PHYS103: General Physics I
Notes

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Contents

Introduction

This document aims to highlight the important content of the PHYS103 course in traditional notes format. These notes are completely open-source, which means anyone is allowed to use these notes for their own personal benefit without having to seek permission from myself.

While these notes are designed for the PHYS103 course, all of the content seen in these notes are equivalent to a one-semester calculus-based introductory physics class taken at many universities. The content in these notes might also have some overlap with the AP Physics C: Mechanics Course. As such, students in the AP Physics C: Mechanics course and students taken any calculus-based introductory physics course might still find the content provided in these notes useful.

Due to the open-source nature of these notes, anyone is allowed to contribute to improving these notes as they see fit. Since I am using GitHub to distribute these notes easily, you must request all changes through the repository website on GitHub, which you can find **here**. If you are interested in contributing to these notes, then there are a few ways that you can do so:

1. **Open and submit an issue on my GitHub repository:** I write all my notes in LaTeX, which is a typesetting language that is really helpful when it comes to typing and rendering math equations quickly and easily. If you do not know how to write LaTeXcode but are still interested in making a change to the notes, you can open an issue by going to the MathNotes repo on GitHub, and clicking on the button labeled "New Issue." From there, you can type out the change that you wish to see in the notes. It would be helpful if you would indicate what course you would like to see changed so that I can understand what you are referring to. I will then update the code to include your issue so that you don't have to worry about writing the code yourself.

2. **Create and submit a pull request:** If you know how to write LaTeX code and you understand how GitHub works, you can submit a pull request where you can write the code that you want to change yourself. I will then review the code and either submit the code to be incorporated into the notes OR provide some comments on your code if I wish for something to be different.

Thank you so much for using these notes. I hope that the information is provided in such a way that it can help you when reviewing content for you homework, quizzes, and exams and just in general when it comes to learning the content for the course. Happy studying!

Chapter 1

Measurement and Vectors

§1.1 Units and Unit Conversions

- In order to effectively understand the physical world around us and be able to make comparisons between physical situations, we need to be able to quanitify different physical situations
- There are many different ways that we can quantify the same unit of measure, and as such it is important to know how to convert between these different measurement types
- To convert between different units, find a relationship between the two units you wish to convert and express them as a 1-to-1 ratio. If you multiply this ratio by the measurement you wish to convert, you are effectively multiplying by one (it is a 1-to-1 ratio afterall) and therefore you are not changing the value of the expression
- To find relationships between different units, feel free to use Google (however it is encouraged to see if you can commit some of these relationships to memory as the class progresses)

§1.2 EXAMPLES: Unit Conversions

The following are some examples of how to convert different units of measurement. Some of these units are completely arbitrary and are simply used to show the process for converting between different units of measurement.

Problem 1. *Convert* $70\frac{miles}{hour}$ *to* $\frac{meters}{second}$

$$\frac{70\ miles}{hour}\cdot\frac{1.609\ km}{1\ mile}\cdot\frac{1\ hr}{3600\ sec}\cdot\frac{1000\ m}{1\ km}\approx 31.286\frac{m}{s}$$

Problem 2. *A 'gry' is $\frac{1}{10}$ of a line which is $\frac{1}{12}$ of an inch. A common length in publishing is a 'point' which is $\frac{1}{72}$ of an inch. Convert* $0.5\ grys^2$ *to* $points^2$.

$$\frac{0.5\ grys^2}{1} \cdot \frac{(0.1\ lines)^2}{(1\ gry)^2} \cdot \frac{(\frac{1}{12}\ inches)^2}{(1\ line)^2} \cdot \frac{(1 point)^2}{(\frac{1}{72}\ inches)^2} = 0.18\ points^2$$

Problem 3. *A lake has 120 acres of water and is 20 feet deep. How many kiloliters of water are in the lake?*

Begin by first computing the volume of the lake using the units given:

$$V = (120\ acres) \cdot (20\ ft) = 2400\ acres \cdot ft$$

Now we can convert the volume to kiloliters:

$$\frac{2400\ acres \cdot ft}{1} \cdot \frac{(1.609\ km)^2}{(1\ mi)^2} \cdot \frac{1\ mi^2}{640\ acres} \cdot \frac{1\ m}{3.28\ ft} \cdot \frac{1\ km}{1000\ m} = 0.00296\ km^3$$

$$\frac{0.00296\ km^3}{1} \cdot \frac{1\ mL}{1 cm^3} \cdot \frac{1\ L}{1000\ mL} \cdot \frac{(1000\ m)^3}{(1\ km)^3} \cdot \frac{(100\ cm)^3}{(1\ m)^3} \cdot \frac{1 kL}{1000 L} = 2.96 \times 10^6 kL$$

Chapter 2

Kinematics

Chapter 3

Forces

Definition 3.0.1: Newton's Second Law

To calculate the **net force** that an object is experiencing, use the following equation:

$$\sum \vec{F} = m\vec{a}$$

Or using calculus:

$$\sum \vec{F} = m\frac{d\vec{v}}{dt}$$

- Many times, we will be looking at forces in 2D, which means that the net force looks like the following:

$$\sum F = F_x\vec{i} + F_y\vec{j}$$

- Since net force has two components, we can solve force problems by analyzing each component of the net force independently because **the horizontal and vertical directions are independent of each other**

Definition 3.0.2: Newton's Second Law in 2D

To find the net force of an object that is experiencing forces in multiple directions, use the following formulas:

$$\sum F_x = m\vec{a_x}$$

$$\sum F_y = m\vec{a_y}$$

Or using calculus:

$$\sum F_x = m\frac{d\vec{v_x}}{dt}$$

$$\sum F_y = m\frac{d\vec{v_y}}{dt}$$

§3.1 Common Forces

There are many common forces that we will talk about in physics:

- **GRAVITY:** Also called the "weight force," this force is the force that an object experiences due to gravity and can be calculated by using $F_g = mg$ where m is the mass of the object and g is the acceleration due to gravity ($9.81\frac{\text{m}}{\text{s}}$ on Earth).
  - NOTE: Some people may also refer to this as the "weight force" and use F_W so be careful with notation as well as *consistent* with your notation
- **NORMAL:** The forces that the suface an object rests on. This forces always acts *perpendicular* to the surface the object is resting on.
- **FRICTION:** If an object is on the ground, the frictional force is the force that always acts opposite to the direction of motion (will talk about frictional force in a later section).
- **TENSION:** The force extertерd by a string or similar type of object that holds an object in place. No clear cut formula for finding this force, but drawing a free-body diagram and isolating the tension force will allow you to find the magnitude of the tension force.
- **SPRING:** The force a spring exterts on an object. Can find the magnitude of the force of a spring by using $F_s = -kx$ where k is the spring constant and x is the distance the string was stretched/compressed.
- **AIRDRAG:** The force of air on an object when an object is in free-fall or moving through the air.
- **CENTRIPETAL:** The force that keeps an object moving in a circle.
  - We've talked about centripetal acceleration, and if you have acceleration then you have force.

