (2006). Among these algorithms, LSGA proposed by Huang and Lim can obtain better results than all other existing heuristics.

As an important tool to design heuristics for NP-hard problems, the backbone (i.e., the shared common parts of all optimal solutions for a NP-hard problem instance) has received widespread attention in recent years. Schneider (2003) proposed a multilevel reduction algorithm for the traveling salesman problem (TSP) by using the intersection of local optimal solutions as the approximate backbone. Zhang and Looks (2005) presented a backbone guided LK algorithm for the TSP. Zhang (2004), Dubois and Seymour (2001), and Valnir (2006) gave backbone guided local search algorithms for the SAT, respectively. Zou, Zhou, Chen, Jiang and Gu (2005) developed the approximate backbone-guided fant (ABFANT) for the quadratic assignment problem (QAP).

In contrast to the backbone, the muscle (the union of local optimal solutions) was introduced in this paper. Firstly, we proved that there is no polynomial time algorithm to obtain the muscle of the AP3 under the assumption that $P \neq NP$. Its basic idea is to map any instance of the AP3 to an instance with a unique optimal solution. Therefore, finding the muscle of the modified instance is equivalent to finding an optimal solution of the original instance. Similarly, we indicated that it is intractable to obtain a fixed fraction of the muscle as well. Secondly, we approximated the muscle with the union of local optimal solutions and proposed a new meta-heuristic, the Approximate Muscle guided Global Optimization (AMGO) for the AP3. It consists of two phases: the sampling phase approximates the muscle by running a multi-restart scheme for AP3; the global optimization phase exploits a recursive procedure to find better solutions by restricting the search in the approximate muscle. In the latter phase, the running time can be dramatically reduced in such a way that we detect the feasible solution and extract poor partial solution. Experimental results on the standard AP3 benchmark indicated that the new heuristic outperforms LSGA and GRASP with Path Relinking in terms of solution quality, especially on those hard instances (instances of size > 18 in Balas and Saltzman Dataset).

The rest of this paper is organized as follows. Section II first describes definitions of the AP3 and the muscle, and computational complexity results of the muscle are given. In Section III, we shall approximate the muscle with the union of local optimal solutions, and propose the AMGO heuristic. Experimental results are reported in Section IV. Finally, we conclude this paper in Section V.

II. Muscle of AP3

A. Preliminaries

In this subsection, we shall give out some useful definitions about the AP3 and the muscle.

Definition 1. Given sets $I = J = K = \{1, 2, \cdots, n\}$ and the cost function $c : I \times J \times K \to \mathbb{R}^+$, where c_{ijk} represents the cost of a triple $(i, j, k) \in I \times J \times K$. A feasible solution to the

This work was supported in part by the National Natural Science Foundation of China under Grant 60673046 and 60673066, the Natural Science Foundation of Liaoning Province under Grant 20051082, and the Gifted Young Foundation of Dalian University of Technology.

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978-1-4244-1823-7/08/$25.00©2008 IEEE