TITLE

AP CALCULUS NOTES

Edited by

Trevor Bushnell

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Chapter 1

Limits and Continuity

§1.1 Introducing Calculus: Can Change Occur at an Instant?

- Traditional algebra uses relationships such as $\frac{\Delta y}{\Delta x}$ to model relationships
  - However, this model falls apart because if $\Delta y = 0$ and $\Delta x = 0$, then the result is $\frac{0}{0}$ which is indeterminant
    - **indeterminant** means that there might be a possible solution, but we cannot determine what that possible solution could be based on the current problem solving method
- We can use the **limit** to allow us to define change that occurs instantaneously in terms of incredibly small average rates in change (for example, doing $\Delta x = 0.000001$ instead of $\Delta x = 0$)
- Calculus uses limits to understand and model more precise/instantaneous change that algebra cannot answer

§1.2 Defining Limits and Using Limit Notation

Definition 1.2.1

Given a function f, the limit of $f(x)$ as x approaches c is a real number R if $f(x)$ can be made arbitrarily close to R by taking x extremely close to c (but *not equal to* c).

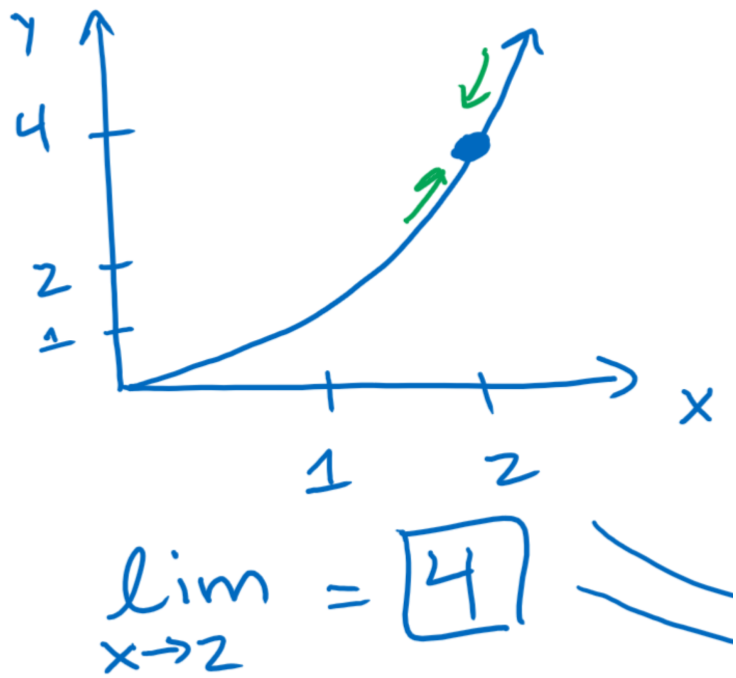
If the limit exists and is a real number, then:

$$\lim_{x \to c} f(x) = R$$

- A limit can be expressed graphically, numerically, or analytically

§1.3 Estimating Limits From Graphs

- ONE SIDED LIMIT: A limit where you approach from a specific direction (either the left or the right)
  - LHL (Left Hand Limit): $\lim_{x \to c^-} f(x) = L$
  - RHL (Right Hand Limit): $\lim_{x \to c^+} f(x) = L$
  - A limit exists if the left hand limit equals the right hand limit (LHL = RHL)
- Using the information provided on a graph can help you interpret the limit of a function



- In the example above, the limit is 4 because the function output as you approach 2 from the left is equal to the function output as you approach 2 from the right
- Because there can be possible issues with scale, graphical representations of limits can possibly be inaccurate and can miss important behaviors of functions if you're too far zoomed out
- A limit can fail to exist at particular values of x if LHL $\neq$ RHL, the function oscillates near x, or if the function is unbounded

x	1.25	1.5	1.75	2.25	2.5	2.75
y	1.5625	2.25	3.0625	5.0625	6.25	7.5625

$$\lim_{x \to 2} = \boxed{4}$$

$$\lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$$

$$\lim_{x \to a} [f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$$

$$\lim_{x \to a} [cf(x)] = c \lim_{x \to a} f(x)$$

$$\lim_{x \to a} [f(x)g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$$

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} \quad \text{if } \lim_{x \to a} g(x) \neq 0$$

$$\lim_{x \to a} [f(x)]^n = \left[\lim_{x \to a} f(x)\right]^n \quad \text{where } n \text{ is a positive integer}$$

$$\lim_{x \to a} c = c \qquad \textbf{8.}\ \lim_{x \to a} x = a$$

$$\lim_{x \to a} x^n = a^n \quad \text{where } n \text{ is a positive integer}$$

$$\lim_{x \to a} \sqrt[n]{x} = \sqrt[n]{a} \quad \text{where } n \text{ is a positive integer}$$

(If n is even, we assume that $a > 0$.)

$$\lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to a} f(x)} \quad \text{where } n \text{ is a positive integer}$$

[If n is even, we assume that $\lim_{x \to a} f(x) > 0$.]

§1.4 Estimating Limit Values From Tables

- Numerical information from tables can be used to estimate Limits

- As seen in the table, the output values for 1.75 and below all seem to be getting closer to 4 while x gets larger, while the output values for 2.25 and greater all have values approaching 4 as x gets smaller. Since the LHL is equal to the RHL, we can conclude that the limit as x approaches 2 is indeed 4

§1.5 Determining Limits Using Algebraic Properties of Limits

- To evaluate a limit, simply substitute the desired value that you wish to find the limit at