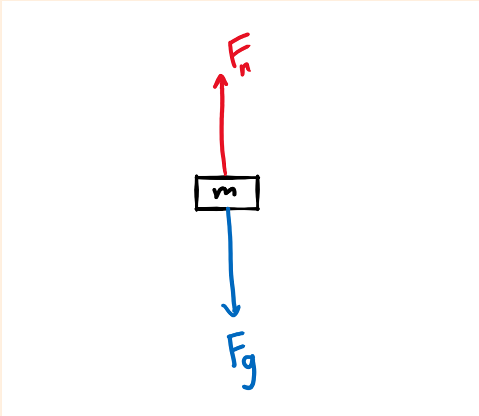
§3.2 Free Body Diagrams (FBD)

A **free body diagram** is a visual representation of all of the forces that are acting on a given object. We represent that object with a simple point and draw arrows (in the proper directions) that represent all of the forces that are acting on an object.

HOW TO DRAW A FREE BODY DIAGRAM:

1. Isolate the object of interest
2. Diagram the forces that act on the object
3. Sum the forces

Problem 4. *A block with mass m is sitting on a table. Draw the FBD of the block and calculate the net force that the block experiences.*



First we can algebraically analyze the forces in the x-direction. Since the object is not moving, its acceleration must be 0. There are also no forces being acted on the block in the x-direction. Therefore:

$$\sum F_x = m\vec{a_x} = 0$$

Now we can algebraically solve for the net force in the y-direction. Since the object is not moving, its acceleration must be 0. Therefore:

$$\sum F_y = m\vec{a_y} = 0$$
$$\sum F_y = F_N - F_g = F_N - mg$$

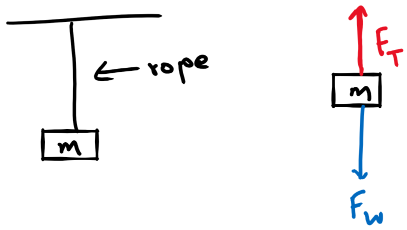
Threfore, the total net force of the block is as follows:

$$\sum F = 0\vec{i} + F_n - mg\vec{j}$$

§3.3 Other Posssible Scenarios and Their FBDs

While many systems are just on the ground, there are many other physical situations where objects are hanging or on an incline. Here are some examples of FBDs of the three common scenarios of objects that are not simply on the ground:

Hanging From a Rope

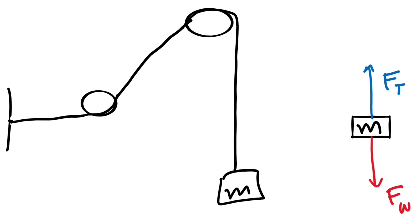


In the FBD pictured above, notice how there is no normal force because a normal force ony exists if an object is resting on a surface (in this case the object is airborne). We can also solve this algebraically:

$$\sum F = m\vec{a} = 0 \quad \text{(object is in equilibrium)}$$
$$\sum F_x = 0 \quad \text{(there are no forces in the } x \text{ direction)}$$
$$\sum F_y = F_T - F_g = F_t - mg$$

Keep in mind that F_T is the *tension force*, which is the force that the rope pulls on the object that it is attached to.

On a Pulley



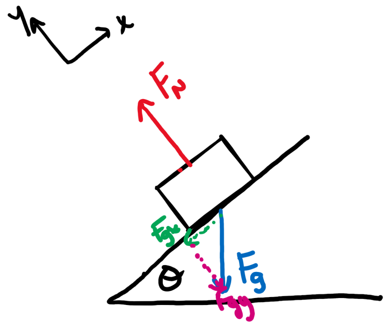
In the FBD pictured above, despite the rope changing direction multiple times across the pulley, it is still the same rope. Therefore, the tension force of the rope will be the same across the entire rope, and we can simply represent the tension force by one vector in the FBD. Algebraically solving this system results in the following:

$$\sum F = m\vec{a} = 0 \quad \text{(object is in equilibrium)}$$
$$\sum F_x = 0 \quad \text{(there are no forces in the } x \text{ direction)}$$
$$\sum F_y = F_T - F_g = F_T - mg$$

On A Ramp

Before drawing the FBD, it is important to note that the easiest way to solve this is to change your coordinate system to match your ramp/inclined plane. To do this, line the x-axis up with the ramp. By doing this, the acceleration is only facing in the direction of the ramp (VS having to find acceleration in both the x and y components and combining them together to find the magnitude).

The FBD looks like the following:



We can then solve this system algebraically:

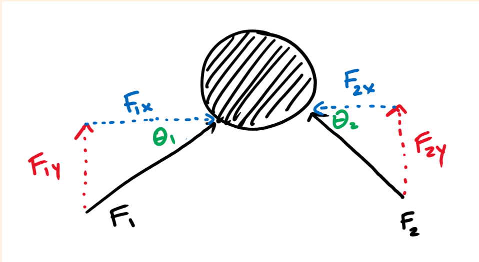
$$\sum \vec{F} = m\vec{a}$$

$$\begin{aligned}
\sum \vec{F}_x &= -F_{gx} \\
-F_{gx} &= m\vec{a} \\
-mg &= m\vec{a} \\
\frac{-mg}{m} &= \vec{a} \\
-g &= \vec{a}
\end{aligned}$$

$$\begin{aligned}
&\sum \vec{F}_y = 0 \quad \text{(object is in equlibrium)} \\
&F_N - F_{gy} = 0 \\
&F_N = F_{gy}
\end{aligned}$$

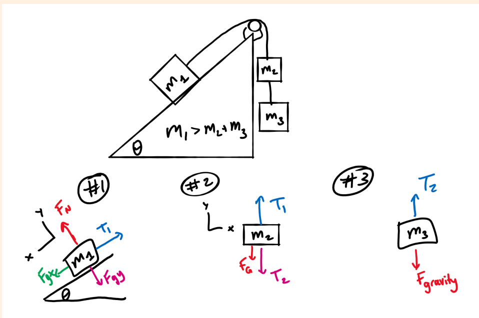
Example Problems

Problem 5. *A hockey puck experiences two forces, $\vec{F_1}$ and $\vec{F_2}$. Find the magnitude and angle of the acceleration of the hockey puck.*



$$\sum F_x = ma_x = F_1 \cos\theta_1 - F_2 cos\theta_2$$
$$\sum F_y = ma_y = F_1 \sin\theta_1 + F_2 \sin\theta_2$$
$$\vec{a} = a_x\vec{i} + a_y\vec{j}$$
$$|\vec{a}| = \sqrt{a_x^2 + a_y^2}$$
$$\phi = \arctan\left(\frac{a_y}{a_x}\right)$$

Problem 6. *A block of mass m_1 is attached to a pulley and sits on a slope with angle of inclination θ. Two additional masses m_2 and m_3 are hanging off the side of the slope and are also attached to the pulley. All three masses are attached to the same string. Find the tension force and the acceleration of the third block.*



For #1:

$$\sum F_x = m_1 a = m_1 g \sin\theta - T_1$$
$$\sum F_y = 0 = F_N - m_1 g \sin\theta$$

