Assignment 1.2

Let d be the lambda: $(\lambda n.(\lambda s.sample([1...s], n)))$.

Let the notation nds be equivalent to $((d\ n)\ s)$.

1. What is the support of X?

The support of X is $\{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$.

2. Is X a discrete or continuous random variable? Explain.

It is discrete. The set of the support of X is a sparse (non-dense).

That is, $\neg(\forall x, y \in S_X : x < y, (\exists z \in S_X : x < z < y))$.

This is trivially true, as the support of X is finite.

It therefore cannot be continuous.

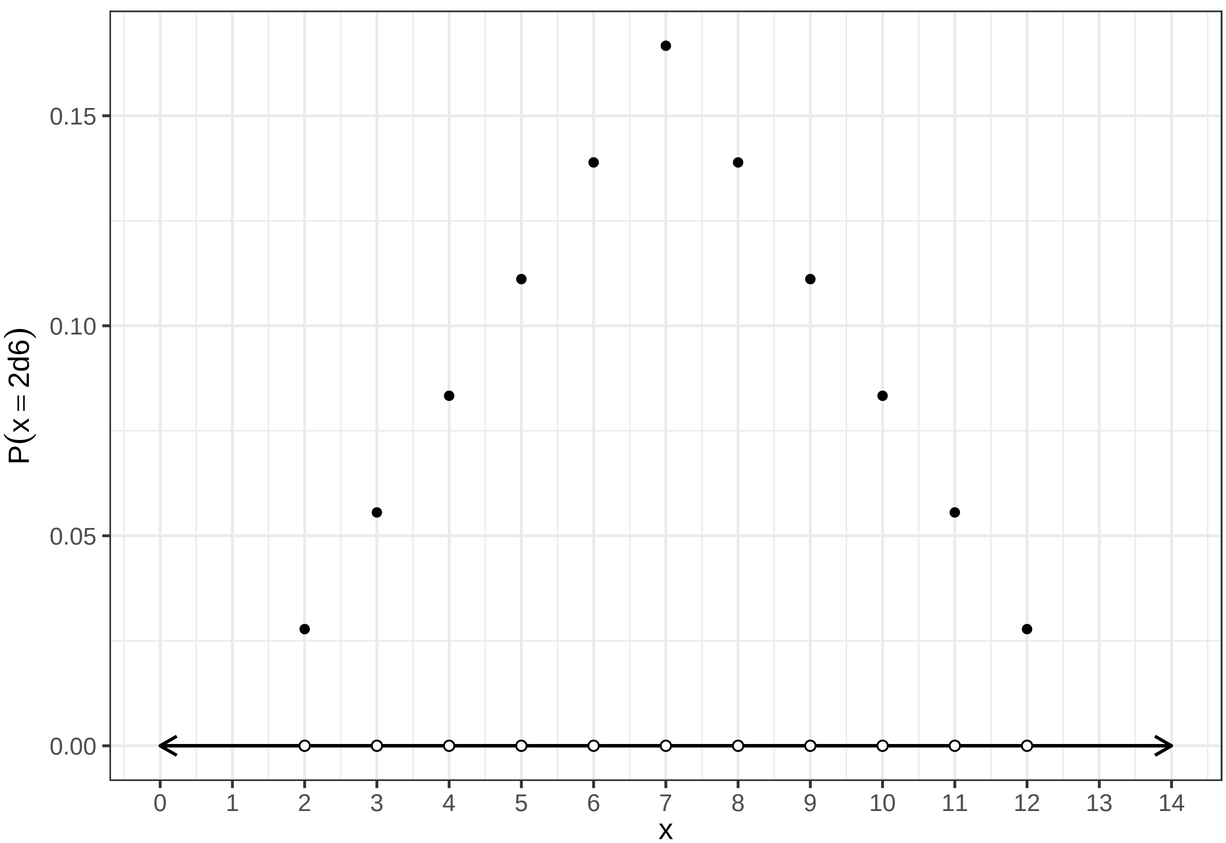
In this case, our general formula for the support of nds is $S_{nds} = [n...ns]$.

3. Based on your answer to the previous question, construct the pdf or pmf of X.

The probability of the value x being chosen from the distribution given by nds (denoted $P(x = nds)$) is:

$$\frac{1}{s^n} \sum_{k=0}^{\lfloor \frac{x-n}{s} \rfloor} (-1)^k \binom{n}{k} \binom{x - sk - 1}{n - 1}$$

... when $x \in S_X$. Otherwise, $P(x = nds) = 0$.

