A Hybrid Genetic Algorithm for Three-Index Assignment Problem

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Abstract- *Three-Index Assignment Problem* **(AP3) is well-known problem which has been shown to be $\mathcal{NP}$-hard. This problem has been studied extensively, and many exact and heuristic methods have been proposed to solve it. Inspired by the classical assignment problem, we propose a new iterative heuristic, called** ***Fragmental Optimization*** **(FO), which solves the problem by simplifying it to the assignment problem. We further hybridize our heuristic with the Genetic Algorithm (GA). Extensive experimental results indicate that our hybrid method to be superior to all previous heuristic methods including those proposed by Balas and Saltzman(1991), Crama and Spieksma(1992), Burkard et al(1996), and Aiex et al(2003).**

1 Introduction

The Three-Index Assignment Problem (AP3), also known as the 3-Dimensional Assignment Problem, has been a subject of extensive research since the sixties[10, 11].

The 0-1 programming model for AP3 is

$$\min \sum_{i\in I, j\in J, k\in K} c_{ijk} x_{ijk}$$

subject to

$$\sum_{j\in J, k\in K} x_{ijk} = 1 \quad , \quad \forall i \in I \tag{1}$$

$$\sum_{i\in I, j\in J} x_{ijk} = 1 \quad , \quad \forall k \in K \tag{2}$$

$$\sum_{i\in I, k\in K} x_{ijk} = 1 \quad , \quad \forall j \in J \tag{3}$$

$$x_{ijk} \in \{0, 1\} \quad , \quad \forall \{i, j, k\} \in I \times J \times K \tag{4}$$

where I, J, K are three disjoint sets with $|I| = |J| = |K| = n$.

In fact, AP3 can be considered as an optimization problem on a complete tripartite graph $K_{n,n,n} = (I \times J \times K; (I \cup J) \times (I \cup K) \times (J \cup K))$, where I, J and K are the three disjoint vertex sets. The cost of choosing triangle $(i, j, k) \in I \times J \times K$ is $c_{i,j,k}$. The objective of AP3 is to choose n disjoint triangles (i, j, k) so that the total cost is minimized. As a result, of all the n^3 entries of $c_{i,j,k}$, only n entries are chosen.

If we line up these n chosen triples, we get 3 permutations of $1, 2, ..., n$. This means that the solution to AP3 can be also represented by 3 permutations. Below provides the permutation based AP3 formulation:

$$\min \sum_{i=1}^{n} c_{I(i),p(i),q(i)} \qquad \text{with } I, p, q \in \pi_N$$

where π_N denotes the set of all permutations on the set of integers $N = \{1, 2, ..., n\}$.

Since we do not care the order of these n triples, we can reorder them such that $I(i) = i$, or $I = (1, 2, 3, ..., n)$. Once this "index permutation" is fixed, the solution to AP3 can be represented by using a pair of permutations (p, q).

AP3 can be formulated as:

Instance	:	a matrix $C = \{c_{i,j,k}\}_{n\times n\times n}$
Solution	:	(p, q), $p, q \in \pi_N$
Objective	:	to minimize $C(p, q) = \sum_{i=1}^{n} c_{i,p(i),q(i)}$

It is quite obvious that AP3 is a straightforward extension of the classical two-dimensional assignment problem (AP2) defined below:

Instance	:	matrix$D = \{d_{i,j}\}_{n\times n}$(bipartite graph)
Solution	:	$q = (q_1, q_2, ..., q_n)$, $q \in \pi_N$
Objective	:	to minimize $D(q) = \sum_{i=1}^{n} d_{i,q(i)}$

Although it is well-known that AP2 can be solved efficiently in polynomial time[8, 9], the AP3 is $\mathcal{NP}$-hard since the 3-D Matching Problem, which is one of the basic $\mathcal{NP}$-hard problems[7] is one of the special case of AP3.

Both exact and heuristic algorithms have been proposed to solve AP3[1, 2, 3, 4, 5, 6, 10, 11]. Among these, Balas and Saltzman(1991)[5] proposed the MAX-REGRET and VARIABLE DEPTH INTERCHANGE heuristics. Crama and Spieksma(1992)[4] studied a special case of AP3 by restricting the cost of edges in any triangle to obey the rule of triangle inequalities. Burkard et al(1996)[2] focused on AP3 with decomposable cost coefficients. However, even for these two special cases, AP3 is still $\mathcal{NP}$-hard. Recently, Aiex et al.(2003)[1] applied GRASP with Path Relinking for AP3 and obtained better results than all other existing heuristics.

The purpose of this paper is to present a hybrid genetic algorithm for AP3. Genetic Algorithm (GA) is one of the most successful evolutionary algorithms based on Darwin's evolution theory of natural selection and "survival of the fittest". Genetic algorithms have shown to be a competitive technique to solve general combinatorial optimization problems. If we can incorporate problem-specific knowledge into GA, the results can be further improved. In this paper, we first propose an iterative heuristic algorithm, named Fragmental Optimization (FO), which utilizes problem-specific knowledge to solve AP3. We then hybridize FO with Genetic Algorithm. Experiments indicate that GA benefited from this hybridization. We test our al-