For #2:

$$\sum F_y = m_2 a = T_1 - T_2 - m_2 g$$

For #3:

$$\sum F_y = m_3 a = T_2 - m_3 g$$

§3.4 Frictional Force

Definition 3.4.1: Frictional Force

There are two different types of frictional forces:

- **STATIC FRICTION:** A frictional force that is opposing the object when a force is applied to an object, but an object is not moving. The magnitude of this force *changes with the magnitude of the force being applied.*

$$F_f \leq \mu_s F_N$$

INSERT A GRAPH HERE

- **KINETIC FRICTION:** A frictional force that opposes the object while the object is in motion. This force *remains constant.*

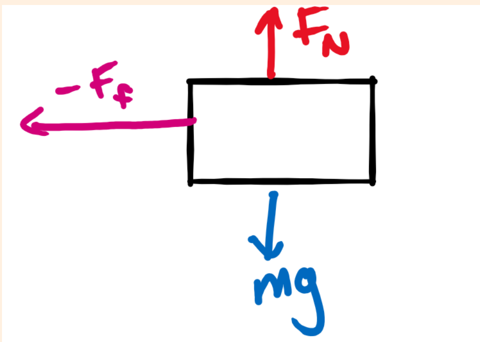
$$F_f = \mu_k F_N$$

In all cases, μ is the coefficient of friction (which is a constant that varies depending on the surface that the object is on).

Another thing to note is that $\mu_k < \mu_s < 1$.

If the system is at rest or has no acceleration, use **static coefficient of friction**. If the system is accelerating, use the **kinetic coefficient of friction**.

Problem 7. *A car is traveling at a velocty of* $1.6\frac{m}{s}$ *with a mass of 1500 kg for a distance of 50 m. Assume that the acceleration is constant. Find the coeffeficient of kinetic friction.*



First, find the sum of the forces in the y-direction:

$$\sum F_y = 0$$
$$F_N - mg = 0$$
$$F_N = mg$$

Now, find the sum of the forces in the x-direction:

$$\sum F_y = ma$$
$$-F_f = ma$$
$$\mu_k F_N = ma$$
$$\mu_k mg = ma$$
$$\mu_k = -\frac{a}{g}$$

Using kinematics equations, we can solves for a:

$$v_f^2 = v_i^2 + 2a\Delta x$$
$$a = \frac{v_f^2 - v_i^2}{2\Delta x}$$
$$a = \frac{\left(0\frac{m}{s}\right)^2 - \left(1.\bar{6}\frac{m}{s}\right)^2}{2 \cdot 50\ m}$$
$$a = -7.7\frac{m}{s^2}$$

Now, we can solve for μ_k:

$$\mu_k = -\frac{a}{g}$$
$$\mu_k = -\frac{-7.7\frac{m}{s^2}}{9.8\frac{m}{s^2}}$$
$$\mu_k = ANSWER$$

§3.5 Air Drag

Definition 3.5.1: Air Drag

$$F_{AD} = \frac{1}{2}C\rho A|v|^2$$

C = coefficient of "air" friction
ρ = the density of the air
A = the cross-sectional area of the object
v = the velocity of the object (SCALAR QUANTITY)

Problem 8. *A bowling ball with mass of 5 kg and radius of 4 cm falls through the air with a C value of 1.2 and air density of 1.23* $\frac{kg}{m^3}$*. What is the terminal velocity of the bowling ball?*

$$\sum F_y = 0 = F_{AD} - mg$$
$$F_{AD} = mg$$
$$\frac{1}{2}C\rho A v^2 = mg$$
$$v = \sqrt{\frac{2mg}{C\rho A}}$$
$$v = \sqrt{\frac{2(5\ kg)\left(9.81\frac{m}{s^2}\right)}{(1.2)\left(1.23\frac{kg}{m^3}\right)(0.04\ m)^2\pi}}$$
$$v \approx 115\frac{m}{s^2}$$

§3.6 Centripetal Forces

- Recall that $a_{centripetal} = \frac{|v|^2}{R}$
- A **centripetal force** is any force that causes a system to begin rotating in a circle
- You would still use Newton's second law, but the acceleration used in Newton's Second Law will be the *centripetal acceleration*

Problem 9. *You are riding on a ferris wheel and your mass is 50 kg. The radius of the ferris wheel is 25 m. You go around the ferris wheel at a rate of 1 revolution per minute. Find* F_N *a) at the bottom of the ferris wheel and b) at the top of the ferris wheel.*

a) (top)

$$\sum F_T = ma_c = -N_T + mg$$
$$m\left(\frac{|v|^2}{R}\right) = -N_T + mg$$
$$N_T = mg - \frac{mv^2}{R}$$

b) (bottom)

$$\sum F_B = ma_c = N_B - mg$$
$$N_B = m\left(g + \frac{v^2}{R}\right)$$

Problem 10. *A car with masss m is going around a turn on an inclined plane inclined at an angle θ.* ***DRAW A FBD*** *and determine the friction force being applied.*

Sum of the forces in the y-direction:

$$\sum F_y = ma_y = F_N - mg\cos\theta$$
$$\sum F_y = ma\sin\theta = F_N - mg\cos\theta$$
$$F_N = m\left(\frac{v^2}{R}\sin\theta - g\cos\theta\right)$$

Sum of the forces in the x-direction:

$$\sum F_x = ma_x = F_f + mg\sin\theta$$
$$\sum F_x = ma\cos\theta = F_f + mg\sin\theta$$
$$F_f = m(a\cos\theta - g\sin\theta)$$
$$F_f = m\left(\frac{v^2}{R}\cos\theta - g\sin\theta\right)$$
$$\mu F_N = m\left(\frac{v^2}{R}\cos\theta - g\sin\theta\right)$$

NOW GO BACK AND TRY THIS PROBLEM AGAIN BUT WITHOUT TILTING THE AXES AND PUT THIS INTO YOUR NOTES

Problem 11. *If the static coefficient of friction is 0.7, then how fast can a 5 m merry-go-round spin by your slide.*

$$\sum F_y = 0 = F_N - mg$$
$$F_N = mg$$

$$\sum F_x = ma = F_f$$
$$\mu_s F_N = ma$$
$$\mu_s mg = ma$$
$$\mu_s g = \frac{v^2}{R}$$
$$v = \sqrt{\mu_s g R}$$

Chapter 4

Work, Energy, and Momentum

§4.1 Introduction to Work

Definition 4.1.1

WORK: How much energy it takes to do a certain physical action.

$$W = \Delta E = \vec{F} \cdot \vec{\Delta x}$$

W = Work done (SI Units: J = N m)
ΔE = Change in energy
$\vec{F}$ = the force applied
Δx = the displacement over which the object was applied the given force $\vec{F}$

