Homework 3

1. ... $5x_1 + 4x_2 = 100$.

a. The price of good x_1 is 5. (No units are given?) In theory the function would be $\$5x_1 + \$4x_2 = \$100$, and the price would be \$5, but since no units are given then the price is just 5.

b. The price of good x_2 is 4.

c. The available income is 100.

d. See Figure 1.

e. See Figure 1.

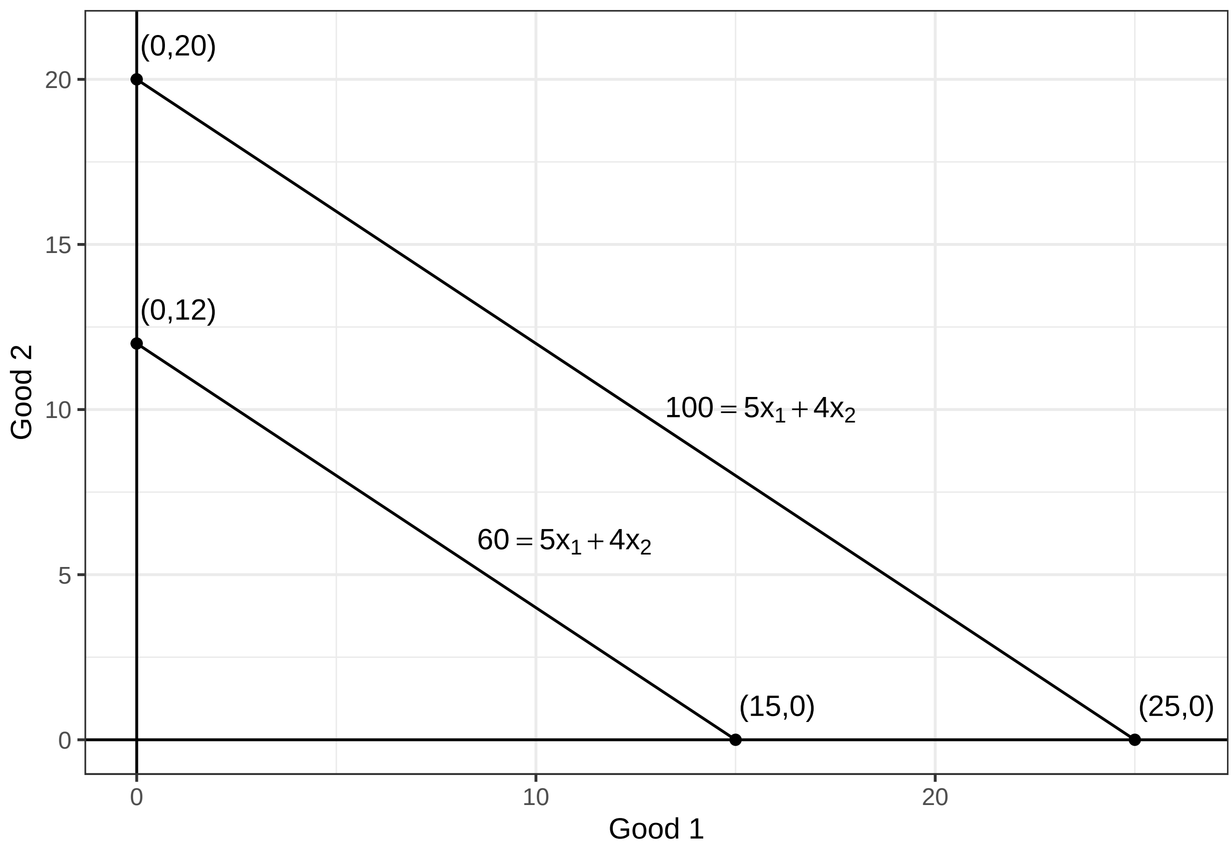

Figure 1: the budget constraint.

2. ... $U = x_1^{\frac{2}{3}} x_2^{\frac{1}{3}}$ ... $p_1 = 1$, $p_2 = 1$, and $Y = 120$.

a. $MU_1 = \frac{\partial U}{\partial x_1} = \frac{2}{3} x_1^{-\frac{1}{3}} x_2^{\frac{1}{3}}$.

b. $MU_2 = \frac{\partial U}{\partial x_2} = \frac{1}{3} x_1^{\frac{2}{3}} x_2^{-\frac{2}{3}}$.

c. $B \doteq (Y = p_1 x_1 + p_2 x_2)$, $(120 = x_1 + x_2)$, $(0 = 120 - x_1 - x_2)$.

d. $\Lambda(U,B) \equiv x_1^{\frac{2}{3}} x_2^{\frac{1}{3}} + \lambda\left(120 - x_1 - x_2\right)$

e. ...

