Multivariable Calculus

Public Notes for Any Multivariable Calculus Course

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Contents

Chapter 1

Vectors and the Geometry of Space

§1.1 3D Coordinate Systems

§1.1.1 About 3D Coordinate Systems

- We are used to working in textbfplanes (1D input and 1D output), and we can model each input and output on a 2D plane
- Multivariable calculus is all about representing points in **space** (2D input and 1D output)
- There are 2 *axes* in 2D space → there are textbf3 axes in 3D space
- There are 4 *quadrants* in 2D space → there are 8 **octants** in 3D space
- There is *one plane* in 2D space (xy-plane) → there are **3 planes** in 3D space (xy-plane, xz-plane, and yz-plane)
- Points in 2D space have *2 coordinates* → points in 3D space have **3 coordinates**
- We went from creating *graphs* in 2D space to creating **surfaces** in 3D space
- If you have a hard time visualizing 3D space, use the room around you and pick a bottom corner in the room
  - Floor is xy-plane
  - Left wall is xz-plane
  - Right wall is yz-plane

§1.1.2 The Distance Formula (3D)

- Distance formula in 3D is very similar to the distance formula in 2D
- Finds the distance between point P_1 and P_2

Definition 1.1.1

o find the distance between two points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$, use the following distance formula:

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

§1.1.3 Equation of a Sphere

We will be representing many surfaces and it is important to understand the different types of surfaces that we will encounter. A basic one is a sphere, which looks like the following:

Proposition 1.1.1

he equation of a sphere with center $C(h, k, l)$ and radius r is

$$(x - h)^2 + (y - k)^2 + (r - l)^2 = r^2$$

§1.2 Vectors

§1.2.1 About Vectors

Definition 1.2.1

vector is a quantity that has both a *direction* and a *magnitude* (length).

Ex: 60mph North is a vector because it has a magnitude (60mph) and a direction (North)

We denote a vector by using either boldface ($\mathbf{v}$) or drawing an arrow over the name of the vector ($\vec{v}$)

- $\vec{O}$ is called the "zero vector and has length 0 and no direction

§1.2.2 Vector Components and Magnitudes

- A vector algebraically looks like this $\rightarrow \vec{a} =< a_1, a_2 >$
  - The coordinates of $\vec{a}$ are the **components** of $\vec{a}$

Proposition 1.2.1

he algebraic representation of a vector between points $A(x_1, y_1, z_1)$ and B(x_2, y_2, z_2) is

$$\vec{a} =< x_2 - x_1, y_2 - y_1, z_2 - z_1 >$$

To find the x-component of a vector, you can use the formula:

$$\vec{a}_x = ||\vec{a}|| \cos\theta$$

To find the y-component of a vector, you can use the formula:

$$\vec{a}_x = ||\vec{a}|| \cos\theta$$

Where θ is the angle between the vector and the x-axis

Proposition 1.2.2

o find the magnitude of the vector $\vec{a}$, use the distance formula (2D in the case of working with vectors having 2 components, 3D in the case of working with vectors with 3 components):

$$||\vec{a}|| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$
$$OR$$
$$||\vec{a}|| = \sqrt{a_1^2 + a_2^2}$$

§1.2.3 Adding Vectors

- TIP TO TAIL METHOD: Put the end of the second vector on the tip of the first vector *without changing the direction of length*
  - The vector connecting the tail of the first vector to the tip of the second vector is the sum of the two vectors
- ALGEBRAICALLY: To add two vectors, add together their components
- To subtract two vectors, subtract their components

§1.2.4 Other Properties of Vectors

- To multiply a vector by a scalar (any number), multiply that scalar by each component of the vector
- Other properties of vectors can be found below:

1. $\mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a}$	**2.** $\mathbf{a} + (\mathbf{b} + \mathbf{c}) = (\mathbf{a} + \mathbf{b}) + \mathbf{c}$
3. $\mathbf{a} + \mathbf{0} = \mathbf{a}$	**4.** $\mathbf{a} + (-\mathbf{a}) = \mathbf{0}$
5. $c(\mathbf{a} + \mathbf{b}) = c\mathbf{a} + c\mathbf{b}$	**6.** $(c + d)\mathbf{a} = c\mathbf{a} + d\mathbf{a}$
7. $(cd)\mathbf{a} = c(d\mathbf{a})$	**8.** $1\mathbf{a} = \mathbf{a}$

§1.2.5 Basis Vectors

- Vectors can also be written in relation to the unit vectors in the x, y, and z directions respectively (known as the **basis vectors**)

§1.2.6 Applications

§1.3 The Dot Product

§1.3.1 Definition of the Dot Product

§1.3.2 What Does the Dot Product Represent?

§1.3.3 Direction Angles and Direction Cosines

§1.4 The Cross Product

§1.4.1 Definition of the Cross Product

§1.4.2 What Does the Cross Product Represent?

§1.4.3 Scalar Triple Products

§1.4.4 Application: Torque

§1.5 Equations of Lines and Planes

§1.5.1 Lines in 3D

§1.5.2 Planes

Chapter 2

Partial Derivatives

Chapter 3

Multiple Integrals

Chapter 4

Vector Calculus