Problem 12. *A couch is pushed with a force 25 N over a distance of 5 m. Calculate the work that is applied to the couch.*

$$W = F_A \cdot \Delta x$$
$$W = (25\ N)(5\ m)$$
$$W = 100\ J$$

Problem 13. ***SEE ATTACHED FIGURE***

a) How much energy is expended by the applied force to move the couch 5 m?
b) How much energy does the frictional force expend to move the couch 5 m?
To solve part a):

$$W_A = F_{Ax} \cdot \Delta x$$
$$W_A = F \cos\theta \cdot \Delta x$$
$$W_A = (50\ N)\cos(25°)(5\ m)$$
$$W_A = 227\ J$$

To solve part b):

$$W_f = -F_f \cdot \Delta x$$
$$W_f = -\mu_k F_N \cdot \Delta x$$
$$W_f = -\mu_k (mg + F_A \sin\theta) \cdot \Delta x$$
$$W_f = -(0.15)((20\ kg)(9.81\frac{m}{s^2}) + (50\ N)\sin(25°))W_f \quad = VALUE$$

§4.2 Energy

Energy comes in many different forms. The main forms of energy (as well as the proofs to get their respective equations) are listed below:

Definition 4.2.1: Kinetic Energy

If an object is moving, the energy that the object expends is equal to the **kinetic energy**.

$$KE = \frac{1}{2}mv^2$$

KE = the kinetic energy expended
m = mass of the object
v = velocity of the object at the given moment where you wish to find the energy

Proof. PROOF OF KINETIC ENERGY HERE ■

Definition 4.2.2: Potential Energy

$$PE = mgh$$

Proof. PROOF OF POTENTIAL ENERGY HERE ■

Definition 4.2.3: Spring Energy

$$SE = \frac{1}{2}kx^2$$

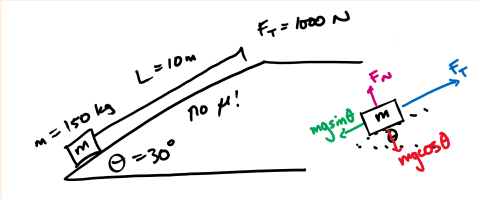
Proof. PROOF OF SPRING ENERGY HERE ■

NOTE: Energy is *positive* if energy is being stored and *negative* if energy is being released.

§4.2.1 The Usefulness of Energy and Work

While we have an equation for the work if we know the applied forces, if we know other elements about the system (how fast) [FINISH THIS LATER]

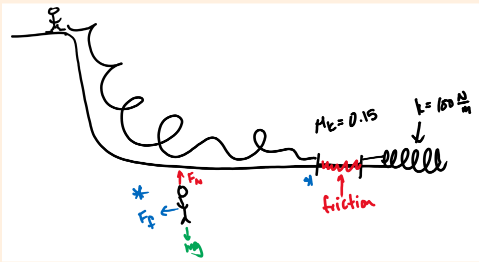
Problem 14. *ADD PROBLEM TEXT LATER*



While this problem could be solved using kinematics and Newton's second law, we can also use the Work-Energy theorem.

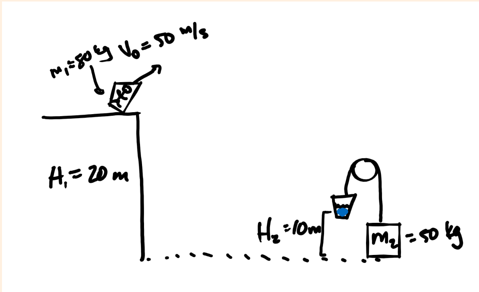
$$
\begin{aligned}
W_{in} &= \Delta KE + \Delta PE \\
F_T \cdot L &= \frac{1}{2}m(v_f - v_i)^2 + mg(h_f - h_i) \\
F_T \cdot L &= \frac{1}{2}mv_f^2 + mgh_f \\
v_f &= \sqrt{\frac{2(F_T L - mgh_f)}{m}} \\
v_f &= \sqrt{\frac{2(F_T L - mg(L\sin\theta))}{m}}
\end{aligned}
$$

Problem 15. *A skiier starts at the top of a mountain and is attached a spring at the bottom of the cliff. The skiier will also experience friction for 15m at the bottom of the slope right before the spring. How far was the spring compressed when the skiier reaches the bottom of the slope?*



$$
\begin{aligned}
W_{in} = 0 &= \Delta PE + \Delta E_s + \Delta E_T \\
0 &= mg(h_f - h_i) + \frac{1}{2}k(x_f - x_i)^2 + \mu_k mgD \\
0 &= -mgh_i + \frac{1}{2}kx_f^2 + \mu_k mgD \\
x_f &= \sqrt{\frac{(mgh_i - \mu_k mgD)^2}{k}} \\
x_f &= \sqrt{\frac{((70\ kg)(9.81\frac{m}{s^2})(50\ m) - (0.15)(70\ kg)(9.81\frac{m}{s^2})(15\ m))^2}{150\frac{N}{m}}}
\end{aligned}
$$

Problem 16. *A stunt artist was launched out of a cannon on a 20m pedestal. He will land in a bucket attached to a pulley. The pulley also has a 50kg mass attached to it, and the bucket is 10m above the ground. Assume the bucket has no mass and the pulley exerts no friction on the string wrapped around the pulley. What is the speed of the stunt artist right when he lands in the bucket?*



$$W = 0 = \Delta KE_1 + \Delta PE_1 + \Delta KE_2 + \Delta PE_2$$

$$0 = \frac{1}{2}m(v_{f1} - v_{i1})^2 + \frac{1}{2}m_2(v_{f2} - v_{i2})^2 + m_1 g(H_{f1} - H_{i1}) + m_2 g(H_{f2} - H_{i2})$$

ANSWER: $41.3\frac{m}{s}$

§4.3 Work and Calculus

- Sometimes, the force being applied to an object over a given distance is variable
- If you are given an equation for what the force is (dependent on distance), then $W = \int_{x_i}^{x_f} F(x)dx$

Problem 17. *If the force applied to an object (as a function of distance) is* $F(x) = 5 - 2x$*, what is the work done on the object if the object moves 3 m?*

$$W = \int_{x_i}^{x_f} F(x)\, dx$$

$$W = \int_0^3 5 - 2x\, dx$$

$$W = (5 - 2(3)) - (5 - 2(0)) = -1 - 5W \quad = -6\ J$$

§4.4 Power

Definition 4.4.1: Power

Power: How much energy is expended over a given time interval.

$$P = \frac{\Delta E}{\Delta t} = \frac{W}{t}$$

SI Units: Watts (W)

Definition 4.4.2: Instantaneous Power

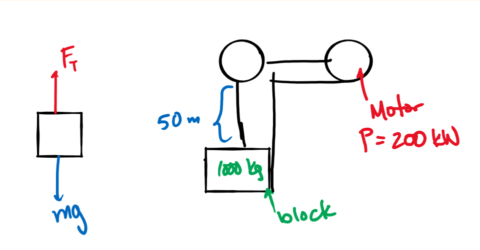
If power is rewritten as a function of time $P(t)$, then:

$$E = \int_{t_1}^{t_2} P(t)dt$$

Likewise, if you want to find the instantaneous power, then:

$$P(t) = \frac{dE}{dt}$$

Problem 18. *A block of mass 1000 kg is attached to a pulley at the top of a building. The pulley is attached to a motor that uses 200 kW. What is the final velocity of the block the moment it reaches the top of the building?*



$$P = \frac{\Delta E}{\Delta t}$$
$$P = \frac{\Delta KE + \Delta PE}{\Delta t}$$
$$P = \frac{\frac{1}{2}mv^2 + mgh}{\Delta t}$$
$$v_f = \sqrt{\frac{2(-mgh + \Delta TP)}{m}}$$

