

# Homework 1

1. Assume that the demand function for a good takes the following explicit form:  $Q = 285 - 5p + 10p_1$ . Where  $Q$  is quantity demanded,  $p$  is the price of the good, and  $p_1$  is the price of a substitute good. Assume  $p_1 = 1.5$ .

- a. Find the inverse demand function.

The function-inverse of  $Q$  with respects to  $p$  is  $p = 2p_1 - \frac{Q}{5} + 57$ . With  $p_1$  held at 1.5, it is  $p = 60 - \frac{Q}{5}$ .

- b. Graph it. Be sure to label all axes, curves, and intercepts!!!

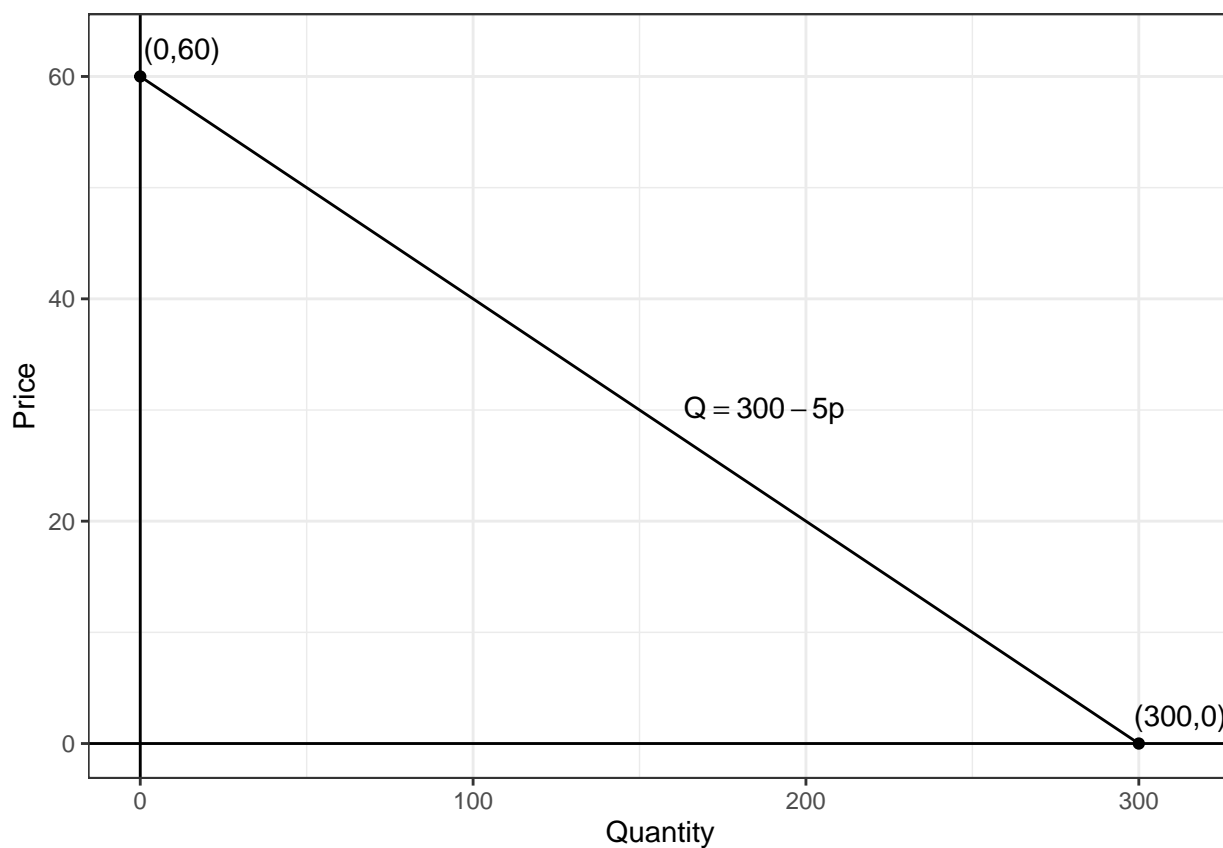


Figure 1: the demand function plotted. The  $Q$  intercept is 300, and the  $p$  intercept is 60.

- c. What price would you charge in order to sell 250 units of the good? I would charge \$10.
2. Given the following demand functions for three individuals:
 
$$Q_1 = 10 - 1.5p$$

$$Q_2 = 12 - 2p$$

$$Q_3 = 5 - p$$
    - a. Find the market demand function. The market demand function is  $Q = \sum_{i \in \{1..3\}} Q_i = 27 - 4.5p$ .

- b. Find the inverse of the market demand function. The function-inverse of  $Q$  is  $p = 6 - \frac{2Q}{9}$ .
- c. Graph your answer for part b. Be sure to label all axes, curves, and intercepts!!!