$$\frac{\partial \Lambda}{\partial x_1} = \frac{\partial U}{\partial x_1} + \frac{\partial\left(\lambda\left(120 - x_1 - x_2\right)\right)}{\partial x_1} = \frac{2}{3} x_1^{-\frac{1}{3}} x_2^{\frac{1}{3}} - \lambda = 0$$

$$\frac{\partial \Lambda}{\partial x_2} = \frac{\partial U}{\partial x_2} + \frac{\partial\left(\lambda\left(120 - x_1 - x_2\right)\right)}{\partial x_2} = \frac{1}{3} x_1^{\frac{2}{3}} x_2^{-\frac{2}{3}} - \lambda = 0$$

$$\frac{\partial \Lambda}{\partial \lambda} = \frac{\partial U}{\partial \lambda} + \frac{\partial\left(\lambda\left(120 - x_1 - x_2\right)\right)}{\partial \lambda} = 120 - x_1 - x_2 = 0$$

f. ...

$$\frac{2}{3} x_1^{-\frac{1}{3}} x_2^{\frac{1}{3}} = \lambda, \quad \frac{1}{3} x_1^{\frac{2}{3}} x_2^{-\frac{2}{3}} = \lambda \text{ (See e. 1\&2.)}$$

$$\frac{1}{3} x_1^{\frac{2}{3}} x_2^{-\frac{2}{3}} = \frac{2}{3} x_1^{-\frac{1}{3}} x_2^{\frac{1}{3}} \text{ (Equality.)}$$

$$\frac{\frac{1}{3} x_1^{\frac{2}{3}} x_2^{-\frac{2}{3}}}{\frac{2}{3} x_1^{-\frac{1}{3}} x_2^{\frac{1}{3}}} = 1$$

$$\frac{x_1}{2x_2} = 1 \text{ (Simplified.)}$$

$$\frac{1}{2} x_1 - x_2 = 0 \text{ (Linear form.)}$$

$$x_1 + x_2 = 120 \text{ (See e. 3.)}$$

$$\begin{bmatrix} \frac{1}{2} & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 120 \end{bmatrix} \text{ (Linear equations.)}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 2 & 2 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 120 \end{bmatrix} \text{ (Solution by matrix inverse.)}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 80 \\ 40 \end{bmatrix} \text{ (Simplified.)}$$

$$x_1 = 80, \quad x_2 = 40 \text{ (Solution.)}$$

3. ...

a. An indifference curve is a continuous set of a fixed utility of some set of goods.

Example: for the utility function $U(x_1, x_2)$, the indifference curve would be all the points such that the result of $U(x_1, x_2)$ is equal to the value of that indifference curve. Emphasis: It is all such points with that utility value.

b. See a. Utility by definition does not change along an indifference curve.

c. ...

Indifference curves:

- do not cross each-other. Since all points on an indifference curve have the same value, and by definition the indifference curve encompasses all points such that utility is equal to that value, for another curve to include a point on that indifference curve would imply that it *is* that indifference curve, and thus not a distinct curve which crosses it.
- are non-strictly downward-sloping. That is to say either their slope is negative, or it is 0. This holds because of our initial stipulation that "more is better than less", and if there were an upward sloping indifference curve that would be violated as there would be a way to get more of both without increasing utility. The one (slight) exception to this is the case of 0 slope; a special case of perfect compliments, where the marginal utility of only one good, without including some of the other, is 0.
- are strictly increasing away from the origin. Any indifference curve which falls further away from the origin at its closest point than another indifference curve, must have higher utility than that indifference curve. This is because of, again, our initial stipulation of "more is better than less". The closest point to the origin for an indifference curve further away from the origin than another, will by definition have more of all goods. It must then have higher utility.
- for a given budget constraint, exactly one indifference curve meets that given budget line as a tangent (or at corner solution). This is because if the budget constraint were to meet an indifference curve at a non-tangent point, it would cross it. If it crosses an indifference curve, then there will be some points on one side of the budget line with higher utility than those points on the other side of the budget line, or even on it. So there exists exactly 1 point on the budget line which meets the indifference curve representing the maximum utility available given that budget constraint.

4. See Figure 2. on the following page.

The optimal consumption bundle is the point at which the budget constraint meets an indifference curve as a tangent. There are no points on the budget line which have a higher utility than that one point, as all other indifference curves would either not meet the budget constraint or fall closer to the origin.

5. The intercepts would remain the same. Numerically:

$$Y = p_1x_1 + p_2x_2 \text{ (Standard constraint.)}$$

$$2Y = 2p_1x_1 + 2p_2x_2 \text{ (Introduced doubling.)}$$

$$Y = \frac{2p_1x_1 + 2p_2x_2}{2}$$

$$Y = \frac{2}{2}(p_1x_1 + p_2x_2)$$

$$Y = p_1x_1 + p_2x_2 \text{ (Original constraint after simplifying.)}$$

Given the exact same constraint, the bundle would not change. The consumer would purchase exactly the same amount of the goods as before the doubling.

