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Network Routing

1. Code Submission
**See appendix

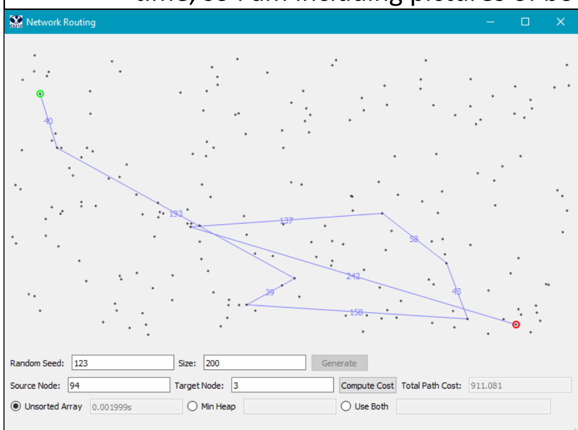
2. Priority Queue's Complexity	
Array	Heap
Insert(): I do not use an insert function with the array implementation, but adding something to the array is O(1) complexity because appending to the end of an array takes constant time. **Deletemin_Array():** The complexity is O(n) because every node is traversed to determine which has the lowest value.	**Insert_heap():** Appending to the end of an array takes O(1) time. Afterwards, checking to make sure that the array is sorted is O(logn) due to visiting a node at each level during the "bubbling up"stage to rebalance the tree. **DecreaseKey():** Mine has worst case scenario Complexity O(n). This is because I implemented my pointerArray in an inverse way *unintentionally*. However, this is the incorrect implementation. To fix this, I would have the indices of the pointerArray be related to the nodeID's, which values would then point to the position of the nodeID in the BinaryHeap array. This would allow me to use the nodeID for a lookup of O(1) time. This would greatly improve speeds. **Deletemin_Heap():** Popping off values from the arrays is O(1) time. However, once the last value becomes gets put onto the front of the array (or top of the tree depending on how you look at it), the "sifting down" will take O(logn) due to having to revisit a node at each layer. Thus, rebalancing the tree. **Swap():** Complexity O(1) **MakeQueue():** O(1) **Sift_down():** O(logn)

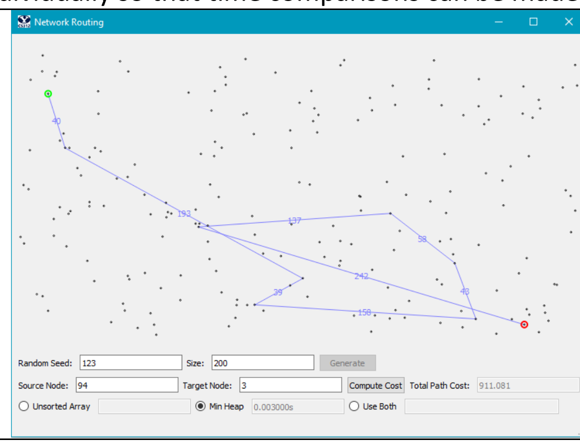
3. Time and Space Complexity	
Array	Heap
Time Complexity: O(n^2). This is because we run "n" times for Dijkstra's algorithm for each node. Then each time we run deletemin is called, which is also O(n) complexity, therefore we have O(n^2) complexity. **Space Complexity:** O(n) Because for each array that is being stored, "n" space is required. But due to the max rule, we reduce to O(n).	**Time Complexity:** The time complexity should be O(nlogn). Because we run for "n" times (in Dijkstra's, but for each run we also use deletemin which *should be* O(logn). Therefore giving us O(nlogn). However, because my implementation is not correct, our worst case scenario will be O(n^2). **Space Complexity:** O(n) Because for each array that is being stored, "n" space is required. But due to the max rule, we reduce to O(n).

4. Screenshots

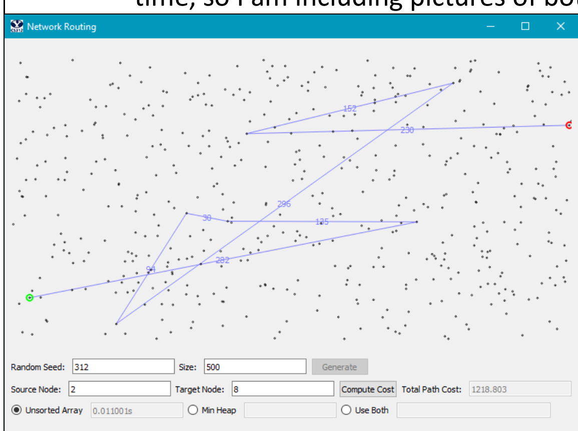
a. I am not including a screenshot of this because there is no path between the two points.