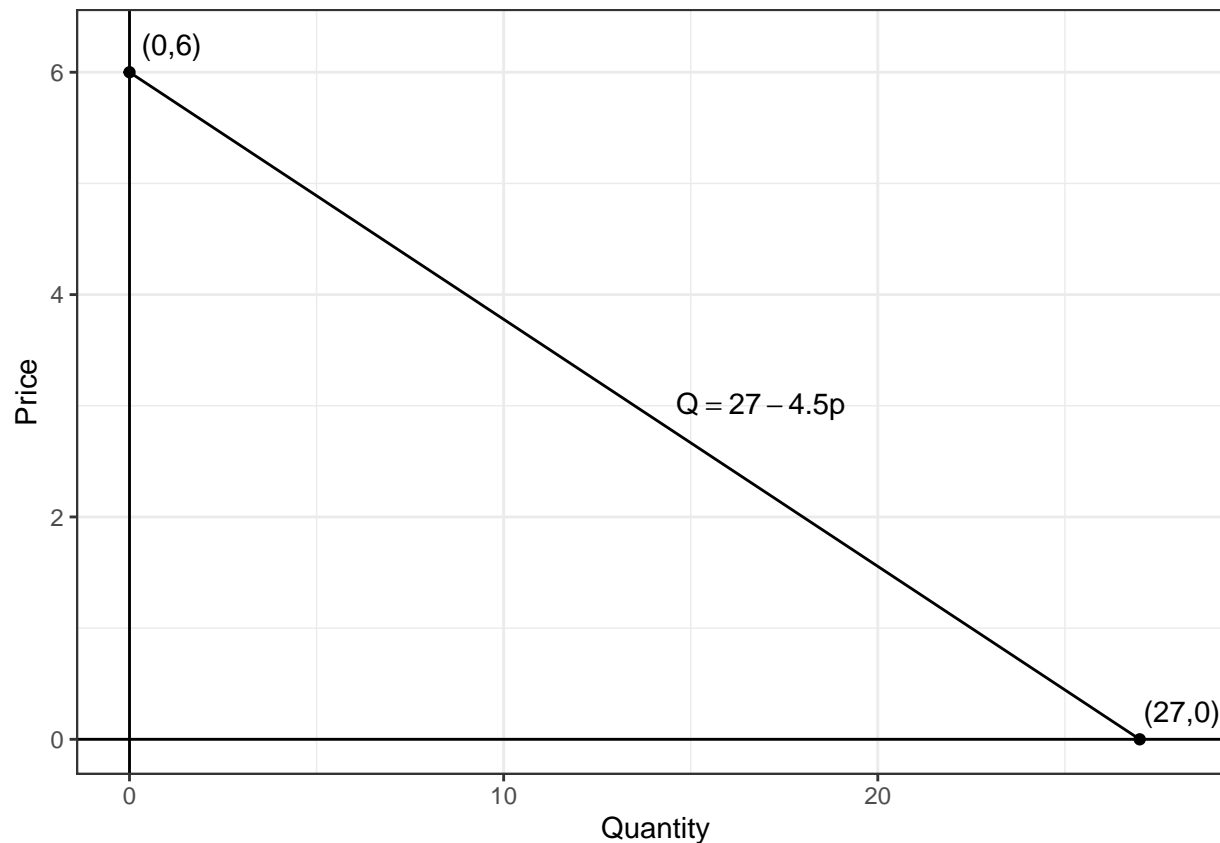


Figure 2: the demand function plotted. The  $Q$  intercept is 27, and the  $p$  intercept is 6.

2. Given the following demand and supply functions:

$$Q_D = 35 - 0.5p$$

$$Q_S = p - 10$$

Find the equilibrium price and quantity.

$$\begin{bmatrix} 1 & \frac{1}{2} \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} Q^* \\ p^* \end{bmatrix} = \begin{bmatrix} 35 \\ -10 \end{bmatrix}$$

$$\begin{bmatrix} Q^* \\ p^* \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 2 & 1 \\ 2 & -2 \end{bmatrix} \cdot \begin{bmatrix} 35 \\ -10 \end{bmatrix}$$

$$\begin{bmatrix} Q^* \\ p^* \end{bmatrix} = \begin{bmatrix} 20 \\ 30 \end{bmatrix}$$

4. ...

a. Imagine the following model represents the market for wind energy in Ohio. Show how the discovery of a large coal deposit might effect the equilibrium price and quantity. Briefly explain your answer.

As cheap coal enters the market, the cost of producing coal power falls. This raises the supply of coal power, lowering its price. Coal power is an alternative good to wind power, and so as the price of coal power falls the demand for wind power falls with it. As the demand for wind power falls, so does its price.

