Homework 1

1. Given the following demand function for a computer, $Q = 100 - 0.4p + 1.2p_a + 3Y$, and $p = 20$, $p_a = 10$, and $Y = 50$, where p is the good's own price, p_a is the price of good a, and Y is income:

a. Calculate elasticity of demand. Explain in one sentence what your answer means.

$$\frac{\partial Q}{\partial p} = -0.4p$$
$$\varepsilon = \frac{\partial Q}{\partial p}\frac{p}{Q}$$
$$\varepsilon = -0.4p\frac{p}{100 - 0.4p + 1.2p_a + 3Y} = \frac{2p}{2p - 15Y - 6p_a - 500}$$
$$\varepsilon = \frac{2(20)}{2(20) - 15(100) - 6(10) - 500} = \frac{40}{40 - 1500 - 60 - 500}$$
$$\varepsilon = -\frac{40}{2020} = -\frac{2}{101} \approx -0.01980198020$$

The sensitivity of demand quantity to change in price. Specifically percentage change.

From now on I will be shorthanding this calculation, I believe I have demonstrated I know how to calculate elasticity.

b. Calculate cross-price elasticity. Explain in one sentence what your answer means.

$$\varepsilon_a = \frac{\partial Q}{\partial p_a}\frac{p_a}{Q} = \frac{3}{101} \approx 0.02970297030$$

The sensitivity of demand quantity to change in the price of an alternative good.

c. Calculate income elasticity. Explain in one sentence what your answer means.

$$\xi = \frac{\partial Q}{\partial Y}\frac{Y}{Q} = \frac{75}{101} \approx 0.7425742574$$

The sensitivity of demand quantity to change in income.

2. a. Given the following demand and supply functions, calculate the change in price paid by consumers, Δp, following the assessment of a specific tax, $\tau = \$15.50$ (starting from $\tau = \$0.00$). Assume the market is in equilibrium to begin with.

$$Q_D = 30,000 - 15p$$
$$Q_S = 5,000 + 35p$$

$$\begin{bmatrix} 1 & 12 \\ 1 & -35 \end{bmatrix}\begin{bmatrix} Q^* \\ p^* \end{bmatrix} = \begin{bmatrix} 30,000 \\ 5,000 \end{bmatrix}$$
$$\begin{bmatrix} Q^* \\ p^* \end{bmatrix} = \begin{bmatrix} 22,500 \\ 500 \end{bmatrix}$$

$$\varepsilon = \frac{\partial Q_D}{\partial p}\frac{p}{Q_D} = 1 + \frac{2000}{p^* - 2000} = -\frac{1}{3}$$

$$\eta = \frac{\partial Q_S}{\partial p}\frac{p}{Q_S} = 1 + \frac{1000}{7p^* - 1000} = \frac{7}{9}$$

$$\Delta p = \frac{\eta}{\eta - \varepsilon}\Delta\tau = \frac{7}{10}\$15.5 = \$10.85$$

$$p^{**} = p^* + \Delta p = \$510.85$$

$$Q^{**} = 30,000 - 15p^{**} = 22,337.25$$

b. How much will be collected in tax revenue?

$$T = Q^{**}\tau = 22,337.25 \cdot \$15.5 = \$346,227.375$$

c. How much of this amount will be paid by consumers?

$$T_{\text{consumer}} = Q^{**}\Delta p = 22,337.25 \cdot \$10.85 = \$242,359.1625$$

3. ...

a. Calculate the change in price paid by the consumer following the assessment of a specific tax of $\tau = \$0.50$.

$$\Delta p = \frac{1.5}{1.5 - (-0.5)} \cdot \$0.50 = \$1.50$$

b. ...

The ad valorem tax is a variable tax rate based on price, such that if the price is zero, then the tax is zero. This fixes the point on the demand curve at zero + the constant demand. However, at higher prices the tax increases, giving a wider gap between the true demand curve and the taxed one. This effect happens all along the line all the way up to the $Q = 0$ intercept. This causes the graph to appear to "rotate" as it were.

4. ...

$$\eta = \frac{\delta Q}{\delta p}\frac{p}{Q} = \frac{1}{6}$$