Multiple linear Regression

Multiple linear regression is an extension of simple linear regression that allows for modeling the relationship between a dependent variable and multiple independent variables. It assumes a linear relationship between the dependent variable and the independent variables.

The equation for multiple linear regression can be written as:

$$y = \beta 0 + \beta 1x1 + \beta 2x2 + \ldots + \beta nxn$$

Where:

- y represents the dependent variable (response variable).

- $x1, x2, \ldots, xn$ represent the independent variables (predictor variables).

- $\beta 0$ is the y-intercept of the line (the value of y when all the independent variables are 0).

- $\beta 1, \beta 2, \ldots$, βn are the slopes of the line (the change in y for a one-unit increase in each independent variable).

In multiple linear regression, the goal is to estimate the values of $\beta 0, \beta 1, \beta 2, \ldots, \beta n$ that minimize the sum of squared differences between the observed values of y and the predicted values based on the linear model. This is typically done using the least squares method.

The estimated values of the β coefficients provide information about the strength and direction of the relationships between the dependent variable and each independent variable. They indicate how much the dependent variable is expected to change for a one-unit increase in each independent variable, while holding the other independent variables constant.

Multiple linear regression allows for analyzing the simultaneous effects of multiple variables on the dependent variable, making it a valuable tool in various fields such as economics, social sciences, and data analysis.

Multiplelar linear Regression:-

For 2D Data:-

$$y = mx + b$$

$$y = \beta_0 + \beta_1 X$$

For 4D Data:-

$$\hat{y} = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3$$

For Now we have 'n' columns

$$\hat{y} = \begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \vdots \\ \hat{y}_n \end{bmatrix} = \begin{bmatrix} \beta_0 & \beta_1 X_{11} & \beta_2 X_{12} & \cdots & \beta_m X_{1m} \\ \beta_0 & \beta_2 X_{21} & \beta_2 X_{22} & \cdots & \beta_m X_{2m} \\ \vdots & \vdots & \vdots & & \vdots \\ \beta_0 & \beta_1 X_{n1} & \beta_2 X_{n2} & \cdots & \beta_m X_{nm} \end{bmatrix}$$

$$\hat{y} = \underbrace{\begin{bmatrix} 1 & X_{11} & X_{12} & \cdots & X_{1m} \\ 1 & X_{21} & X_{22} & \cdots & X_{2m} \\ \vdots & & & & \\ 1 & X_{n1} & X_{n2} & \cdots & X_{nm} \end{bmatrix}}_{(n \times (m+1))}^{x} \underbrace{\begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \vdots \\ \beta_m \end{bmatrix}}_{((m+1)\times 1)}^{\beta}$$

$$\boxed{\hat{y} = X\beta} \text{ ——— } (i)$$

y-metrics Actual value

$$y_i = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

y-metric predict value

$$\hat{y}_i = \begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \vdots \\ \hat{y}_n \end{bmatrix}$$

$$E = e = y - \hat{y}$$

$$e = \begin{bmatrix} y_1 - \hat{y}_1 \\ y_2 - \hat{y}_2 \\ \vdots \quad \vdots \\ y_n - \hat{y}_n \end{bmatrix}$$

Simple linear Regression Error function is

$$E = \sum (y_i - \hat{y}_i)^2$$

For multiple linear Regression the loss, Error function is

$$E = e^T . e$$

$$\left[(y_1-\hat{y}_1) \, (y_2-\hat{y}_2) \, .. \, (y_n-\hat{y}_n) \right] \begin{bmatrix} (y_1-\hat{y}_1) \\ (y_2-\hat{y}_2) \\ \vdots \\ (y_n-\hat{y}_n) \end{bmatrix}$$

$$E = e^T . e = (y - \hat{y})^T (y - \hat{y})$$

$$\text{-----} \left\{ \begin{array}{l} [A + B]^T = A^T + B^T \; \& \\ [A - B]^T = A^T - B^T \end{array} \right.$$

$$\boxed{(y^T - \hat{y}^T)(y - \hat{y}^T)} \text{ ——— } (ii)$$

Put eq (i) in equation (ii)

$$[y^T - (X\beta)^T](y - X\beta)$$

$$y^T (y - X\beta) - (X\beta)^T (y - X\beta)$$

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$$y^T y - \underline{y^T X\beta} - \underline{(X\beta)^T y} + (X\beta)^T X\beta$$

prove this are equal.

$$y^T X\beta = (X\beta)^T y$$

Now = $y = A$ & $(X\beta) = B$

$$A^T B = B^T A \quad \ldots\ldots \quad (A^T B)^T = A B^T$$

$$(A^T B) = (A^T B)^T$$

Now:- .... $A^T B = C$

$$C = C^T$$

$$y^T X\beta = (y^T X\beta)^T$$

Now:- $\underset{(1\times n)}{[\,\ldots\, y_n]} \underset{(n\times(m+1))}{\begin{bmatrix} 1 & x & \ldots & x_{1n} \\ 1 & \ldots & \ldots & \\ 1 & \ldots & \ldots & x_{mn} \end{bmatrix}} \underset{((m+1)\times 1)}{\begin{bmatrix} \beta_0 \\ \vdots \\ \beta_n \end{bmatrix}}$

Matrix multiplication

$$(1\times \underline{n}) \; [\underline{n}\times(m+1)] \; [(m+1)\times 1]$$

$$[1\times \underline{(m+1)}] \; [\underline{(m+1)}\times 1]$$

$$[1\times 1]$$

$$[1]^T = [1]$$

Therefore :- $\boxed{y^T X\beta = (X\beta)^T y}$

New : Equation:-

$$E = y^T y - 2y^T X\beta + \beta^T X^T \cdot X\beta \quad \text{--- (iii)}$$

Derivate:-

$$\frac{dE}{d\beta} = \frac{dE}{d\beta}\left[y^T y - 2y^T X\beta + \beta^T X^T \cdot X\beta\right]$$

$$= \frac{d}{d\beta} y^T y - \frac{d}{d\beta} 2y^T X\beta + \frac{d}{d\beta} \beta^T X^T \cdot X\beta$$

$$= 0 - 2y^T X + 2X^T \cdot X\beta^T$$

$$= -2y^T \cdot X + 2X^T \cdot X\beta^T$$

$$\cancel{2}X^T \cdot X\beta^T = \cancel{2}y^T \cdot X$$

$$\beta^T = (y^T X)(X^T \cdot X)^{-1}$$

Apply transpose Both Side.

$$[\beta^T]^T = [X^T y^T]^T \left[[X^T \cdot X]^{-1}\right]^T$$

Nothing change due to Square matric. e.g.
$m \times n = n \times m$
$2 \times 2 = 2 \times 2$

$$\beta = [X^T \cdot y][X^T \cdot X]^{-1}$$

$$\boxed{\beta = [X^T \cdot X]^{-1}[X^T \cdot y]}$$

$$\begin{bmatrix} \beta_0 \\ \beta_1 \\ | \\ \beta_n \end{bmatrix}_{[(m+1)\times 1]} = \left[X^T X\right]^{-1} \left[X^T \cdot Y\right]$$

Now metric multiplication

$[\cancel{(m+1)\times 1}]$. — R.H.S.

$$[(m+1)\cdot(m+1)]\ [(m+1)\cdot n]\ [n\times 1]$$

$$[(m+1)\cdot n]\ [n\times 1]$$

$$[[m+1]\cdot 1]$$

$\therefore$ therefore LHS = R.H.S.

$$\boxed{\beta = (X^T X)^{-1} (X^T Y)}$$

multiple
linear equation line formula-

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 - - x_n \beta_m$$