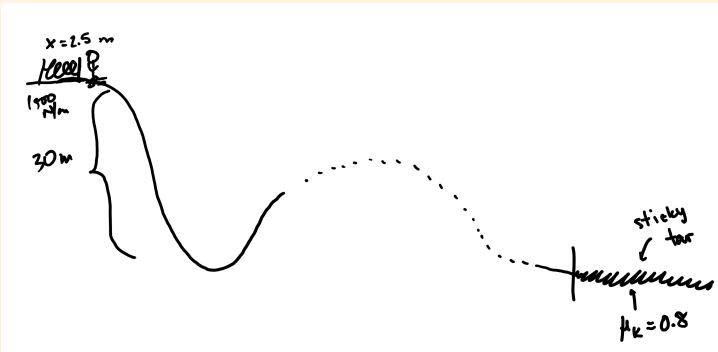
Problem 19. *A truck drives up a ramp with an angle of inclination of* $15°$*. If the truck drives 200 meters in 20 seconds, what is the power of the truck (in kW) after the truck has moved 200 meters? Assume that 10% of the total energy is lost to the environment.*

$$
\begin{aligned}
P &= \frac{\Delta E}{t} \\
P &= \frac{\Delta KE + \Delta PE}{t} + 10\%\ energy \\
P &= \frac{\frac{1}{2}mv^2 + mgh}{t} + 10\%\ energy \\
P &= \left[\frac{\frac{1}{2}m\left(\frac{\Delta x}{t}\right)^2 + mgL\sin\theta}{t}\right]\frac{1}{0.9} \\
P &= 98.3\ kW
\end{aligned}
$$

Problem 20. *A person with mass 80 kg is on a scooter on top of a hill that is 30 m tall. The person is pushed off by a spring with a spring coefficient of 1500 N/m. The spring is compressed 2.5 m. The person will then go down a hill and launch into the air, then land in a pit of tar with a coefficient of kinetic friction of 0.8.*

a) Find the velocity of the person right after the spring pushes him off.

b) Find the distance the person will travel after he lands in the tar.



a)

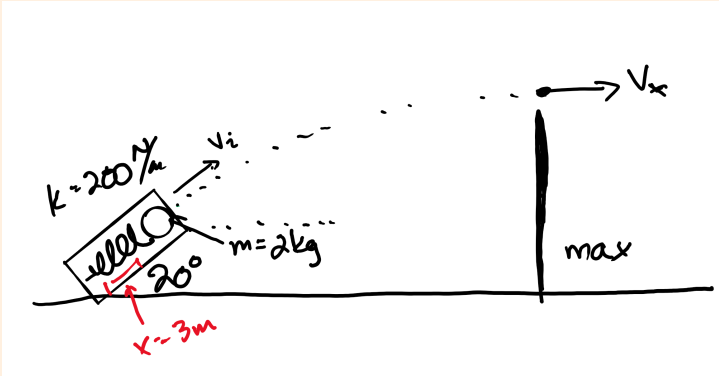
$$
\begin{aligned}
W &= \Delta KE + \Delta E_s \\
0 &= \frac{1}{2}mv_i^2 + \frac{1}{2}k(-x_i)^2
\end{aligned}
$$

$$
\begin{aligned}
v_i &= \sqrt{\frac{kx_i^2}{m}} \\
v_i &= 10.8\frac{m}{s}
\end{aligned}
$$

b)

$$
\begin{aligned}
W &= -\frac{1}{2}m(v_i)^2 + mg(-h_i) + F_f d \\
0 &= -\frac{1}{2}mv_i^2 + -mgh_i + \mu_k F_N d \\
d &= \frac{mgh_i + \frac{1}{2}mv_i^2}{\mu_k mg} \\
d &= 44.9\ m
\end{aligned}
$$

Problem 21. *A ball with mass 2 kg is placed inside a cannon that is launched using a spring with spring constant 200 N/m. The spring is compressed 2 m and is compressed all the way to the ground (so that $PE_i = 0$). The cannon is at an angle of $20°$. What is the maximum height that the ball is launched?*



We first need the velocity right when the ball leaves the spring. We can set up a mini Work-Energy problem:

$$W = \Delta KE + \Delta PE + \Delta E_s$$
$$0 = \frac{1}{2}mv_i^2 + mgL\sin\theta + \frac{1}{2}k(-(L^2))$$
$$\frac{1}{2}mv_i^2 = \frac{1}{2}kL^2 - mgl\sin\theta$$
$$v_i = \sqrt{\frac{2\left(\frac{1}{2}kL^2 - mgl\sin\theta\right)}{m}}$$

Now we can use the x-component of that velocity to get the final velocity in the overall Work-Energy problem.

$$W = \Delta KE + \Delta PE + \Delta E_s$$
$$0 = \frac{1}{2}mv_x^2 + mgh_f + \frac{1}{2}k(-(x^2))$$
$$mgh_f = \frac{1}{2}mv_x^2 - \frac{1}{2}kx^2$$
$$h_f = \frac{\frac{1}{2}mv_x^2 - \frac{1}{2}kx^2}{mg}$$

§4.5 Non-Conservative Energy

Not all energy is conserved and remained in the system - sometimes the energy is lost to the environment as thermal energy and sound energy

Problem 22. *A skiier skiis down a ramp in two parts. On the first part, the angle of elevation is $30°$ and the kinetic coefficient of friction is 0.2. Once the skiier reaches the bottom of the slope, the new coefficient of kinetic friction is 0.3. The* *slope is 70m tall.*

a) What is the velocity of the skiier at the bottom of the slope?

b) What is the acceleration of the skiier while he is on the slope?

c) What is the distance the skiier will slide once he hits the bottom of the slope?