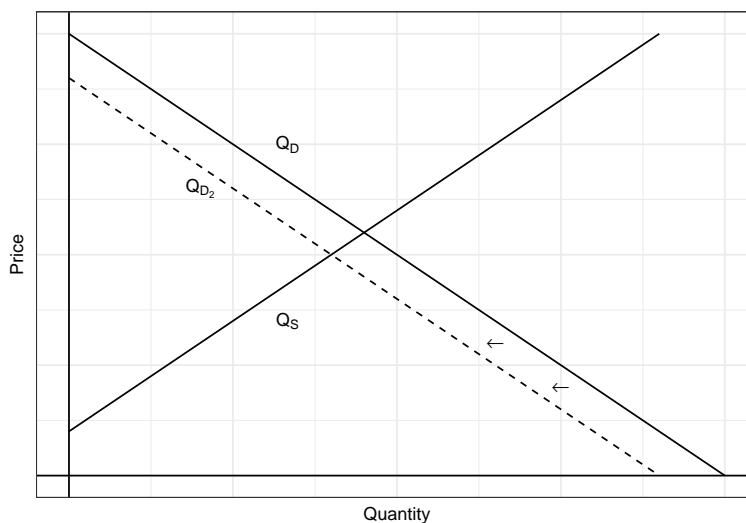


Figure 3: the demand for wind energy falls as cheap coal enters the market.

b. Imagine the following model represents the market for coal-generated electricity. Assume there is a government-mandated maximum level of total allowable carbon emissions. Show how the improvement of low-cost carbon-sequestration technology may affect the equilibrium price and quantity. Briefly explain your answer. Hint: There may be more than one correct answer to this one!!!

The introduction of a new technology which makes coal power more effective will lower the costs of production of coal power. This will increase supply, and lower price.

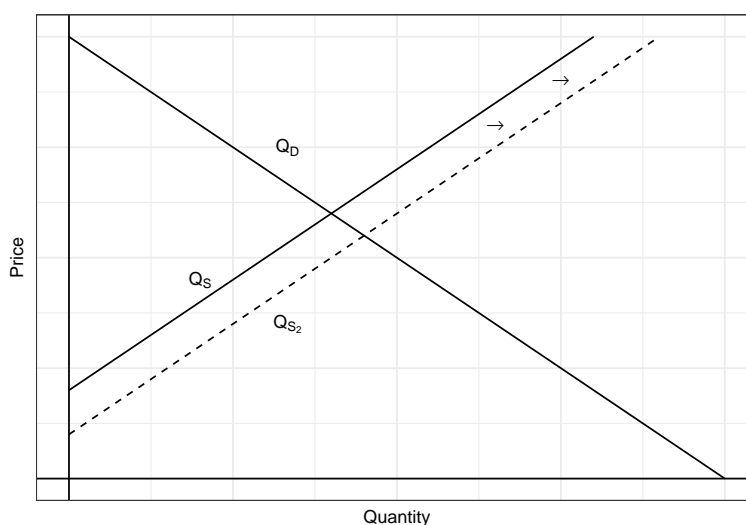


Figure 4: the supply of coal power rises as technology lowers production costs.

5. Imagine the demand function for a good takes the following form:  $Q_D = 200 - 2p + 4p_1 - 3p_2$ . Assume  $p_1 = 10$  and  $p_2 = 5$ . The supply function is:  $Q_S = 4p - 75$ .

a. In the demand function, looking at the coefficients for  $p_1$  and  $p_2$ , which is the complement good and which is the substitute good? (Call them good 1 and good 2.) Briefly explain your answer.

Good 1 is a substitute and good 2 is a complement. As the price of good 1 rises, people opt-out of that good into this good. As the price of good 2 rises, people purchase more of this good to go with it.  $\frac{\delta Q_D}{\delta p_1} = 4 > 0$ , and  $\frac{\delta Q_D}{\delta p_2} = -3 < 0$ .

- b. What are the equilibrium price and quantity,  $p^*$  and  $Q^*$ ?

$$\begin{bmatrix} 1 & 2 \\ 1 & -4 \end{bmatrix} \cdot \begin{bmatrix} Q^* \\ p^* \end{bmatrix} = \begin{bmatrix} 225 \\ -75 \end{bmatrix}$$

$$\begin{bmatrix} Q^* \\ p^* \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 4 & 2 \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 225 \\ -75 \end{bmatrix}$$

$$\begin{bmatrix} Q^* \\ p^* \end{bmatrix} = \begin{bmatrix} 128 \\ 50 \end{bmatrix}$$

- c. How much excess demand would there be if the government decided to set a price ceiling at  $p = 25$ ?,  $p^*$  and  $Q^*$ ?

There would be an excess demand of 150 units.

- d. How much excess supply would there be if the government decided to set a price floor at  $p = 75$ ?,  $p^*$  and  $Q^*$ ?

There would be an excess supply of 150 units.

6. Given the following demand function:  $Q = 88 - 0.4p$ , calculate the elasticity of demand when  $p = 20$ . Explain in one sentence what your answer means.

$$\frac{\delta Q}{\delta p} \frac{p}{Q} = \frac{\delta(88 - 0.4p)}{\delta p} \frac{p}{88 - 0.4p} = -0.4 \cdot \frac{p}{88 - 0.4p}$$

$$-0.4 \cdot \frac{p}{88 - 0.4p} \Big|_{p=20} = -0.4 \cdot \frac{20}{88 - 0.4 \cdot 20} = -0.4 \cdot \frac{20}{80} = -0.4 \cdot \frac{1}{4} = -\frac{1}{10}$$

Interpretation: for a 1% change in price, quantity will change by 0.1%.