Question 1: Solve the following system of differential equations:

$$y_1'(t) = 5y_1(t) - 7y_2(t)$$
$$y_2'(t) = 2y_1(t) - 4y_2(t)$$

Step #1: Create a matrix equation comprising of the two differential equations:

$$\mathbf{y}' = A\mathbf{y}$$
$$\begin{bmatrix} y_1'(t) \\ y_2'(t) \end{bmatrix} = \begin{bmatrix} -5 & -7 \\ -2 & -4 \end{bmatrix} \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix}$$

Step #2: Calculate the determinant

Step #3: For each eigenvalue, calculate the eigenvectors:

In order to find the associated eigenvectors, we simply substitute λ into $(A - I\lambda) = \vec{0}$

Step #4:

Step #5: Calculate the general solution:

Step #6: Given initial conditions $\mathbf{y} = \begin{bmatrix} 16 \\ 6 \end{bmatrix}$ solve for c_1 and c_2 to find the specific solution

$$\begin{aligned} 16 &= 7c_1 + c_2 \\ 6 &= 2c_1 + c_2 \\ (16 - 6) &= (7c_1 - 2c_2) + (c_2 - c_2) \\ 10 &= 5c_1 \\ c_1 &= 2 \end{aligned}$$

$$\begin{aligned} 6 &= 2(2) + c_2 \\ 6 &= 4 + c_2 \\ 2 &= c_2 \end{aligned}$$

Step #7: State the answer

Way #1: Express as a system of equations:

$$y_1(t) = 14e^{3t} + 2e^{-2t}$$
$$y_2(t) = 4e^{3t} + 2e^{-2t}$$

Way #2: Express as an array:

$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = 2 \begin{bmatrix} 7e^{3t} \\ 2e^{3t} \end{bmatrix} + 2 \begin{bmatrix} e^{-2t} \\ e^{-2t} \end{bmatrix}$$