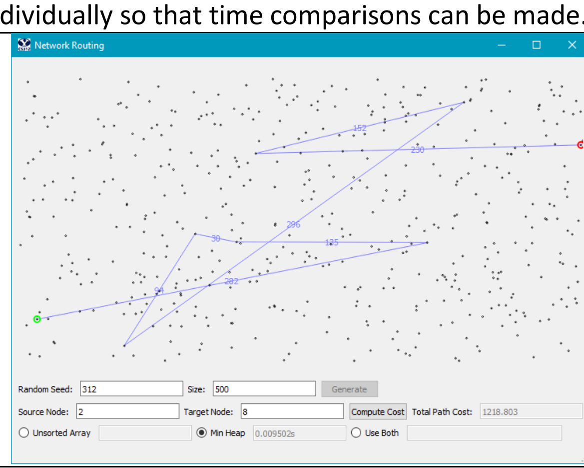
b. The GUI was not working for me when I did "use both". It would not report the Min Heap time, so I am including pictures of both individually so that time comparisons can be made.





c. The GUI was not working for me when I did "use both". It would not report the Min Heap time, so I am including pictures of both individually so that time comparisons can be made.





5. Empirical Analysis

Node Count	Array Time	Heap Time
100	0.003987	0.001994
100	0.003950	0.003986
100	0.003985	0.004989
100	0.004984	0.004984
100	0.000997	0.00202
Average:		
1000	0.138628	0.03590
1000	0.104744	0.035903
1000	0.127666	0.035904
1000	0.103722	0.033934
1000	0.103704	0.038879
Average:		
10,000	12.257217	2.387613
10,000	11.144193	3.111657
10,000	10.987611	2.690777
10,000	11.250908	2.596055
10,000	11.976964	2.983012
Average:		