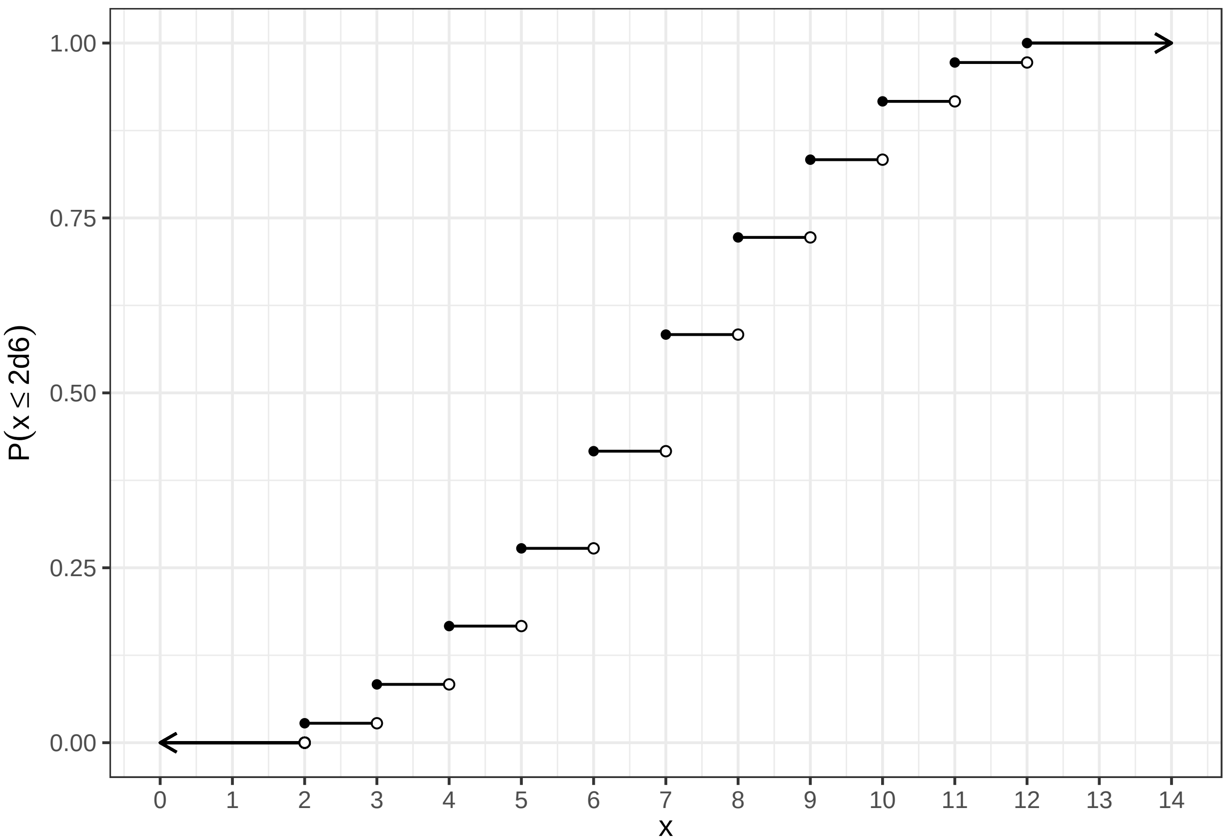


4. Construct the cdf of X.

This is very similar to the previous question.

The cumulative density of the value x being less than or equal to a value from the distribution given by nds (denoted $P(x \leq nds)$) is just the sum of probabilities below and equal to x.

$$P(x \leq nds) \doteq \sum_{k=0}^{x} P(k = nds)$$



5. Compute $E[X]$ and $V[X]$.

A slight modification on the notation you give in the notes gives us our function E.

$$E(nds) \doteq \sum_{x \in S_{nds}} xP(x = nds)$$

$$E(2d6) = \sum_{x \in \{2,3,4,5,6,7,8,9,10,11,12\}} xP(x = 2d6)$$

The expectation when rolling $2d6$ is 7.

The variance is trivial to calculate.

Variance is normally expressed as $E((X + E(X))^2)$.

This - due to the fact that dice rolls are iid - can be simplified to $E(X^2) + E(X)^2$

The expression would be:

$$V(nds) \doteq \left(\sum_{x \in S_{nds}} x^2 P(x = nds) \right) - \left(\sum_{x \in S_{nds}} x P(x = nds) \right)^2$$

In this case:

$$V(2d6) \doteq \left(\sum_{x \in \{2,3,4,5,6,7,8,9,10,11,12\}} x^2 P(x = 2d6) \right) - \left(\sum_{x \in \{2,3,4,5,6,7,8,9,10,11,12\}} x P(x = 2d6) \right)^2$$

The variance when rolling $2d6$ is 5.8333333.

If you are reading the pdf, go ahead and switch over to the Rmd if you want to play with the parameters of this document. You can change the number of dice and the faces on each die to arbitrary natural numbers.

(Setting them to non-naturals will produce undefined behavior.)