To solve part a):

$$\begin{aligned}
W &= \Delta KE + \Delta PE + \Delta E_s + E_{TH} \\
0 &= \frac{1}{2}m(v_f^2 - 0) + mg(0 - h_i) + F_f \cdot \frac{h_i}{\sin\theta} \\
v_f^2 &= 2\left(gh_i - g\cos\theta\mu_k\frac{h_i}{\sin\theta}\right) \\
v_f &= \sqrt{2\left(gh_i - g\cos\theta\mu_k\frac{h_i}{\sin\theta}\right)} \\
v_f &= \sqrt{2gh_i\left(1 - \frac{\mu_k}{\tan\theta}\right)} \\
v_f &= 36.374\frac{m}{s}
\end{aligned}$$

To solve part b):

$$\begin{aligned}
v_f^2 &= v_i^2 + 2aL \\
a &= \frac{v_f^2}{2L} \\
a &= \frac{m}{s^2}
\end{aligned}$$

To solve part c):

$$\begin{aligned}
W &= \Delta KE + \Delta PE + \Delta E_s + E_{TH} \\
0 &= \frac{1}{2}m(-v_i^2) + F_f D \\
D &= \frac{v_i^2}{2\mu_k g} \\
D &= 240.619\ m
\end{aligned}$$

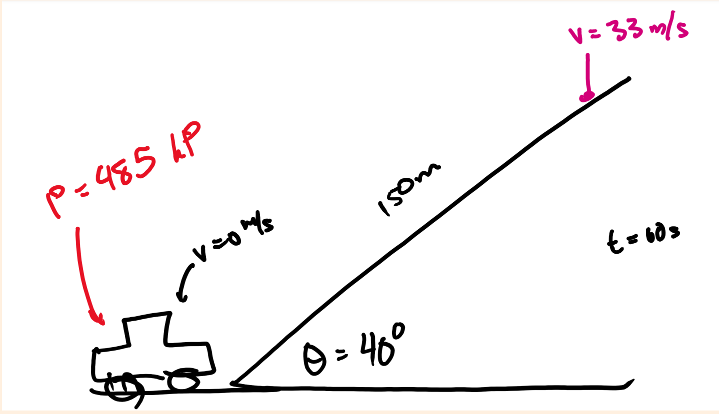
Problem 23. *A rock with mass 10kg falls 5m onto a spring with spring constant 100 N/m. What is the the distance that the spring compresses due to the rock falling onto the spring?*

INCLUDE FIGURE HERE

$$\begin{aligned}
W &= \Delta KE + \Delta PE + \Delta E_s + E_{TH} \\
0 &= \Delta PE + \Delta E_s \\
0 &= mg(0 - (h + y)) + \frac{1}{2}k(y^2 - 0)
\end{aligned}$$

FINISH THIS PROBLEM LATER

Problem 24. *INSERT TEXT ABOUT THE PROBLEM HERE*



$$W = \Delta KE + \Delta PE + \Delta E_s + E_{TH}$$
$$E_{TH} = W - \Delta KE - \Delta PE$$
$$E_{TH} = Pt - \Delta KE - \Delta PE$$
$$E_{TH} = Pt - \frac{1}{2}mv_f^2 - mgd\sin\theta$$

§4.6 Center of Mass

- A lot of times we are looking at an entire system with a bunch of different objects spread throughout
- If this is the case, we want to know where all of the mass for every single object (on average) is concentrated
- Finding the point where most of the mass is cnocentrated is called the **center of mass**

Definition 4.6.1: Center of Mass (1D)

If there are multiple masses $m_1, m_2 \ldots, m_n$ that are all in 1D space, then the center of mass can be determined using the following equation:

$$m_c = \frac{\sum_{i=0}^{n} m_i x_i}{\sum_{i=0}^{n} m_i}$$

Where x_i is the position value for the indicated mass.

Definition 4.6.2: Center of Mass (2D)

If there are multiple masses $m_1, m_2 \ldots, m_n$ that are all in 2D space, then the center of mass can be determined using the following equations:

$$m_{xc} = \frac{\sum_{i=0}^{n} m_i x_i}{\sum_{i=0}^{n} m_i}$$
$$m_{yc} = \frac{\sum_{i=0}^{n} m_i y_i}{\sum_{i=0}^{n} m_i}$$

Where x_i is the x-position of the indicated mass and y_i is the y-position of the indicated mass.

§4.7 Introduction to Momentum

Definition 4.7.1: Momentum

$$\vec{p} = m\vec{v}$$

momentum is a vector (it's simply a scaled version of velocity) and the SI units are $\mathrm{kg}\frac{\mathrm{m}}{\mathrm{s}}$.

- linear momentum is **always conserved**
- kinetic energy is only conserved in an **elastic collision**
  - *ELASTIC COLLISION*: Where two bodies bounce off of each other
  - *INELASTIC COLLISION:* When two bodies stick together after the collision
- FUN FACT: We can get Newton's second law through differentiating the definition of momentum because $\frac{d}{dt}p = \frac{d}{dt}(mv) \implies F_{net} = ma$

Definition 4.7.2: Impulse

$$\vec{J} = \vec{F}t = \Delta\vec{p} = m(\vec{v}_f - \vec{v}_i)$$

impulse is how much is applied to an object over a given period of time

- NOTE: Impulse is the **area under a Force vs time graph**

Problem 25. *If a baseball comes in to the left at $50\frac{m}{s}$ and gets hit back to the right with a velocity of $80\frac{m}{s}$ at an angle of $30°$. If the right direction is positive, what is the time interval during which the collision of the ball occurred?*

$$F_{avg}t = m(v_f - v_i)$$
$$F_{avg} = \frac{m(v_f - v_i)}{t}$$
$$F_{avg} = \frac{m < v_{fx} - v_{ix}, v_{fy} - v_{iy} >}{t}$$
$$F_{avg} = 328.6\vec{i} + 80\vec{j} = 251.6\angle 18.5°$$

Problem 26. *A cue ball comes and towards the 6-ball with a speed of $2\frac{m}{s}$. The 6-ball goes down at an angle of $30°$ with a velocity magnitude of $3\frac{m}{s}$. What is the final velocity of the cue ball after the collision?*

$$\begin{aligned}
\vec{p}_i &= \vec{p}_f \\
m\vec{v}_{ci} &= m\vec{v}_{cf} + m\vec{v}_6 \\
v_{cf} &= v_{ci} - v_6 \\
v_{cf} &= 2\vec{i} - (2.6\vec{i} - 1.5\vec{j}) \\
v_{cf} &= -0.6\vec{i} + 1.5\vec{j} \\
v_{Cf} &= 1.62\angle 62.5° \frac{m}{s}
\end{aligned}$$

NOTE: You can also choose to solve for the momentums in each direction independently (so say that $p_{xi} = p_{xf}$ and then also $p_{yi} = p_{yf}$ and then solve both individually).

