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CSCI 3104, Algorithms **Profs. Grochow & Layer**
Explain-It-Back 9 **Spring 2019, CU-Boulder**

A finance colleague asks for your help in developing software that will help her automate some of the buy and sell orders that she receives. Simplifying things a bit, she describes buy orders as target asset and a dollar amount to spend and sell orders as target asset and an amount of the asset to sell. As you develop this application you see a funny pattern. The US dollar (USD) to Pound sterling rate is 0.77 (GBP), the GBP to Canadian dollar (CAD) rate it 1.75, and the CAD to USD rate is 0.75. You get very excited by this observation and immediately stop work on the automated buy/sell tool and start implementing a shortest path algorithm. After a few tests you are confident in your idea, now you pitch this new method to your friend.

Dear Colleague,

I understand that you asked me to help you with something specific, but amid studying what you gave me, I found a golden nugget that led me to make something different. I was looking at the exchange rates you gave me, and I found what we in the business like to call, "A Negative Sum Cycle." Let me explain.

To understand this, we need to change the way we look at our currency and its corresponding exchange rate. Instead of thinking of them as arbitrary numbers, it would help if we mapped them out on to some sort of graph. Now when I say a map, I literally mean a map. Just like we have a map of the world, each country would represent a 'vertex' or point on the map. Then we would need lines that connect those vertices or points. Those lines would be called edges and would represent the exchange rates. The last thing we need to define is a path. Which is just a route you would take from vertex (or country) to another. Now if we were to travel from one vertex to another, we would have to convert our money. We would do this by simply multiplying our money by whatever the exchange rate is. This is a simple business procedure, and I'm sure you're familiar with it. I know that's a bit of a set up, but it will be worth it, I promise!

A common algorithm problem is to look at graphs, or maps like these, and study the different path options. For example, if you wanted to change your US dollars, to Yen, would it be more beneficial to first change your US dollars to Canadian dollars, and then to Yen? Or should you just convert directly? There are all sorts of algorithms to find the optimal paths between x and y. One thing to keep track of though, is to make sure that you don't go back to the same vertices that you've already visited. For most purposes, it wouldn't make sense to change your US dollars to Canadian Dollars, then back to US, then to Chinese Yen. Most of the time, that would result in you ending up with less money. Most of the time. But this happens to be where I found the gold nugget.

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This loop is what I was referring to in the first paragraph; a negative sum cycle. It's the idea that taking a path which returns you to where you started usually results in a loss of money/time. So in our programming, we usually throw an error. However, with a little bit of tweaking that involves taking the negative logs of the exchange rate, we can make our algorithm display a message whenever we have a positive sum cycle. Meaning that when we end up back with US dollars, we have more money than what we started with.

With the numbers you gave me, it turns out that we have a positive sum cycle. When you exchange your money from US dollars to British pounds, and then from pounds to Canadian dollars, and then back to US dollars, you end up with about 1.011 times the amount you started with. So, if you had $1,000 dollars, you could gain $11 dollars without doing anything but a little bit of trading. (Which is a lot easier than buying and selling all sorts of stuff.)

Now this is just the beginning, I can have my algorithm run over larger data sets (ie. Exchange rates) and we can see if there are any more profitable paths to take. I'm very glad that you decided to include in your task for the buying and selling of goods, but I would love to change our focus to a more safe and quicker way of earning money; arbitrage!

Please get back to me with any questions or concerns, thanks!

Best regards,
Trevor Buck