Appendix

```
from CS312Graph import *
import time
import numpy as np
import collections

class NetworkRoutingSolver:
    def __init__( self):
        pass

    def initializeNetwork( self, network ):
        assert( type(network) == CS312Graph )
        self.network = network
        self.dist = [np.inf] * len(self.network.nodes)
        self.prev = [None] * len(self.network.nodes)
        self.binaryHeap = []
        self.pointerArray = []

    def getShortestPath( self, destIndex ):
        self.dest = destIndex
        path_edges = []
        total_length = 0

        node = self.network.nodes[self.dest]
        while node.node_id != self.source:
            prevNode = node
```

```
            node = self.network.nodes[self.prev[node.node_id]] # <--- this
will not work for HEAP because here is where "prev" is used
            for edge in node.neighbors:
                if edge.dest.node_id == prevNode.node_id:
                    path_edges.append( (edge.src.loc, edge.dest.loc,
'{:.0f}'.format(edge.length)) )
                    total_length += edge.length

        return {'cost':total_length, 'path':path_edges}

    #######################
    # Time Complexity: O(1)
    #######################
    def makequeue(self):
        # populate heap array

        # SET VALUES TO DIST AND PREV
        startNode = self.network.nodes[self.source]
        self.insert_heap(0, startNode)
        for edge in startNode.neighbors:
            self.insert_heap(edge.length, edge.dest)

        return

    #######################
    # Time Complexity: O(logn)
    #######################
    def insert_heap(self, distance, nodeIn):
        index = len(self.binaryHeap)
        self.binaryHeap.append(distance)
        self.pointerArray.append(nodeIn.node_id)
        while (index != 0):
            p = self.parent(index)
            if (self.binaryHeap[p] > self.binaryHeap[index]):
                self.swap(p, index)
            index = p

    def parent(self, i):
        return (i - 1)//2

    def left_child(self, i):
        return 2*i + 1

    def right_child(self, i):
        return 2*i + 2

    #######################
    # Time Complexity: O(1)
    #######################
    def swap(self, i, j):
        self.binaryHeap[i], self.binaryHeap[j] = self.binaryHeap[j],
self.binaryHeap[i]
        self.pointerArray[i], self.pointerArray[j] = self.pointerArray[j],
self.pointerArray[i]

    #######################
    # Time Complexity: O(n)
```

```
    ########################
    def deletemin_array(self, tempNodes):
        # return lowest value of array
        lowestDistance = np.inf
        lowestIndex = -1
        nodeId = -1

        for i in range(len(tempNodes)):
            if lowestDistance > self.dist[tempNodes[i].node_id]:
                lowestDistance = self.dist[tempNodes[i].node_id]
                lowestIndex = i
                nodeId = tempNodes[i].node_id
        if lowestIndex != -1:
            del tempNodes[lowestIndex]

        return nodeId

    ########################
    # Time Complexity: O(n) should be O(logn
    ########################
    def decreaseKey(self, nodeIdToDecrease, updateDistance):
        for i in range(len(self.pointerArray)):                # <---------
------- CAN BE SPED UP HERE
            if nodeIdToDecrease == self.pointerArray[i]:
                self.binaryHeap[i] = updateDistance
                # change the value in heap to updateDistance
                # check if parent is bigger, if so, swap.
                index = i
                while (index != 0):
                    p = self.parent(index)
                    if (self.binaryHeap[p] > self.binaryHeap[index]):
                        self.swap(p, index)
                    index = p
                break
        return

    #########################
    # Time Complexity: O(logn)
    #########################
    def sift_down(self):
        # check right or left and replace with whichever one is smaller
        # continue until either null child or at bottom
        # if none less, then end
        index = 0
        changed = True
        while(changed):
            # if both null, break
            if (self.right_child(index) > len(self.binaryHeap)-1 and
self.left_child(index) > len(self.binaryHeap)-1):
                break
            # null right, check left. if less, swap
            elif (self.right_child(index) > len(self.binaryHeap)-1):
                if (self.binaryHeap[self.left_child(index)] <
self.binaryHeap[index]):
                    self.swap(self.left_child(index), index)
                    index = self.left_child(index)
                else:
```

```
                    break
                # if left < right, check left. if less, swap
                elif (self.binaryHeap[self.left_child(index)] <
self.binaryHeap[self.right_child(index)]):
                    if (self.binaryHeap[self.left_child(index)] <
self.binaryHeap[index]):
                        self.swap(self.left_child(index), index)
                        index = self.left_child(index)
                    else:
                        break
                # if left > right, check right. if less, swap
                elif (self.binaryHeap[self.left_child(index)] >
self.binaryHeap[self.right_child(index)]):
                    if (self.binaryHeap[self.right_child(index)] <
self.binaryHeap[index]):
                        self.swap(self.right_child(index), index)
                        index = self.right_child(index)
                    else:
                        break
                changed = False
        return

    ########################
    # Time Complexity: O(logn)
    ########################
    def deletemin_heap(self):
        # pop the top off of both arrays
        if (len(self.binaryHeap) == 0):
            return
        else:

            distance = self.binaryHeap.pop(0)
            i = self.pointerArray.pop(0)
            node = self.network.nodes[i]

            # set the last value to the first
            if (len(self.binaryHeap) != 0):
                bHtoAppend = self.binaryHeap.pop(len(self.binaryHeap)-1)
                self.binaryHeap.insert(0, bHtoAppend)
                pAtoAppend = self.pointerArray.pop(len(self.pointerArray)-1)
                self.pointerArray.insert(0, pAtoAppend)

                # sift down until ordered properly
                self.sift_down()

        return node, distance

    # used to populate arrays
    ###############################
    # Time Complexity Array: O(n^2)
    # Time Complexity Heap: worst case O(n^2) should be O(nlogn)
    ###############################
    def computeShortestPaths( self, srcIndex, use_heap=False ):
        # if not use heap, do priority queue
        self.source = srcIndex
        if use_heap == False:
            t1 = time.time()
```

```
            # set start distance as 0 and prev as None
            self.dist[self.source] = 0
            i = self.source
            self.prev[self.source] = None

            tempNodes = self.network.nodes[:]
            del tempNodes[self.source]

            for indx in range(len(self.network.nodes)):
                node = self.network.nodes[i] # begin with starting node
                for edge in node.neighbors: # for all edges(u, v) in E:
                    alt = self.dist[i] + edge.length
                    if (self.dist[edge.dest.node_id] > alt):
                        self.dist[edge.dest.node_id] = alt
                        self.prev[edge.dest.node_id] = edge.src.node_id
                i = self.deletemin_array(tempNodes)
            t2 = time.time()
        else:
            t1 = time.time()

            # else use heap
            self.source = srcIndex
            # set start distance as 0 and prev as None
            self.dist[self.source] = 0
            self.prev[self.source] = None

            self.makequeue()  # using dist-values as keys
            while len(self.binaryHeap) != 0:
                node, distance = self.deletemin_heap()

                for edge in node.neighbors:
                    alt = self.dist[edge.src.node_id] + edge.length
                    if (self.dist[edge.dest.node_id] == np.inf):
                        self.insert_heap( alt, edge.dest)
                        #self.dist[edge.dest.node_id] = edge.length
                        self.prev[edge.dest.node_id] = node.node_id
                    old = self.dist[edge.dest.node_id]
                    if (old > alt):
                        self.dist[edge.dest.node_id] = alt
                        self.prev[edge.dest.node_id] = edge.src.node_id
                        self.decreaseKey(edge.dest.node_id,
self.dist[edge.src.node_id] + edge.length)
            t2 = time.time()
        return (t2-t1)
```