Chapter 5

Rotational Kinematics, Torque, and Angular Momentum

§5.1 Rotational Kinematics

- While objects can move in a straight line, objects can ALSO move in a circle
  - Either an object is orbiting some point or something spins around some center
- Instead of having position, velocity, and acceleration, we will have *new* terms for moving in a circle instead
  - **Angular position (θ):** How much the object has rotated around the positive x-axis
  - **Angular velocity (ω):** How fast an object is going in a given direction at any point in the circle
    - $\frac{d\theta}{dt} = \omega(t)$
    - $\theta(t) = \int \omega \, dt$
  - **Angular acceleration (α):** How fast an object changes it's velocity at any point in the circle of motion
    - $\frac{d\omega}{dt} = \alpha(t)$
    - $\omega(t) = \int \alpha \, dt$
- the kinematics equations from translational (standard) motion are the EXACT SAME equations in rotational kinematics, but we simply use the rotational kinematics variables instead of the translational kinematics variables

Definition 5.1.1: Rotational Kinematics Equations

To calculate different values of

$$\omega_f = \omega_i + \alpha t$$
$$\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2$$
$$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$$

- it is important to note that values are **positive** if things move *in the counterclockwise direction* and values are **negative** if things move *in the clockwise direction*
- If you know the distance between the object moving in rotation and the center of rotation (in other words: the radius of the circle of rotation), then you can relate the rotational kinematics values with the translational kinematics values

Definition 5.1.2: Rotational Kinematics ↔ Translational Kinematics

$$S = \Delta\theta R$$

Where S is the arc that the object travels around through a given angle θ and R is the radius (the distance between the object and the center of rotation).

$$v = \omega R$$

Where R is the radius (the distance between the object and the center of rotation).

$$a_{tangential} = \alpha R$$

Where R is the radius (the distance between the object and the center of rotation). Note that this is the acceleration that is PENPENDICULAR to the acceleration that points towards the center of the circle.

$$a_{centripetal} = \frac{v^2}{R} = \frac{\omega^2 R^2}{R} = \omega^2 R$$

Where R is the radius (the distance between the object and the center of rotation). Note that this is the acceleration that points towards the CENTER of the circle.

Problem 27. *A grinder rotates at 850RPM. Once the grinder turns off, it stops 3 minutes later. What is the angular acceleration and what is the final anglular position of the grinder?*

First, convert RPM to radians/second:

$$850\frac{rev}{min} \cdot \frac{2\pi \ radians}{1 \ rev} \cdot \frac{1 \ minute}{60 \ seconds} = 28.\bar{3}\frac{radians}{s}$$

Now find angular acceleration:

$$\alpha = \frac{\Delta\omega}{t} = \frac{\omega_f - \omega_i}{t} = \frac{-28.3\frac{radians}{s}}{60\ s} = -0.49\frac{radians}{s^2}$$

Now find the final angular position:

$$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$$
$$\theta_f = \frac{\omega_f^2 - \omega_i^2}{2\alpha} + \theta_i$$
$$\theta_f = \frac{-(28.\bar{3}\frac{radians}{s})^2}{2(-0.49\frac{radians}{s^2})}$$
$$\theta_f = 817.235\ radians$$

Problem 28. *An object rotates and has a function that defines the angluar acceleration to be $\alpha(t) = 5t^3 - 4t$. You know that the initial rotational velocity is 5 $\frac{radians}{s}$ and the initial angular position is 2 radians.*

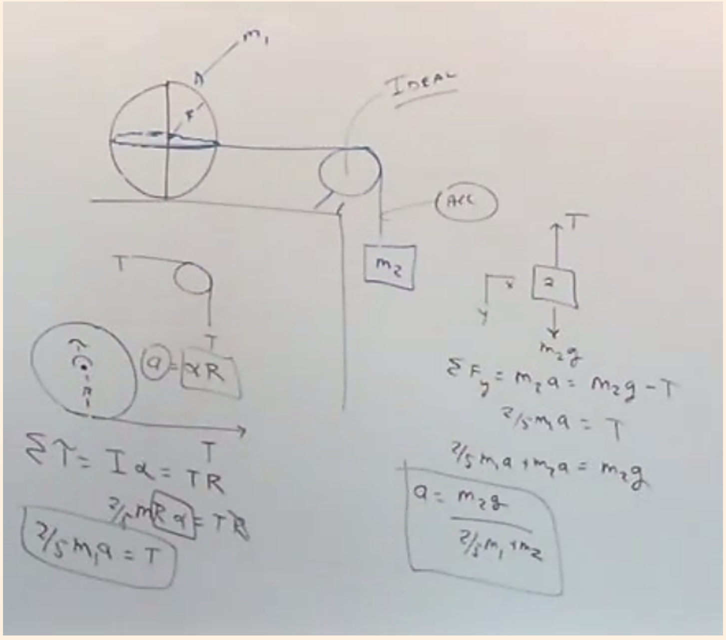
$$\omega(t) = \int_0^t \alpha(t)\, dt$$
$$\omega(t) = \int_0^t 5t^3 - 4t\, dt$$
$$\omega(t) = \frac{5}{4}t^4 - 2t^2 + 2$$

§5.2 Moment of Inertia

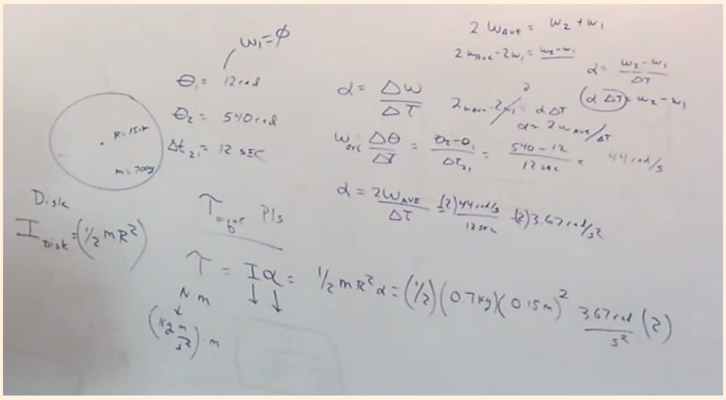
Intertia is the distribution of mass around the object's center of mass. This term allows us to differentiate between a disk and a chair that are both 5kg because up to this point we have tretaed those two objects as the same object.

§5.3 Force, Energy, and Momentum in Rotation

- In the **linear world**, $KE = \frac{1}{2}mv^2$, but in the **rotational world** $KE = \frac{1}{2}I\omega^2$
- In the **linear world**, $F = ma$, but in the **rotational world** $\tau = I\alpha$
  - It shuold be noted that the above is Newton's Second Law just applied to rotation. Individual torques are $\tau_A = F_A r \sin\theta$ where r is the distance between the center of rotation and the point where the force is applied.
- In the **linear world**, $W = F \cdot d$, but in the **rotational world** $W = \tau \cdot \theta$
- In the **linear world**, $P = Fv$, but in the **rotational world** $P = \tau\omega$
- In the **linear world**, $p = mv$, but in the **rotational world** $L = I\omega$



Problem 29.



Problem 30.