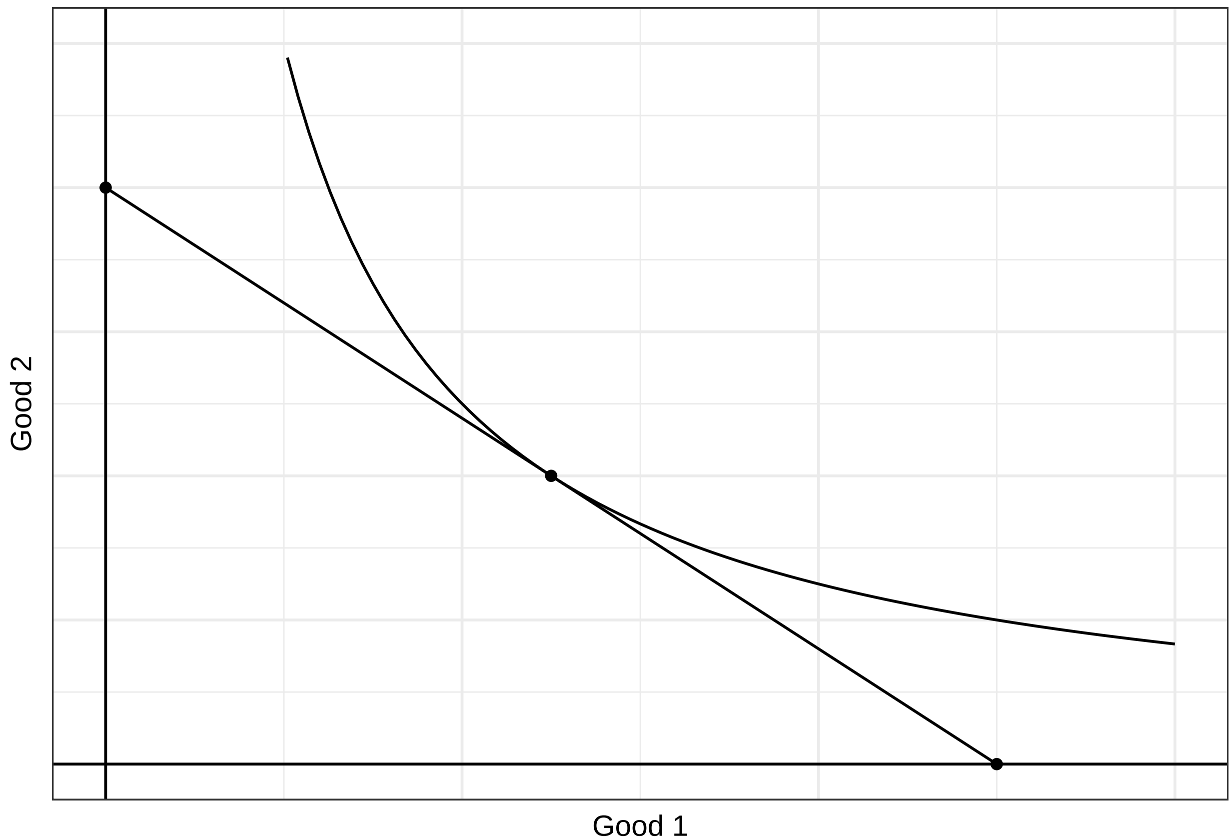

Figure 2: consumer optimization for two goods.

BONUS.

$$U = 200 - (x_1 - 10)^2 - (x_2 - 10)^2 \text{ (Utility.)}$$

$$\$40 = \$2x_1 + \$1x_2 \text{ (Constraint.)}$$

$$\Lambda = 200 - (x_1 - 10)^2 - (x_2 - 10)^2 + \lambda (40 - 2x_1 - x_2) \text{ (Lagrangian.)}$$

$$\frac{\partial \Lambda}{\partial x_1} = 20 - 2x_1 - 2\lambda = 0 \text{ (First order condition 1.)}$$

$$\frac{\partial \Lambda}{\partial x_2} = 20 - 2x_2 - \lambda = 0 \text{ (First order condition 2.)}$$

$$\frac{\partial \Lambda}{\partial \lambda} = 40 - 2x_1 - x_2 = 0 \text{ (First order condition 3.)}$$

$$10 - x_1 = 20 - 2x_2 \text{ (From first order conditions 1\&2.)}$$

$$2x_2 - x_1 = 10 \text{ (Linear form.)}$$

$$2x_1 + x_2 = 40 \text{ (First order condition 3 in linear form.)}$$

$$\begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 40 \end{bmatrix} \text{ (Linear equations.)}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \frac{1}{5} \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 40 \end{bmatrix} \text{ (Solution by matrix inverse.)}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 14 \\ 12 \end{bmatrix} \text{ (Simplified.)}$$

$$x_1 = 14, \quad x_2 = 12 \text{ (Solution.)}$$