§5.4 Smooth Rolling Motion

$$\sum F_x = ma_{COM} = F_f$$
$$\sum \tau_{NET} = I\alpha = \tau_{APP} - \tau_f = \tau_{APP} - F_f R$$

§5.5 Newton's Second Law for Rotation

$$\tau = I\alpha$$
$$F_f R = I\alpha$$

§5.6 Angular Momentum and Impulse

- We've been used to *linear momentum* but there is also **angular momentum** that functions under the same properties

Definition 5.6.1: Angular Momentum

$$L = I\omega$$

In every case in this class, **angular momentum will always be conserved**

- Angular momentum is the reason that an ice skater changes their speed
  - When the skater holds out their arms, then the

Problem 31 (Merry Go Round)**.**

$$\begin{aligned} L_i &= L_f \\ I_m\omega_i &= I_m\omega_f + I_{\|}\omega_f \\ \omega_f &= \frac{I_m\omega_i}{I_m + I_{\|}} \end{aligned}$$

Problem 32 (Merry Go Round with Dog)**.**

$$\begin{aligned} L_i &= L_f \\ I_m\omega_i + I_D\omega_i &= I_m\omega_f + I_D(\omega_f - \omega_D) \end{aligned}$$

- Additionally, we can get an angular impulse similarly to how we get a linear impulse

Definition 5.6.2: Angular Impulse

$$J_\angle = \tau_{AVG}\Delta T = I\Delta\omega$$

Problem 33 (Grindstone Problem)**.**

$$\begin{aligned} L_i &= L_f \\ I_1\omega_i &= I_1\omega_f + I_2\omega_f \\ I_1\omega_i &= \omega_f(I_1 + I_2) \\ \omega_f &= \frac{I_1}{I_1 + I_2}\omega_i \\ \omega_f &= \frac{0.5M_1R_1^2}{0.5M_1R_1^2 + 0.5M_2R_2^2}\omega_i \end{aligned}$$

Chapter 6

Statics

- In reality, you can sum forces but you can't really sum torques, so instead we sum *moments* of torque at a given point
  - This is all about statics

Problem 34 (Beeg Statics Problem)**.**

$$\sum F_x = 0 = T_2 - T_{1x}$$
$$\sum F_y = 0 = T_1 y - mg + N - W$$

$$\sum M_P = 0 = -mg\frac{L}{2} + N\frac{3}{4}L + -WL$$
$$\sum M_P = 0 = -mg\frac{1}{2} + N\frac{3}{4} + -W$$
$$N = \frac{4}{3}(mg + N)$$

$$T_{1y} = mg - N + W$$

- **INDETERMINANT STRUCTURE:** Has more supports than it needs to be stable
- **NEWTON PHAPSON:**
- **GUASS-DIEDEL:**
- **SHEAR:** There is a cutting motion occuring (you may not actually cut through something)
- **COMPRESSION:** Push (opposite of tension which is pull)
- **ELASTIC DEFORMATION:** We bend something, but when we let go of it then it comes back to its original shape (example: rubber band)
- **INELSATIC DEFORMATION:** You start off with a shape and when you stretch it the object does not return to its oroginal shape (permanently longer)
  - The distance that the object gets stretched by is called the "creep"

Problem 35 (The Two Scales Problem)**.**

$$\sum F_x = 0 = 0$$
$$\sum F_y = 0 = N_1 + N_2 - m_1 g - m_2 g$$

$$\sum M_{N_1} = 0 = -m_1 g \frac{L}{4} - m_2 g \frac{L}{2} + N_2 L$$
$$\sum M_{N_1} = 0 = -m_1 g \frac{1}{4} - m_2 g \frac{1}{2} + N_2$$
$$N_2 = \frac{m_1 g}{4} + \frac{m_2 g}{2}$$

Problem 36 (Tom on the Ladder)**.**

$$\sum F_x = R_W - F_f$$
$$\sum F_y = N_g - mg - W_T$$
$$\sum M_g = 0 = R_W L \sin\theta + W_T L \frac{2}{3} \cos\theta + mg \frac{L}{2} \cos\theta$$
$$\sum M_g = 0 = R_W \sin\theta + W_T \frac{2}{3} \cos\theta + mg \frac{1}{2} \cos\theta$$
$$R_W = \frac{\frac{2}{3} W_T \cos\theta + \frac{mg}{2} \cos\theta}{\sin\theta}$$

Problem 37 (QUIZ #2 QUESTION)**.** ***DRAW THE PROBLEM HERE***

$$\sum F_x = 0 = R_x - T_x$$
$$\sum F_y = 0 = R_y - mg - T_y$$

$$\sum M_R = 0 = -mg \frac{L}{2} \cos\theta = T_y L \cos\theta + T_x \sin\theta$$
$$= -\frac{mg\cos\theta}{2} - T\sin\phi\cos\theta + T\cos\phi\sin\theta$$
$$= \frac{\frac{mg\cos\theta}{2}}{\cos\phi\sin\theta - \sin\phi\cos\theta}$$

Chapter 7

Fluids

§7.1 General Fluid Statics Terms

- **FLUID:** A substance that accompanies chips and salsa
- **DENSITY:** $m = \rho V$
  - The denisity of water is $\rho_{water} = 1000 \frac{\mathrm{kg}}{\mathrm{m}^3} = 1 \frac{\mathrm{g}}{\mathrm{cm}^3}$
- **COMMON VOLUME UNITS:** Liter, Cubic Meter
  - 1 Liter $= 1000\ \mathrm{cm}^3$
  - 1000 Liter $= 1\ \mathrm{m}^3$
- **PRESSURE:** $P = \frac{F}{A_\perp}$
  - PRESSURE AT SEA LEVEL: 14.7 psi $= 1.01 \times 10^5$ Pa $= 101$ KPa

Problem 38 (Basic Pressure Problem)**.** *A room is* $3.5\ m \times 4.2\ m \times 2.4\ m$*. If the density of air is* $1.21 \frac{kg}{m^3}$*, what is the force weight in the room and the pressure at the floor of the room?*

$$\begin{aligned} W &= mg \\ W &= \rho_{air} V_{room} g \\ W &= (1.21)(3.5)(4.2)(2.4)(9.81) \\ W &= 418\ N \end{aligned}$$

$$\begin{aligned} P &= \frac{F}{A} \\ P &= \frac{\rho_{air} lwhg}{wl} \\ P &= \rho_{air} hg \\ P &= (1.21)(2.4)(9.81) \\ P &= 28.44\ Pa \end{aligned}$$

Definition 7.1.1: General Pressure Equations

The pressure at a given reference point R is given by the following equation:

$$P = P_R \pm \rho H g$$

If you want to find the **absolute pressure** at a point R, then you can use the following equation:

$$P = P_{atm} \pm \rho H g$$

The **guage pressure** is the pressure experience without considering atmospheric pressure, which is given by the following equation:

$$P - P_{atm} = \rho H g$$

Problem 39. *There is a pipe that is in the shape of a square D without the top. Water is put in the pipe so that the water is at the same height on both sides of the pipe. Oil is placed on the left side of the pipe and the height of the water rises on the right. What is the density of the oil?*

$$P_{atm} + \rho_{oil} g D = P_{atm} + \rho_{water} g y_1$$
$$\rho_{oil} = \rho_{water} \frac{y_1}{D}$$

- A force on an imcompressible fluid is additive - that is if you enact a force down on a fluid, you add the created pressure to the reference pressure of the point in the fluid you wish to analyze.

§7.2 Archimedes' Principle

Definition 7.2.1: Archimedes' Principle

An object that is submerged in water is equal to the weight of the displaced fluid. The weight of the displaced fluid is called the **buoyant force**.

$$F_B = V \rho g$$

Problem 40 (The Concrete Canoe)**.**

$$\sum F = 0 = F_B - mg$$
$$F_B = mg$$
$$\rho_{water} V_{disp} g = \rho_{obj} V_{obj} g$$